
Trailer to door assignment in a synchronous cross-dock operation

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Abstract: With the advent of lean supply-chain management, cross-docking has become increasingly popular among shippers. Operating a cross-dock facility requires assigning receiving and shipping doors to trailers. The assignment determines the amount of freight handling within the cross-dock facility and therefore its efficiency. Due to the immense complexity of optimal formulations, dock-door assignment has been typically solved using heuristics. This paper discusses the existing approaches, and suggests a new optimal formulation, and a new heuristic approach for assigning cross-dock doors to trailers. In the process, the reader gains insight into several important issues related to dock door assignment.

Keywords: cross-dock; freight consolidation; freight; door assignment; linehaul; LTL breakbulk; QAP; quadratic assignment problem.

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1 Introduction

Freight transportation using trailers is one of the major components of most supply chains. Cross-docks are transshipment facilities to which trailers arrive with freight that should be sorted and loaded directly onto outbound trailers (or staged in the load position while waiting for the outbound trailer). The freight to each destination is consolidated from several origins. This consolidation of freight is called cross-docking. This operation is prevalent in large carriers of ground and overnight service.

Cross-docks are attractive for two main reasons (Gue and Kang, 2001):

- Cross-docking is a way to reduce inventory holding costs
- For Less-Than-Truckload (LTL) and small package carriers, cross-docking is a way to reduce transportation costs.

Foulds and Lue (2006) mention cross-docking as one of the value-added factors of third party logistics providers, and recommend increasing the use of cross-docking in general.

For ensuring efficient operation, all existing cross-docks are divided into load and unload positions called load/unload doors, or receiving/shipping doors. Such a door is the space required for the trailer when it parks against the dock for loading or unloading. These spaces are marked and designed in order to enable efficient planning and performing of the dock's operation. The efficiency of the cross-dock operation is enhanced by obtaining optimal assignment of doors. For example, Bartholdi and Gue (2000) reported increasing productivity by 11% by optimising dock door assignment. The arrangement of inbound and outbound doors and the assignment of destinations to outbound doors is a difficult combinatorial optimisation problem (Aickelin and Adewunmi, 2006).

Some formulations consider many more factors like staffing, routing, scheduling, and locating cross-docks in the transportation network, (e.g., Peck, 1983; Yano et al., 1998; Gümüs and Bookbinder, 2004; Lim et al., 2006). However, the dock door assignment problem is typically concentrated and managed in the cross-dock facility by the facility manager. The problem of assigning the doors is a complex NP problem (Li et al., 2004; Bozer and Carlo, 2006; Stickel and Furmans, 2006; Deshpande et al., 2007). Thus, this paper is focused only on the dock-doors' assignment and its effect on the workload at the cross-dock facility. This characterises the cross-dock operational management decisions.

A good solution to this problem will be to make cross-dock door assignment dynamic and proactive enough to respond quickly to changes in freight flow patterns (Aickelin and Adewunmi, 2006). Therefore, a very simple heuristic procedure is suggested that could be calculated in real time. Using the heuristic small or local changes in the freight flow pattern yield small changes in the door to trailer assignment.

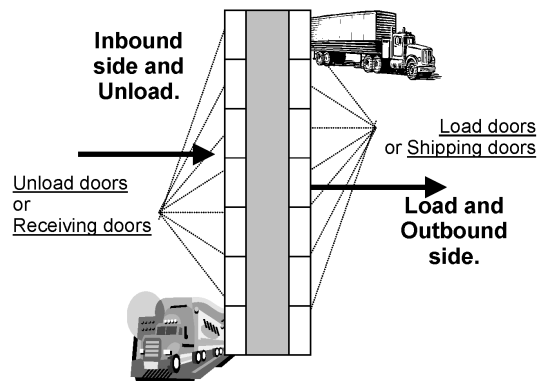
The rest of the paper proceeds as follows: Section 2 presents the classic single dock and its existing formulations and discusses their advantages and drawbacks. Section 3 presents the suggested exact model and proposes a way of using this formulation with small clusters of origins and destinations. Section 4 presents the suggested heuristic. Section 5 illustrates the suggested technique on a small case study. Section 6 concludes the paper.

