Improved Approximation Algorithms for Maximum Lifetime Problems in Wireless Networks

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Abstract

A wireless ad-hoc network is a collection of transceivers positioned in the plane. Each transceiver is equipped with a limited battery charge. The battery charge is then reduced after each transmission, depending on the transmission distance. One of the major problems in wireless network design is to route network traffic efficiently so as to maximize the *network lifetime*, i.e., the number of successful transmissions. In this paper we consider Rooted Maximum Lifetime Broadcast/Convergecast problems in wireless settings. The instance consists of a directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, node capacities $\{b(v) : v \in V\}$, and a root r. The goal is to find a maximum size collection $\{T_1, \ldots, T_k\}$ of Broadcast/Convergecast trees rooted at r so that $\sum_{i=1}^{k} w(\delta_{T_i}(v)) \leq b(v)$, where $\delta_T(v)$ is the set of edges leaving v in T. In the Single Topology version all the Broadcast/Convergecast trees T_i are identical. We present a number of polynomial time algorithms giving constant ratio approximation for various broadcast and convergecast problems, improving previously known result of $\Omega(\lfloor 1/\log n \rfloor)$-approximation by [6]. We also consider a generalized Rooted Maximum Lifetime Mixedcast problem, where we are also given an integer $\gamma \geq 0$, and the goal is to find the maximum integer k so that k Broadcast and γk Convergecast rounds can be performed.

1 Introduction

Wireless ad-hoc networks received a lot of attention in recent years due to massive use in a large variety of domains, from life threatening situations, such as battlefield or rescue operations, to more civil applications, like environmental data gathering for forecast prediction. The network is composed of nodes located in the plane, communicating by radio. A transmission between two nodes is possible if the receiver is within the transmission range of the transmitter. The underlying physical topology of the network depends on the distribution of the nodes as well as the transmission power assignment of each node. Since the nodes have only a limited initial power charge, energy efficiency becomes a crucial factor in wireless networks design.

The transmission range of node v is determined by the power $p(v)$ assigned to that node. It is customary to assume that the minimal transmission power required to transmit to distance d is d^{ϕ}, where the *distance-power gradient* ϕ is usually taken to be in the interval $[2, 4]$ (see [19]). Thus, node

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v receives transmissions from u if $p(u) \geq d(u,v)^\phi$, where $d(u,v)$ is the Euclidean distance between u and v. There are two possible models: symmetric and asymmetric. In the symmetric model, also referred to as the undirected model, there is an undirected communication link between two nodes $u, v \in T$, if $p(u) \geq d(u,v)^\phi$ and $p(v) \geq d(v,u)^\phi$, that is if u and v can reach each other. The asymmetric variant allows directed (one way) communication links between two nodes. Krumke et al. [13] argued that the asymmetric version is harder than the symmetric one. This paper addresses the asymmetric model.

Ramanathan and Hain [21] initiated the formal study of controlling the network topology by adjusting the transmission range of the nodes. Increasing of the transmission range allows more distant nodes to receive transmissions but leads to faster battery exhaustion, which results in a shorter network lifetime. We are interested in maximizing the network lifetime under two basic transmission protocols, data broadcasting and data gathering (or convergecast). *Broadcasting* is a network task when a source node r wishes to transmit a message to all the other nodes in the network. In *convergecast* there is a destination node r, and all the other nodes wish to transmit a message to it. Here we consider convergecast *with aggregation*, meaning that a node uses aggregation mechanism to encode the data available at that node before forwarding it to the destination. We consider the case of unidirectional antennas, hence the message is transmitted to every node separately. Each node v, has an initial battery charge $b(v)$. The battery charge decreases with each transmission. The network lifetime is the number of rounds performed from network initialization to the first node failure due to battery depletion.

We assume that all the nodes share the same frequency band, and time is divided into equal size slots that are grouped into frames. Thus, the study is conducted in the context of TDMA. In TDMA wireless ad-hoc networks, a transmission scenario is valid if and only if it satisfies the following three conditions:

1. A node is not allowed to transmit and receive simultaneously.

2. A node cannot receive from more than one neighboring node at the same time.

3. A node receiving from a neighboring node should be spatially separated from any other transmitter by at least some distance D.

