

Experimental Bounds on Quantum Gravity from Fermi GRB Observations

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on behalf of the Fermi LAT & GBM Collaborations

Experimental search for quantum gravity

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Outline of the Talk:

- Brief motivation & narrowing down the scope
- Vacuum birefringence: helicity dependence of v_{ph}
- Vacuum dispersion: energy dependence $v_{\text{ph}}(E)$
- Pulsars/AGN/GRBs: why, and how we set the limits
- ◆ 3 different types of limits from the short bright GRB 090510 at $z = 0.903$ (Abdo et al. 2009, Nature, 462, 331)
- ◆ New analysis: 3 methods, 4 GRBs (Vasileiou et al. 2013)
- ◆ Limits on stochastic LIV (Vasileiou et al. 2014; submitted)
- Future prospects: the Cherenkov Telescope Array
- Conclusions

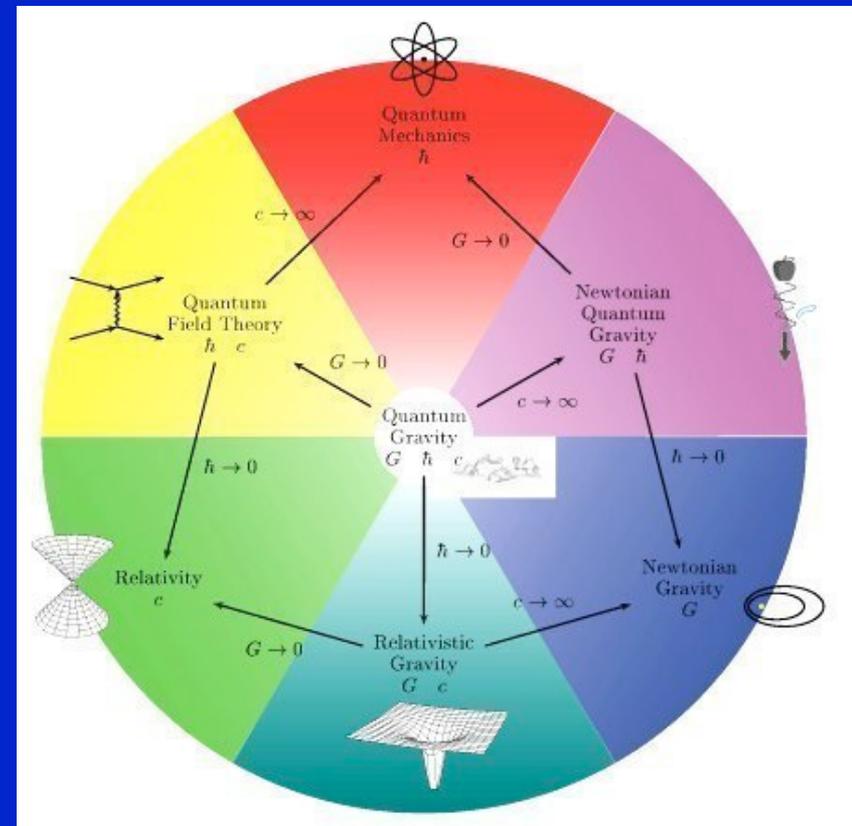
Quantum Gravity: a physics holy grail

- **Motivation:** to unify in a self-consistent theory Einstein's general relativity that dominates on large scales & Quantum theory that dominates on small scales (Stecker's talk)
- Quantum effects on space-time structure expected to become strong near the Planck scale:

$$l_{\text{Planck}} = (\hbar G/c^3)^{1/2} \approx 1.62 \times 10^{-33} \text{ cm}$$

$$E_{\text{Planck}} = M_{\text{Planck}} c^2 = (\hbar c^5/G)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

- Many models / ideas out there: experimental constraints needed



Astrophysics as a test bed:

- **Advantage:** large energies and distances available for free
- **Disadvantage:** uncontrolled experimental setup / conditions
 - ◆ Vacuum birefringence: constrained by polarization
 - ◆ Vacuum dispersion: by short timescale variability
 - ◆ Pair production threshold: attenuation on the EBL
 - ◆ Electron LIV: synchrotron radiation from the Crab nebula
 - ◆ Space-time fuzziness: blur sources, broaden spectral lines
 - ◆ UHECR / ν LIV: energy spectrum / arrival time from GRBs
 - ◆ Massive gravitons: supernovae cooling
 - ◆ Cosmic string: gravitational lensing, gravity waves
 - ◆ Early universe: CMB polarization, 21 cm HI line surveys...

Vacuum energy dispersion: parameterization

- Some quantum-gravity (QG) models allow or even predict (e.g. Ellis et al. 2008) Lorentz invariance violation (LIV)
- We directly constrain a simple form of LIV - dependence of the speed of light on the photon energy: $v_{ph}(E_{ph}) \neq c$
- This may be parameterized through a Taylor expansion of the LIV terms in the dispersion relation:

$$c^2 p_{ph}^2 = E_{ph}^2 \left[1 + \sum_{k=1}^{\infty} s_k \left(\frac{E_{ph}}{E_{QG,k}} \right)^k \right], \quad \text{where } E_{QG,k} \leq E_{\text{Planck}} \text{ is naturally expected}$$

- $s_k = -1, 0, 1$ stresses the model dependent sign of the effect

- The most natural scale for LIV is the **Planck scale**

$$l_{\text{Planck}} \approx 1.62 \times 10^{-33} \text{ cm}; \quad E_{\text{Planck}} = M_{\text{Planck}} c^2 \approx 1.22 \times 10^{19} \text{ GeV}$$

Vacuum energy dispersion: parameterization

- The photon propagation speed is given by the group velocity:

$$c^2 p_{ph}^2 = E_{ph}^2 \left[1 + \sum_{k=1}^{\infty} s_k \left(\frac{E_{ph}}{E_{QG,k}} \right)^k \right], \quad v_{ph} = \frac{\partial E_{ph}}{\partial p_{ph}} \approx c \left[1 - s_n \frac{(1+n)}{2} \left(\frac{E_{ph}}{E_{QG,n}} \right)^n \right]$$

- Since $E_{ph} \ll E_{QG,k} \lesssim E_{Planck} \sim 10^{19} \text{ GeV}$ the lowest order non-zero term, of order $n = \min\{k \mid s_k \neq 0\}$, dominates
- Usually $n = 1$ (linear) or 2 (quadratic) are considered
- We focus here on $n = 1$, since only in this case are our limits of the order of the Planck scale
- We try to constrain both possible signs of the effect:
 - ◆ $s_n = 1, v_{ph} < c$: higher energy photons propagate slower
 - ◆ $s_n = -1, v_{ph} > c$: higher energy photons propagate faster
- We stress: here $c = v_{ph}(E_{ph} \rightarrow 0)$ is the low energy limit of v_{ph}

Vacuum Birefringence: Polarization

- Helicity (left or right circular polarization) dependence of the photon propagation speed: $c - v_{\text{ph,L}}(E) \approx v_{\text{ph,R}}(E) - c$
- Rotates the position angle θ of linearly polarized radiation:

$$\Delta\phi_{\text{R,L}} = 2\Delta\theta = \omega\Delta t_{\text{R,L}} \approx \omega\Delta v_{\text{R,L}}D/c^2 \approx E^{n+1}D(1+n)/\hbar c(E_{\text{QG}^*,n})^n$$
- $\Delta E/E \gtrsim 0.2-1 \Rightarrow \Delta\theta(E_2) \sim 2\Delta\theta(E_1)$
 $\Delta\theta(E_1) \gtrsim 1 \Rightarrow$ depolarization
- \Rightarrow linear pol. constrains $E_{\text{QG}^*,n} = \xi_{1*} E_{\text{Planck}}$



- ◆ Galaxy at $D \sim 0.3$ Gpc, optical:



$$P \sim 10\% \Rightarrow \xi_{1*} > 5 \times 10^3 \text{ (Gleizer \& Nozameh 01)}$$

- ◆ Crab nebula (Galactic SNR; $D \approx 2$ kpc)

$$\text{X/\gamma-rays: } P \sim 46\% \text{ (INTEGRAL 150-300 keV)}$$

$$\Rightarrow \xi_{1*} > 1.1 \times 10^9 \text{ (99\% CL; Maccione et al. 2008)}$$



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- \Rightarrow linear pol. constrains $E_{\text{QG}^*,n} = \xi_{1*} E_{\text{Planck}}$
- ◆ **Gamma-Ray Bursts:** ($z \sim 1$; $D \sim$ several Gpc):
 - ◆ Optical: $P \sim 10\% \Rightarrow \xi_{1*} > 5 \times 10^6$ (Fan et al. 2007)
 - ◆ X/ γ -ray: $P \sim 50-80\%$ (IKAROS/GAP; 70-300 keV)



$$\Rightarrow \xi_{1*} > 10^{15}$$

(Toma et al. 2012)

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◆ X/ γ -ray: $P \sim 50-80\%$ (IKAROS/GAP; 70-300 keV)

$\Rightarrow \xi_{1*} > 10^5$

(Toma et al. 2012)

Unreliable

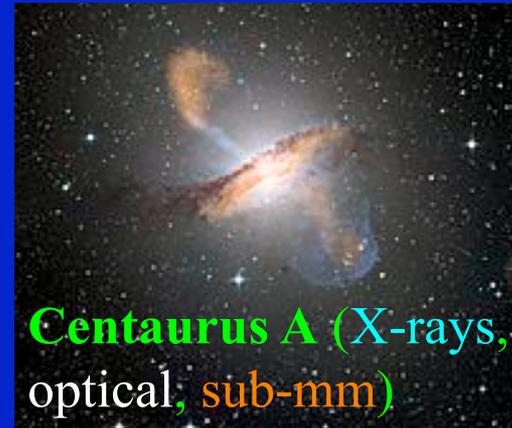


Vacuum dispersion: different sources

property	For better constraints	Pulsars	Active Galactic Nuclei (AGN)	Gamma-Ray Bursts (GRB)
Distance	larger	Galactic	Extragalactic	Cosmological
Variability Time	shorter	$\gtrsim 0.1$ ms	\gtrsim minutes	\gtrsim a few ms
Photon energies	higher	$\lesssim 400$ GeV	\lesssim TeV	\lesssim tens of GeV
# useful sources	larger	1	a few	a few
Best source		Crab pulsar (VERITAS/Fermi)	PKS 2155-304 (HESS)	GRB 090510
Relative strength of results		OK for $n = 1$ Weak for $n = 2$	Good for $n = 1$ ~Best for $n = 2$	Best for $n = 1$ ~Best for $n = 2$

Great, Good, OK

Crab nebula (X-ray)



