Novel Algorithms for the Network Lifetime Problem in Wireless Settings

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Abstract. A wireless ad-hoc network is a collection of transceivers positioned in the plane. Each transceiver is equipped with a limited, non-replenishable battery charge. The battery charge is then reduced after each transmission, depending on the transmission distance. One of the major problems in wireless network design is to route network traffic efficiently so as to maximize the *network lifetime*, i.e., the number of successful transmissions. This problem is known to be NP-Hard for a variety of network operations. In this paper we are interested in two fundamental types of transmissions, broadcast and data gathering.
We provide polynomial time approximation algorithms, with guaranteed performance bounds, for the maximum lifetime problem under two communication models, omnidirectional and unidirectional antennas. We also consider an extended variant of the maximum lifetime problem, which simultaneously satisfies additional constraints, such as bounded hop-diameter and degree of the routing tree, and minimizing the total energy used in a single transmission.

1 Introduction

Wireless ad-hoc networks gained much appreciation in recent years due to massive use in a large variety of domains, from life threatening situations, such as battlefield or rescue operations, to more civil applications, like environmental data gathering for forecast prediction. The network is composed of numerous transceivers (nodes) located in the plane, communicating by radio. A transmission between two nodes is possible if the receiver is within the transmission range of the transmitter. The underlying physical topology of the network is dependent on the distribution of the wireless nodes (location) as well as the transmission power (range) assignment of each node. Since the nodes have only a limited, non-replenishable initial power charge (battery), energy efficiency becomes a crucial factor in wireless networks design.

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The transmission range r_v of node v is determined by the power assigned to that node, denoted by $p(v)$. It is customary to assume that the minimal transmission power required to transmit to distance d is d^α, where the *distance-power gradient* α is usually taken to be in the interval $[2, 4]$ (see [1]). Thus, node v receives transmissions from u if $p(u) \geq d(u, v)^\alpha$, where $d(u, v)$ is the Euclidean distance between u and v. There are two possible models: symmetric and asymmetric. In the symmetric model, also referred to as the undirected model, there is an undirected communication link between two nodes $u, v \in T$, if $p(u) \geq d(u, v)^\alpha$ and $p(v) \geq d(v, u)^\alpha$, that is if u and v can reach each other. The asymmetric variant allows directed (one way) communication links between two nodes. Krumke et al. [2] argued that the asymmetric version is harder than the symmetric one. This paper addresses the asymmetric model.

Ramanathan and Hain [3] initiated the formal study of controlling the network topology by adjusting the transmission range of the nodes. Intuitively, an increase to the transmission range assignment allows more distant nodes to receive transmissions. But at the same time, it causes a quicker battery exhaustion, which results in a shorter network lifetime. We are interested in maximizing the network lifetime under two basic transmission protocols, data broadcasting and data gathering. **Data broadcasting**, or in short broadcast, is a network task when a source node s wishes to transmit a message to all the other nodes in the network. **Data gathering -** a less popular, nevertheless important network task, is also known as convergecast. Opposite to broadcast, there is a destination node d, and all the other nodes wish to transmit a message to it. We consider data gathering *with aggregation*.

Each node v, has an initial battery charge $b(v)$. The battery charge decreases with each transmission. The network lifetime is the time from network initialization to the first node failure due to battery depletion. It is possible to look at two formulations of the maximum network lifetime problem. In the *discrete* version, node v can transmit at most $\lfloor b(v)/d^\alpha \rfloor$ times to distance d. Whereas, the *fractional* variant states that a transmission from node v to distance d is valid for b/d^α time units. For example, for $b(v) = 15$, $d = 2$, and $\alpha = 2$, the discrete version of the problem would allow $\lfloor 15/4 \rfloor = 3$ *separate* transmissions, while the fractional formulation determines that node v can have a valid transmission for $15/4 = 3.75$ time units. Most of the current research addresses the fractional formulation. The discrete version was introduced by Sahni and Park [4]. They provided a number of heuristics without guaranteed performance bounds. This paper studies the discrete version, which seems to be more problematic.

An additional consideration in wireless networks design, is the type of the antenna used for communication. In this paper we consider two types of communication antennas, *omnidirectional* and *unidirectional*. For a node $u \in \mathcal{V}$ equiped with an omnidirectional antenna, a single message transmission to the most distant node in a set of nodes X is sufficient so that all the nodes in X receive the message. While, if u uses a unidirectional antenna, then it has to transmit to each of the nodes in X separately.

The paper is organized as follows. In the rest of the section, we introduce our model, discuss previous work and outline our contribution. In Sections 2 and 3 we present our results for the unidirectional and omnidirecitonal antenna types, respectively.

1.1 The Model

Graph Preliminaries. Here we provide some graph theory related definitions used in this paper.

