Magnetized Relativistic Outflows: effects of strong time dependence

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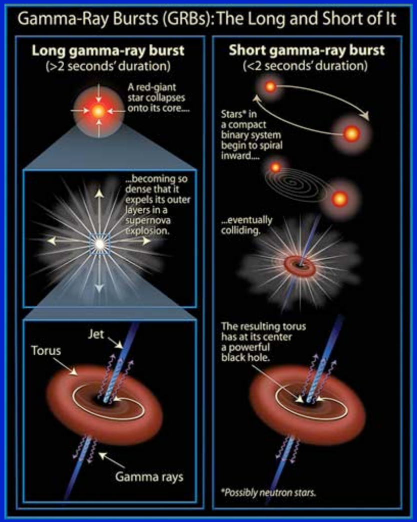
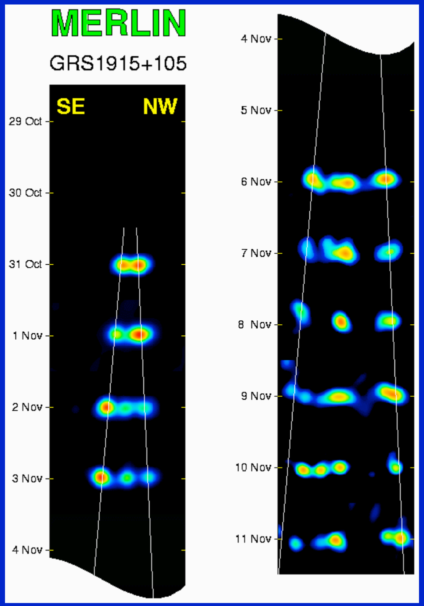
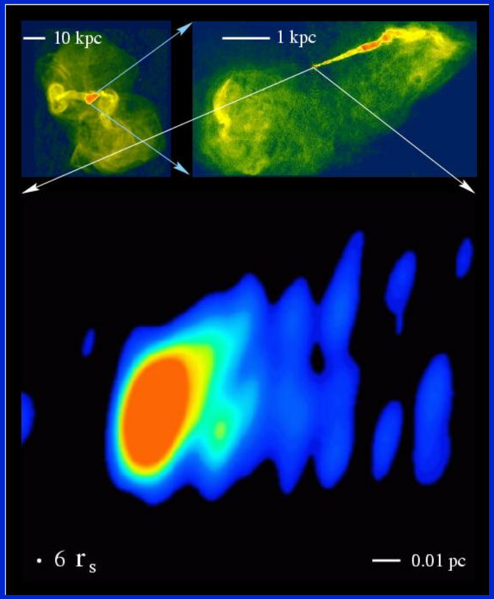
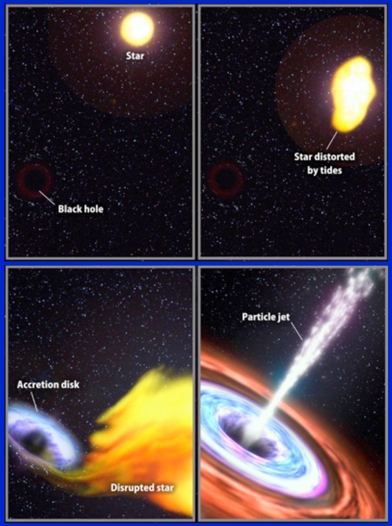








13th Marcel Grossmann meeting, Stockholm, July 5, 2012

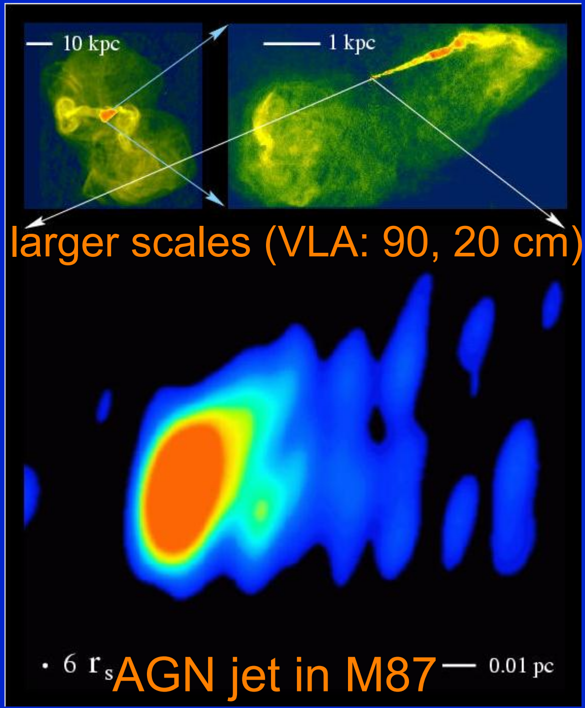
Outline of the talk:

- Motivation & comparison to thermal acceleration
- Steady magnetic acceleration
- The σ problem & possible solutions
- A new solution: impulsive magnetic acceleration
  - **A single shell accelerating into vacuum**
  - **A single shell expanding into an external medium**
  - **Many shells: acceleration + internal shock efficiency**
- Implications for GRBs

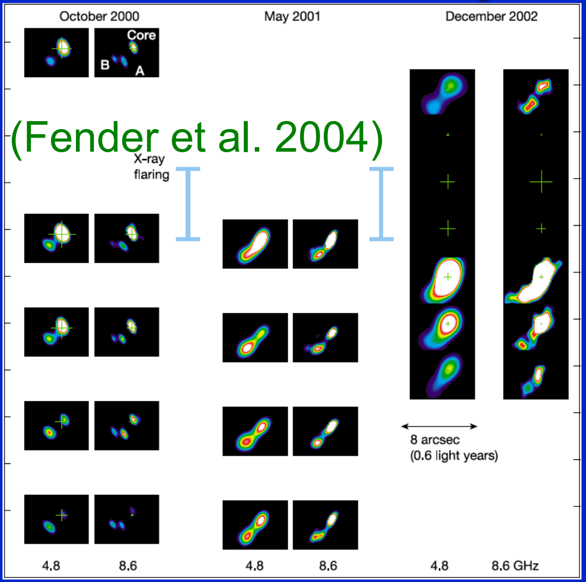
Relativistic Magnetic Acceleration:

- Relativistic ($v \approx c$) outflows/jets are very common in astrophysics & involve strong gravity at the source: PWN (NS), GRBs, AGN (SMBH), μ-quasars (BH/NS)
- Most models assume a steady flow for simplicity, despite observational evidence for time variability

Crab Nebula: X-ray in blue, optical in red



AGN jet in M87 (VLBA @ 43 GHz)



Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)

Relativistic Magnetic Acceleration:

Is the acceleration magnetic?

✔ ? ✔ ?

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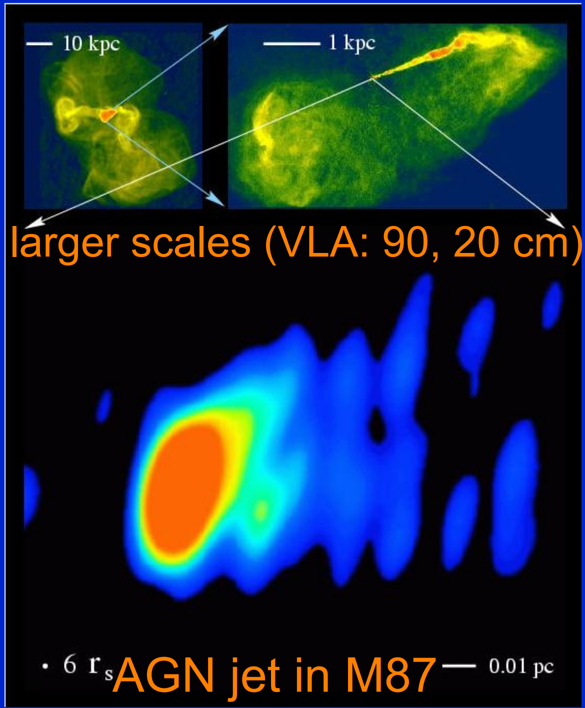

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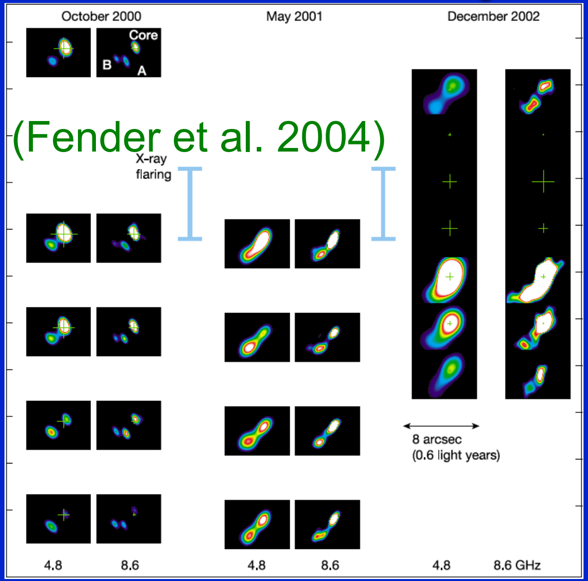


Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)

Relativistic Magnetic Acceleration:

- Magnetic acceleration of jets: energy is transported to large distances form the source by Poynting flux
- Option 1: The initial magnetic energy is converted into the kinetic energy of plasma, which is then dissipated in internal shocks & produces radiation
- Option 2: The flow remains highly magnetized far from the source & magnetic reconnection events directly accelerate particles that produce radiation

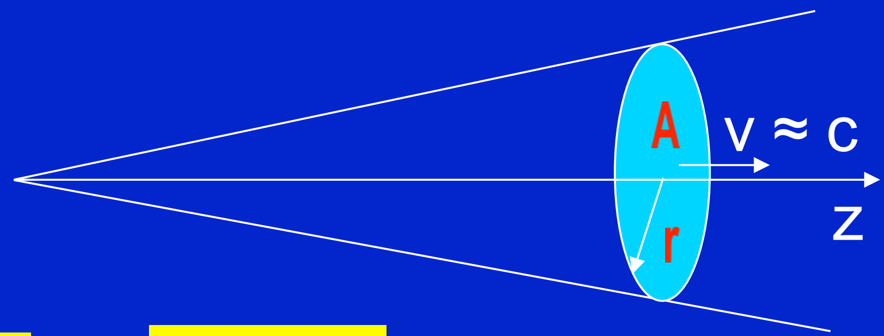
Thermal vs. Magnetic Acceleration:

- Most of the acceleration is in the supersonic regime

Key difference between thermal and magnetic steady state acceleration of relativistic supersonic flows:

- **Thermal**: fast, robust & efficient
- **Magnetic**: slow, delicate & less efficient

Thermal acceleration: conical flow



Relativistic EoS: $p \propto \rho^{4/3}$

Mass conservation: $A\Gamma\rho c = \text{const} \rightarrow \rho \propto 1/r^2\Gamma$

$A \propto r^2$

Energy conservation: $A\Gamma^2(\rho c^2 + 4p)c = \text{const}$

Bernoulli equation: $(1 + 4p/\rho c^2)\Gamma = \text{const}$

for a conical flow

$p >> \rho c^2$

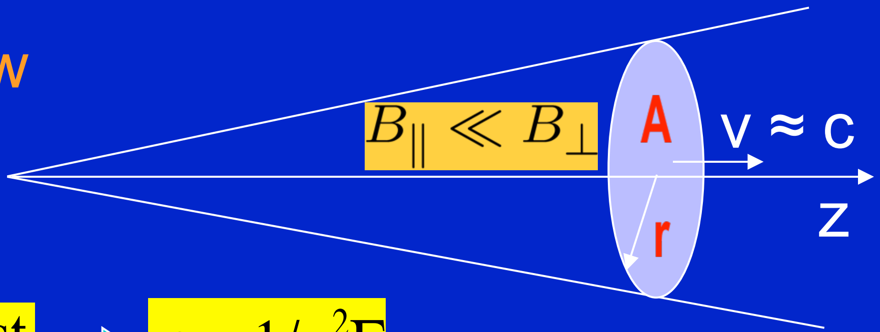
$\Gamma = \rho/p \propto \rho^{-1/3} \propto \Gamma^{1/3} r^{2/3} \rightarrow \Gamma \propto r \propto z$

Very fast acceleration !!!

Ideal MHD acceleration: conical flow

for a conical flow

Ideal MHD (flux freezing): $B = \Gamma B' \propto 1/r$



Mass conservation: $A\Gamma\rho c = \text{const}$ → $\rho \propto 1/r^2\Gamma$

$A \propto r^2$

Energy conservation: $A\Gamma^2(\rho c^2 + B'^2/4\pi)c = \text{const}$

Bernoulli equation: $(1+\sigma)\Gamma = \text{const}$

$$\sigma = \frac{B'^2}{4\pi\rho c^2} \propto \frac{1}{\Gamma}$$

$$\Gamma = \text{const}$$

No acceleration!!!

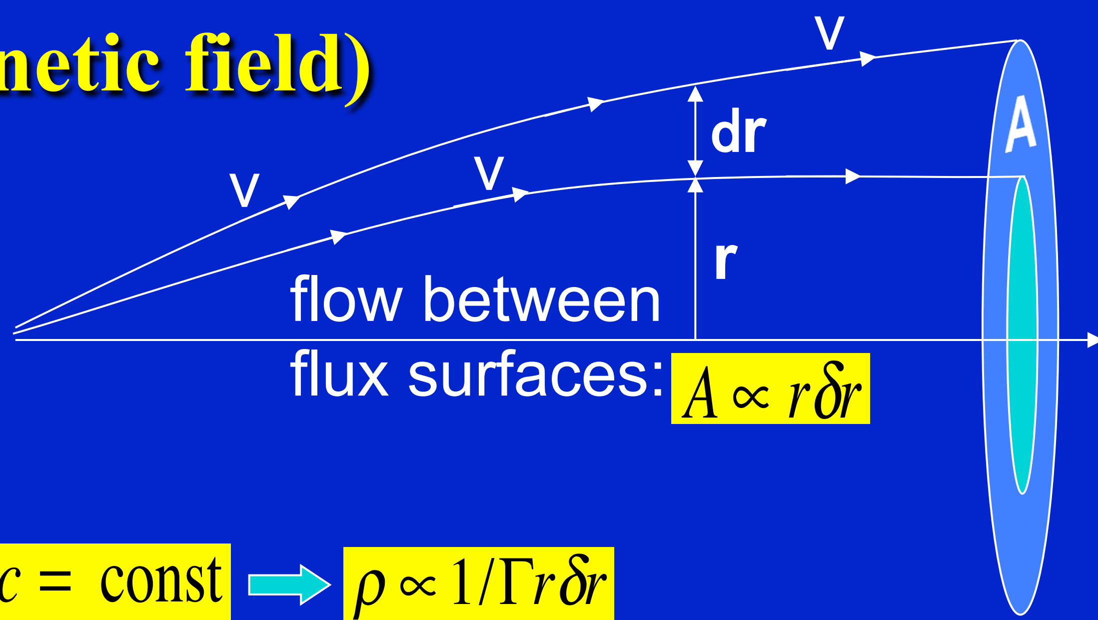
Fluid element volume: $V \propto r^2$

Its magnetic field: $B \propto r^{-1}$

Its electromagnetic energy:

$E_{em} \propto B^2 V = \text{const}$

Ideal MHD acceleration: non-conical flow (toroidal magnetic field)



$A \propto r\delta r$

Ideal MHD: $B = \Gamma B' \propto 1/\delta r$

Mass conservation: $A\Gamma\rho c = \text{ const}$ → $\rho \propto 1/\Gamma r\delta r$

Energy conservation: $A\Gamma^2(\rho c^2 + B'^2/4\pi)c = \text{ const}$

Bernoulli equation: $(1+\sigma)\Gamma = \text{ const} = C_0$

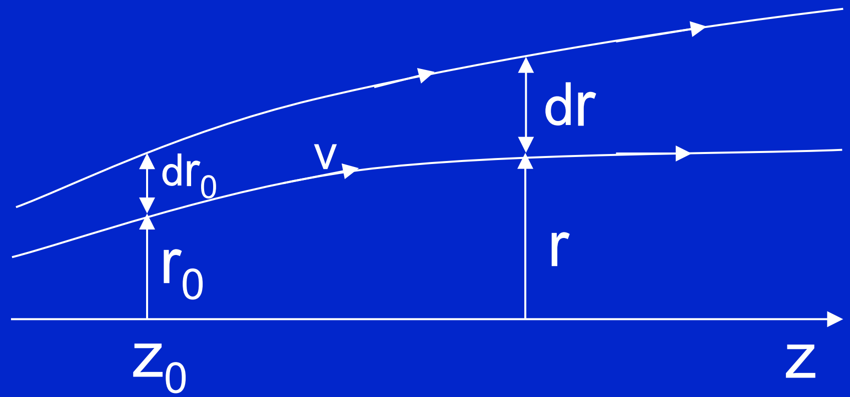
$$\sigma = \frac{B'^2}{4\pi\rho c^2} = C_1 \frac{r}{\Gamma\delta r}$$

$$\Gamma = C_0 - C_1 \frac{r}{\delta r}$$

$\Gamma = C_0 - C_1 r/\delta r$ → r/δr should decrease for acceleration!!! (stream lines must diverge faster than conical)

- Power-law stream lines: $z = z_0(r/r_0)^\alpha \Rightarrow r/\delta r = r_0/\delta r_0$ $\Rightarrow$ no acceleration