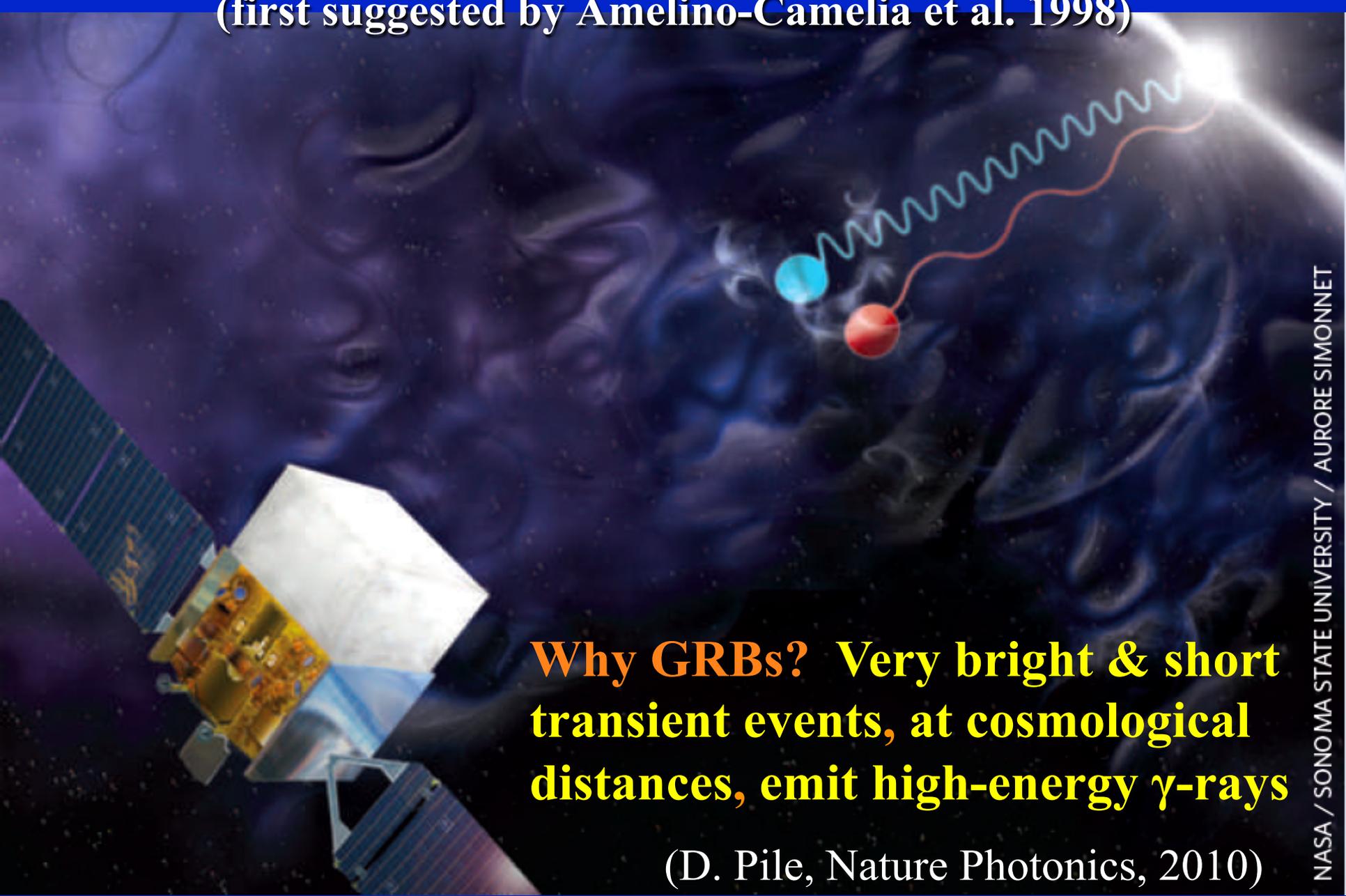
Centaurus A (X-rays, optical, sub-mm)

GRB (artist's concept)



Probing Vacuum dispersion Using GRBs

(first suggested by Amelino-Camelia et al. 1998)



Why GRBs? Very bright & short transient events, at cosmological distances, emit high-energy γ -rays

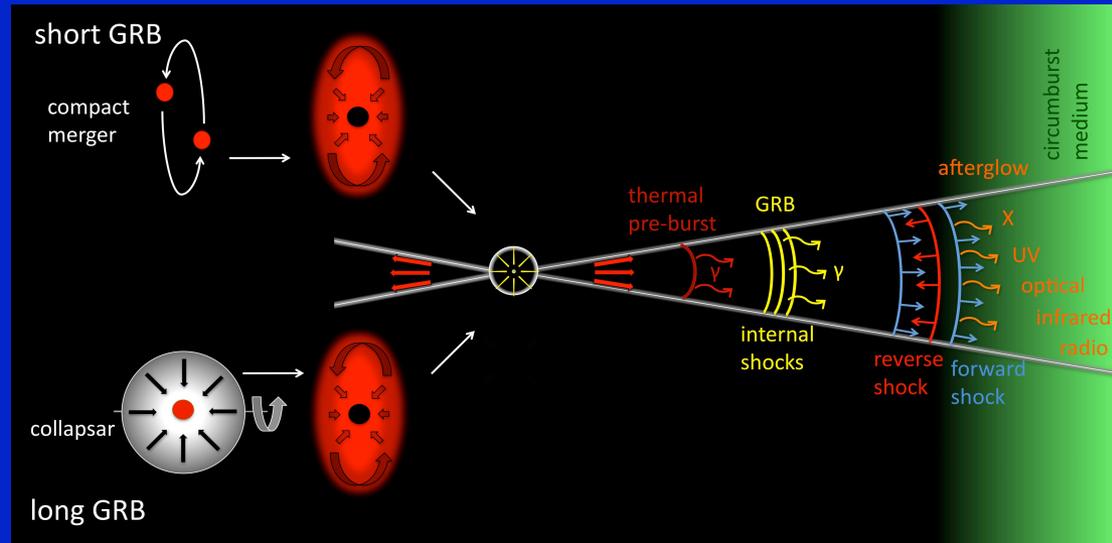
(D. Pile, Nature Photonics, 2010)

GRB Theoretical Framework:

■ Progenitors:

- ◆ Long: massive stars
- ◆ Short: binary merger?

■ Jet Acceleration: fireball or magnetic?



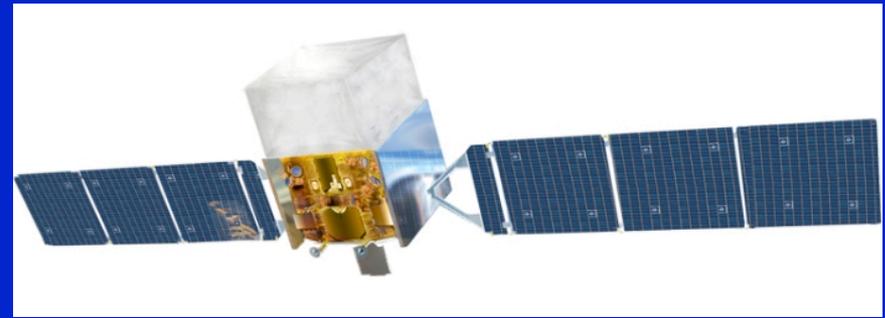
■ γ -rays: internal shocks? emission mechanism?

■ Deceleration: the outflow decelerates (by a reverse shock for $\sigma \lesssim 1$) as it sweeps-up the external medium

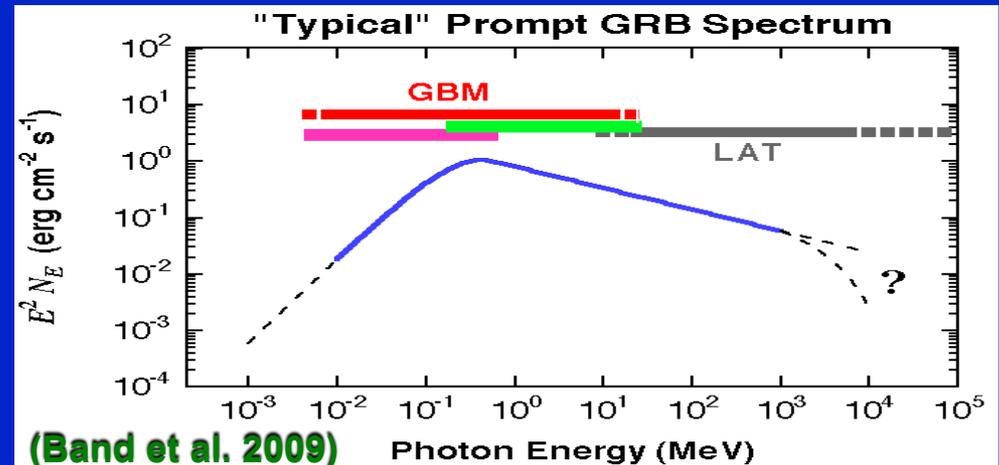
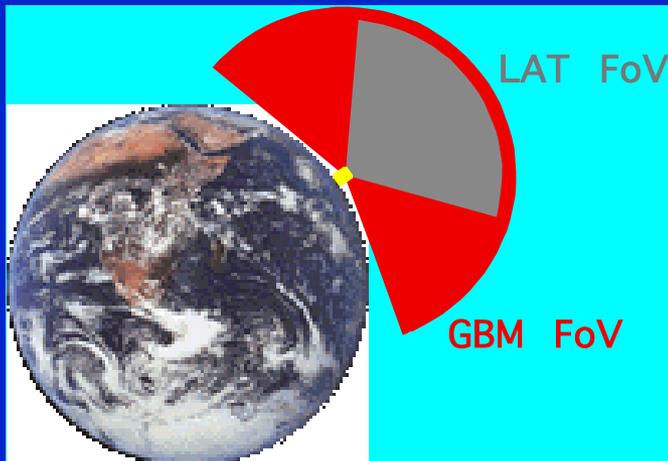
■ Afterglow: from the long lived forward shock going into the external medium; as the shock decelerates the typical frequency decreases: X-ray \rightarrow optical \rightarrow radio

Fermi Gamma-ray Space Telescope

(launched on June 11, 2008)



- Fermi GRB Monitor (GBM): 8 keV – 40 MeV
(12×NaI 8 – 10³ keV, 2×BGO 0.15 – 40 MeV), full sky
- Comparable sensitivity + larger energy range than its predecessor - BATSE
- Large Area Telescope (LAT): 20 MeV – >300 GeV FoV
~ 2.4 sr; up to 40× EGRET sensitivity, ≪ downtime