- For any graph H, let $V(H)$ and $E(H)$ be the node and edge sets of H, respectively.
- In a directed graph H, let $\delta_H(v)$ be the set of outgoing edges from v in $V(H)$.
- For a weighted graph H, with a weight function w, we alternately use the notation $w(e)$ and $w(u,v)$, to specify the weight of edge $e=(u,v) \in E(H)$. The weight of H is given by $C(H)=\sum_{e \in E(H)} w(e)$.
- The weight function w of graph H is said to be uniform, if $\forall e \in E(H),\ w(e)=w_0$, for some non-negative value w_0.
- The cube of graph H, denoted H^3, contains an edge (u,v) if there is a path from u to v in H with at most 3 edges.
- A Hamiltonian circuit $h=(u_1,u_2,\ldots,u_{n+1}=u_1)$ in graph H, where $u_i \in V(H)$ for $1 \leq i \leq n$, is a graph cycle that visits each node in $V(G)$ exactly once and also returns to the starting node. The weight of h is given by $C(h)=\sum_{i=1}^{n} w(u_i,u_{i+1})$, where w is the weight function of H.
- Given an undirected graph H, let $MST(H)$ be a minimum spanning tree of H.

Network Model. We have n wireless nodes $\mathcal{V}$ positioned in a Euclidean plane. The wireless network is then modeled by a complete, weighted, and undirected graph $G_{\mathcal{V}}$ with a weight function $w:\mathcal{V}\times\mathcal{V}\rightarrow \mathbb{R}$, $w(u,v)=d(u,v)^{\alpha}$. It is easy to verify that the weight function obeys the weak triangle inequality with coefficient $2^{\alpha-1}$, i.e., for any $u,v,w \in \mathcal{V}$, $w(u,w) \leq 2^{\alpha-1}(w(u,v)+w(v,w))$.

Both types of messages, broadcast or convergecast, are propagated by using a directed spanning tree of $G_{\mathcal{V}}$, called a *transmission tree.* A broadcast message, originating in $s \in \mathcal{V}$, is propagated by an arborescence T_s rooted at s, also called a *broadcast tree.* In the case of a convergecast to $d \in \mathcal{V}$, the messages from all nodes are propagated by a reversed arborescence T_d rooted at d, also called a *convergecast tree.* In the case of a broadcast message, a node may be required to transmit to multiple recipients (its children in the broadcast tree), while a convergecast message is transmitted once to the parent in the convergecast tree.[4]

Every node $v \in \mathcal{V}$ has an initial battery charge $b(v)$. After each message propagation, its residual energy decreases. The energy decrease depends on the

[4] We consider data gathering with aggregation, which means that each node v combines the messages sent by the nodes in a subtree rooted at v into one message, and then propagates it to its parent.

recipient nodes location, as well as the antenna type used, either omnidirectional or unidirectional. Formally, the power consumption of $v \in \mathcal{V}$ due to a transmission tree T is,

$$\beta_T(v) = \begin{cases} \max\limits_{e \in \delta_T(v)} w(e), & \text{omnidirectional,} \\ \sum\limits_{e \in \delta_T(v)} w(e), & \text{unidirectional.} \end{cases}$$

Note that the reverse of a broadcast tree is a convergecast tree. Due to this symmetry property, and in an attempt to keep the definitions simple, from this point, we refer to the broadcast transmission protocol only. Although there is symmetry in definitions, nevertheless not all the results work well for both cases. We provide explicit statements whenever the results are relevant for convergecast as well. In this paper we assume $\alpha = 2$ for simplicity, though our results can be easily extended to any constant value of α.

Problems Definition. The general maximum lifetime broadcast (MLB) problem is defined as follows. **The input** to the MLB problem is graph $G_{\mathcal{V}}$, initial battery charges $b : \mathcal{V} \to \mathbb{R}$, and a sequence of m source nodes $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where $s_i \in \mathcal{V}$, for $1 \leq i \leq m$. Each of the source nodes has one broadcast message to transmit to all the other nodes. **The output** is a sequence of broadcast trees $\mathcal{T}_B = \{T_1, T_2, \ldots, T_k\}$, where T_i is rooted at s_i, for $1 \leq i \leq m$, so that for all $v \in \mathcal{V}$, $\sum_{i=1}^{k} \beta_{T_i}(v) \leq b(v)$. **Our objective** is to maximize k. Intuitively, given a sequence of source nodes, we wish to maximize the number of successful broadcast message propagations, while satisfying the battery constraint. That is, all the nodes have enough battery charge to support message propagation in a sequence of broadcast trees.

There are two possible relaxations of the general maximum lifetime broadcast problem. **The first relaxation** is to set $s_i = s$, for all $s_i \in \mathcal{S}$, that is one source node s generates all broadcast messages. **The second relaxation** is to require that all the broadcast trees would be an orientation of one undirected tree. In this paper we consider the following three problems.