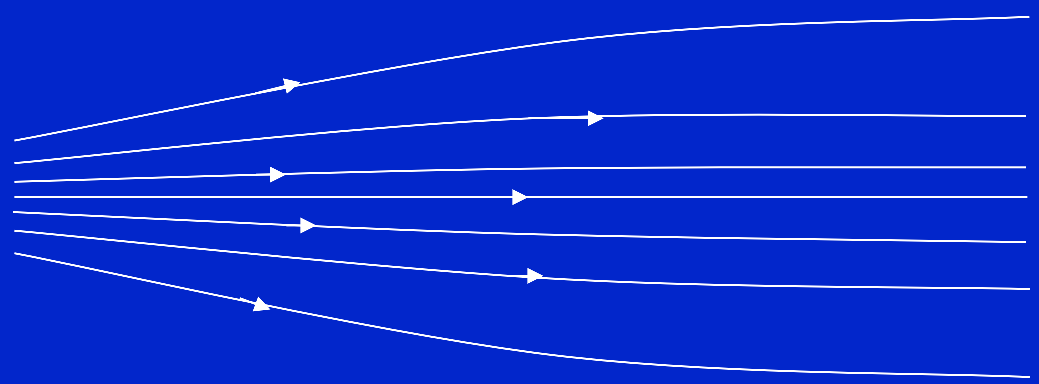
- Varying $\alpha = \alpha(r_0)$:



$$r/\delta r = (r_0/\delta r_0)[1-\alpha' r_0 \alpha^{-2} \ln(z/z_0)]^{-1}$$

decreases if $\alpha' = d\alpha/dr_0 < 0$

Can this be satisfied? It requires causal contact across the jet



Ideal MHD acceleration: numerical + analytic results (Komissarov 2009; Lyubarsky 2009)

- **Unconfined** flows quickly lose lateral causal contact, become quasi-spherical (locally conical) & stop accelerating when $\Gamma_\infty \sim \sigma_0^{1/3}$ & $\sigma_\infty \sim \sigma_0^{2/3} \gg 1$ (Goldreich & Julian 1970)

- **Weak confinement**: $p_{ext} \propto z^{-\alpha}$ with $\alpha > 2 \Rightarrow$ lose lateral causal contact, become conical & stop accelerating later:
  causal contact: $$\theta_{jet} < \theta_{Mach} \approx \sin\theta_{Mach} = \frac{(\Gamma\beta)_{ms}}{\Gamma\beta} \approx \frac{\sigma^{1/2}}{\Gamma} \sim \frac{\sigma_0^{1/2}}{\Gamma^{3/2}}$$
  $\sigma_\infty \sim (\sigma_0\theta_{jet})^{2/3}$
  $\Gamma_\infty \sim \sigma_0^{1/3}\theta_{jet}^{-2/3}$
  $\sigma > (\Gamma\theta_{jet})^2 \Rightarrow$ efficient conversion: $\Gamma_\infty\theta_{jet} < 1$

- **Strong confinement**: $p_{ext} \propto z^{-\alpha}$ with $\alpha < 2 \Rightarrow$ stay in causal contact $\Gamma \propto z^{\alpha/4}$ and reach $\Gamma_\infty \sim \sigma_0$, $\sigma_\infty \sim 1$, $\Gamma_\infty\theta_{jet} \sim 1$

The σ-problem: for a "standard" steady ideal MHD axisymmetric flow

- $\Gamma_\infty \sim \sigma_0^{1/3}$ & $\sigma_\infty \sim \sigma_0^{2/3} \gg 1$ for a spherical flow; $\sigma_0 = B_0^2/4\pi\rho_0 c^2$
- However, PWN observations (e.g. the Crab nebula) imply $\sigma \ll 1$ after the wind termination shock – the σ problem!!!
- A broadly similar problem persists in relativistic jet sources
- Jet **collimation** helps, but not enough: $\Gamma_\infty \sim \sigma_0^{1/3}\theta_{jet}^{-2/3}$, $\sigma_\infty \sim (\sigma_0\theta_{jet})^{2/3}$ & $\Gamma\theta_{jet} \lesssim \sigma^{1/2}$ (~1 for $\Gamma_\infty \sim \Gamma_{max} \sim \sigma_0$)
- Still $\sigma_\infty \gtrsim 1$ $\Rightarrow$ inefficient internal shocks, $\Gamma_\infty\theta_{jet} \gg 1$ in GRBs
- Sudden drop in external pressure can give $\Gamma_\infty\theta_{jet} \gg 1$ but still $\sigma_\infty \gtrsim 1$ **(Tchekhovskoy et al. 2009)** $\Rightarrow$ inefficient internal shocks

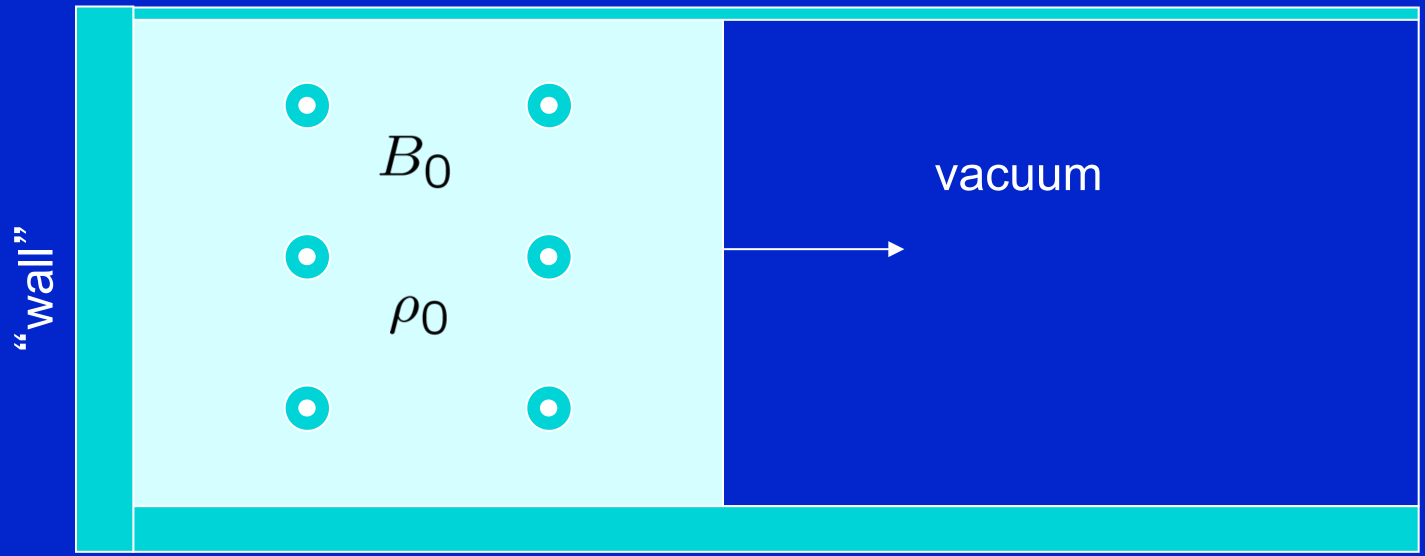
Alternatives to the "standard" model

- ~~Axisymmetry~~: non-axisymmetric instabilities (e.g. the current-driven kink instability) can tangle-up the magnetic field (Heinz & Begelman 2000)
- If $\langle B_r^2 \rangle = \alpha \langle B_\phi^2 \rangle = \beta \langle B_z^2 \rangle \;;\;\; \alpha, \beta = \text{const}$ then the magnetic field behaves as an ultra-relativistic gas: $p_{\text{mag}} \propto V^{-4/3}$
  $\Rightarrow$ magnetic acceleration as efficient as thermal
- ~~Ideal~~ MHD: a tangled magnetic field can reconnect (Drenkham 2002; Drenkham & Spruit 2002)
  magnetic energy $\Rightarrow$ heat (+radiation) $\Rightarrow$ kinetic energy
- ~~Steady-state~~: **effects of strong time dependence**
  (JG, Komissarov & Spitkovsky 2011; JG 2012a, 2012b)

Impulsive Magnetic Acceleration: a single shell expanding into vacuum

(JG, Komissarov & Spitkovsky 2011, MNRAS; 411, 1323)

- **Impulsive magnetic acceleration** (Contopoulos 1995, "plasma gun" - unsteady source; Lyutikov 2010; Levinson 2010)
- Highly magnetized cold plasma shell expanding into vacuum



(useful **case study**)