2 Existing models of the dock-door assignment problem

The most popular configuration of a cross-dock facility is rectangular in shape with several tens of load/unload doors on each of the longer sides of the rectangle (the terms unload/load doors are interchangeable in this paper with receiving/shipping doors and inbound/outbound doors). For management purposes, many cross-dock sites use to designate one side of the dock to inbound trailers and the other side to outbound trailers (Tsui and Chang, 1990; Magableh et al., 2005). However, other studies present different or more general scheme of operation (e.g., Bartholdi and Gue, 2004).

Figure 1 depicts an example of the two-sided inbound/outbound cross-dock configuration. This configuration dedicates one side of the dock for inbound freight, and the other side of the dock to outbound freight. This popular configuration is simple and minimises the chances of shipping items to a wrong destination.

Figure 1 The configuration of a single two-sided inbound/outbound cross-dock



In a typical cross-dock the inbound (i.e., the content of the incoming trailers) and the destination of every incoming package are known (Gue and Kang, 2001). The objective of the daily cross-dock's operation plan is to assign the incoming trailers to the inbound door positions and the outgoing trailers to the outbound doors position, in order to minimise handling. i.e., the total distance travelled by the freight itself or by the forklifts.

Note that this problem is separated from the problem of minimising the travel distance of the trailers. Many recent studies are concerned with the broader transportation network (e.g., Bas et al., 2005; Chen et al., 2006; Gümüs and Bookbinder, 2004) and their solutions could assist in getting the input ('from-to' matrix) to the door assignment problem (tackled in this paper.)

It is important to differentiate between cross-docks and sortation systems (or distribution centres). Any receiving and shipping system that uses a conveyor as a major freight moving tool ceases to be a cross-dock. The operation and the characteristics of the dock change when a conveyor is in place. For example, unloading is always done at the feeding end of the conveyor so that one receiving door serves a lot of arriving trailers (Yonghui et al., 2006; McWilliams, 2005). Thus, dock-door assignment problem is totally different in cross-docks than the door assignment in conveyor-based systems.

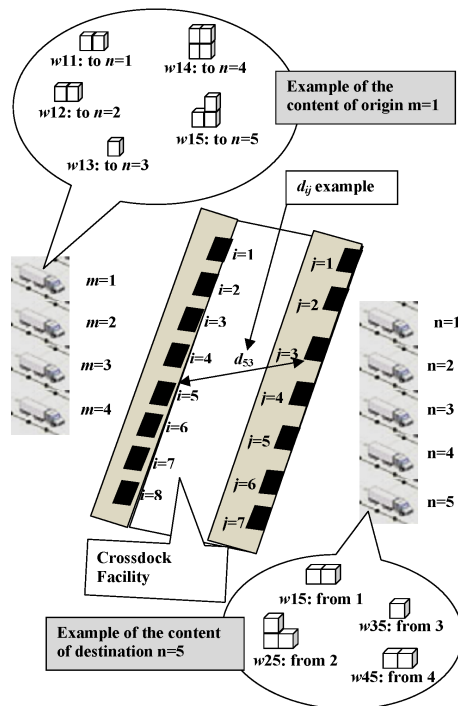
2.1 The classic formulation of the door assignment problem

For the purpose of presenting the traditional formulations we offer the following *notation scheme*:

- I : Number of receiving doors (inbound unload doors)
- J : Number of shipping doors (outbound load doors)
- M : Number of origins ($I \geq M$) each inbound trailer is considered an origin
- N : Number of Destinations ($J \geq N$) destination is a city, or a hub or a cross-dock
- d_{ij} : The distance between receiving door i , and destination door j
- x_{mi} : 0/1 indicator for assignment of an origin m to a receiving door i
- y_{nj} : 0/1 indicator for assignment of a destination n to a shipping door j
- w_{mn} : The number of forklift trips necessary for carrying the freight from origin m to destination n . This number is a function of the freight volume sent from m to n .

The notation (which is the basis for the following formulation) is illustrated in Figure 2.