Problem 1. [Single Source Maximum Lifetime Broadcast (SSMLB)]
Input: Graph $G_{\mathcal{V}}$, initial battery charges $b : \mathcal{V} \to \mathbb{R}$, and a source node $s \in \mathcal{V}$.
Output: A sequence of broadcast trees $\mathcal{T}_B = \{T_1, T_2, \ldots, T_k\}$, so that T_i is rooted at s, and for all $v \in \mathcal{V}$, $\sum_{i=1}^{k} \beta_{T_i}(v) \leq b(v)$.
Objective: Maximize k.

Problem 2. [Single Source/Topology Maximum Lifetime Broadcast (SSTMLB)]
Input: Graph $G_{\mathcal{V}}$, initial battery charges $b : \mathcal{V} \to \mathbb{R}$, and a source node $s \in \mathcal{V}$.
Output: A directed spanning tree T of $G_{\mathcal{V}}$ rooted at s, and an integer k, $1 \leq k \leq m$, so that for all $v \in \mathcal{V}$, $k\beta_T(v) \leq b(v)$.
Objective: Maximize k.

Problem 3. [Single Topology Maximum Lifetime Broadcast (STMLB)]
Input: Graph $G_{\mathcal{V}}$, initial battery charges $b : \mathcal{V} \to \mathbb{R}$, and a sequence of m source nodes $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$, where $s_i \in \mathcal{V}$.
Output: An undirected spanning tree T of $G_{\mathcal{V}}$ and an integer k, $1 \le k \le m$, so that for all $v \in \mathcal{V}$, $\sum_{i=1}^{k} \beta_{T_i}(v) \le b(v)$, where T_i, $1 \le i \le k$, is a broadcast tree rooted at s_i, and is obtained by orienting the edges of T.
Objective: Maximize k.

The analogous problems for convergecast, SSMLC, SSTMLC, and STMLC are defined in a similar way.

1.2 Previous Work

Numerous studies were conducted in the area of maximizing the network lifetime under various transmission protocols. In addition to broadcast and convergecast, it is common to find references to multicast and unicast[5] as well. Different formulations of the maximum lifetime problem are due to the single/multiple source/topology relaxations. These relaxations, mixed together with the antenna type, have impact on the complexity of the problem.

As mentioned previously, to the best of our knowledge, there is no reference to the discrete version of the maximum lifetime problem, except for [4]. Instead, we survey the state of current results for the fractional case, grouped in accordance to the communication model used.

Omnidirectional Model Orda and Yassour [5] gave polynomial-time algorithms for broadcast, multicast and unicast in the case of **single source/single topology**, which improved previous results by [6]. Segal [7] improved the running time of the MLB problem for the broadcast protocol and also showed an optimal polynomial-time algorithm for convergecast with aggregation. Additional results may be found in [8, 6]. By allowing **single source/multiple topology**, the broadcast and multicast become NP-Hard [5], while convergecast and unicast have polynomial-time optimal solutions. In [5], the authors establish an $O(\log n)$ and $O(k^{\varepsilon})$ approximation algorithms for broadcast and multicast, respectively, where k is the size of the multicast destination set and ε is any positive constant. The same paper shows an optimal solution for the unicast case by using linear programming and max-flow algorithms. Liang and Liu [9] prove that the convergecast problem without aggregation is NP-Complete for general costs. An easier version, with aggregation, does have a polynomial solution [10] in $O(n^{15} \log n)$ time. To counter the slowness of the algorithm, Stanford and Tongngam [11] proposed a $(1 - \varepsilon)$-approximation in $O(n^3 \frac{1}{\varepsilon} \log_{1+\varepsilon} n)$ time based on Garg and Könemann [12] algorithm for packing linear programs. They also propose several heuristics and evaluate their performance by simulation.

[5] *Multicast* is a more general case of broadcast. A source node is required to transmit to a set of nodes; *unicast* is more specific, a source node is required to transmit to a single node.

Table 1. Current results for the fractional case

Single Source - Omnidirectional Model		
Topology	*Broadcast*	*Convergecast (with agg.)*
Single	OPT [5, 7, 6]	OPT [7]
Multiple	$6(1-\varepsilon)$ approx. (follows from [11] and [16])	OPT [10]
Single Source - Unidirectional Model		
Topology	*Broadcast*	*Convergecast (with agg.)*
Single	NP-Hard [5]	OPT [7]
Multiple	OPT [5]	OPT [10]

Generally, a common approach to solving the fractional problem is to use various LP formulations that reduce the problem to one of finding the maximum multicommodity flow in a network. See also [13–15].