Constraining LIV Using GRBs

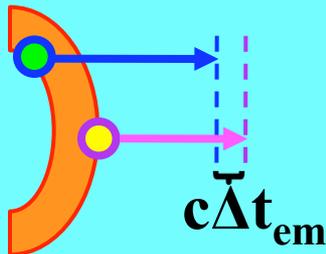
- A high-energy photon E_h would arrive after (in the sub-luminal case: $v_{ph} < c$, $s_n = 1$), or possibly before (in the super-luminal case, $v_{ph} > c$, $s_n = -1$) a low-energy photon E_l emitted together
- The time delay in the arrival of the high-energy photon is:

$$\Delta t_{LIV} = s_n \frac{(1+n)}{2H_0} \frac{E_h^n - E_l^n}{E_{QG,n}^n} \int_0^z \frac{(1+z')^n}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}} dz'$$

(Jacob & Piran 2008)

- The photons E_h & E_l do not have to be emitted at exactly the same time & place in the source, but we must be able to limit the difference in their effective emission times, i.e. in their arrival times to an observer near the GRB along our L.O.S

source



$$\Delta t_{obs} = \Delta t_{em} + \Delta t_{LIV}$$



observer

Constraining LIV Using GRBs

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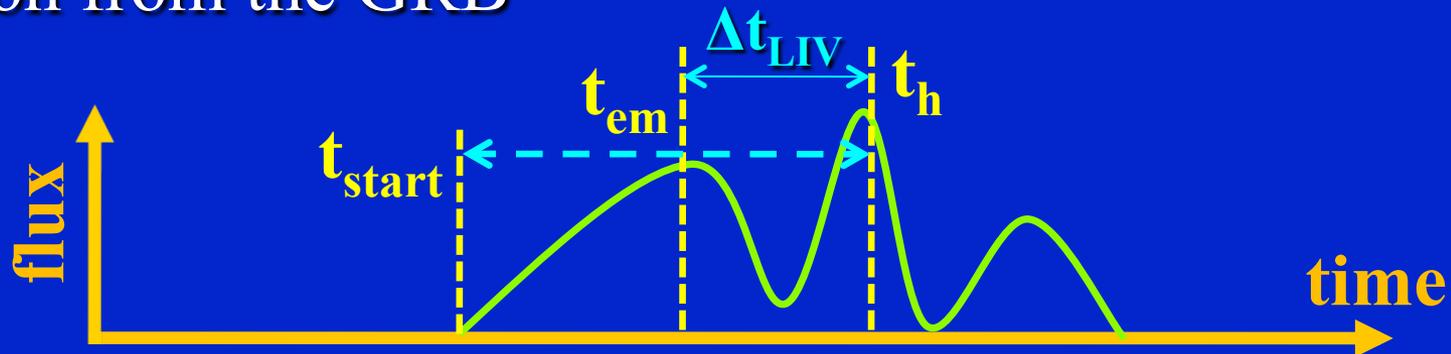
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- Our limits apply to any source of energy dispersion on the way from the source to us, and may constrain some (even more) exotic physics ($\Delta t_{LIV} \rightarrow \Delta t_{LIV} + \Delta t_{exotic}$)

Method 1

- Limits only $s_n = 1$ - the sub-luminal case: $v_{ph} < c$, & positive time delay, $\Delta t_{LIV} = t_h - t_{em} > 0$ (here t_h is the actual measured arrival time, while t_{em} would be the arrival time if $v_{ph} = c$)
- We consider a single high-energy photon of energy E_h and assume that it was emitted after the onset time (t_{start}) of the relevant low-energy (E_l) emission episode: $t_{em} > t_{start}$
- $\rightarrow \Delta t_{LIV} = t_h - t_{em} < t_h - t_{start}$
- A conservative assumption: $t_{start} =$ the onset of any observed emission from the GRB



Limits on LIV: GRB080916C ($z \approx 4.35$)

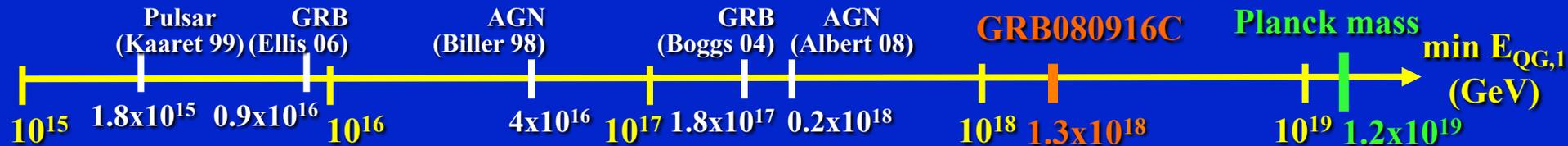
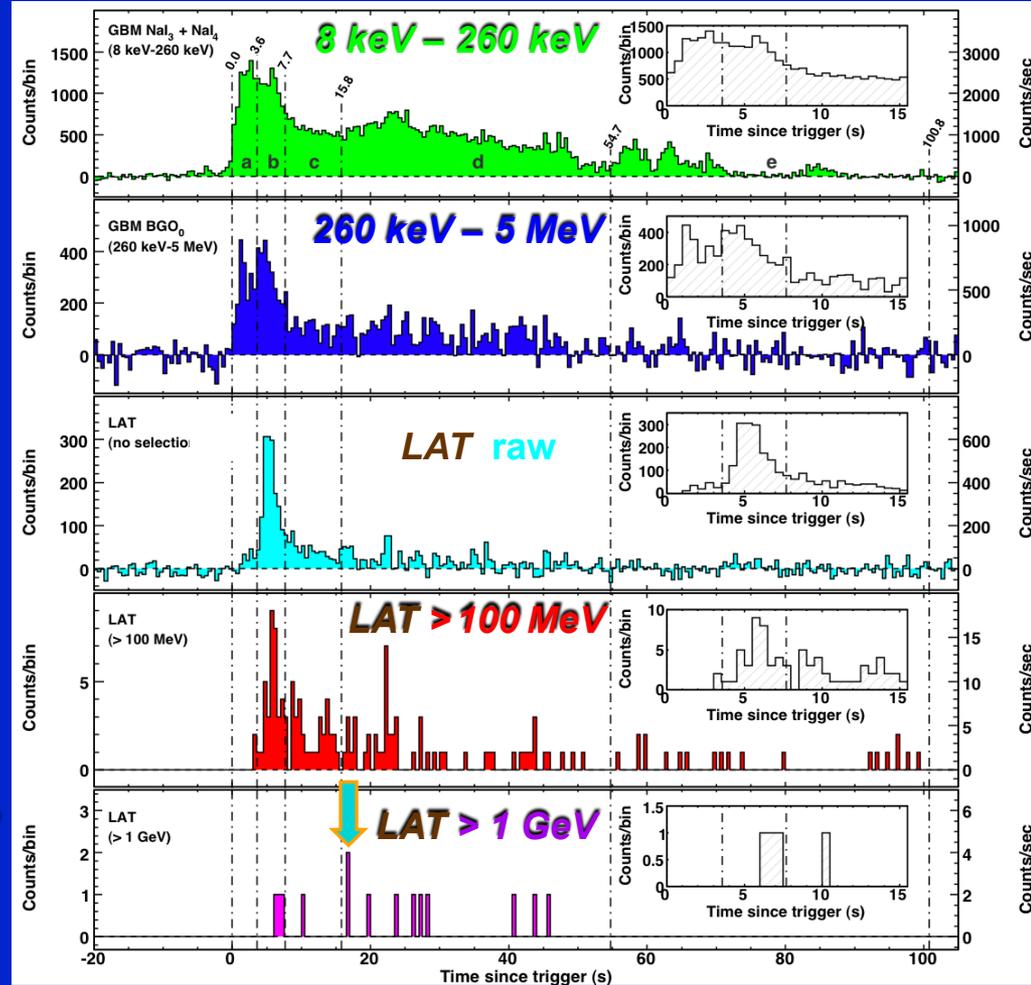
(Abdo et al. 2009, Science, 323, 1688)

■ GRB080916C: highest energy photon (13 GeV) arrived 16.5 s after low-energy photons started arriving (=the GRB trigger)

➔ conservative lower limit:

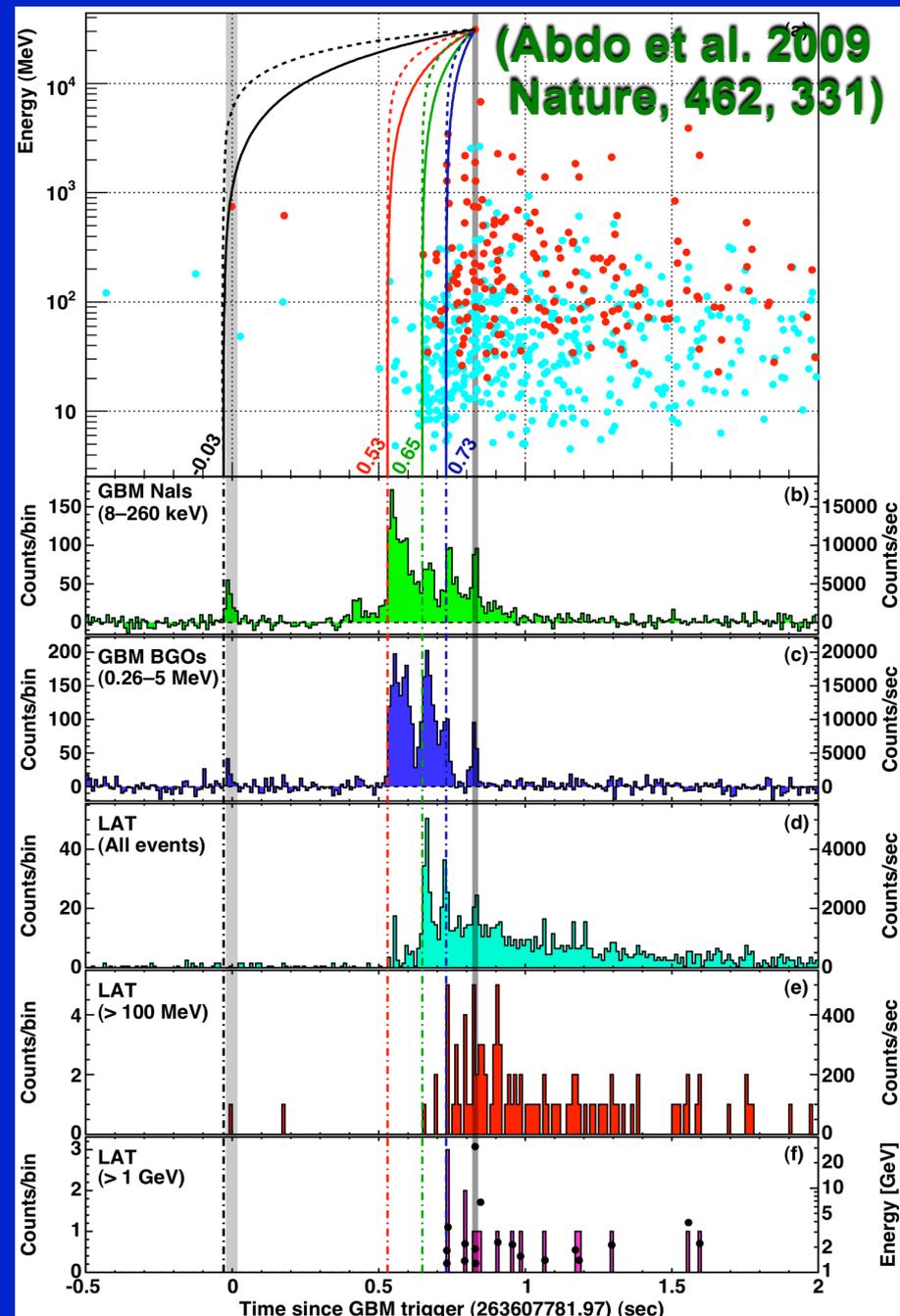
$$E_{QG,1} > 1.3 \times 10^{18} \text{ GeV} \\ \approx 0.11 E_{\text{Planck}}$$