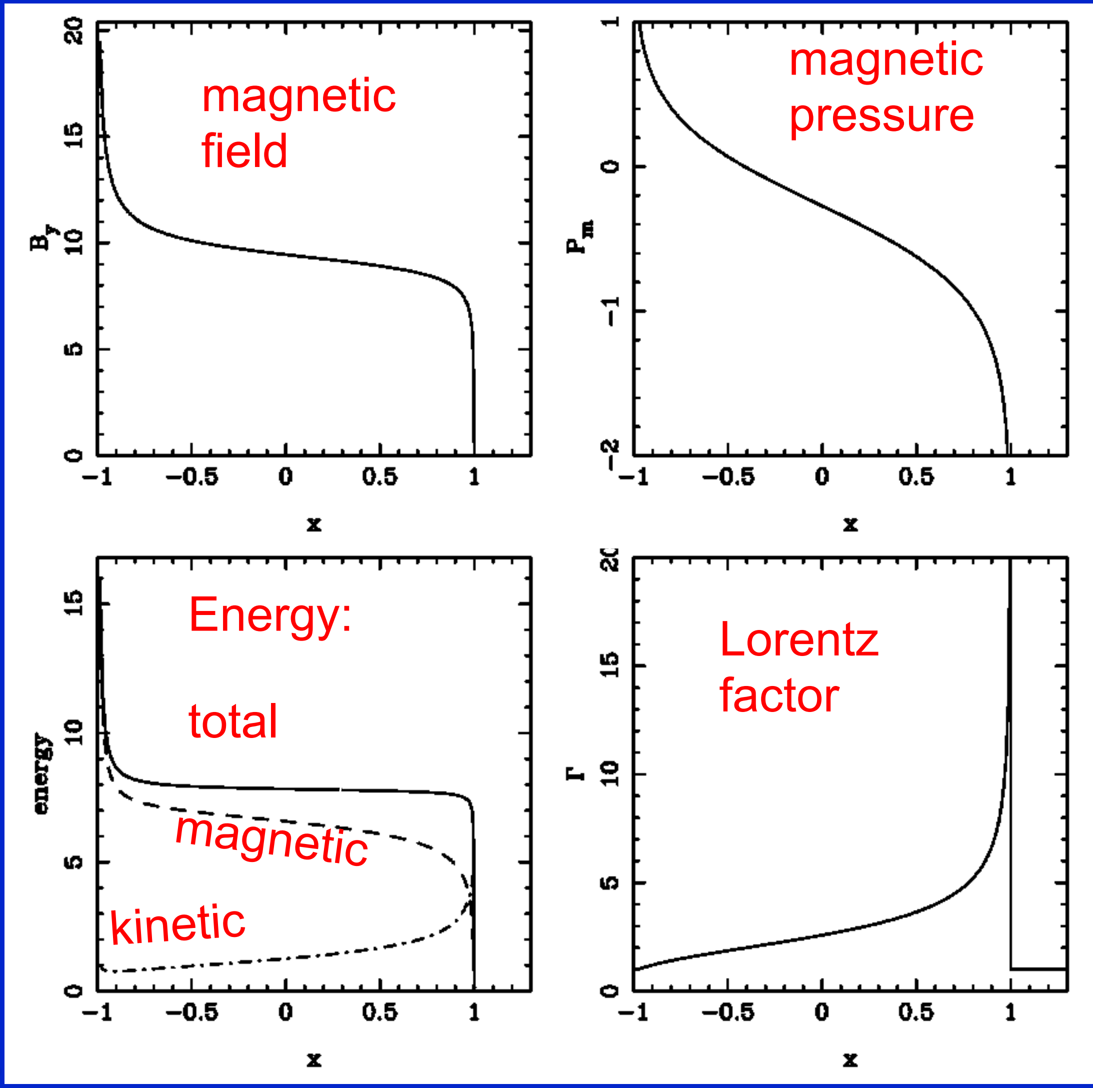
Initial value of magnetization parameter: $\sigma_0 = \frac{B_0^2}{4\pi\rho_0 c^2} >> 1$

1. Self-similar rarefaction wave

- Solution at $t = 1$ when the rarefaction just reaches the wall ($c = 1$)
- Self-similar solution: simple rarefaction wave
- At the boundary with vacuum: $\Gamma \approx 2\sigma_0$
- However, the mean value is: $\langle\Gamma\rangle_E \approx (\sigma_0)^{1/3}$
- $\langle E/M\rangle \sim \langle\sigma\Gamma\rangle \approx$ **const** $\sim \sigma_0$ & this fast acceleration requires causal contact:

$\Gamma \approx u < u_{ms} = \sigma^{1/2} \sim (\sigma_0/\Gamma)^{1/2}$

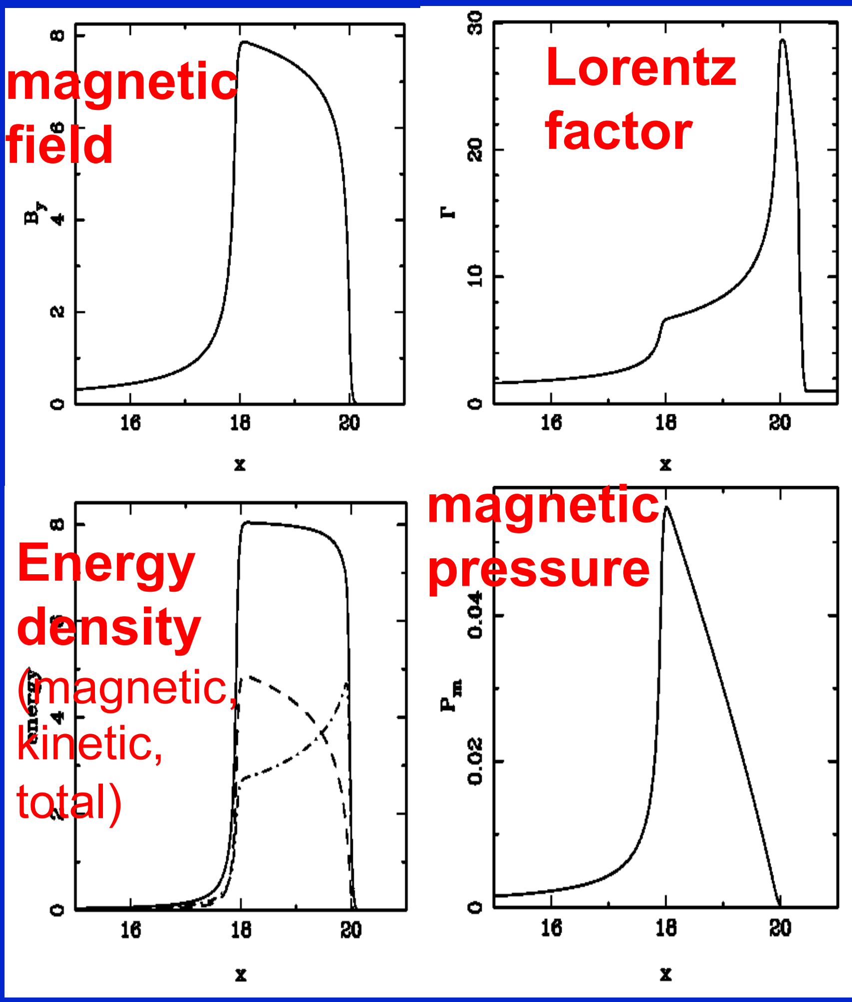
$\Rightarrow \Gamma \lesssim (\sigma_0)^{1/3}$ $\quad u = \Gamma\beta$



initial width = 1; a wall at $x = -1$; $s_0 = 30$

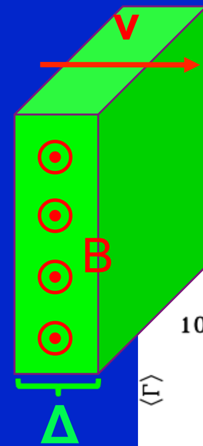
2. After separation from the wall:

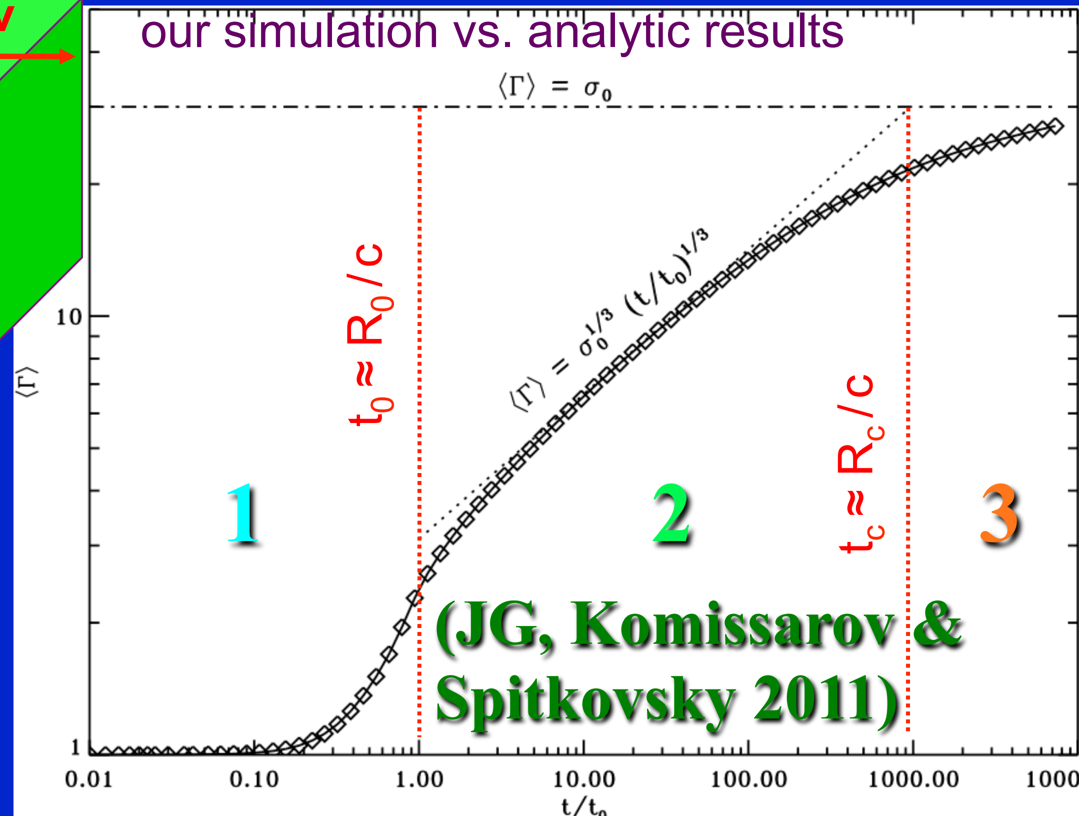
- A second rarefaction wave forms
- Solution at $t = 20$ after the shell has separated from the ($c = 1$):
- the shell width, energy, mass & momentum hardly change
- The Lorentz factor $\langle\Gamma\rangle_E$ grows as magnetic energy & momentum are transferred to the plasma
- Once the shell separates from the wall Γ_{CM} remains constant (no external force) while $\langle\Gamma\rangle_E$ grows since the front part carries most of the energy in the lab frame