Figure 2 Example illustrating the notation for Section 2.1 (interpreting origins and destinations as trailers) (see online version for colours)



The model by Tsui and Chang (1990, 1992) is by far the most cited model for cross-docks' door assignment. This model suggests the following optimisation formulation:

$$\text{Minimise } \sum_j \sum_i \sum_n \sum_m w_{mn} d_{ij} x_{mi} y_{nj}$$

Subject to:

$$\sum_m x_{mi} = 1 \quad \text{for } i = 1, 2, \dots, I \quad (1)$$

$$\sum_i x_{mi} = 1 \quad \text{for } m = 1, 2, \dots, M \quad (2)$$

$$\sum_n y_{nj} = 1 \quad \text{for } j = 1, 2, \dots, J \quad (3)$$

$$\sum_j y_{nj} = 1 \quad \text{for } n = 1, 2, \dots, N \quad (4)$$

$$x_{mi} = 0 \text{ or } 1 \quad \text{for all } m, i$$

$$y_{nj} = 0 \text{ or } 1 \quad \text{for all } n, j.$$

The objective function sums the distance travelled by the freight on the dock, between the unload door and the load door. For every origin-destination combination (m, n) the volume of freight is given as the coefficient w_{mn} . Each (m, n) combination is assigned a pair of load and unload doors (i, j) for which $x_{mi}y_{nj} = 1$, with given distance d_{ij} . Thus, the objective function sums the distance times the freight volume.

Constraint set (1) guarantees that each receiving door is assigned only one origin.

Constraint set (2) guarantees that each origin is assigned only one receiving-door.

Constraint set (3) guarantees that each shipping door is assigned only one destination.

Constraint set (4) guarantees that each destination is assigned only one shipping door.

Since the above formulation uses equalities, it may require dummy origins if $I > M$, and dummy destinations in case $J > N$. This formulation also assumes that each origin and each destination is assigned exactly one door. This could be realised even if origins or destinations require two trailers instead of one (a pair of trailers could be unloaded one after the other from the same door, and the same is true for loading). However, the sort is done during a relatively short time-window that suffices for loading two trailers sequentially through the same door. Loading more than two trailers sequentially takes too much time. Thus, typically volume of more than two trailers justifies additional door assignment to the origin/destination (the above model does not allow for this).

The above formulation is called a bilinear program (Vaish and Shetty, 1977) having high complexity. This is a special case of the Quadratic Assignment Problem (QAP), and like all QAP problems, this bilinear problem is NP complete (Tsui and Chang, 1990, 1992). However, if we arbitrarily fix all the x_{mi} origins to unload-doors assignments, the problem becomes a standard assignment problem, which is easy to solve. Also, if all the y_{nj} (destinations to load-doors assignments) are arbitrarily fixed, the problem becomes

a standard assignment problem (Spivey and Thrall, 1970). Based on this insight, Tsui and Chang (1990) propose a heuristic starting from an arbitrary door assignment for the origins, and solve a standard assignment problem for outbound doors to destinations. Then fix the destinations assignment, and solve the standard assignment problem for the origins. The algorithm continues to switch sides until convergence occurs. However, Tsui and Chang did not prove or show that the procedure actually converges. Even if it does converge, it is likely to find a local optimum. Tsui and Chang (1992) proposed a branch and bound approach for solving this problem having exponential growth in run time.

The above model totally disregards the weight and capacity issue and the fact that many trailers could come from a single origin and many can go to the same destination. These issues are treated in the next model.

2.2 The capacity model for cross-dock

The weight and capacity are important factors and more than one trailer could be coming from the same origin and going to the same destination. One of the first works that included these considerations was done by Peck (1983). Peck proposes a 'greedy' heuristics to improve productivity by assigning incoming and outgoing trailers to dock doors for minimising the total distance material handlers travel during the transfer operations. A simulation of an LTL terminal showed 5–15% reduction in its transfer distance due to the heuristic.

Capacity considerations led also to the following model proposed by Gue (1999).

Notation scheme:

I : Set of inbound (strip) trailers

J : Set of outbound (destination) trailers

x_{ij} : The weight (in pounds) of freight in inbound trailer i bound for destination trailer j

d_{ij} : The distance between trailer i and trailer j

b_j : The total weight of freight bound for destination trailer j

c : The capacity (in pounds) of an inbound (strip) trailer

ξ : The minimum number of trailers that must contain freight for any destination.

The formulation is:

$$\text{Minimise } \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \quad (5)$$

$$\text{Subject to } \sum_{i \in I} x_{ij} = b_j \quad \forall j \in J \quad (6)$$

$$\sum_{j \in J} x_{ij} = c \quad \forall i \in I \quad (7)$$

$$x_{ij} \leq b_j / \xi \quad \forall i \in I, j \in J \quad (8)$$

$$x_{ij} \geq 0 \quad \forall i \in I, j \in J. \quad (9)$$

In this transportation type formulation the freight is treated like general commodity (where the b_j is analogous to demand and c to the supply). Thus, it does not solve the original problem where each unit of freight has specific origin and destination.