Unidirectional Model The authors in [5] show that for broadcast, the problem is NP-Hard in the case of **single source/single topology** and has a polynomial solution in the case of **single source/multiple topology**. They also show that it is NP-Hard in both of these cases for multicast. To the best of our knowledge, this is the only paper to address the unidirectional communication model. Note that for convergecast there is no difference between the two models (omnidirectional and unidirectional), as the node is required to transmit to its parent in the convergecast tree only. Therefore, the results from [7] and [10] hold.

A summary of the results for the fractional case under the omnidirectional model is given in Table 1 (OPT represents that the problem can be solved optimally). The result for single source/multiple topology in case of broadcast is derived from the simple fact that when the Garg-Könemann $(1-\varepsilon)$-approximation algorithm uses λ-approximation minimum length columns it produces a $\lambda(1-\varepsilon)$ approximation to the packing LP defined by [11] if used for broadcasting. We can choose a 6-approximation by Ambühl [16] as the λ-approximation algorithm for the minimum energy broadcast problem. The 6-approximation can be improved by using the result in [17].

1.3 Our contribution

We study the discrete version of the maximum lifetime problem under broadcast/convergecast transmissions. We provide polynomial time approximation algorithms, with guaranteed performance bounds, for the maximum lifetime problem under two communication models, omnidirectional and unidirectional antennas. We also consider an extended variant of the maximum lifetime problem, which simultaneously satisfies additional constraints. In particular, our main contributions are:

1. Under the unidirectional model, we state the NP-Hardness of the SSMLB and SSTMLB problems. We provide an $O(\log n)$-approximation to the SSTMLB

Table 2. Our contribution in the discrete case

Single Source - Unidirectional Model		
Topology	*Approx.*	*Remarks*
Single	$O(\log n)$	
Multiple	1	battery violation by $O(\log(nk^*))$, k^* is OPT
Multiple Source - Omnidirectional Model		
Topology	*Approx.*	*Remarks*
Single	2	with additional bi-criteria
Multiple	$O(\rho^2)$	with $n/\rho + \log \rho$ hop-diameter, and additional bi-criteria

problem. Then, for the SSMLB problem we find a sequence of broadcast trees of optimal length k^*, so that the battery constraint is violated by at most $O(\log(nk^*))$ times. That is, the energy consumed by node v is at most $O(\log(nk^*))b(v)$.

2. Under the omnidirectional model, we develop two approximation algorithms for the STMLB problem. We assume uniform initial battery charges and present a 2-approximation algorithm by using the $MST(G)$ as the broadcast tree. This immediately yields constant bounds for the total energy consumed in a single transmission and the maximum degree. We then construct a broadcast tree which is a $O(\rho^2)$-approximation to the problem. In addition, it has a bounded hop-diameter $n/\rho + \log \rho$, where $1 \leq \rho \leq n$, a constant maximum degree, and the energy consumed in a single transmission is at most ρ times the optimum for a broadcast transmission. We argue that the tradeoff between the maximum lifetime and the hop-diameter is optimal. That is, our multi-criteria approximation is tight.
3. Finally, we show that the results for the STMLB problem, can be applied for the STMLC problem as well.

To the best of our knowledge, these are the first theoretic results for the discrete formulation of the problem. Our results are summarized in Table 2.

2 Unidirectional Communication Model

The unidirectional model implies that each node is charged for every outgoing edge in the transmission tree. The power consumption of $v \in \mathcal{V}$ due to a single message transmission, in a directed tree T, is $\beta_T(v) = \sum_{e \in \delta_T(v)} w(e)$.

In this section we consider two variants of the MLB problem under the single source relaxation. First the more general case is addressed, where multiple topologies are allowed, which is the SSMLB problem. Then, we show that by doing slight modifications to the proposed algorithms, we establish a similar result in the case of single topology relaxation, namely the SSTMLB problem. We slightly modify the original problems, by allowing a violation of the battery constraint by γ. That is, we require that the energy consumption of every $v \in \mathcal{V}$ is at most $\gamma b(v)$.

Assuming P$\neq$NP, both the single and the multiple topology cases cannot achieve a $1/\gamma$-approximation algorithm for any constant $\gamma > 0$, since deciding whether even one tranmission is possible is equivalent to the so called **Degree Constrained Arborescence** problem. This implicates that the SSMLB and SSTMLB problems are NP-Hard (take $\gamma = 1$).

Note that in the single topology case, k transmissions with initial battery charges $\{\gamma b(v) : v \in \mathcal{V}\}$ imply $\lfloor k/\gamma \rfloor$ transmissions for initial battery charges $\{b(v) : v \in \mathcal{V}\}$. Indeed, since we are using the same arborescence, the power consumption of every node in every message propagation is identical and there are k message propagations, then for the original charges $\{b(v) : v \in \mathcal{V}\}$ the number of propagations is at least $\lfloor b(v)/(\gamma b(v)/k) \rfloor = \lfloor k/\gamma \rfloor$. Unfortunately, for the multiple topology case, we do not have a method to convert the battery violation to a standard approximation.