■ This improved upon the previous limits of this type, reaching 11% of E_{Planck}



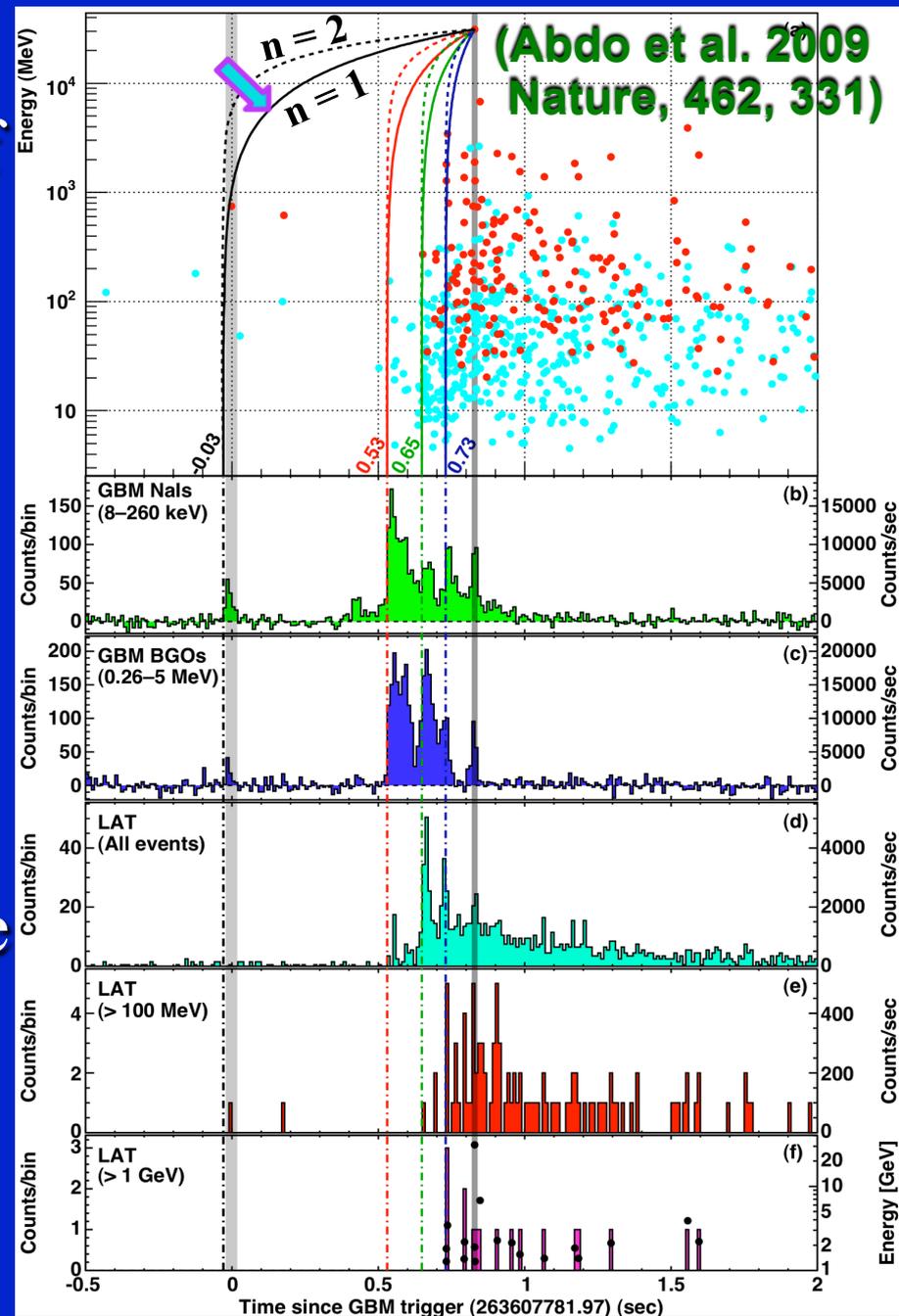
GRB090510: L.I.V

- A short GRB (duration ~ 1 s)
- Redshift: $z = 0.903 \pm 0.003$
- A ~ 31 GeV photon arrived at $t_h = 0.829$ s after the trigger
- We carefully verified it is a photon; from the GRB at $>5\sigma$
- We use the $1-\sigma$ lower bounds on the measured values of E_h (28 GeV) and z (0.900)
- Intrinsic spectral lags known on timescale of individual pulses: weak effect expected



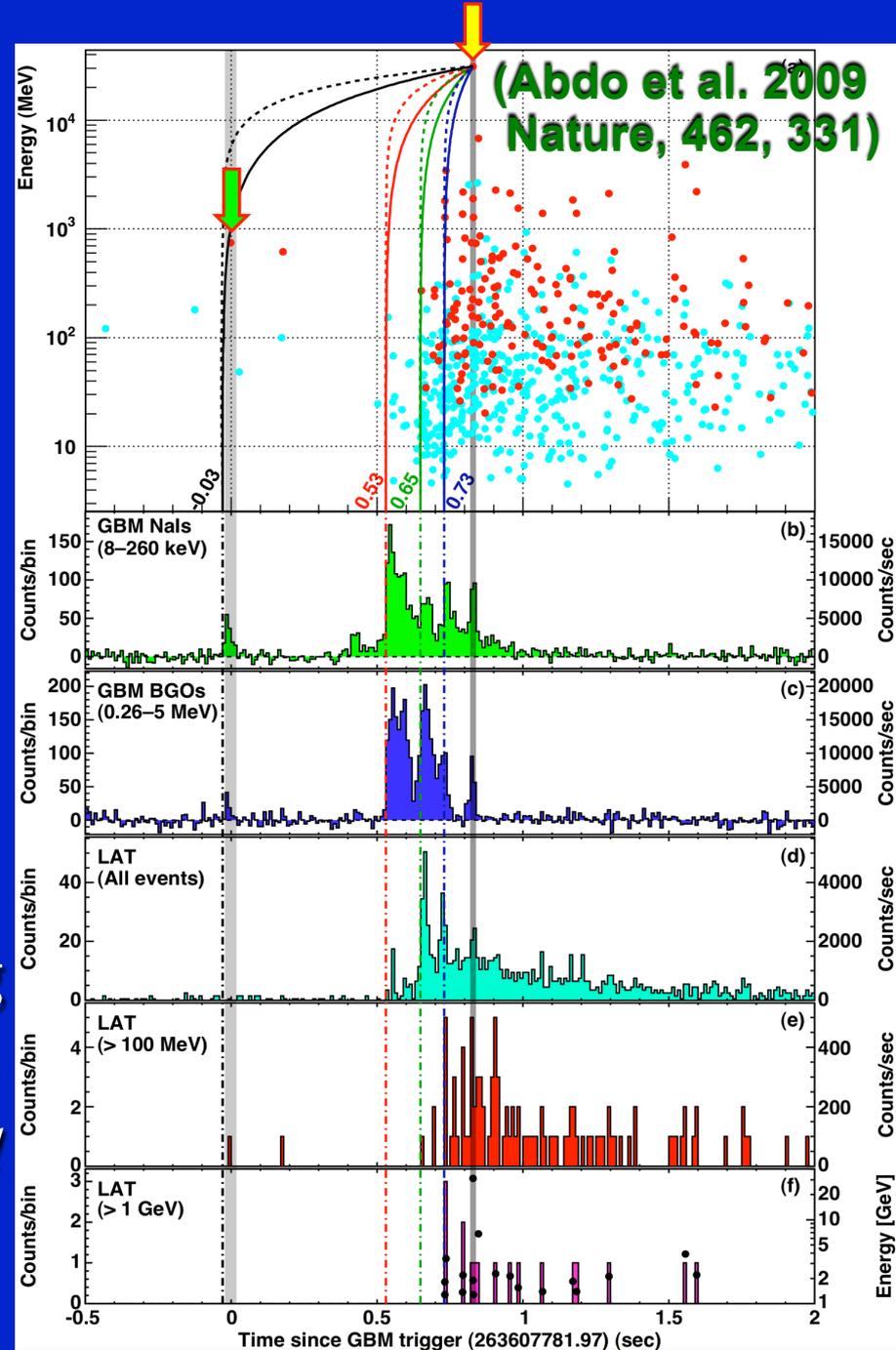
GRB090510: L.I.V

- Method 1: different choices of t_{start} from the most conservative to the least conservative
- $t_{\text{start}} = -0.03$ s precursor onset
→ $\xi_1 = E_{\text{QG},1}/E_{\text{Planck}} > 1.19$
- $t_{\text{start}} = 0.53$ s onset of main emission episode → $\xi_1 > 3.42$
- For any reasonable emission spectrum a ~ 31 GeV photon is accompanied by many γ 's above 0.1 or 1 GeV that “mark” its t_{em}
- $t_{\text{start}} = 0.63$ s, 0.73 s onset of emission above 0.1, 1 GeV
→ $\xi_1 > 5.12$, $\xi_1 > 10.0$



GRB090510: L.I.V

- Method 2: least conservative
- Associating a high energy photon with a sharp spike in the low energy lightcurve, which it falls on top of
- Limits both signs: $s_n = \pm 1$
- Non-negligible chance probability ($\sim 5-10\%$), but still provides useful information
- For a 0.75 GeV photon during precursor: $|\Delta t| < 19 \text{ ms}$, $\xi_1 > 1.33$
- For the 31 GeV photon (*shaded vertical region*) $\rightarrow |\Delta t| < 10 \text{ ms}$ and $\xi_1 = E_{\text{QG},1}/E_{\text{Planck}} > 102$