Also, a careful look at this model shows that it assumes that the door assignment is already determined and given, and the flow of material between these doors is optimised: the four indexes $\{m, i, j, n\}$ become two indexes $\{i, j\}$, and the trailer-to-door assignment is now assumed to be pre-assigned by the supervisors and the variables are the weights to be carried between input and output trailer pairs.

Thus, Gue (1999) proposed heuristic pair swap and heuristic search techniques for the door assignment. For each door assignment, the optimal weights to be carried from each inbound to each outbound trailer was computed via the above fast LP. As mentioned above, this is only true when the freight is the same general commodity, and no freight has a specific destination.

2.3 Drawbacks of the existing models

Summarising the main drawback of the existing formulations we observe that:

- The door assignment formulations (e.g., Tsui and Chang, 1990) ignore the trailer capacity and weights issues.
- The weights formulations (equations (5)–(9)) are ignoring the door assignment issue and the fact that each good has fixed origin-destination which cannot be changed.

The first model by Tsui and Chang (1990) has some problematic assumptions. For example, they assume that one receiving door is assigned to each origin and one shipping door is assigned to each destination. However, this is unrealistic in most real life cases: for example it often happens that the freight volume shipped to a large destination amounts to several trailers and may be assigned to several shipping doors. The same is true for large origins. Typically, constraints of the dock operation necessitate simultaneous unloading and loading of these trailers. So each origin, and each destination, may have multiple doors assigned to it.

Also, in some cases it may be desirable to allow inbound trips to collect freight from more than one origin on its way to the dock. In other cases, it may be desirable to allow shipping trips to more than one destination (i.e., loading freight of two or more destinations onto one trailer, in separate bulks).

The second model (equations (5)–(9)), is not only leaving out the door assignment to trailers to be given by an outside source, it also prevents shipments from having specific destinations and treats them as a commodity that could be manipulated at will as long as the destinations get their demand (similar to the transshipment models). However, the most prevalent situation in cross-docks is that pallets, parcels, packages and freight do have a specific destination. For such cases the above model is useless.

3 The proposed exact model

3.1 Considerations and new notations for the proposed model

The proposed model essentially uses the notation presented in Section 2.1 with some additions. For example, w_{mn} is the weight of freight that was loaded on an incoming

trailer m and must be unloaded in the cross-dock (at door i) in order to be reloaded and delivered to a destination n . The incoming trailer m should be directed to one of the cross-dock's receiving doors, say door i .

There are more ways than one to measure the capacity of a trailer. In this paper the chosen measure of trailer's capacity is its weight, but it could have been volume, or a combination of weight and volume, etc.

The trailer m has a finite capacity C_m , and therefore its load must be smaller than C_m : $\sum_n w_{mn} \leq C_m$. After trailer m is unloaded, the freight w_{mn} is sent to the load door/s of destination n . In case that the total freight to destination n justifies two or more shipping doors, the freight w_{mn} may be split to two or more shipping doors assigned to the destination n . A freight that was unloaded at a receiving door i and then transferred to a shipping door j , passes over a distance of d_{ij} .

The trailers at any shipping door also have a finite capacity. We will assume for simplicity, that each shipping trailer and incoming trailer has the same capacity C , and therefore we must have $\lceil (\sum_{m=1}^M w_{mn}) / C \rceil$ trailers (and load doors) having the same destination n . As we said, in the case of more than one shipping door to a destination n , w_{mn} must be split among the $\lceil (\sum_{m=1}^M w_{mn}) / C \rceil$ doors. This split could be described as percentages of w_{mn} . However, the percentages are variables that are unknown in advance but must be related to doors. So for any combination of n and m (w_{mn}) and possible receiving door i , we define percentages $\alpha_{m,i,j,n}$ for the j shipping doors. The percentages are such that a weight of $(\alpha_{m,i,j,n} w_{mn})$ will be transferred from the receiving door i to the shipping door j , a weight of $(\alpha_{m,i,k,n} w_{mn})$ will be transferred from the receiving door i to the shipping door k and so on. This split was not allowed in Tsui and Chang (1990, 1992) and most previous exact formulations.

Of course the proposed model allows the possibility of receiving doors and shipping doors to stay free from any assignment if this policy leads to optimality.

For the purpose of presenting the formulation we offer the following notation scheme.