However, if nodes use unique signature sequences (i.e., a joint TDMA/CDMA scheme), then the second and third conditions may be dropped, and the first condition only characterizes a valid transmission scenario. Thus, our MAC layer is based on TDMA scheduling [3, 5, 11], such that collisions and interferences do not occur.

Many papers considered *fractional* version of the problem when splitting of packets into fractional portions is allowed. This versions admits an easy polynomial time algorithm via linear programming, c.f., [2, 7, 10, 12, 18, 26]. As data packets are usually quite small, there are situations where splitting of packets into fractional ones neither desirable nor practical. We consider a model where data packets are considered as units that cannot be split, i.e., when the packet flows are of *integral* values only. This discrete version was introduced by Sahni and Park [20].

In many *Network Design* problems one seeks a subgraph H with prescribed properties that minimizes/maximizes a certain objective function. Such problems are vastly studied in Combinatorial Optimization and Approximation Algorithms. Some known examples are Max-Flow, Min-Cost k-Flow, Maximum b-Matching, Minimum Spanning/Steiner Tree, and many others. See, e.g., [22, 9].

Formally, we obtain the following problem, which is the "wireless variant" of the classic arborescence packing problems. An *out-arborescence*, or simply an *arborescence*, is a directed tree that has a path from a root s to every node; an *in-arborescence* is a directed tree that has a path from every node to s. For a graph $H = (V, I)$ and a node $v \in V$, let $\delta_H(v) = \delta_I(v)$ denote the set of edges leaving v in H, and let $\delta_H^{in}(v) = \delta_I^{in}(v)$ denote the set of edges entering v in H. We consider variants of the following problem:

Rooted Maximum Lifetime Broadcast

Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, battery capacities $\{b(v) : v \in V\}$, and a root $r \in V$.

Objective: Find a maximum size collection $\mathcal{T} = \{T_1, \ldots, T_k\}$ of out-arborescences in G rooted at r that satisfies the *energy constraints*

$$\sum_{i=1}^{k} w(\delta_{T_i}(v)) \le b(v) \quad \text{for all } v \in V \ . \tag{1}$$

In the Rooted Maximum Lifetime Convergecast we seek a maximum size collection of in-arborescences, that are directed to r. In the Rooted Maximum Lifetime Mixedcast, we are also given an integer $\gamma \geq 0$, and the goal is to find the maximum integer k so that k Convergecast and γk Broadcast rounds can be performed. We observe that the problem is APX-hard, and the broadcast version is hard even for unit weights (however, Convergcast version with unit weights is polynomially solvable). In fact, for Broadcast, even determining whether $k \geq 1$ is NP-complete [6]. Hence it seems that an approximation ratio of the type $\lfloor k/\rho \rfloor$ is the best one can expect. We give algorithms when ρ is a constant, improving the ratio $\rho = O(\log n)$ established in [6].

More generally, we characterize the class of Maximum Network Lifetime problems as follows. In these problems, every node v has a limited battery capacity $b(v)$, and a transmission energy $w(vu)$ to any other node u is known. In transmission round i, we choose a subnetwork H_i with given properties, and every node transmits one message to each one of its neighbors in H_i; in many applications, each H_i is an arborescence (see [6]). The goal is to maximize the *lifetime* of the network, that is to find a maximum length feasible sequence $H_1, H_2, \ldots, H_k$ of subnetworks; feasibility means that every graph H_i satisfies the required properties, and that for every node v the total transmission energy during all rounds is at most $b(v)$. This is the *Multiple Topology* version of the problem. In the *Single Topology* variant, all the networks H_i are identical, c.f., [6] for more details. We note that in [16] was given a constant ratio approximation algorithm for the case when each H_i should be an st-path, for given $s, t \in V$. Here we consider the case when each H_i is an arborescence rooted from/to the root r.

In a more general setting, we might also be given a *cost-function* c on the edges, which can be distinct from the *weight-function* w, and wish to minimize the total cost $\sum_{i=1}^{k} c(H_i)$ of the communication subnetworks. We call this variant Min-Cost Maximum Network Lifetime.