Method 3: DisCan (Scargle et al. 2008)

- Based on lack of smearing of the fine time structure (sharp narrow spikes in the lightcurve) due to energy dispersion
- Constrains both possible signs of the effect: $s_n = \pm 1$
- Uses all LAT photons during the brightest emission episode (obs. range 35 MeV – 31 GeV); no binning in time or energy
- Shifts the arrival time of photons according to a trail energy dispersion (linear in our case), finding the coefficient that maximizes a measure of the resulting lightcurve variability
- We found a symmetric upper limit on a linear dispersion:
 $|\Delta t/\Delta E| < 30 \text{ ms/GeV}$ (99% CL) $\rightarrow E_{\text{QG},1} > 1.22 E_{\text{Planck}}$
- Remains unchanged when using only photons < 1 or 3 GeV (a very robust limit)

Limits on LIV from Fermi GRBs (2009)

GRB	duration or class	# of events > 0.1 GeV	# of events > 1 GeV	method	Lower Limit on $M_{QG,1}/M_{Planck}$	Valid for $S_n =$	Highest photon Energy	redshift
080916C	long	145	14	1	0.11	+1	~ 13 GeV	~ 4.35
090510	short	> 150	> 20	1	1.2, 3.4, 5.1, 10	+1	~ 31 GeV	0.903
				2	102	± 1		
				3	1.2	± 1		
090902B	long	> 200	> 30	1	0.068	+1	~ 33 GeV	1.822
090926	long	> 150	> 50	1, 3	0.066, 0.082	+1	~ 20 GeV	2.1062

- **Method 1:** assuming a high-energy photon is not emitted before the onset of the relevant low-energy emission episode
- **Method 2:** associating a high-energy photon with a spike in the low-energy light-curve that it coincides with
- **Method 3:** DisCan (dispersion cancelation; very robust) – lack of smearing of narrow spikes in high-energy light-curve

Newer Analysis of the same 4 GRBs:

(Vasileiou, Jacholkowska, Piron, Bolmont, Couturier, Granot, Stecker, Cohen-Tanugi & Longo 2013, PRD, 87, 122001)

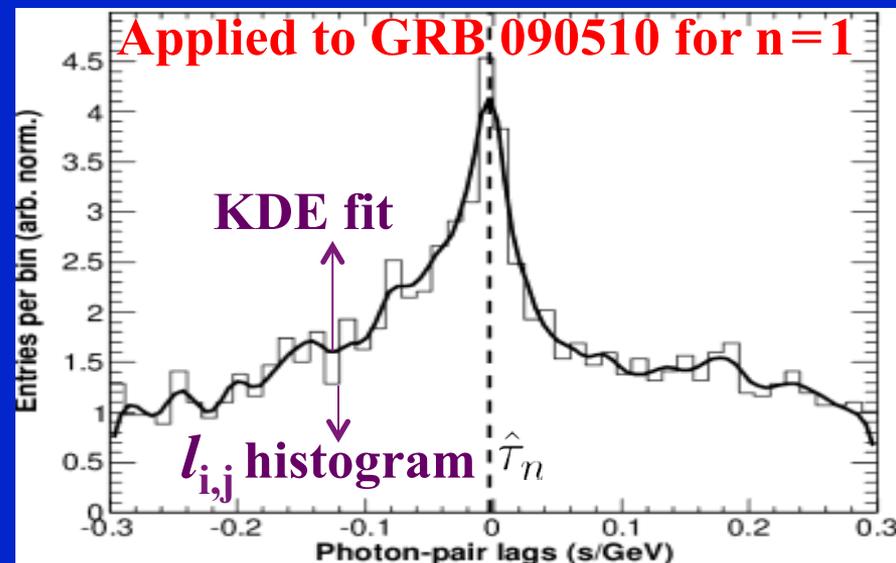
- Use 3 different analysis methods: complimentary in sensitivity & improves reliability of results
 - ◆ PairView (PV)
 - ◆ Sharpness Maximization Method (SMM)
 - ◆ Maximum Likelihood (ML)
- Use same 4 brightest Fermi/LAT GRBs with known redshifts
- The new analysis methods improve the sensitivity/LIV limits

Method A: PairView

- calculate spectral lags $l_{i,j}$ between all photon pairs in a dataset
- The $l_{i,j}$ distribution peaks approximately at the true value τ_n
- This peak serves the best estimate $\hat{\tau}_n$ of the LIV parameter τ_n
- If data has no lag, there will still be a peak but at zero
- Peak width/height depend on the statistical strength of the dataset: many GeV photons in a bright pulse will give the strongest signal

$$l_{i,j} \equiv \frac{t_i - t_j}{E_i^n - E_j^n} \quad k_n \equiv \int_0^z \frac{(1+z')^n}{\sqrt{\Omega_\Lambda + \Omega_M(1+z')^3}} dz'$$

$$\tau_n \equiv \frac{\Delta t}{(E_h^n - E_l^n)} \simeq s_\pm \frac{(1+n)}{2H_0} \frac{1}{E_{QG}^n} \times k_n$$

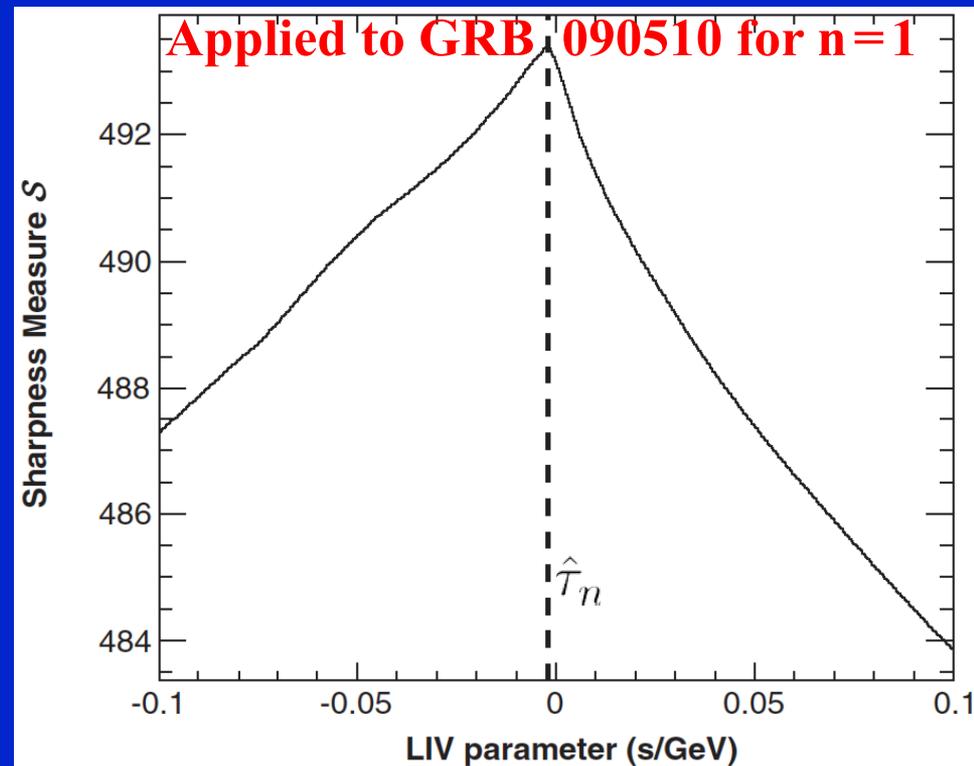


Method B: Sharpness Maximization

- Similar to DisCan (method 3 of Abdo et al. 2009, Nature, 462, 331; dispersion smears sharp lightcurve features \Rightarrow cancelling it will make the lightcurve sharper). **New improvement:**
- Averages arrival time differences of ρ neighboring photons
- Sharpness measure:

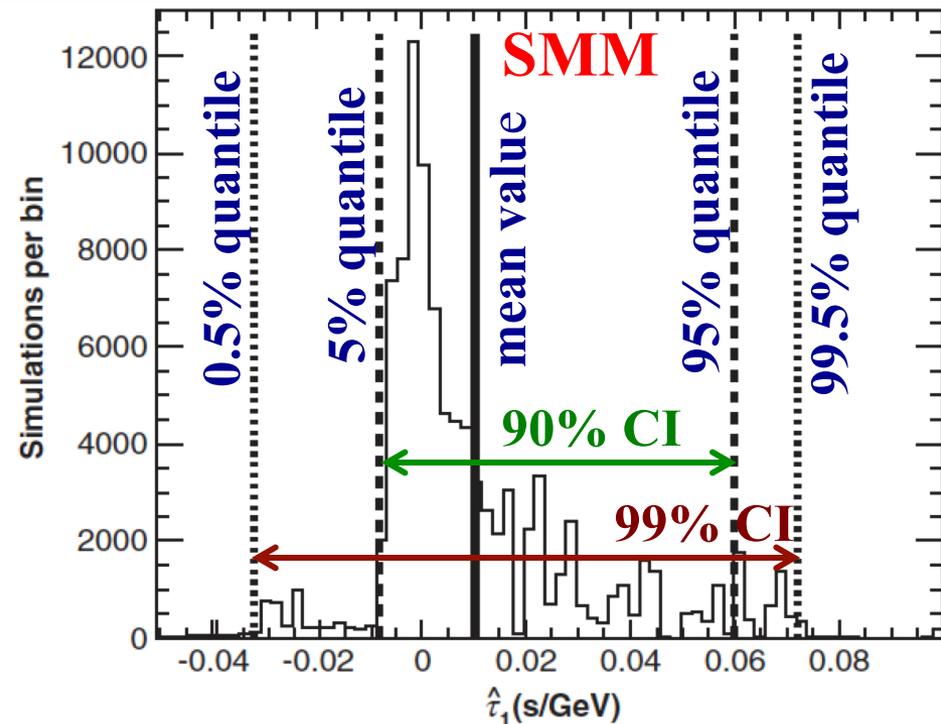
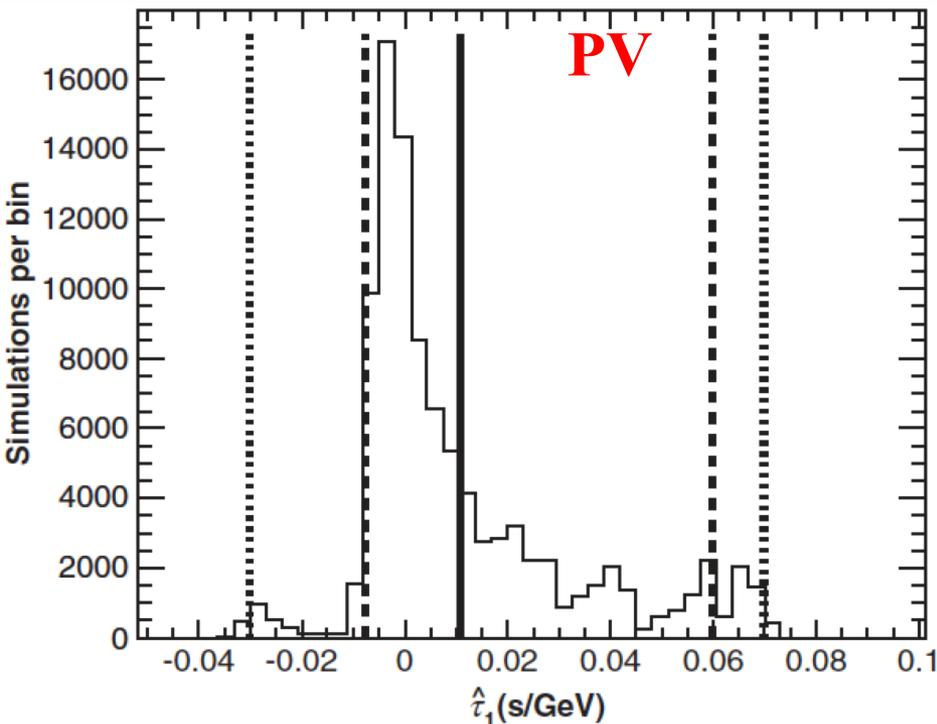
$$S(\tau_n) = \sum_{i=1}^{N-\rho} \log \left(\frac{\rho}{t'_{i+\rho} - t'_i} \right)$$