3.2 Notation scheme

- I : Number of available receiving doors (inbound unload doors), $i = 1, 2, \dots, I$
- J : Number of available shipping doors (outbound load doors), $j = 1, 2, \dots, J$
- M : Number of incoming trailers from all the origins during one day ($I \geq M$)
- N : Number of destinations (doors) ($J \geq N$)
- K : Number of outgoing trailers to all the destinations during one sort (a sort is usually a time slot of about 3–5 hours). Since at least one trailer is needed for the load of one destination ($J \geq K \geq N$)
- d_{ij} : The distance between receiving door i , and shipping door j
- x_{mi} : 0/1 indicator for assignment of trailer m to receiving door i
- y_{jn} : 0/1 indicator for assignment of shipping door j to destination n
- w_{mn} : The weight of bundle of goods (or the number of forklift trips necessary for carrying the freight) that was loaded on the incoming trailer m and must be delivered at destination n

$\alpha_{m,i,j,n}$: The portion of w_{mn} (that was on trailer m) unloaded at the receiving door i , and transported to the shipping door j . All combinations having receiving door i that was not assigned to trailer m have $0 = \alpha_{m,i,j,n}$. The same is true for all combinations having shipping door j that is not assigned to n . However, if m is assigned to i , and if j is assigned to the destination n we have $0 \leq \alpha_{m,i,j,n} \leq 1$

- C: The maximum capacity of any trailer, i.e., the freight weight of FTL
- D: The maximum capacity of any load door during a sort.

3.3 The proposed model

$$\text{Minimise } \sum_j \sum_i \sum_n \sum_m x_{m,i} w_{m,n} \alpha_{m,i,j,n} d_{i,j} y_{j,n} \tag{10}$$

$$\text{Subject to: } \sum_m x_{m,i} \leq 1 \quad \text{for } i = 1, 2, \dots, I \tag{11}$$

$$\sum_i x_{m,i} = 1 \quad \text{for } m = 1, 2, \dots, M \tag{12}$$

$$\sum_n y_{j,n} \leq 1 \quad \text{for } j = 1, 2, \dots, J \tag{13}$$

$$\sum_j y_{j,n} = \left\lceil \left(\sum_{m=1}^M w_{m,n} \right) / D \right\rceil \quad \text{for } n = 1, 2, \dots, N \tag{14}$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{m,i} y_{j,n} \alpha_{m,i,j,n} = 1 \quad \text{for all } m, n \tag{15}$$

$$\sum_i \sum_n \sum_m x_{m,i} w_{m,n} \alpha_{m,i,j,n} y_{j,n} \leq C \quad \text{for } j = 1, 2, \dots, J \tag{16}$$

- $x_{m,i} = 0$ or 1 , for all m, i
- $y_{j,n} = 0$ or 1 , for all n, j
- $0 \leq \alpha_{m,i,j,n} \leq 1$, for all m, i, j, n .

The only difference in the objective function from Tsui and Chang (1990) is the addition of $\alpha_{m,i,j,n}$. This difference enables finding the optimal flow in the case of several different shipping doors to a single destination. Note that the expressions: w_{mn} and d_{ij} , are known coefficients. Still, the objective function sums the products (multiplications) of three variables each (x_{mi} , y_{jn} , $\alpha_{m,i,j,n}$).

Constraint set (11) guarantees that each receiving door is assigned to only one trailer or none. Constraint set (12) guarantees that each incoming trailer is assigned to one and only one receiving-door. Constraint set (13) guarantees that each shipping door is assigned to only one destination or none. Constraint set (14) guarantees that each destination is assigned to enough shipping doors that fulfil the delivery requirements.

Constraint set (15) guarantees that the sum of the portions ($\alpha_{m,i,j,n}$) of each w_{mn} among the shipping doors is 1. Constraint set (16) guarantees that in any shipping door the load is up to FTL.

The proposed formulation is non-linear mixed integer optimisation. Note that in addition to the non-linear objective function constraints (15) and (16) are multiplications of variables and therefore are non-linear. The complexity and non-linearity of the above formulation makes it impractical for real size problems, and, lead us to suggest the following heuristic approach for large systems (10 doors or more on each side).