While most of the problems considered in this paper deal with the case of convergecast with aggregation, we also deal with the problem Partial Level Aggregation Convergecast where we want to find a tree T of G directed towards the root node r that satisfies the energy constraints $\sum w(\delta_T(v)) \leq \frac{b(v)}{level(v)}$ for all $v \in V$, where $level(v)$ is the length of the longest path between v and its descendant in T.

1.1 Previous work

The authors in [17] show that for Broadcast, the problem is NP-Hard in the case of Single Source/Single Topology and has a polynomial solution for fractional version in the case of Single Source/Multiple Topology. They also show that it is NP-Hard in both of these cases for multicast. Segal [23] improved the running time of the solution for the Broadcast protocol and also showed an optimal polynomial time algorithm for the integral version of Single Topology Convergecast; the latter algorithm simply finds, using binary search, the largest integer k so that the graph $G - \{vu \in E : w(vu) > b(v)/k\}$ contains an arborescence directed to the root. For Multiple Topology Convergecast fractional version Kalpakis et al. [12] does have a polynomial solution in $O(n^{15} \log n)$ time. To counter the slowness of the algorithm, Stanford and Tongngam [24] proposed a $(1-\varepsilon)$-approximation in $O(n^3 \frac{1}{\varepsilon} \log_{1+\varepsilon} n)$ time based on the algorithm of Garg and Könemann [8] for packing linear programs. Elkin et al. [6] gave an $\Omega(\lfloor 1/\log n \rfloor)$-approximation for the discrete version of Multiple Topology Convergecast problem. Regarding the case without aggregation, some partial results were given in [25], [1] and [14]. The paper [25] considered the conditional aggregation where data from one node can be compressed in the presence of data from other nodes. Liang and Liu [14] present a number of heuristics for different types of aggregation problems. Buragohain et al. [1] proved the hardness of optimal routing tree problem for non-aggregate queries.

1.2 Our results

We give the first constant ratio approximation algorithm for Rooted Maximum Lifetime Broadcast/Convergecast/Mixedcast.

Theorem 1.1: Rooted Maximum Network Lifetime admits a polynomial time algorithm that finds a solution of value $\ell \geq \lfloor k/\beta^2 \rfloor$ where:
Single Topology: $\beta = 1$ for Convergecast, $\beta = 5$ for Broadcast and $\beta = 6$ for Mixedcast.
Multiple Topology: $\beta = 4$ for Convergecast, $\beta = 6$ for Broadcast, and $\beta = 10$ for Mixedcast.
Furthermore, for Min-Cost Maximum Network Lifetime the algorithm computes $\ell \geq \lfloor k/\beta^2 \rfloor$ arborescences so that their total cost is at most the total cost of k optimal arborescences.

For Broadcast, even checking whether $k \geq 1$ is NP-complete [6]. Thus we cannot guarantee any approximation ratio for such a problem, and an approximation of the form $\lfloor k/\rho \rfloor$ is the best one can expect. However, as Single Topology Convergecast is in P [23], for Multiple Topology Convergecast checking whether $k \geq 1$ can be done in polynomial time. This implies:

Corollary 1.2: Multiple Topology Convergecast version of Maximum Network Lifetime admits a $1/31$-approximation algorithm.

Proof: We return the better solution among two algorithms. The first algorithm is as in Theorem 1.1 that returns a solution of size $\ell \geq \lfloor k/16 \rfloor$. The second algorithm is the one that checks whether $k \geq 1$, and if so, returns a single arborescence.

Now, if $k = 0$ then no solution exists. If $1 \leq k \leq 31$, then we return a single arborescence. Finally, if $k \geq 32$, then we return at least $\ell \geq \lfloor k/16 \rfloor$ arborescences. Following this, the worst case is for $k = 31$ where we return a single arborescence. Thus, the approximation ratio is $1/31$. □

The following table summarizes the ratios for variants of Maximum Network Lifetime.

Single Topology			Multiple Topology		
Convergecast	*Broadcast*	*Mixedcast*	*Convergecast*	*Broadcast*	*Mixedcast*
k	$\lfloor k/25 \rfloor$	$\lfloor k/36 \rfloor$	$\max\{\lfloor k/16 \rfloor, 1\}$	$\lfloor k/36 \rfloor$	$\lfloor k/100 \rfloor$

Table 1: Summary of the ratios for variants of Maximum Network Lifetime. Except the polynomial solvability of the Single Topology Convergecast that was proved in [23], the other ratios are proved in this paper.