Although the input to the SSMLB problem, is a weighted, undirected graph $G_{\mathcal{V}}$, we can alternatively look at the directed version $G'_{\mathcal{V}}$, i.e., for every edge $e = (u, v) \in E(G_{\mathcal{V}})$, create the instances $(u, v), (v, u)$ in $E(G'_{\mathcal{V}})$. The weight of the directional edge is the same as of the original one. In the rest of the section we prove the next theorem, which summarizes our main results for the unidirectional model.

Theorem 1. *Given a weighted, directed graph $G'_{\mathcal{V}}$ and a source node $s \in \mathcal{V}$, let k_1^* and k_2^* be the number of successful message propagations in the optimal solutions of the SSTMLB and SSMLB problems, respectively. Then, (i) there exists a broadcast tree T rooted at s, so that for all $v \in \mathcal{V}$, $(k_1^*/\log n)\beta_T(v) \leq b(v)$; (ii) there exists a sequence of broadcast trees $\mathcal{T}_B = \{T_1, T_2, \ldots, T_{k_2^*}\}$, each rooted at s, and for all $v \in \mathcal{V}$, $\sum_{i=1}^{k_2^*} \beta_{T_i}(v) \leq (\log{(nk_2^*)})b(v)$.*

2.1 Weight Scaling Reduction

We start by showing a simple scaling of weights, which allows us to manipulate the input graph $G'_{\mathcal{V}}$. If for some node $v \in \mathcal{V}$ and constant $c > 0$, we set $b(v) \leftarrow b(v)/c$ and for every outgoing edge $e \in \delta_{G'_{\mathcal{V}}(v)}$, set $w(e) \leftarrow w(e)/c$, we obtain a similar instance to our problem. Note that an instance with uniform weights[6] is easily transformed into an instance with *unit* weights (all weights being 1), by applying the weight scaling reduction described above.

2.2 The SSMLB problem

We start with the multiple topology case of the MLB problem under the single source relaxation and prove part (ii) of Theorem 1.

[6] Though graph $G'_{\mathcal{V}}$ does not necessarily has uniform weights, nevertheless we use this scaling in future developments.

A directed graph H is *k-edge-outconnected from s* if it contains k-edge disjoint paths from s to any other node. By Edmond's Theorem [18], a graph is k-edge-outconnected from s if, and only if, it contains k edge-disjoint spanning arborescences rooted at s. Let us introduce the following decision problem.

Problem 4 (Bound Constrained k-Outconnected Subgraph (BCkOS)).
Input: A directed graph G with a weight function w, bounds $b : V(G) \to \mathbb{R}$, a source node $s \in V(G)$, and a positive integer k.
Question: Does G have a k-edge-outconnected spanning subgraph H, so that for all $v \in V(G)$, $\beta_H(v) \leq b(v)$.

Given a positive integer k, the problem of finding a sequence of broadcast trees of length k in $G'_{\mathcal{V}}$ can be reduced to the BCkOS problem as follows. As an edge in $E(G'_{\mathcal{V}})$ may be used several times, we add $k-1$ copies of each edge to the graph.[7] Call this graph $G^k_{\mathcal{V}}$. Then we solve the BCkOS problem for $G^k_{\mathcal{V}}$.

To solve the SSMLB problem, we need to search for the maximum value of k, for which the BCkOS returns a positive answer given $G^k_{\mathcal{V}}$. This can be done by a simple binary search in the range $\{1, \ldots, K\}$, where $K = \max_{e \in \delta_{G_{\mathcal{V}}}(s)} b(s)/w(e)$. The upper bound is due to the source node battery constraint. The BCkOS problem is NP-hard even for uniform weights and $k = 1$. We therefore consider the optimization problem that seeks to minimize the factor of the weight-degree bounds violation.

Problem 5 (Weighted-Degree Constrained k-Outconnected Subgraph (WDCkOS)).
Input: A directed graph G with a weight function w, bounds $b : V(G) \to \mathbb{R}$, a source node $s \in V(G)$, and a positive integer k. Graph G has a k-edge-outconnected spanning subgraph H^* satisfying, for all $v \in V(G)$, $\beta_{H^*}(v) \leq b(v)$.
Output: Find a k-edge-outconnected spanning subgraph H of G, so that for all $v \in V(G)$, $\beta_H(v) \leq \gamma \cdot b(v)$.
Objective: Minimize γ.

Clearly, guaranteeing a factor of γ for the WDCkOS problem also guarantees a γ violation in our case. Let the Degree Constrained k-Outconnected Subgraph (DCkOS) problem be the restriction of WDCkOS problem to instances with unit (or uniform) weights; in this case the bounds $b(v)$ are just the degree constraints, and thus assumed to be integral. The following statement follows from Theorems 1 and 4 in [19] ($d_H(v)$ is the outdegree of v in H).