initial width = 1;
a wall at $x = -1$; $s_0 = 30$

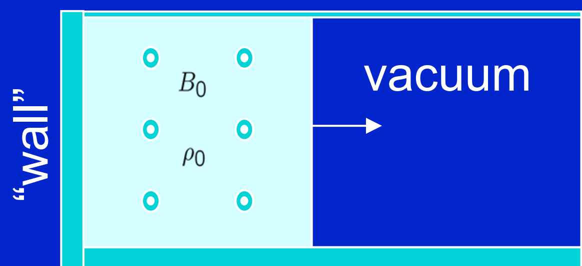
Impulsive Magnetic Acceleration: $\Gamma \propto R^{1/3}$





Useful **case study**:

Initial value of magnetization parameter: $\sigma_0 = \dfrac{B_0^2}{4\pi\rho_0 c^2} >> 1$

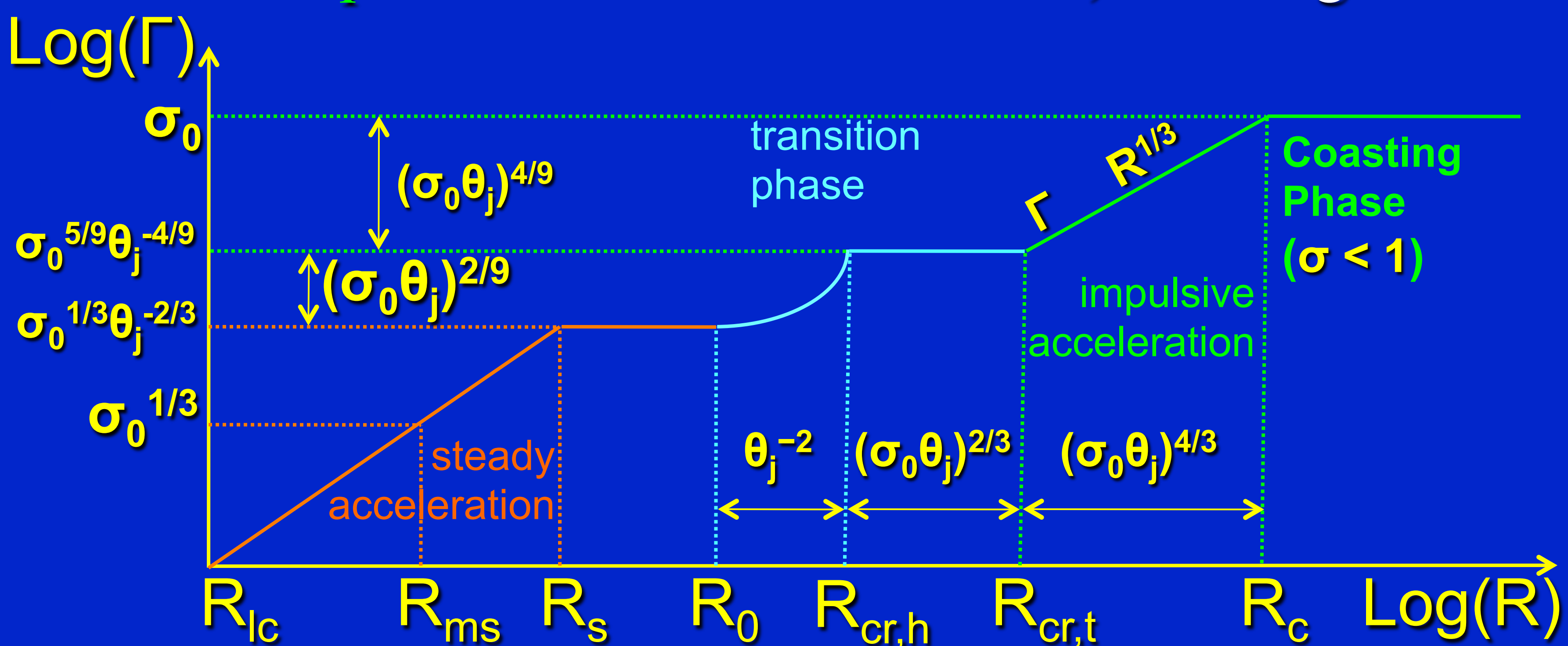


1. $\langle\Gamma\rangle_E \approx \sigma_0^{1/3}$ by $R_0 \sim \Delta_0$
2. $\langle\Gamma\rangle_E \propto R^{1/3}$ between $R_0 \sim \Delta_0$ & $R_c \sim \sigma_0^2 R_0$ and then $\langle\Gamma\rangle_E \approx \sigma_0$
3. At $R > R_c$ the sell spreads as $\Delta \propto R$ & $\sigma \sim R_c/R$ rapidly drops

- **Complete conversion of magnetic to kinetic energy!**
- **This allows efficient dissipation by shocks at large radii**