A related problem for which we can give a constant ratio approximation is minimizing the battery capacity b so that for $b(v) = b$ for all $v \in V$ at least k rounds of communications can be performed. Formally, this problem can be stated as follows:

Minimum Battery Rooted Lifetime k-Convergecast/Broadcast/Mixedcast
Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, a root $r \in V$, and an integer k.
Objective: Find a minimum battery capacity b so that if $b(v) = B$ for all $v \in V$, then there exists a collection $\mathcal{T} = \{T_1, \ldots, T_k\}$ of k out/in-arborescences in G rooted at r so that (1) holds.

Theorem 1.3: Minimum Battery Rooted Lifetime k-Broadcast/Convergecast/Mixedcast admits a β-approximation algorithm, where β is as in Theorem 1.1.

For Partial Level Aggregation Convergecast we gave a number of optimal and approximate solutions and discuss the possibilities to extend them to more general case.

2 Weighted Degree Constrained Network Design

Our results are based on a recent result due to Nutov [15] for directed Weighted Degree Constrained Network Design problems with intersecting supermodular demands. In Degree Constrained Network Design problems (without weights) one seeks the cheapest subgraph H of a given graph G that satisfies both prescribed connectivity requirements and degree constraints. One such type of problems are the Matching/Edge-Cover problems, which are solvable in polynomial time, c.f., [22]. For other degree constrained problems, even checking whether there exists a feasible solution is NP-complete, hence one considers bicriteria approximation when the degree constraints are relaxed.

The connectivity requirements can be specified by a set function f on V, as follows.

Definition 2.1: For an edge set of a graph H and node set S let $\delta_H(S)$ ($\delta_H^{in}(S)$) denote the set of edges in H leaving (entering) S. Given a set-function f on subsets of V and a graph $H = (V, F)$, we say that H is *f-connected* if

$$|\delta_H^{in}(S)| \geq f(S) \quad \text{for all } S \subseteq V. \tag{2}$$

Several types of f are considered in the literature, among them the following known one:

Definition 2.2: A set function f on V is *intersecting supermodular* if for any $X, Y \subseteq V$, $X \cap Y \neq \emptyset$

$$f(X) + f(Y) \leq f(X \cap Y) + f(X \cup Y) . \tag{3}$$

In [15] are considered network design problems with *weighted-degree* constraints. The *weighted degree* of a node v in a graph H with edge-weights $\{w(e) : e \in F\}$ is $w(\delta_H(v)) = \sum_{e \in \delta_H(v)} w(e)$.

Directed Weighted Degree Constrained Network (DWDCN)
Instance: A directed graph $G = (V, E)$ with edge-costs $\{c(e) : e \in E\}$ and edge-weights $\{w(e) : e \in E\}$, a set-function f on V, and degree bounds $\{b(v) : v \in V\}$.
Objective: Find a minimum cost f-connected subgraph H of G that satisfies the *weighted degree constraints*

$$w(\delta_H(v)) \leq b(v) \quad \text{for all } v \in V \ . \tag{4}$$

The function f is usually not given explicitly, but is assumed to admit an evaluation oracle (or other relevant oracles). Since for most functions f even checking whether DWDCN has a feasible solution is NP-complete, one considers bicriteria approximation algorithms. Assuming that the problem has a feasible solution, an (α, β)-approximation algorithm for DWDCN either computes an f-connected subgraph $H = (V, F)$ of G of cost $\leq \alpha \cdot \mathsf{opt}$ that satisfies $w(\delta_H(v)) \leq \beta \cdot b(v)$ for all $v \in V$, or correctly determines that the problem has no feasible solution. Note that even if the problem does not have a feasible solution, the algorithm may still return a subgraph that violates the degree constraints (4) by a factor of β.