- ρ is optimized using simulations (to maximize the sensitivity)
- $t'_i = t_i - \tau_n E_i^n$ is the de-dispersed arrival time of the i^{th} photon whose measured arrival time is t_i for a trial value of τ_n
- best estimate $\hat{\tau}_n$ maximizes $S(\tau_n)$



Methods A, B: Confidence Intervals

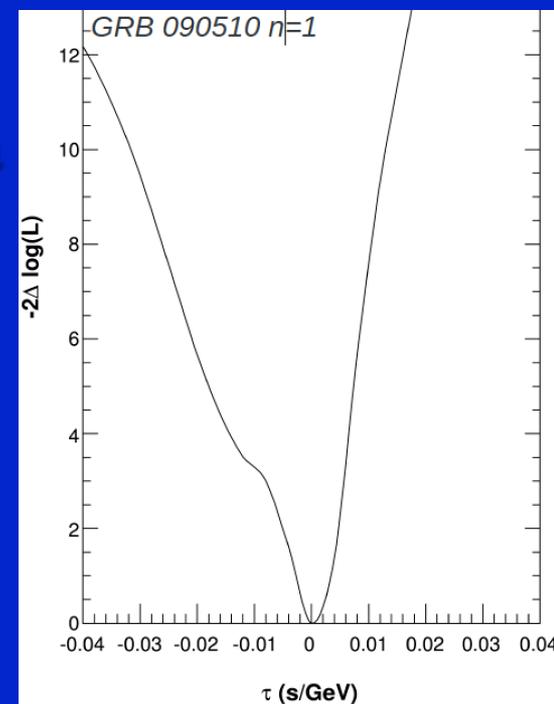
- Apply the method on many data sets derived from the actual one by randomizing association between photon time/energy
- For each randomized data set we produce a $\hat{\tau}_{n,rand}$
- The distribution f_r of $\hat{\tau}_{n,rand}$ is used to approximate the PDF of the error $\epsilon \equiv \hat{\tau}_n - \tau_n$



Method C: Maximum Likelihood

- Existing method previously used in LIV studies with AGN (Martinez & Errando 2009; Abramowski et al. 2011).

- Model the GRB lightcurve for the case of no LIV
 - Lightcurve template:** obtained from low energy photons below a threshold energy, $E < E_{th}$, where LIV effects are negligible
 - Spectral template:** from fit to all data (time-averaged spectrum)
 - Compute likelihood L of detecting the high-energy photons ($E > E_{th}$) in the data given our template & trial value of τ_n
 - Our best estimate $\hat{\tau}_n$ for τ_n is that which maximizes L
- Confidence interval produced by applying the method on simulated data sets



Accounting for GRB Intrinsic Effects:

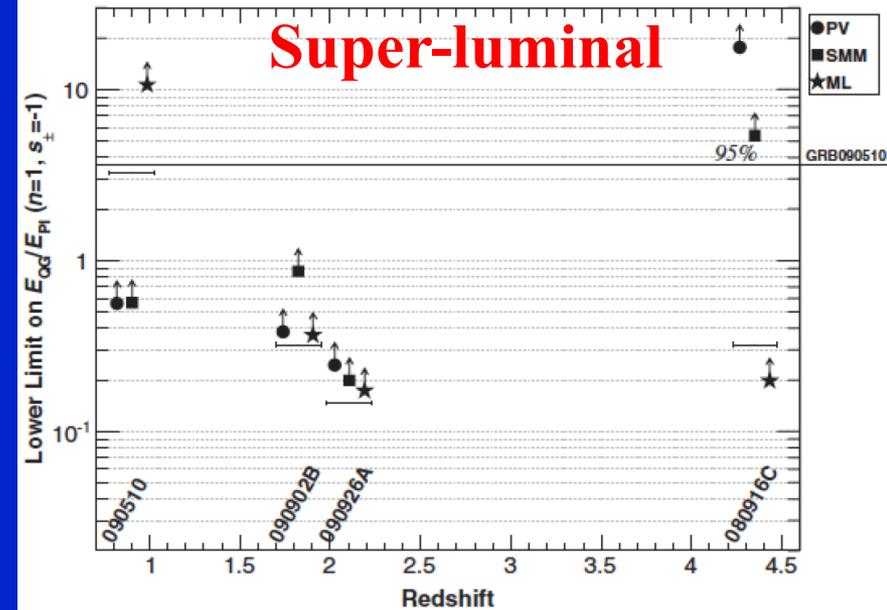
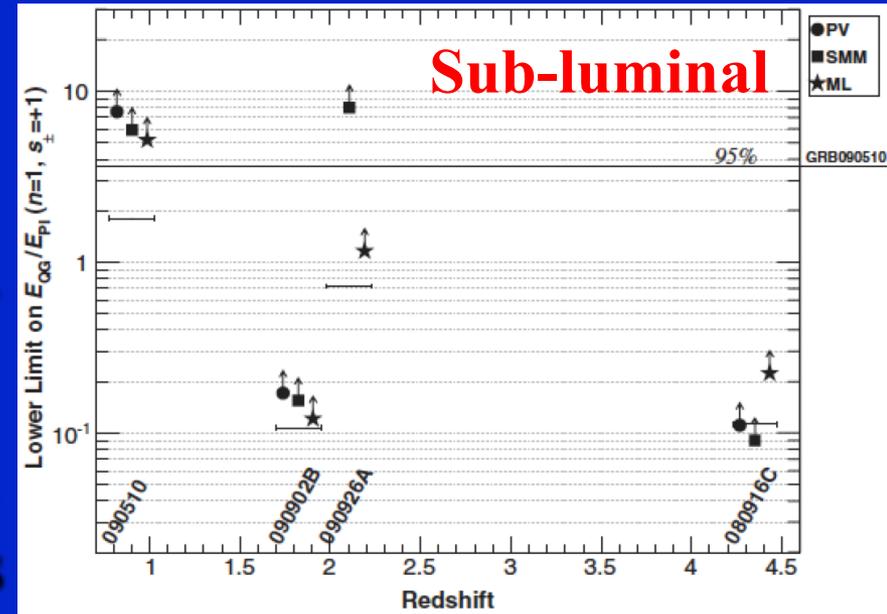
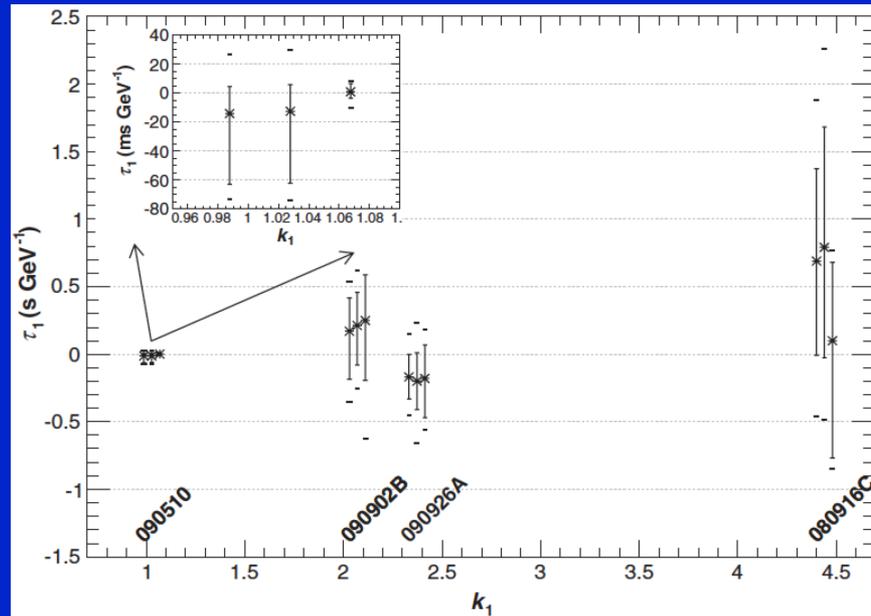
$$\tau_n = \tau_{\text{GRB}} + \tau_{\text{LIV}}$$

- * τ_n = the **total dispersion**, which our methods constrain
- * τ_{LIV} = **LIV-induced dispersion**: the one relevant for our limits
- * τ_{GRB} = **intrinsic dispersion** (treated as a nuisance parameter)

- Model GRB effects (τ_{GRB})? No reliable model available yet
⇒ instead we choose to model τ_{GRB} conservatively:
- Assume observations dominated by GRB-intrinsic effects
 - ◆ τ_{GRB} PDF chosen to match the options for τ_n allowed by our data
 - ◆ E.g. if data has large positive dispersion – model τ_{GRB} to allow this
- This choice for modeling τ_{GRB} gives:
 - ◆ Symmetric CIs on τ_{LIV} , which correspond to the worst case (yet reasonable) scenario for GRB-intrinsic effects
 - ◆ Most conservative (least stringent) overall limits on τ_{LIV}

All 3 Methods: Results (95% CL, n = 1)