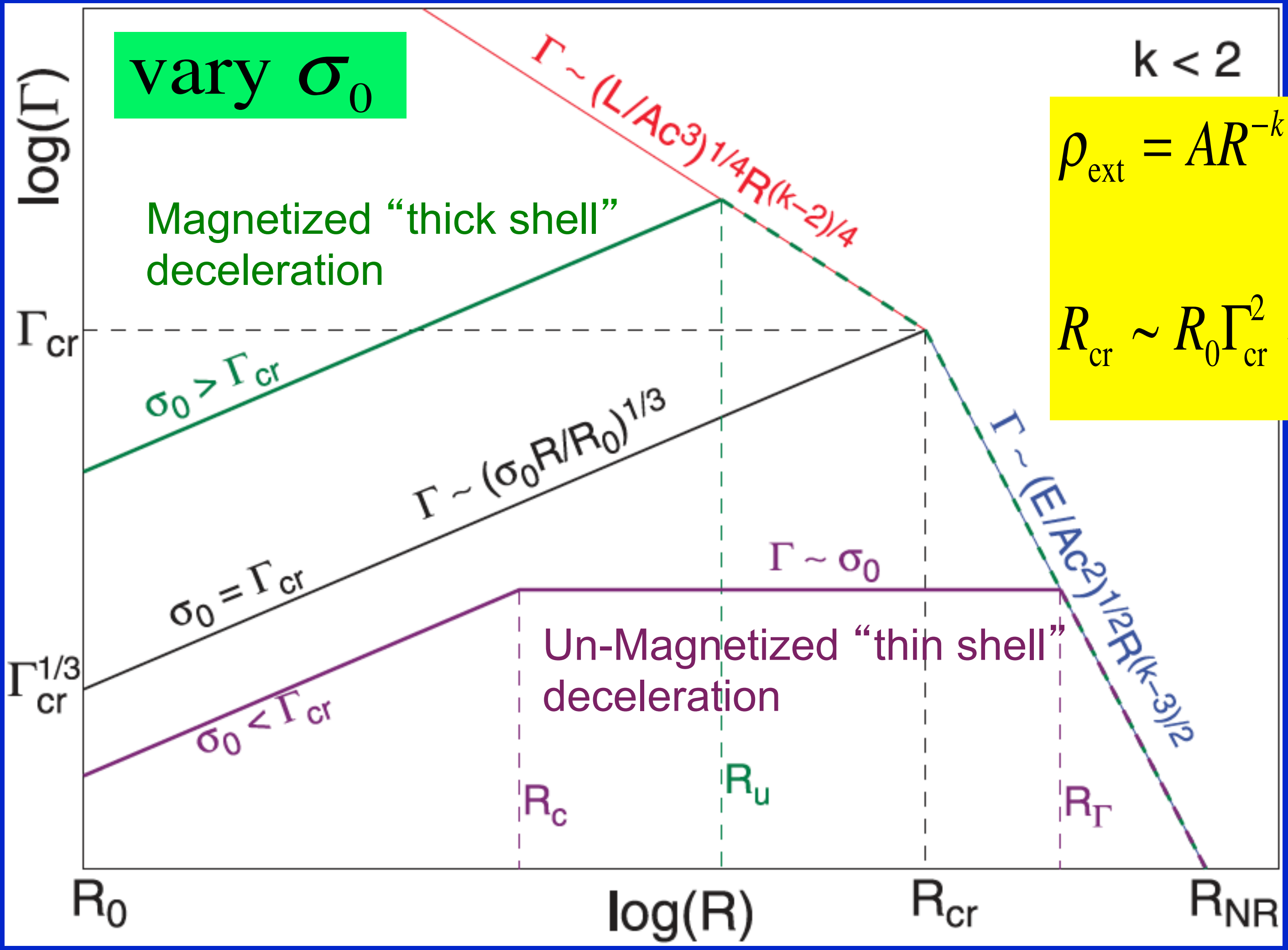
1st Steady then Impulsive Acceleration

- Our test case problem may be directly relevant for giant flares in SGRs (active magnetars); however:
- In most astrophysical relativistic (jet) sources (GRBs, AGN, μ-quasars) the variability timescale ($t_v \approx R_0/c$) is long enough ($>R_{ms}/c$) that steady acceleration operates & saturates (at R_s)
- Then the impulsive acceleration kicks in, resulting in $\sigma < 1$



Impulsive Magnetic Acceleration: single shell propagating in an external medium

acceleration & deceleration are tightly coupled (JG 2012)



$$\rho_{ext} = AR^{-k}$$

$$R_{cr} \sim R_0 \Gamma_{cr}^2 \sim \left(\frac{ER_0}{Ac^2}\right)^{\frac{1}{4-k}}$$

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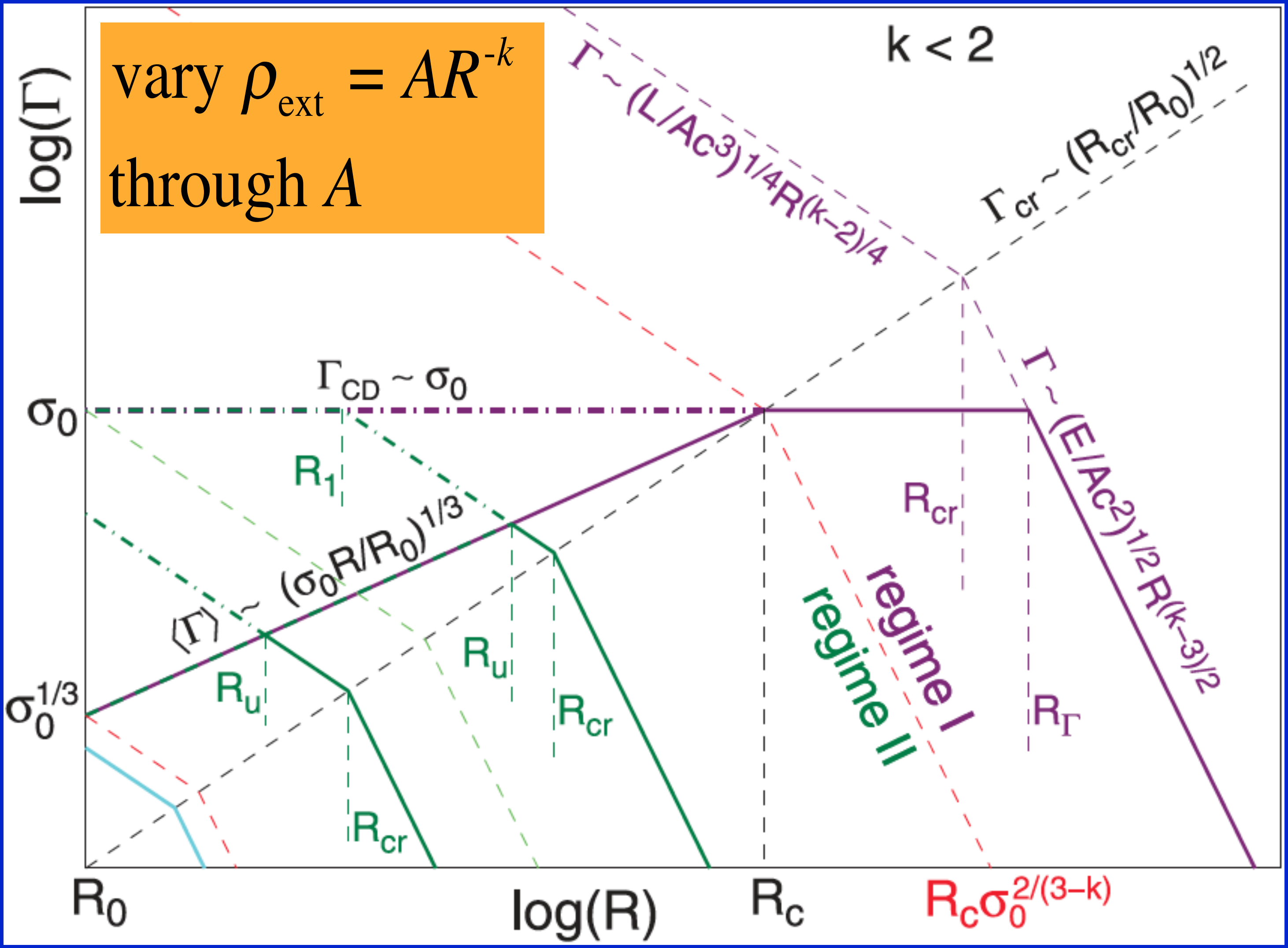


Dynamical Regimes:

I. "Thin shell", low-σ: strong reverse shock, peaks at $\gg T_{GRB}$

II. "Thick shell", high-σ: weak or no reverse shock, $T_{dec} \sim T_{GRB}$

III. like II, but the flow becomes independent of σ_0

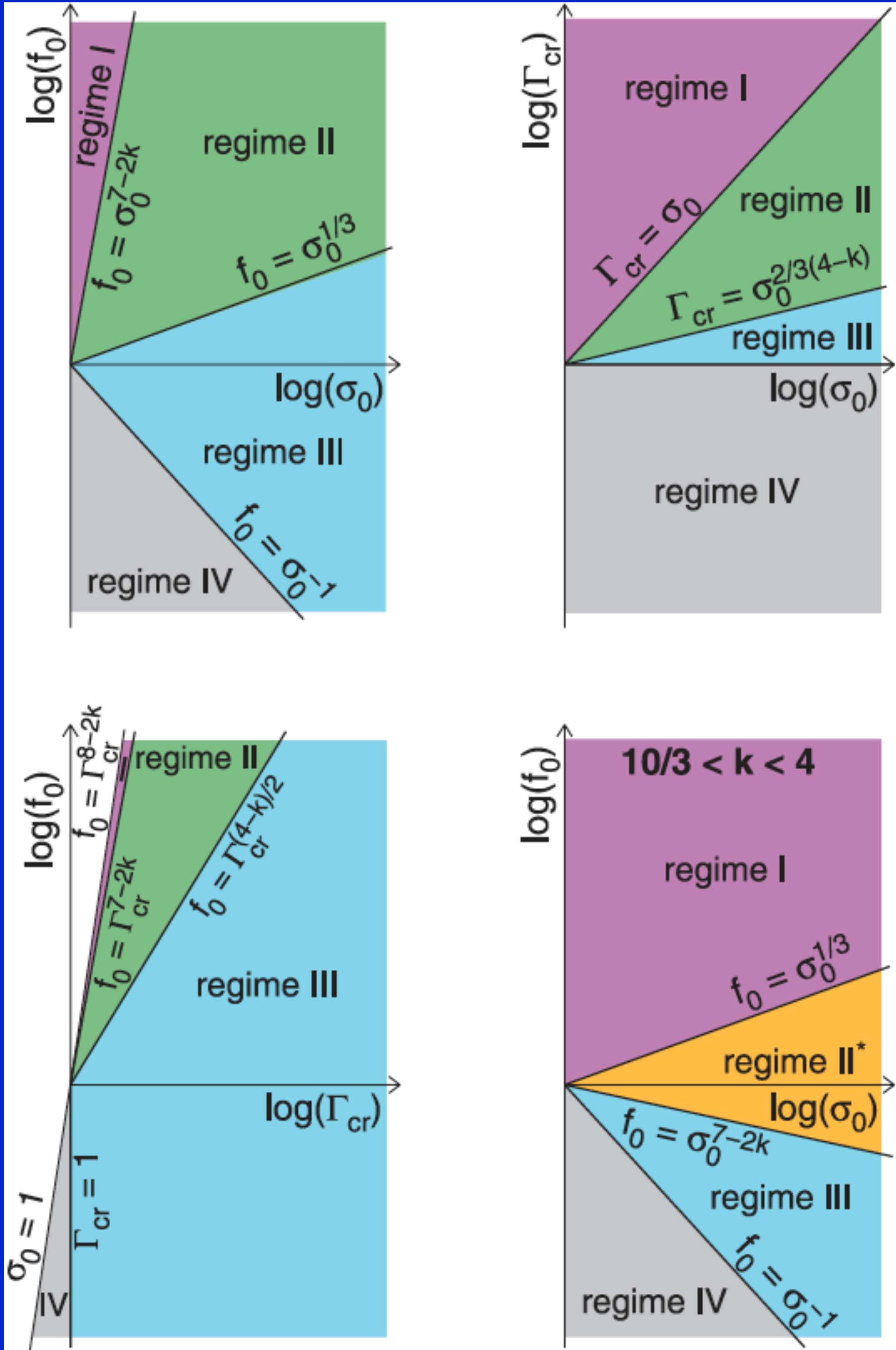
IV. a Newtonian flow (if ρ_{ext} is very high, e.g. inside a star)