4 The proposed heuristic solution

This section presents the proposed heuristic and the next section illustrates it on a case study. The case study that follows illustrates the stages explained below using detailed numerical computations. So if anything may seem to be unclear in this section, the case study in Section 5 should clarify it. The following are the stages of the proposed algorithm:

4.1 Phase 1: Initialisation

- 1 The 'from-to' table of freight quantities to be delivered is always a known input (w_{mn} coefficients). Choose the three largest weight transfer (w_{mn}) volumes in the 'from-to' table. Assign the origins and destinations of these three chosen volumes to the groups of assigned origins and assigned destination respectively.
- 2 Assign the trailer m of the largest (w_{mn}) from step 1 to the middle of the cross-dock on the inbound side, and the corresponding trailer(s) n to the middle of the outbound side.
- 3 Arbitrarily assign the other two inbound trailers (chosen in step one) on each side of the middle trailer of step 2.
- 4 Note that step 2 assigned door/s to the first destination, leaving two other destinations to be assigned in this phase. The two remaining destinations could have only two arrangements (assignments to outbound doors) on the two sides of the centrally assigned largest destination (from step 2). Compute the sum of (weight \times distance) resulting from each one of the two alternatives arrangements (of the remaining two destinations). Choose the arrangement with the minimal sum.
- 5 Define I as iteration counter and assign $I = 0$.

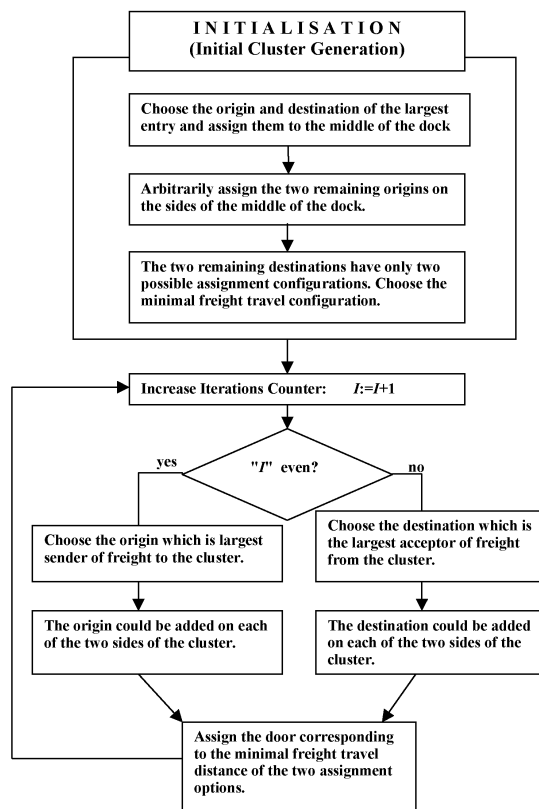
4.2 Phase 2: Iterations

- 1 $I := I + 1$; (increase the iteration conter)
 - If I is even go to step 3. ($I =$ the number of iteration)
 - Else: update the 'from-to' table to represent the current cluster of assigned inbound trailers. The update is done by summing the rows corresponding to the cluster into one row. Each entry in the resulting row is a sum of the column entries of the cluster rows.

- 2 Choose the biggest entry (volume) from the assigned cluster's row while ignoring the cluster's column. Break ties arbitrarily. Add the corresponding destination to the assigned outbound cluster. Compare locating the destination to the right and to the left of the assigned outbound destinations cluster, and choose the assignment that minimises the sum of (distance \times volume) between the trailer and the inbound side of the assigned cluster. If all m trailers and n destinations are assigned – then finish, Otherwise go to step 1 (of phase 2).
- 3 Update the 'from-to' table to represent the current cluster of assigned outbound trailers using one column having each entry as the sum (in the same row) of the entries of the columns corresponding to the outbound trailers.
- 4 Choose the biggest entry (volume) from the assigned cluster's column while ignoring the cluster's row. Break ties arbitrarily. Add the corresponding incoming trailer to the assigned inbound cluster. Compare locating the trailer to the right and to the left of the assigned inbound cluster, and choose the assignment that minimises the sum of (distance \times volume) between the trailer and the outbound side of the assigned cluster. If all m trailers and n destinations are assigned – then finish, otherwise go to step 1 (of phase 2).

The main logic and steps of the proposed heuristic algorithm are depicted in Figure 3, and are illustrated in detail in Section 5.