For an edge set I, let $x(I) = \sum_{e \in I} x(e)$. Let opt denote the optimal value of the following natural LP-relaxation for DWDCN that seeks to minimize $c \cdot x$ over the following polytope P_f:

$$\begin{array}{rcllr}
x(\delta_E^{in}(S)) & \geq & f(S) & \text{for all } S \subset V & \text{(Cut Constraints)} \\
\sum\limits_{e \in \delta_E(v)} x(e)w(e) & \leq & b(v) & \text{for all } v \in V & \text{(Weighted Degree Constraints)} \\
0 \leq x(e) & \leq & 1 & \text{for all } e \in E &
\end{array}$$

Similarly, we may consider the version of DWDCN where the Cut Constraints are on edges leaving S, namely we have $x(\delta_E(S)) \geq f(S)$ for all $S \subset V$. We assume that f is intersecting supermodular. Let us fix parameters α and β as follows:

- Single Topology Convergecast, or DWDCN with Cut Constraint on the edges entering S and $0,1$-valued f:
  $\alpha = \beta = 1$.

- Single Topology Broadcast, or DWDCN with Cut Constraint on the edges leaving S and $0,1$-valued f:
  $\alpha = 2$ and $\beta = 5$.

- Multiple Topology Convergecast, or DWDCN with Cut Constraint on the edges entering S:
  $\alpha = 1$ and $\beta = 4$.

- Multiple Topology Broadcast, or DWDCN with Cut Constraint on the edges leaving S:
  $\alpha = 3$ and $\beta = 6$.

The following result has been proved in [15].

Theorem 2.3: [15] DWDCN with intersecting supermodular f admits a polynomial time algorithm that computes an f-connected graph H of cost $\leq \alpha \cdot \mathsf{opt}$ so that the weighted degree of every $v \in V$ is at most $\beta b(v)$.

3 Proof of Theorems 1.1 and 1.3

A graph H is *k-edge-outconnected from r* if it has k-edge-disjoint paths from r to any other node; H is *k-edge-inconnected from r* if it has k-edge-disjoint paths from every node to r. DWDCN problem includes as a special case the Weighted Degree Constrained k-Outconnected Subgraph problem, by setting $f(S) = k$ for all $\emptyset \neq S \subseteq V \setminus \{r\}$, and $f(S) = 0$ otherwise. For $k = 1$ we get the Weighted Degree Constrained Arborescence problem. Our problems are equivalent to the problem of packing maximum number k of edge-disjoint trees rooted at r so that their union H satisfies (4). By Edmond's Theorem [4], this is equivalent to requiring that H is k-edge-outconnected from r (or k-edge in-connected to r, in the case of convergecast) and satisfies (4). This gives the following problem:

Weighted-Degree Constrained k-Outconnected Subgraph
Instance: A directed graph $G = (V, E)$ with edge-weights $\{w(e) : e \in E\}$, degree bounds $\{b(v) : v \in V\}$, and a root $r \in V$.
Objective: Find a k-edge-outconnected from r spanning subgraph H of G that satisfies the weighted degree constraints (4) so that k is maximum.

In the Weighted-Degree Constrained k-Inconnected Subgraph problem, we require that H is k-edge-inconnected to r.

Let τ^* denote the optimal value of the natural LP-relaxation for Weighted-Degree Constrained k-Outconnected Subgraph that seeks to minimize $c \cdot x$ over the following polytope P_k:

$$\begin{array}{rcll} x(\delta_E^{in}(S)) & \geq & k & \text{for all } \emptyset \neq S \subseteq V \setminus \{r\} \qquad \text{(Cut Constraints)} \\ \sum_{e \in \delta_E(v)} x(e)w(e) & \leq & b(v) & \text{for all } v \in V \qquad \text{(Weighted Degree Constraints)} \\ 0 \leq x(e) & \leq & 1 & \text{for all } e \in E \end{array}$$

Namely, $P_k = P_f$ for

$$f(S) = \begin{cases} k & \text{if } \emptyset \neq S \subseteq V \setminus \{r\} \\ 0 & \text{otherwise} \end{cases}$$

In the Convergecast case the Cut Constraints are on edges leaving S, namely we have $x(\delta_E(S)) \geq f(S)$ for all $S \subset V$, with f as defined above. In both cases, the function f is intersecting supermodular, hence Theorem 2.3 applies.