II*. if ρ_{ext} drops very sharply

$$\sigma_0 = B_0^2 / 4\pi\rho_0 c^2$$

$$f_0 = \rho_0 / \rho_{ext}(R_0)\ , \quad \rho_{ext} = AR^{-k}$$

$$\Gamma_{cr} \sim (f_0\sigma_0)^{1/(8-2k)}$$



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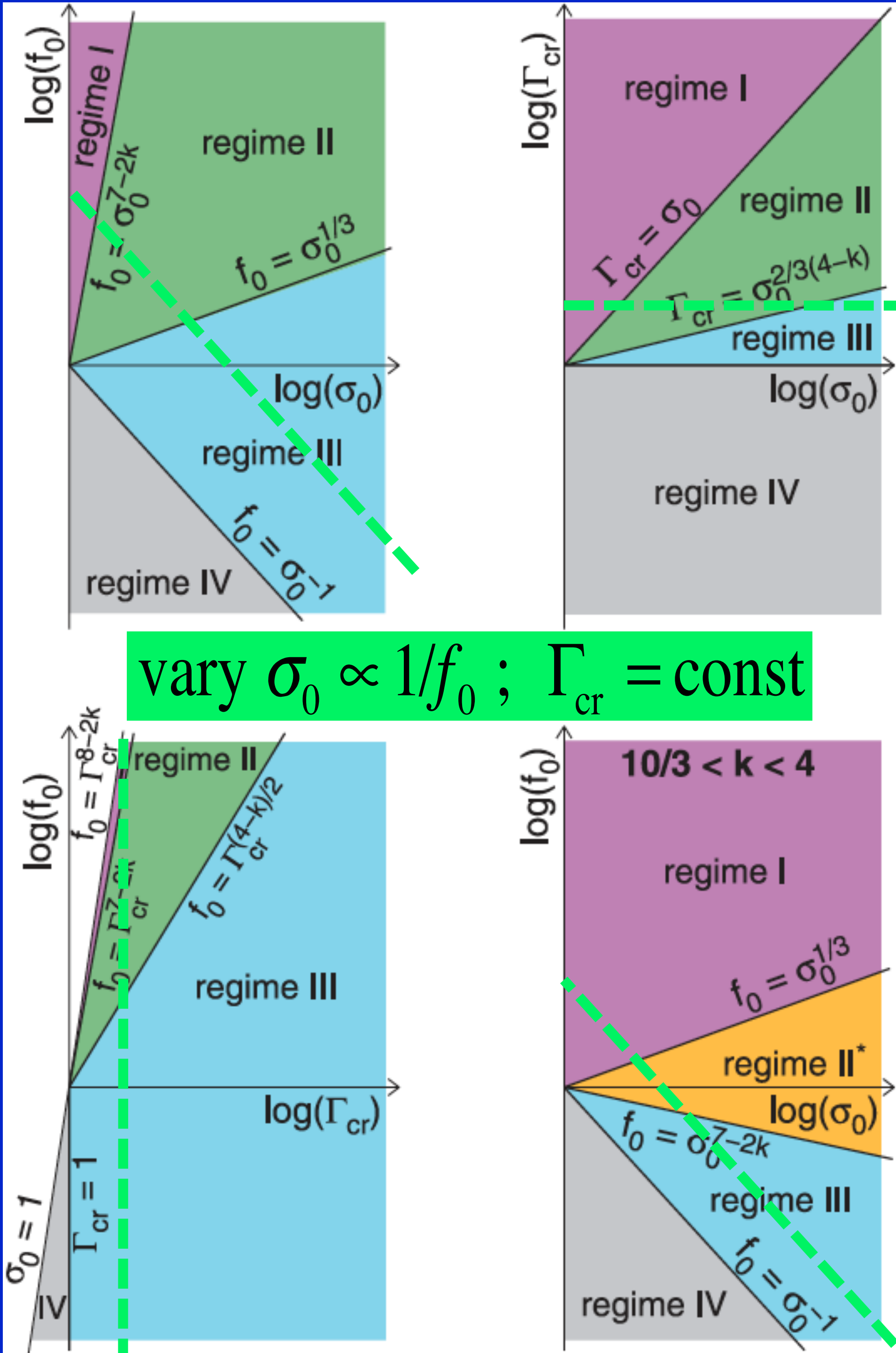

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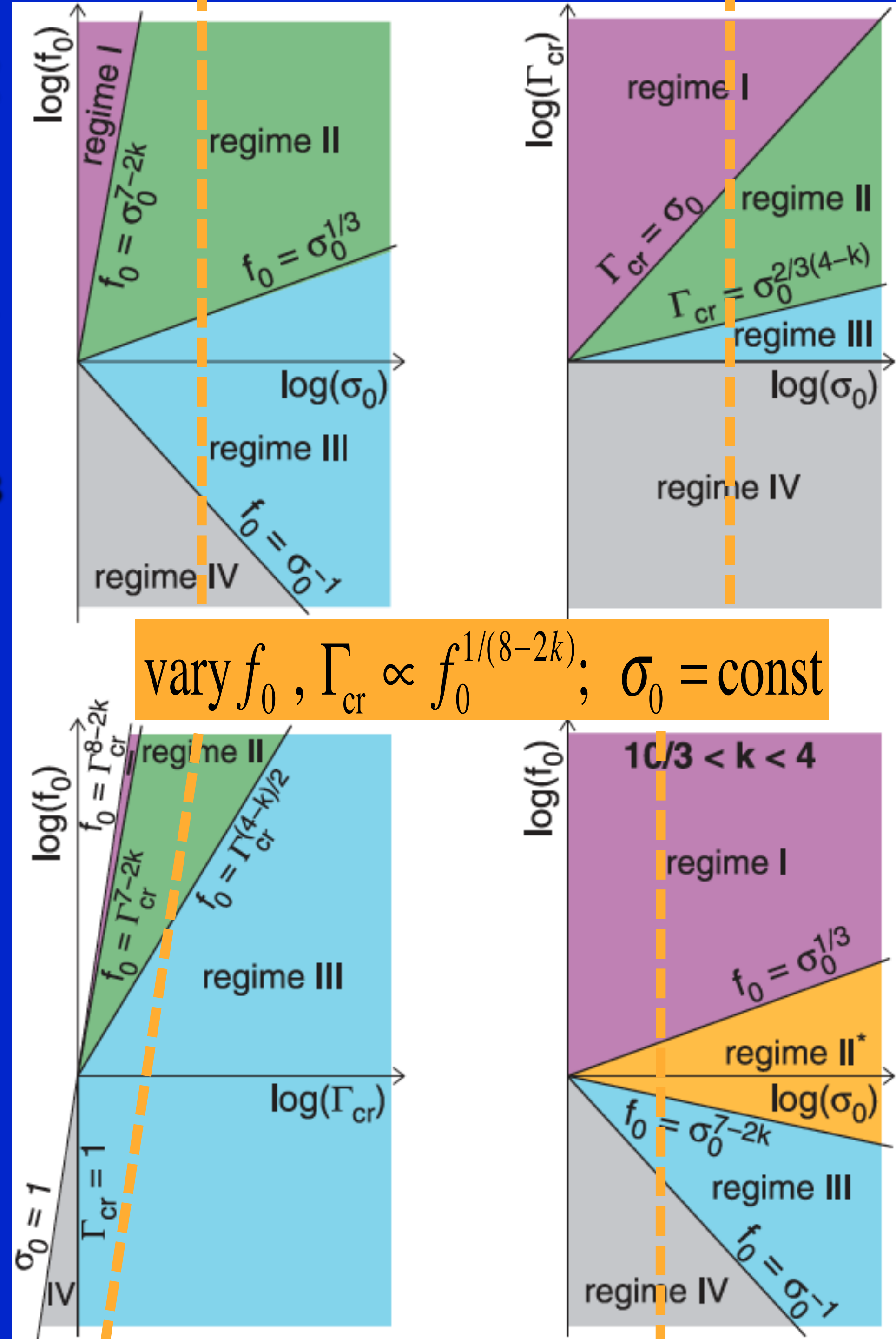


Many sub-shells: acceleration, collisions (JG 2012b)