Theorem 2 ([19]). *There exists a polynomial time algorithm that given an instance of* DCkOS *finds a k-edge-outconnected spanning subgraph H of G so that $d_H(v) \leq b(v) + 2$ if $k = 1$ and $d_H(v) \leq b(v) + 4$ if $k \geq 2$.*

[7] Instead of adding $k-1$ copies of an edge, we may assign to every edge capacity k, and consider the corresponding "capacited" problems; this will give a polynomial algorithm, rather than a pseudo-polynomial one. For simplicity of exposition, we will present the algorithm in terms of multigraphs, but it can be easily adjusted to the terms of capacitated graphs.

It is easy to verify that DCkOS admits a 3-approximation algorithm for $k = 1$ and a 5-approximation algorithm for $k \geq 2$. For every node v with $b(v) = 0$, remove from G the edges leaving v, and then compute a k-edge-outconnected from s spanning subgraph H of G using the algorithm as in Theorem 2. Then $d_H(v) = 0$ for every $v \in V(G)$ with $b(v) = 0$. For every $v \in V$ with $b(v) \geq 1$ we have $d_H(v) \leq b(v) + 2 \leq 3b(v)$ if $k = 1$, and $d_H(v) \leq b(v) + 4 \leq 5b(v)$ if $k \geq 2$.

The following lemma (proof is omitted due to lack of space), in conjunction with the $O(1)$-approximation to DCkOS, proves part (ii) of Theorem 1.

Lemma 1. *An α-approximation algorithm for the DCkOS problem implies an $\alpha \cdot O(\log(kn))$-approximation algorithm for the WDCkOS problem.*

2.3 The SSTMLB Problem

The single topology case of the MLB under the single source relaxation is to find a spanning arborescence T of $G_{\mathcal{V}}$ rooted at s, so that the number of transmissions is maximized under the battery constraints. The problem can be reduced, similar to the multiple topology case, to that of finding a 1-edge-outconnected from s (namely, an arborescence rooted at s) spanning subgraph H of G, satisfying the constraints $k \cdot \beta_H(v) \leq b(v)$ for all $v \in \mathcal{V}$. By setting $B(v) \leftarrow b(v)/k$, we obtain the weighted-degree constraints $\beta_H(v) \leq B(v)$. This defines an instance of the WDCkOS problem with $k = 1$. Thus, we can compute in polynomial time a 1-outconnected from s spanning subgraph H of G so that for every $v \in V(G)$ we have $\beta_H(v) \leq \gamma \cdot B(v) = b(v)/k$, namely, $k \cdot \beta_H(v) \leq \gamma \cdot b(v)$. This means that we can guarantee k transmissions using H with battery capacities $\gamma \cdot b(v)$. Consequently, we can guarantee $\lfloor k/\gamma \rfloor$ transmissions with the original battery capacities $b(v)$, which proves part (i) of Theorem 1.

3 Omnidirectional Communication Model

In this section we consider the omnidirectional model. This model defines that the transmission of some node $v \in \mathcal{V}$ is received by *all* the nodes within the transmission range of v. Therefore, the power consumption of node $v \in \mathcal{V}$ due to a single message transmission, in a directed tree T, is $\beta_T(v) = \max_{e \in \delta_T(v)} w(e)$. We assume uniform initial battery charges, that is for all $v \in \mathcal{V}$, $b(v) = B$. Without loss of generality we may assume $B = 1$.

Recall the STMLB problem. We look for a spanning tree T of $G_{\mathcal{V}}$, so that the number of broadcast messages routed by using its orientations is maximized. We call T the broadcast backbone. In this section we show two different constructions of T, each satisfying additional multi-criteria constraints. In the end, we state that T can be used for convergecast (the STMLC problem) as well.

We are given a weighted, undirected graph $G_{\mathcal{V}}$, and a sequence $\mathcal{S}$ of m source nodes. Let $\langle T^*, k^* \rangle$ be an optimal solution for the SSMLB problem. We start by deriving an upper bound on k^*.

Lemma 2. *Let $e^* = (u, v)$ be the longest edge in T^*. Then $k^* \leq 2/w(e^*)$.*

Proof. Let T_i, $1 \leq i \leq k^*$, be a broadcast tree rooted at s_i, and obtained by orienting the edges of T^*. Note that either u transmits to v ($(u, v) \in E(T_i)$) or v transmits to u ($(v, u) \in E(T_i)$), but not both. Out of the k^* broadcast trees, let k_u be the number of trees in which u transmits to v. Without loss of generality, let $k_u \geq k^*/2$ (otherwise we take v). Since e^* is the longest edge in T^*, we can lower bound the total power consumption of u, $\sum_{i=1}^{k^*} \beta_{T_i}(u) \geq k_u w(e^*) \geq w(e^*)k^*/2$. Due to the power consumption constraint, $\sum_{i=1}^{k^*} \beta_{T_i}(u) \leq B = 1$. As a result, $k^* \leq 2/w(e^*)$. □

3.1 Multi-Criteria Broadcast Backbone

In this section we show that if we take T to be $MST(G_{\mathcal{V}})$, then we obtain a 2-approximation algorithm for the STMLB problem, as well as additional multi-criteria.