We now explain how Weighted-Degree Constrained k-Outconnected/k-Inconnected Subgraph is related to our problem. We may assume that we know the maximum number k of trees, by applying binary search in the range $0, \ldots, nq$ where

$$q = \max_{v \in V} \frac{b(v)}{\min\{w(e) : e \in \delta_E(v), w(e) > 0\}} .$$

Indeed, if G contains an arborescence of weight 0, then k is infinite. Otherwise, every arborescence contains a node v that uses an edge $e \in \delta_G(v)$ with $w(e) > 0$. As there are n nodes, this implies the bound $k \leq nq$. As an edge of G may be used several times, add $k - 1$ copies of each edge of G. Equivalently, we may assign to every edge capacity k, and consider the corresponding "capacitated" problems; this will give a polynomial algorithm, rather than a pseudo-polynomial one. For simplicity of exposition, we will present the algorithm in terms of multigraphs, but it can be easily adjusted to capacited graphs.

As has been mentioned, for the Convergecast case checking whether $k \geq 1$ can be done in polynomial time [23]. Now we observe that Theorem 2.3 implies Theorem 1.3, as well as a "pseudo-approximation" algorithm for Weighted-Degree Constrained k-Outconnected/k-Inconnected Subgraph:

Corollary 3.1: For Weighted-Degree Constrained k-Outconnected/k-Inconnected Subgraph there exists a polynomial time algorithm that either correctly establishes that the polytope P_k is empty, or finds a k-outconnected subgraph H that violates the energy constraints by a factor at most β, namely

$$\sum_{e \in \delta_H(v)} w(e) \leq \beta \cdot b(v) \quad \text{for all } v \in V \ . \tag{5}$$

The above corollary immediately implies Theorem 1.3. We show how to derive from it also Theorem 1.1. Assuming we know k (binary search), The algorithm as in Theorem 1.1 is as follows:

1. Set $b(v) \leftarrow b(v)/\beta$ for all $v \in V$, where β is as in Corollary 3.1.
2. Compute a k-outconnected from r in the case of broadcast, and k-inconnected to r in the case of convergecast, spanning subgraph H of G using the algorithm as in Lemma 3.1.
   In the case of Mixedcast, H is the union of a k-inconnected to r and γk-outconnected from r spanning subgraphs H_{in} and H_{out}.

For the approximation ratio, all we need to prove is that if the original instance admits a k-outconnected/k-inconnected subgraph, then the new instance with weighted-degree bounds $b(v)/\beta$ admits an ℓ-outconnected/ℓ-inconnected spanning subgraph with $\ell = \lfloor k/\beta^2 \rfloor$, and is of low cost. This is achieved via the following lemma.

Lemma 3.2: Let $H_k = (V, F)$ be a k-outconnected from r (k-inconnected to r) directed graph with costs $\{c(e) : e \in F\}$ and weights $\{w(e) : e \in F\}$. Then for any $\ell \leq k$ the graph H_k contains an ℓ-outconnected from r (an ℓ-inconnected to r) spanning subgraph H_ℓ so that $c(H_\ell) \leq c(H_k) \cdot (\alpha \ell / k)$ and so that $w(\delta_{H_\ell}(v)) \leq w(\delta_{H_k}(v)) \cdot (\beta \ell / k)$ for all $v \in V$.

Proof: Consider the Weighted Degree Constrained ℓ-Outconnected Subgraph (Weighted Degree Constrained ℓ-Inconnected Subgraph) problem on H_k with degree bounds $b(v) = w(\delta_{H_k}(v)) \cdot (\ell/k)$. Clearly, $x(e) = \ell/k$ for every $e \in F$ is a feasible solution of cost $c(H_k) \cdot (\ell/k)$ to the LP-relaxation $\min\{c \cdot x : x \in P_\ell\}$. By Theorem 1.1, our algorithm computes a subgraph H_ℓ as required. □

Substituting $\ell = \lfloor k/\beta^2 \rfloor$ in Lemma 3.2 and observing that $\alpha \cdot \lfloor k/\beta^2 \rfloor / k \leq 1$ in all cases, we obtain:

Corollary 3.3: Let H be a k-outconnected from r (k-inconnected to r) directed graph with edge weights $\{w(e) : e \in F\}$. Then H contains a subgraph H' so that H' is $\lfloor k/\beta^2 \rfloor$-outconnected from r ($\lfloor k/\beta^2 \rfloor$-inconnected to r), $c(H') \leq c(H)$, and $w(\delta_{H'}(v)) \leq w(\delta_H(v))/\beta$ for all $v \in V$.