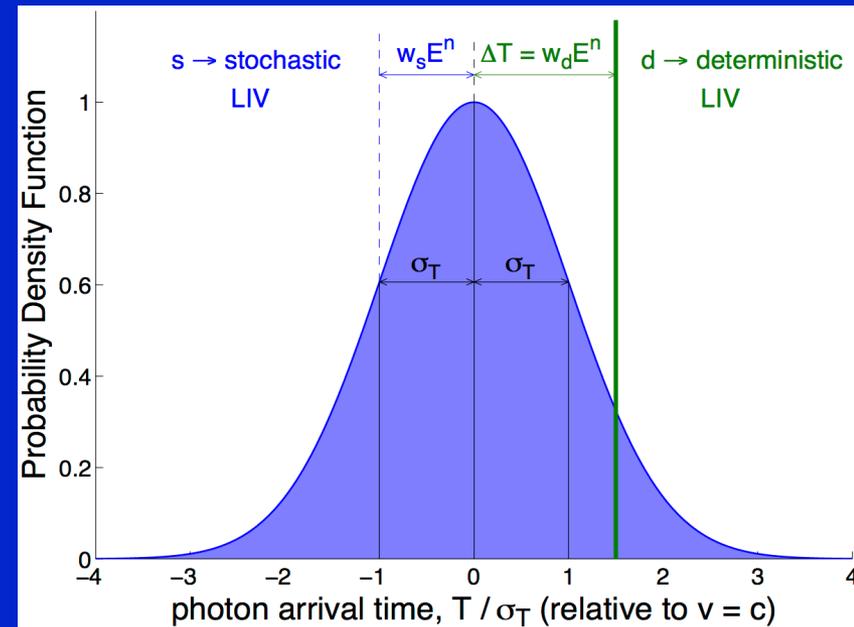
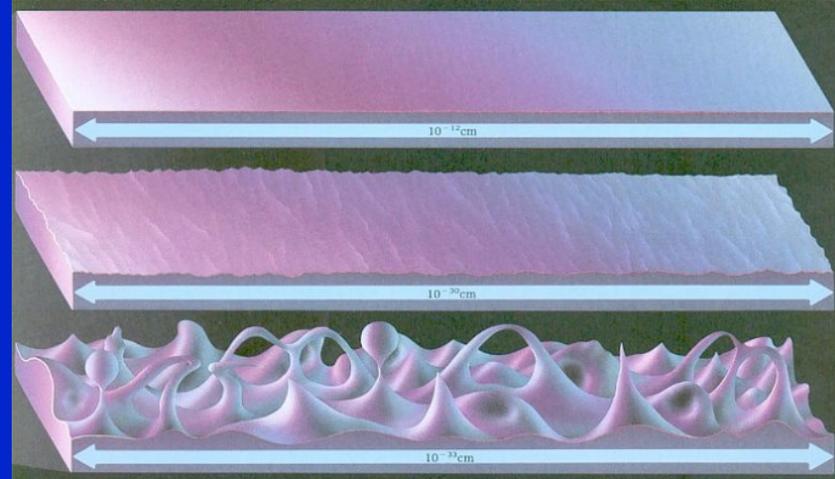
- ~2 times stricter than the best previous limits (horizontal lines)
- Horizontal bars: mean limits over 3 methods, accounting for GRB intrinsic effects
- Neglecting intrinsic effects can lead to unrealistically strict limits



Very New: Limits on Stochastic LIV

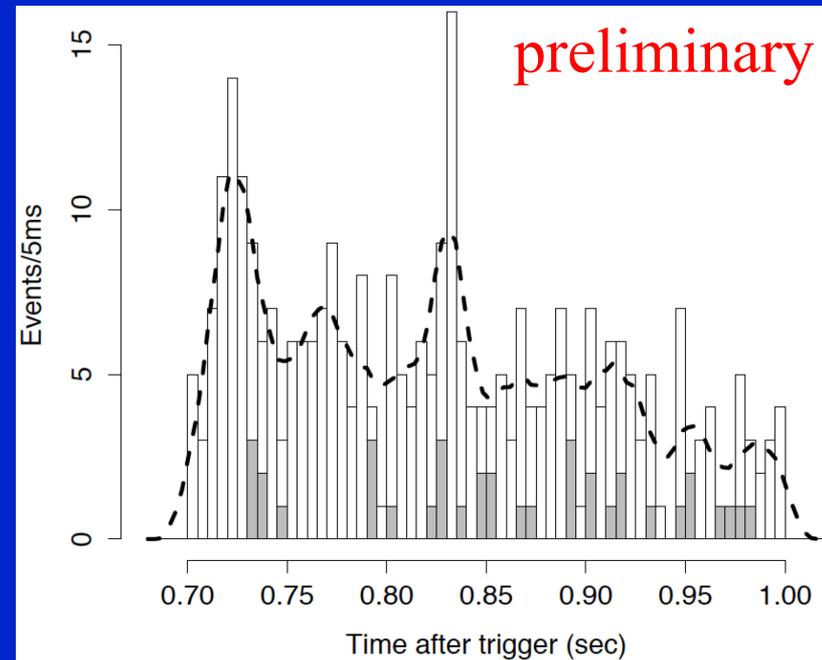
(Vasileiou, Granot, Piran & Amelino-Camelia)

- The concept of spacetime foam: suggests LIV may be stochastic
- Photons of same energy emitted together arrive at different times according to some PDF
- Differs from **deterministic** LIV where E_{ph} uniquely determines v_{ph} & $v_{\text{ph}} - c$ has the same sign:
- We considered a Gaussian PDF:
$$v(E) = c + \delta v(E), \quad \delta v = G(0, \sigma_v)$$
$$\sigma_v(E) = (E/\xi_{s,n_s} E_{\text{Planck}})^{n_s} c$$



Data Analysis: Maximum Likelihood

- We generalized this existing method to stochastic LIV
- $E < E_{th}$ used for emission template; $E > E_{th}$ used for likelihood
- We chose $E_{th} = 300 \text{ MeV}$ (negligible LIV + enough photons $< E_{th}$)
- Time interval: $0.7-1.0 \text{ s}$ (brightest, most variable, highest E_{ph} & relatively stable emission spectrum; 316γ 's $< E_{th}$, 37γ 's $> E_{th}$)
- Optimized lightcurve reconstruction method with simulations
 - ◆ KDE with fixed 6 ms bandwidth



Data Analysis: Maximum Likelihood

- $\sigma_T(E) = T_c \sigma_\nu(E)/c = wE$, $w(z) = \sigma_T(E)/E = T_c/\xi_{s,1} E_{\text{Planck}}$

stochastic LIV parameter

(measured in **s/GeV**):

$$w(z) = \frac{1}{\xi_{s,1} E_{\text{Pl}} H_0} \int_0^z \frac{(1+z')}{\sqrt{\Omega_\Lambda + \Omega_M(1+z')^3}} dz'$$

- **Likelihood**: product of probabilities

for all high-energy photons ($E > E_{\text{th}}$):

$$\mathcal{L}(w) = \prod_{i=1}^N P(E_i, t_i | w, f)$$

- For each photon, a convolution is done to account for all possible emission times with the appropriate probability

$$P_{\text{LIV}}(\Delta t, E | w) = G(\Delta t | 0, \sigma_{\text{LIV}} = wE_i)$$

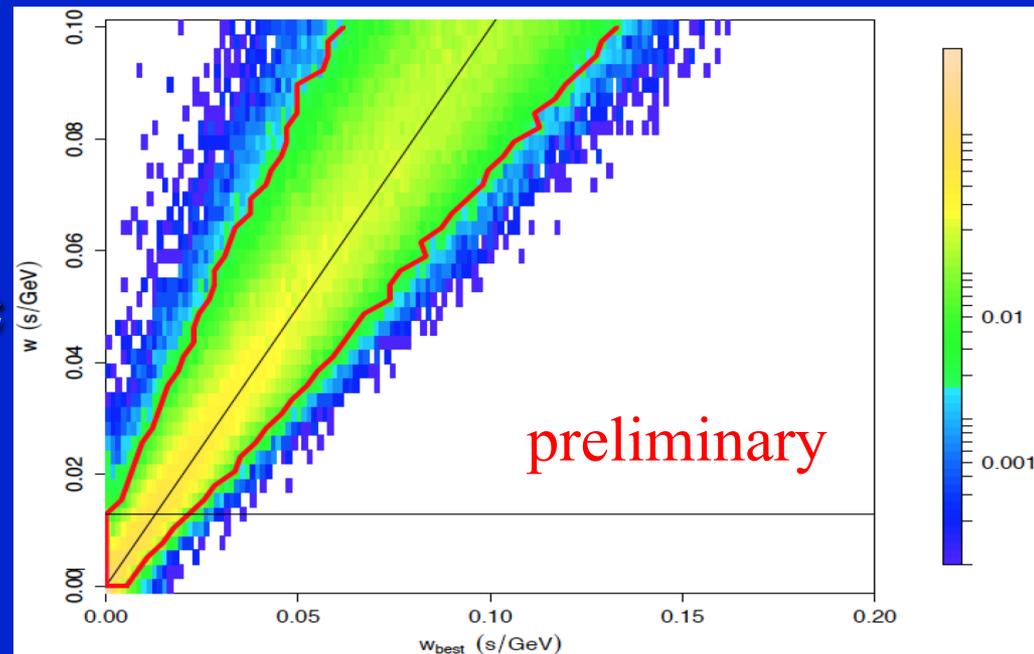
$$P(E_i, t_i | w, f) = \int_{-\infty}^{\infty} G(t'_i - t_i | 0, wE_i) f(t'_i) dt'_i$$

- **Altogether**:

$$\mathcal{L}(w) = \prod_{i=1}^N P_i(w) \propto \prod_{i=1}^N \frac{1}{\sqrt{2\pi wE_i}} \int_{-\infty}^{\infty} f(t_i - \tau) \exp\left[-\frac{1}{2} \left(\frac{\tau}{wE_i}\right)^2\right] d\tau$$