Lemma 3. *Let k be the maximum value, so that for all $v \in \mathcal{V}$, $\sum_{i=1}^{k} \beta_{T_i}(v) \leq b(v)$, where T_i, $1 \leq i \leq k$, is a broadcast tree rooted at s_i, and is obtained by orienting the edges of $MST(G_{\mathcal{V}})$. Then $k \geq k^*/2$.*

Proof. Let $e' = (u', v')$ be the longest edge in $MST(G_{\mathcal{V}})$. Since the longest edge in any minimum spanning tree is not greater than the longest edge of any spanning tree, $w(e') \leq w(e^*)$. Clearly, nodes u', v' have the largest possible power consumption $w(e')$ in any broadcast tree T_i, $1 \leq i \leq k$. Therefore, $k > 1/w(e')$. From Lemma 2, $k^* \leq 2/w(e^*)$. We conclude $k \geq k^*/2$. □

Note that using $MST(G_{\mathcal{V}})$ as the broadcast backbone, also provides some additional valuable multi-criteria guarantees, as concluded in the next theorem.

Theorem 3. *Given a weighted, undirected graph $G_{\mathcal{V}}$, and a sequence of m source nodes $\mathcal{S}$. Setting $T = MST(G_{\mathcal{V}})$; (i) provides us with k successful broadcast message propagations, where $k \geq k^*/2$; (ii) T has a bounded degree of 6; (iii) the total energy consumption in one broadcast tree is at most c times of the optimum, where $6 \leq c \leq 12$.*

Proof. (i) From Lemma 3, $k \geq k^*/2$; (ii) the maximum degree of $MST(G_{\mathcal{V}})$ is at most 6, since the minimum spanning tree of $G_{\mathcal{V}}$ is identical to the Euclidean minimum spanning tree on the node set $\mathcal{V}$, and the latter has a bounded degree of 6; (iii) in [20] the authors prove that for any node set in the plane, the total energy required by broadcasting from any node is at least $\frac{1}{c} \sum_{e \in E(MST(G_{\mathcal{V}}))} w(e)$, where $6 \leq c \leq 12$. Therefore the total energy consumption in one broadcast tree is of a constant factor from the best possible.

3.2 Bounded Hop-Diameter Multi-Criteria Broadcast Backbone

Our construction is based on a Hamiltonian circuit. Sekanina [21] showed that the cube of any tree, with at least 3 vertices, is Hamiltonian. Andrea and Bandelt [22] give a linear time algorithm for the construction of the Hamiltonian circuit in

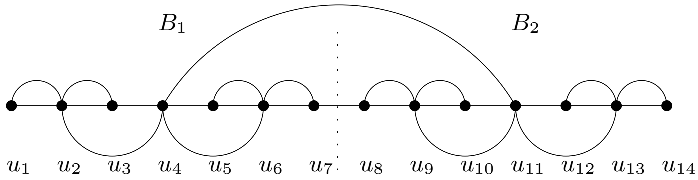

Fig. 1. Bounded hop-diameter broadcast backbone for $h = (u_1, u_2, \ldots, u_{14})$ and $\rho = 7$. There are $14/2 = 7$ node sequences $U_1 = \{u_1, u_2, \ldots, u_7\}$ and $U_2 = \{u_8, u_2, \ldots, u_{14}\}$. The center nodes of U_1 and U_2 are u_4 and u_{11}, respectively. Each of the trees B_1, B_2 spans the corresponding nodes in U_1 and U_2, respectively.

T^3, given T. They also show that the weight of the Hamiltonian circuit is at most $(\frac{3}{2}\tau^2 + \frac{1}{2}\tau)$ times the weight of the tree, where τ is the weak triangle inequality parameter (under our assumption that $\alpha = 2$, $\tau = 2^{\alpha-1} = 2$). Moreover, it can be shown that the longest edge in the Hamiltonian circuit is at most $O(1)$ times the longest edge in T. The following theorem applies the above to $MST(G_\mathcal{V})$.