Except the Mixedcast part, Theorem 1.1 is easily deduced from Corollaries 3.1 and 3.3. For Mixedcast, note that $\beta = \beta_{in} + \beta_{out}$, where β_{in} and β_{out} are the parameters in Theorem 1.1 for Convergecast and Broadcast, respectively. From Lemma 3.2, we obtain that if H_k is k-outconnected from r and γk-inconnected to r, then H_k contains two spanning subgraphs: H_{out} that is ℓ-outconnected from r and H_{in} that is $\gamma\ell$-inconnected to r satisfying:

$$\begin{aligned} w(\delta_{H_{in}}(v)) + w(\delta_{H_{out}}(v)) &\leq w(\delta_H(v)) \cdot \beta_{out} \cdot (\ell/k) + w(\delta_H(v)) \cdot \beta_{in} \cdot (\gamma\ell/\gamma k) \\ &= w(\delta_H(v)) \cdot (\ell/k) \cdot (\beta_{out} + \beta_{in}) = w(\delta_H(v)) \cdot (\beta\ell/k) \ . \end{aligned}$$

Then, similarly to Corollary 3.3, we deduce that if H is k-outconnected from r and γk-inconnected to r, then H contains a subgraph H' so that H' is $\lfloor k/\beta^2 \rfloor$-outconnected from r and $\lfloor \gamma k/\beta^2 \rfloor$-inconnected to r, $c(H') \leq c(H)$, and $w(\delta_{H'}(v)) \leq w(\delta_H(v))/\beta$ for all $v \in V$.

4 Partial Level Aggregation Convergecast

In the problem of Partial Level Aggregation Convergecast we want to find a tree T of G directed towards the root node r that satisfies the energy constraints $\sum w(\delta_T(v)) \leq \frac{b(v)}{level(v)}$ for all $v \in V$, where $level(v)$ is the length of the longest path between v and its descendant in T. We consider the following cases.

- **Uniform initial batteries.** In this case, we note that the optimal solution is achieved by the tree of minimal depth (in regard to tree's root). We can find such tree by choosing every vertex to serve as the root, building the BFS tree starting at the chosen vertex, and picking up the tree of minimal depth. The total time complexity of the proposed algorithm is $O(|V|(|V| + |E|))$. Notice, that the problem becomes NP-complete when we are aiming to find the tree of maximal depth (HAMILTONIAN PATH).

- **Arbitrary initial batteries.** The simplest way to do is to use the above mentioned algorithm and to obtain $B_{\max}/B_{\min}$ approximate solution, where $B_{\max} = \max_{v \in V} b(v)$ and $B_{\min} = \min_{v \in V} b(v)$. We mention that for the case of complete graph, the optimal solution is the star, rooted at the node with minimal battery charge.

We can slightly change the definition of the problem in order to introduce the notion of weighted edges. To reflect this change, we transform the energy constraint to be $\sum w(\delta_T(v)) \leq \frac{b(v)}{w(e_1)+w(e_2)+\ldots+w(e_h)}$, where $w(e_i)$ is the weight of the edge e_i located on the most energy consumed path between v and its descendant in T. For arbitrary initial batteries and arbitrary edge weights we will use the construction based on Hamiltonian circuit and presented at Elkin et al. [6] where G is the complete graph. The authors at [6] have shown how to construct a spanning tree T of G that has a bounded hop-diameter of $O(n/\rho + \log \rho)$ with $w(e^*_T) = O(\rho^2 w(e^*))$, where e^*_T and e^* are the longest edges in T and the MST of G, respectively. The tightness of the tradeoff has been also established in [6]. Following this, the best approximation factor that can be given for this problem is $\Omega(n)$ which is achieved by $\rho = 1$.

5 Conclusions

At this paper we consider a number of broadcast and convergecast problems in wireless settings under the criterion of maximizing the lifetime of the underlying wireless backbone. For unidirectional antennas, we present constant factor approximate solutions improving previously known results as well as extending them and dealing with different aggregation cases. One of the main open questions is obtaining a non-trivial algorithm for the case of omnidirectional antennas.

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