Figure 3 General logic of the proposed greedy heuristic algorithm



5 Illustrative case study

We chose an arbitrary illustrative small case study of paints tooling and plumbing distribution operation. The operation includes seven incoming trailers, and a dock of ten receiving doors and ten shipping doors (overall 20 doors). There are seven different destinations, and due to the distance between destinations a trailer that serves a specific destination cannot serve any other destination. The objective of the facility manager is to minimise the transfer distances of freight. Since all the freight must move from unload side to the load side, the distance is measured only along the dock. That is, for two doors that directly face each other the distance is zero, if the door is next to the directly facing door – the distance is one, and generally the distance counts how many doors to the side the freight is moved. The facility manager wants to have an intuitive heuristic to be used daily to assign the doors according to the ‘from-to’ matrix of volumes for the specific day. The typical ‘from-to’ matrix (given in % of a trailer’s capacity), and the first cluster are depicted in Table 1.

Table 1 The ‘from-to’ matrix: the volume is given as percentage of a trailer’s capacity

Incoming trailer #	Destination							Total
	1	2	3	4	5	6	7	
1	0	5	0	70	0	5	20	100
2	5	0	15	55	5	20	0	100
3	0	0	0	80	0	10	10	100
4	30	0	25	0	30	10	5	100
5	0	5	0	75	0	10	10	100
6	0	15	0	65	0	0	20	100
7	5	0	5	45	5	40	0	100
Total	40	25	45	390	40	95	65	700

The first cluster is chosen in the initialisation stage, by simply choosing the highest three entries of the ‘from-to’ matrix. These entries correspond to origins 1, 3, 5 and destination 4. This concentration of load for destination 4 is typical for large urban centres. Also, it is typical in synchronous operation that tight timing constraints limit the number of trailers that could be loaded (one after the other) per door.

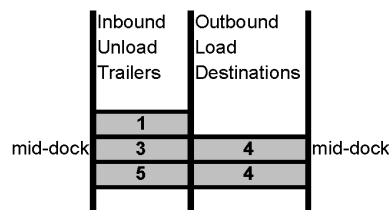
In this case study, due to timing constraints; no more than two full trailers could be loaded (one after the other) on each load door. Thus, destination 4, that receives almost four full trailers, must have two separate load doors. We shall now proceed formally according to the algorithm steps presented in Section 4.

5.1 Phase 1: Initialisation

- 1 The three largest entries in Table 1 are highlighted: {3–4, 5–4, 1–4}. These entries compose the chosen initial cluster.
- 2 The largest entry is 3–4, so trailer 3 is assigned to the middle door of the receiving dock (to be unloaded) and destination 4 is assigned to the middle door of the shipping dock (to be loaded).

- 3 The next two entries are: 5–4 and 1–4. So incoming trailer 5 is arbitrarily assigned to the receiving door on one side of the incoming trailer 3 (assigned in step 2), and trailer 1 is assigned on the other side of the incoming trailer 3.
- 4 In this example the initial cluster has only one destination (the max is 3). The amount of volume sent by the incoming trailers: 3, 5, 1 – is 2.25. It exceeds the maximum of two trailers, and therefore another load door is assigned to destination 4 (against incoming trailer 5 (that sends more volume to destination 4 than trailer 1)). This is illustrated in Figure 4.

Figure 4 The assignment at the end of the initialisation phase



5.2 Phase 2: Iterations

The first iteration assigns a destination to the assigned cluster. This destination is determined by the largest volume sent by the cluster to an unassigned destination. In other words, we shall choose a destination other than 4 that receives the most volume from origins 1, 3, and 5.

Iteration 1

Step 1: $I=1$; The updated volumes of the incoming cluster (determined in phase 1) could be seen in the first row of Table 2.

Table 2 The first iteration assigns destination 7 to the destination cluster

Incoming trailer #	Destination							Total
	1	2	3	4	5	6	7	
{1,3,5}	0	10	0	225	0	25	40	300
2	5	0	15	55	5	20	0	100
4	30	0	25	0	30	10	5	100
6	0	15	0	65	0	0	20	100
7	5	0	5	45	5	40	0	100
<i>Total</i>	40	25	45	390	40	95	65	700

Step 2: In the case study, the incoming cluster {1, 3, 5} sends the most (40% of a trailer) to destination 7. This could be seen in Table 2. Thus destination 7 has to be assigned a load door. There are only two possible configurations for this assignment as seen in Figure 5, and one could be chosen.