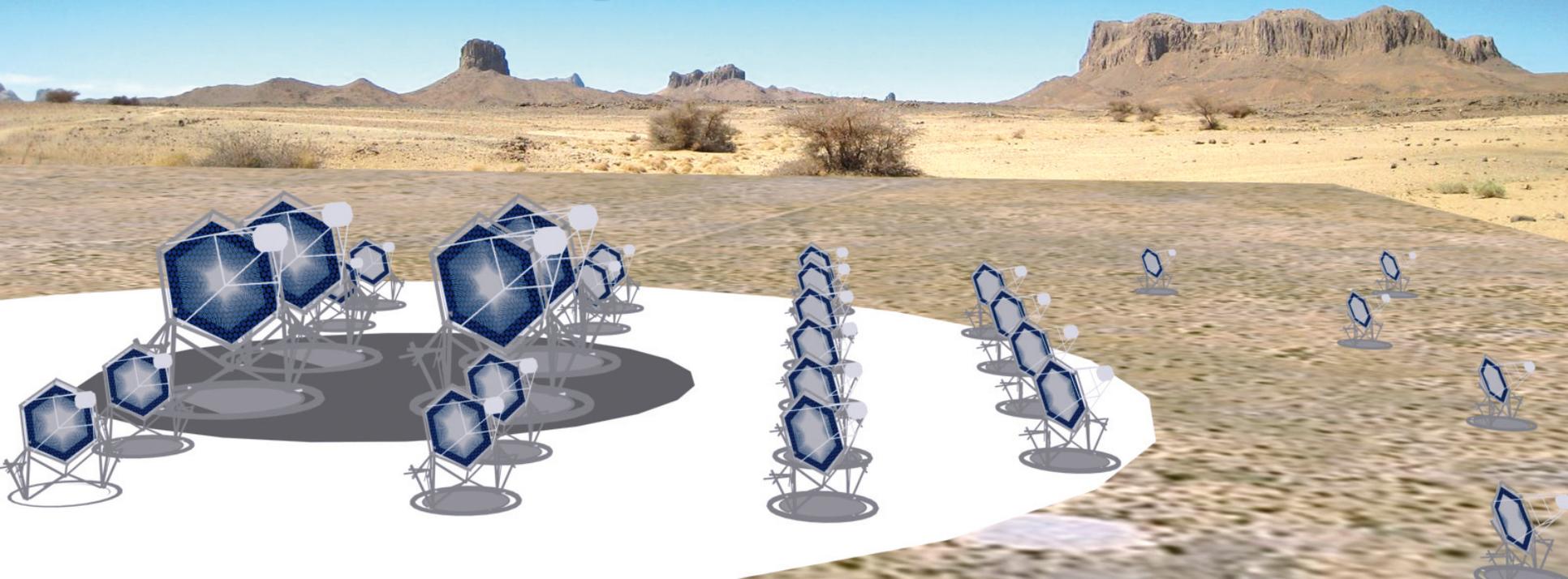
Initial Results & Confidence Intervals:

- Our best estimate for w that maximizes $L(w)$: $w_{\text{best}} = 0 \text{ s/GeV}$
- **Confidence Interval**: Feldman-Cousin method (computationally expensive, but provides proper coverage & is less sensitive to biases)
 - ◆ Use artificial lightcurve close to detected one + inject a known w
 - ◆ Many simulations (random realizations) for each trial value of w
 - ◆ ML applied to each realization $\Rightarrow w_{\text{best}}(w) \Rightarrow$ global confidence belt
 - ◆ \Rightarrow derive Confidence Interval for w using w_{best} from the actual data
- CI on $w \Rightarrow$ CI on $\xi_{s,1}$
- We obtain a Planck-scale limit (the 1st for stochastic or fuzzy LIV)

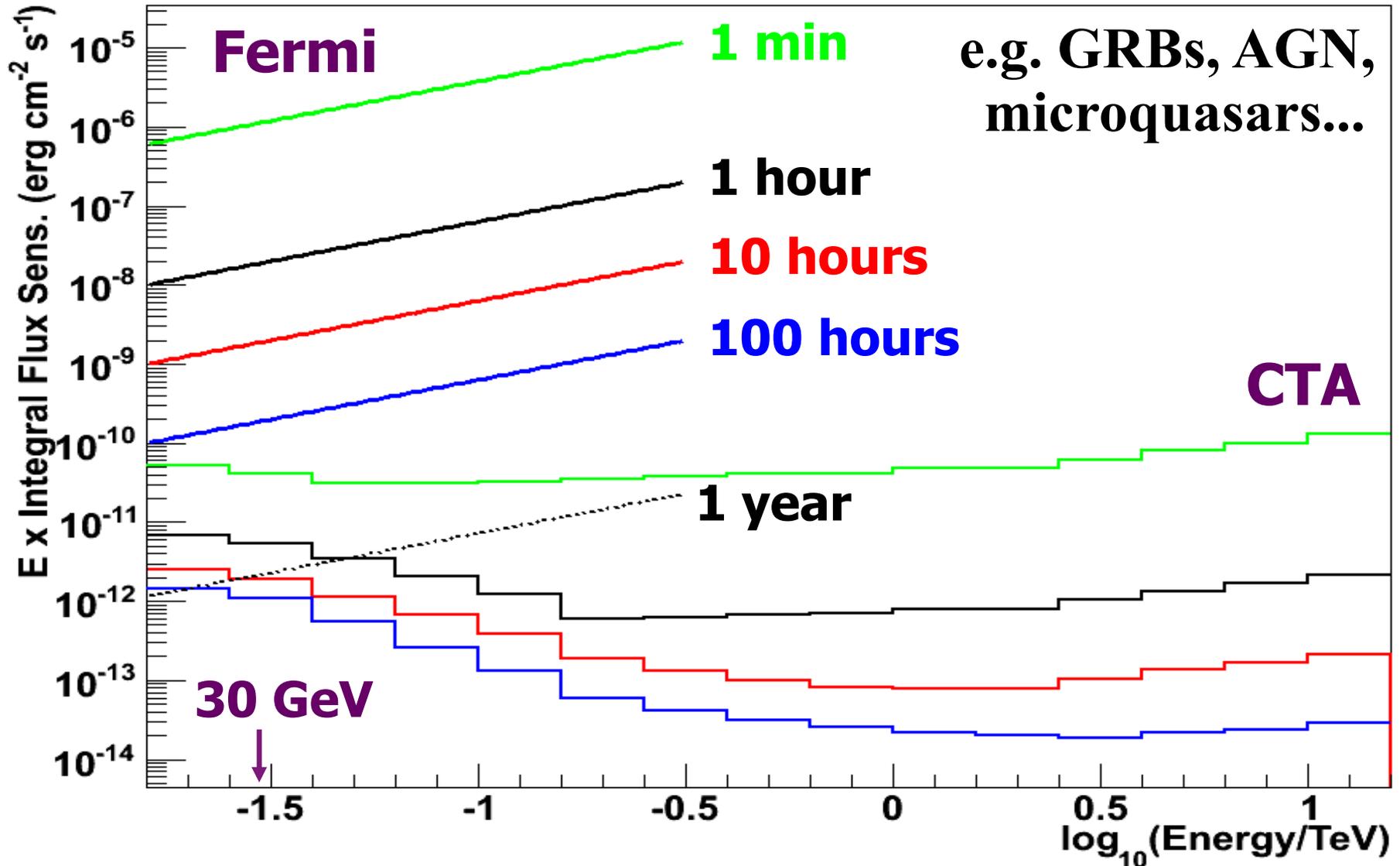


Future – Cherenkov Telescope Array

- **Energy range: ~ 20 GeV to ~ 500 TeV**
 - ◆ an order of magnitude more sensitive than current instruments around 1 TeV (~ 150 M€ price tag), better angular/energy resolution
 - ◆ > 1000 members in 27 countries
 - ◆ Preparatory Phase 2011-2014, construction 2015-2019?
- **2 sites (southern + northern hemispheres)**
- **Hundreds of telescopes of 3 different sizes**

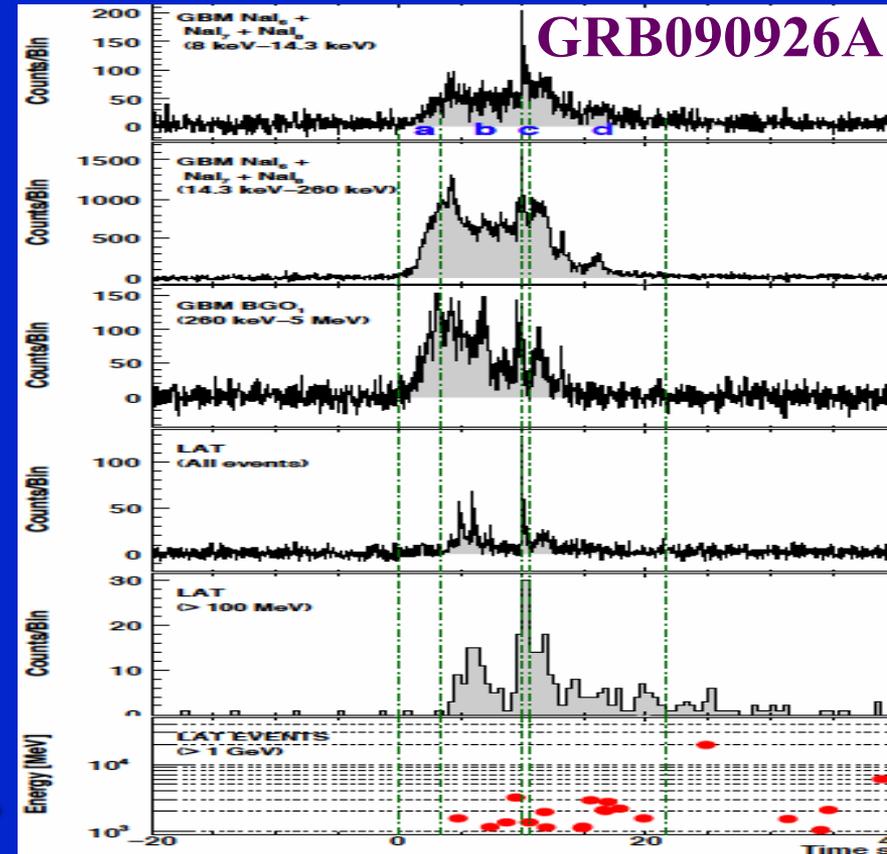


A bigger difference for transient sources



Prospects for LIV studies with CTA GRBs

- Method 1: it may be difficult to do much better
 - ◆ Our current limit $|\Delta t/\Delta E| < 30 \text{ ms/GeV}$ would require $E_h > 1 \text{ TeV}$ for a response time of 30 s
 - ◆ at $> 1 \text{ TeV}$ intrinsically fewer photons + EBL
- Method 3: might work best
 - ◆ Sharp bright spikes up to high energies exist also well within long GRBs
 - ◆ $t_{\text{var}} \sim 0.1 \text{ s}$ & $E_h \sim 0.1 \text{ TeV}$ could do ~ 30 times better
- A short GRB in CTA FoV (survey mode) would be great **10 ms, 1 TeV: $>10^3$ times better**



Conclusions:

- Astrophysical tests of QG can help – look for them
- GRBs are very useful for constraining LIV
- Bright **short** GRBs are more useful than long ones
- $E_{\text{QG},1}/E_{\text{Planck}} \gtrsim \mathbf{a\ few}$ even when conservatively accounting for possible intrinsic source effects
- **New Planck scale limits on stochastic / fuzzy LIV**
- Quantum-Gravity Models with linear ($\mathbf{n = 1}$) photon energy dispersion are disfavored