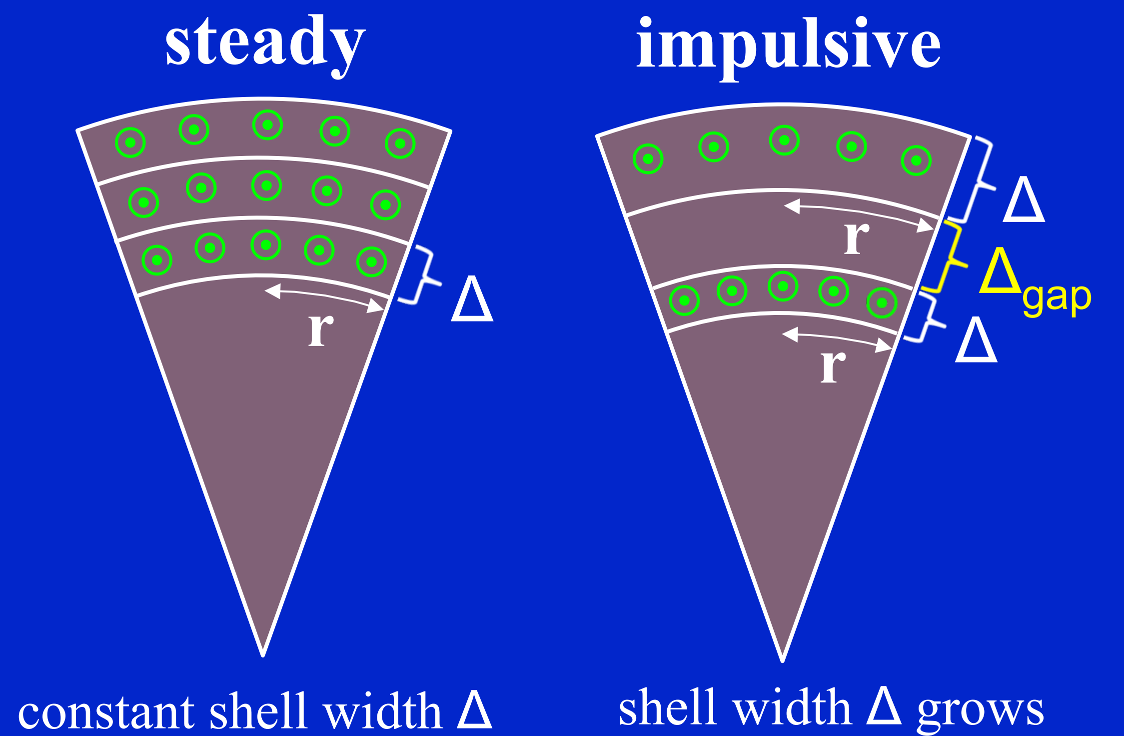
Flux freezing (ideal MHD):

$\Phi \sim B\, r\, \Delta = \textbf{constant}$

$E_{EM} \sim B^2 r^2 \Delta \propto 1/\Delta$

$$\frac{\text{total energy}}{\text{rest energy}} = (1+\sigma)\Gamma$$

acceleration $(\Gamma\uparrow) \Leftrightarrow \sigma\downarrow$



- For a long lived variable source (e.g. AGN), each sub shell can expand by $1+\Delta_{gap}/\Delta_0 \Rightarrow \sigma_\infty = (E_{total}/E_{EM,\infty} - 1)^{-1} \sim \Delta_0/\Delta_{gap}$
- For a finite # of sub-shells the merged shell can still expand

A. Infinite pulse train & no energy losses

- Initial quasi-steady acceleration saturates at Γ_0, σ_0 , Δ_0, Δ_{gap}
- In **planar symmetry**: linear momentum is conserved $\Rightarrow$ final merged state with $\Gamma_{CM} \sim \Gamma_0\sigma_0^{1/2} \ll \Gamma_{max} \sim \Gamma_0\sigma_0$ for $\Delta_{gap} \gtrsim \Delta_0$ (JG, Komissarov & Spitkovsky 2011, Komissarov 2012)
- $\Rightarrow E'_{thermal}/Mc^2 \sim \Gamma_{max}/\Gamma - 1 \sim \sigma_0^{1/2} \gg 1$ i.e. magnetic energy is converted mostly to thermal energy as the shells collide
- Planar symmetry: no thermal acceleration ($\Gamma \propto A^{1/2} = \text{const}$)
- In a **conical** flow (more realistic!): $\Gamma \propto A^{1/2} \propto r \propto z$ thermal energy is quickly converted into bulk kinetic energy
- Bernoulli eq.: $\Gamma(1+\sigma) = \text{const}$, ideal MHD: $E_{EM}\Delta = \text{const} \Rightarrow$

  $\sigma_\infty = [\Delta_{gap}/\Delta_0+(1+\Delta_{gap}/\Delta_0)/\sigma_0]^{-1} \sim \Delta_0/\Delta_{gap}$

  $\Gamma_\infty/\Gamma_0 = (1+\sigma_0)/(1+\sigma_\infty) \sim \sigma_0/(1+\Delta_0/\Delta_{gap})$

B. Infinite pulse train & radiative losses

- Radiation carries both energy & momentum so even in planar symmetry plasma linear momentum is not conserved
- Energy budget: a fraction f_{rad} of the total energy is radiated

$\Rightarrow \Gamma_\infty(1+\sigma_\infty) = (1-f_{rad})\Gamma_0(1+\sigma_0)$, but still $E_{EM}\Delta = \text{const} \Rightarrow$

$$\sigma_\infty = [(1+1/\sigma_0)(1+\Delta_{gap}/\Delta_0)(1-f_{rad}) - 1]^{-1}$$
$$\sim [\Delta_{gap}/\Delta_0 - f_{rad}(1+\Delta_{gap}/\Delta_0)]^{-1}$$

$$\Gamma_\infty/\Gamma_0 = (1+\sigma_0)(1-f_{rad}) - \sigma_0/(1+\Delta_{gap}/\Delta_0)$$

$$\Gamma_\infty/\Gamma_0\sigma_0 \sim 1 - f_{rad} - 1/(1+\Delta_{gap}/\Delta_0)$$

$$f_{rad} \leq \sigma_0/[(1+\sigma_0)(1+\Delta_0/\Delta_{gap})] = f_{rad}/\varepsilon_{rad}$$

e.g.

$\Delta_0/\Delta_{gap} \sim 1/2$

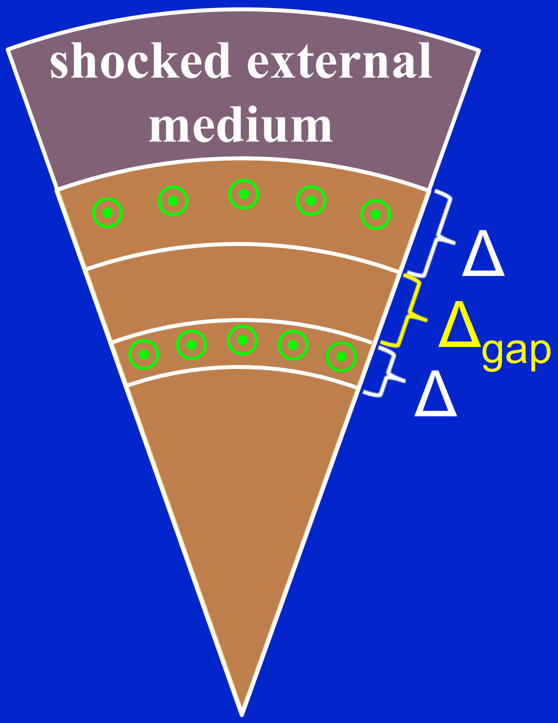
$\varepsilon_{rad} \sim 1/2$

$f_{rad} \sim 1/3$

$\Gamma_\infty/\Gamma_0\sigma_0 \sim 1/3$

N sub-shells: external medium interaction (JG 2012b)

- Leading sub-shell sweeps-up the external medium and clears the way for subsequent sub-shells
- Later sub-shells have a longer time to accelerate and collide with other sub-shells before being influenced by the external medium
  - enables a low-σ thick shell (strong reverse shock, $T_{dec} \sim T_{GRB}$)
  - enables the outflow to reach higher Lorentz factors



Conclusions:

- Magnetic acceleration is generally slower, more delicate & less efficient than thermal acceleration
- The σ-problem: some deviation from a "standard" steady, ideal MHD axisymmetric flow is required by observations
- Strong time dependence in highly magnetized relativistic outflows can efficiently convert magnetic to kinetic energy & lead to efficient internal shock dissipation in the flow
- GRB, AGN, μ-Q: quasi-steady ⇒ impulsive acceleration
- Interaction with external medium: unmagnetized thin shell (strong reverse shock, peaks at $T_{dec} \gg T_{GRB}$) or magnetized thick shell (weak/no reverse shock; afterglow $T_{dec} \sim T_{GRB}$)
- Sub-shells can lead to a low-magnetization thick shell & enable the outflow to reach higher Lorentz factors