Figure 5 Two alternative assignments of destination 7

<u>Assignment A</u>		<u>Assignment B</u>	
Inbound Unload Trailers	Outbound Load Destinations	Inbound Unload Trailers	Outbound Load Destinations
1	7	1	
3	4	3	4
5	4	5	4
			7

The comparison of the volumes time distance products of the two assignments with rectilinear simplification and length units equal to the door width:

$$\text{Travel Distance in Assignment A: } (20 \times 0) + (10 \times 1) + (10 \times 2) = 0 + 10 + 20 = 30$$

$$\text{Travel Distance in Assignment B: } (20 \times 3) + (10 \times 2) + (10 \times 1) = 60 + 20 + 10 = 90.$$

Thus, assignment A (with minimal travel distance) is selected and the second iteration begins.

The second iteration adds an assignment of an incoming trailer to the assigned cluster. This inbound trailer is determined by the largest volume sent to the cluster by an unassigned inbound trailer. A door will be assigned to the trailer, based on the minimal travel distance of freight.

Iteration 2

The second iteration assigns a receiving door to the incoming trailer that sends the largest volume to the assigned cluster.

Step 1: $I = 2$; If I is even then go to step 3 (so step 3 is next).

Step 3: This step updates the cluster by adding the effect of the last added origins. The updated volumes of the incoming cluster could be seen in the fourth column of Table 3.

Step 4: The outbound cluster $\{4, 7\}$ receives the most (85% of a trailer) from trailer 6. This could be seen in Table 3. Thus the inbound trailer 6 has to be assigned a receiving door. There are two configurations to choose from: the first is shown to the right of Table 3, the second and less favourable is assigning trailer 6 next to inbound trailer 5.

Table 3 The second iteration assigns trailer 6 to the unloading cluster

Incoming trailer #	Destination						Total	Inbound unload trailers	Outbound load destinations
	1	2	3	{4,7}	5	6			
{1,3,5}	0	10	0	265	0	25	340		
2	5	0	15	55	5	20	100	6	
4	30	0	25	5	30	10	105	1	7
6	0	15	0	85	0	0	120	3	4
7	5	0	5	45	5	40	100	5	4
Total	40	25	45	480	40	95	790		

Each iteration assigns either an inbound trailer or a destination. At this point there are five unassigned destinations and three unassigned incoming trailers. So after eight more iterations we arrive at the final solution as shown in Figure 6.

Figure 6 The final door assignment of the proposed heuristic solution

Dock I/B Door#	Inbound Unload Trailer	Outbound Load Destination	Dock O/B Door#
1			1
2	6	2	2
3	1	7	3
4	3	4	4
5	5	4	5
6	7	6	6
7	2	5	7
8	4	1	8
9		3	9
10			10

Interestingly, the facility manager said that he prefers a heuristic that makes it possible to trace the logic of each step, over an method that is hard to trace (even if it is optimal). Thus, the suggested method was found to be very appealing.

6 Discussion and conclusion

The proposed heuristic algorithm assigns at least one door to a trailer per iteration. Thus, the iterations stop either all the doors or all trailers are assigned. Thus, defining x as the computed number of destination trailers ($x \geq n$) the maximal number of iterations is: $\text{Min}\{m + x, i + j\}$. Defining $N = \text{Min}\{m + x, i + j\}$

In each iteration the actions are:

- Update of assigned shipped volume for a recently assigned trailer ($O(N)$)
- Finding the minimal entry in a row or a column ($O(N)$)
- Computing the total distance \times volume for two alternative assignments ($2 \times O(N)$).

So having N iterations, each having a complexity of $O(N)$, leads to the overall complexity of $O(N^2)$. This is the smallest known complexity for algorithms dealing with cross-dock door assignment.

To compare the proposed heuristic to other possible solutions we assigned the incoming (inbound) trailers to doors arbitrarily and used the assignment LP formulation to find the optimal assignment of the outbound trailers. A comparison of 20 cases ('from-to matrices') of 10 inbound by 10 outbound trailers and the same number of doors, shows average savings of about 10% (9.5%) in the total distance travelled.

To conclude, this paper discusses the existing formulations of the single cross-dock door assignment problem, and suggests a new formulation. A heuristic algorithm is presented based on growing an initial cluster. The algorithm is illustrated via a case study and its complexity is shown to be the smallest of all known comparable algorithms $O(N^2)$. A comparison to other heuristic assignment shows average savings

of about 10%. While previous formulations were explicitly bilinear, the current formulation shows that decisions on how to split a load among a group of same destination trailers – makes the problem even more complex, and it is not a bilinear problem anymore.

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