Theorem 4 ([22]). *Let $h = (u_1, u_2, \ldots, u_{n+1} = u_1)$, where $u_i \in \mathcal{V}$ for $1 \le i \le n$, be the Hamiltonian circuit as a result of applying the construction in [22] on $MST(G_\mathcal{V})$. Define e^*_{MST} and e^*_h to be the longest edges in $MST(G_\mathcal{V})$ and h, respectively. Then $C(h) = O(C(MST(G_\mathcal{V}))$ and $w(e^*_h) = O(w(e^*_{MST}))$.*

Next, we describe the construction of the broadcast backbone T_h, based on the Hamiltonian circuit $h = (u_1, u_2, \ldots, u_{n+1} = u_1)$ from Theorem 4. Let ρ be an integer parameter, $1 \le \rho \le n$. The node set of T_h is $\mathcal{V}$. We divide the sequence of nodes $U_h = \{u_1, u_2, \ldots, u_n\}$ into n/ρ consecutive sequences U_i with ρ nodes each, so that $U_i = \{u_{\rho(i-1)+1}, u_{\rho(i-1)+2}, \ldots, u_{\rho i}\}$, $1 \le i \le n/\rho$.

The *center node* of a sequence $U = \{x_1, x_2, \ldots, x_j\}$, denoted $c(U)$, is the median node with an index $\lfloor \frac{j+1}{2} \rfloor$. There are two types of edges in T_h, $E(T_h) = E_1 \cup E_2$. The first type of edges connects the center nodes of every two adjacent node sequences, $E_1 = \{(c(U_i), c(U_{i+1}))\}_{i=1}^{n/\rho-1}$. The second type of edges, E_2, induces n/ρ complete binary trees $B_1, \ldots, B_{n/\rho}$. Each tree B_i, $1 \le i \le n/\rho$ spans the nodes in U_i and is rooted at $c(U_i)$. The tree B_i is constructed recursively. The children of $c(U_i)$ are the center nodes in subsequences $U_i^1 = \{v_{\rho(i-1)+1}, \ldots, v_{\rho(i-1)+\frac{\rho-1}{2}}\}$ and $U_i^2 = \{v_{\rho(i-1)+\frac{\rho+3}{2}}, \ldots, v_{\rho i}\}$. We then continue to construct a complete binary tree in each of the subsequences, U_i^1, U_i^2, in a similar way. Note that each tree B_i has $\log \rho$ levels (see example in Figure 1).

Denote by $e^*_{T_h}$ and e^*_h the longest edges in T_h and h, respectively. The next lemma shows some valuable bounds for T_h (the proof is omitted due to lack of space).

Lemma 4. *The graph T_h is a spanning tree of $G_\mathcal{V}$ and has a bounded hop-diameter of $O(n/\rho + \log \rho)$, a bounded degree of 4, and it holds $C(T_h) = O(\rho C(h))$ and $w(e^*_{T_h}) = O(\rho^2 w(e^*_h))$.*

Note that the tradeoff between the approximation of the longest edge and the hop-diameter bound presented in Lemma 4 is optimal. Consider the unweighted n-path: any tree of hop-diameter at most D for it, contains an edge with an interval length of at least $(n-1)/D$, and so its squared length is at least $(n-1)^2/D^2$. Since the longest edge of the n-path has a squared length of 1, we get an increase of the longest edge by a factor of at least $\Omega(n^2/D^2)$. Finally, substitute $D = n/\rho$ to obtain $\Omega(\rho^2)$.

Similar to the first construction, the broadcast backbone T_h satisfies multiple constraints according to Lemma 4. We can therefore derive the next theorem.

Theorem 5. *Given a weighted, undirected graph $G_{\mathcal{V}}$, and a sequence of m source nodes $\mathcal{S}$. Setting $T = T_h$; (i) provides us with k successful broadcast message propagations, where $k \geq k^*/2\rho^2$; (ii) T has a bounded hop-diameter of $n/\rho + \log \rho$; (iii) T has a bounded degree of 4; (iv) the total energy consumption in one broadcast tree is at most $O(\rho)$ times of the optimum.*

Proof. Conditions (ii) and (iii) are immediate from Lemma 4. From the same lemma in conjunction with Theorem 4, $w(e^*_{T_h}) = O(\rho^2 w(e^*_{MST}))$. By following similar arguments as in the proof of Lemma 3, we obtain (i). Combining Theorem 4 and Lemma 4 also yields the bound $C(T_h) = O(\rho C(MST(G_{\mathcal{V}})))$. Following the same arguments as in Theorem 3 condition (iv) follows. □

3.3 Applicability to the STMLC Problem.

The two constructions for the broadcast backbone may be used for convergecast, which will result in similar asymptotic bounds. The similarity follows from Lemma 2, which can be applied for convergecast transmissions, since it does not rely on any broadcast specific characteristics. This results in the same approximation ratios for the network lifetime (number of successful message propagations). The hop-diameter and degree bounds follow immediately from the constructions. Finally, we have to show that the total power consumption bound also holds. In [23], the authors showed that the total power consumption needed for one convergecast propagation is at least $C(MST)$.

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