GRB Jet Dynamics

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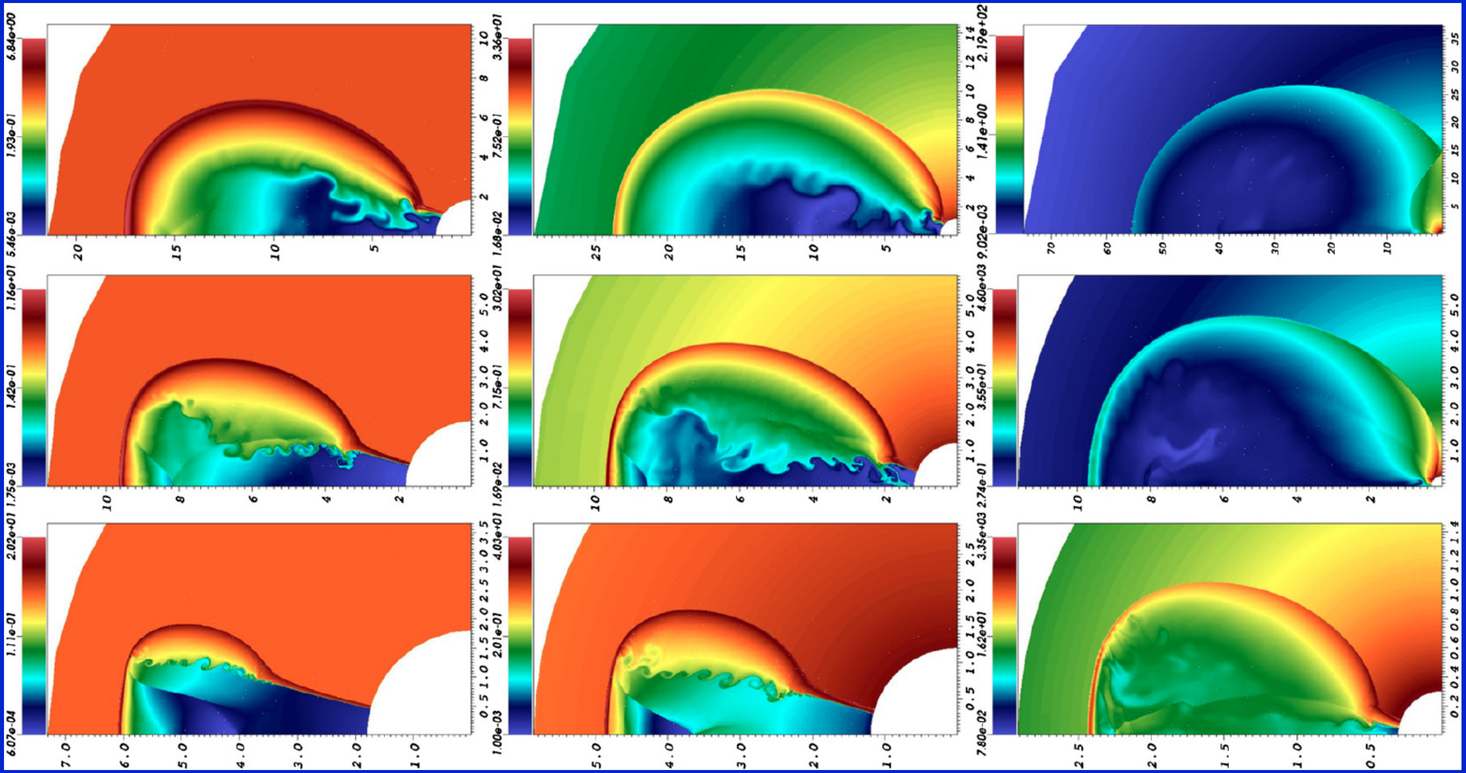


Future Directions of Relativistic Jets, Skokloster, Sweden, 31 August 2013

Outline of the talk:

- Background, jet angular structure & evolution stages
- Magnetic acceleration: overview & recent results
- Jet propagation inside the progenitor star
- Jet dynamics during the afterglow:
  - Recent numerical & analytic results: finally agree
  - Simulations of an afterglow jet propagating into a stratified external medium: $\rho_{ext} \propto R^{-k}$ for $k = 0, 1, 2$
  - Implications for GRBs: **jet breaks, radio calorimetry**

Differences between GRB jets & other Astrophysical Relativistic Jets:

- GRB jets are not directly angularly resolved
  - Typically at $z \gtrsim 1$ + early source size $\lesssim 0.1$ pc
  - Only a single radio afterglow (GRB 030329) was marginally resolved after 25 days (+monitored for years)
  - The jet **structure** is **constrained indirectly**
- GRB jets are **Impulsive:** most observations are long after the source activity
- GRBs are transient events, making the observations much more difficult

Observational Evidence for Jets in GRBs

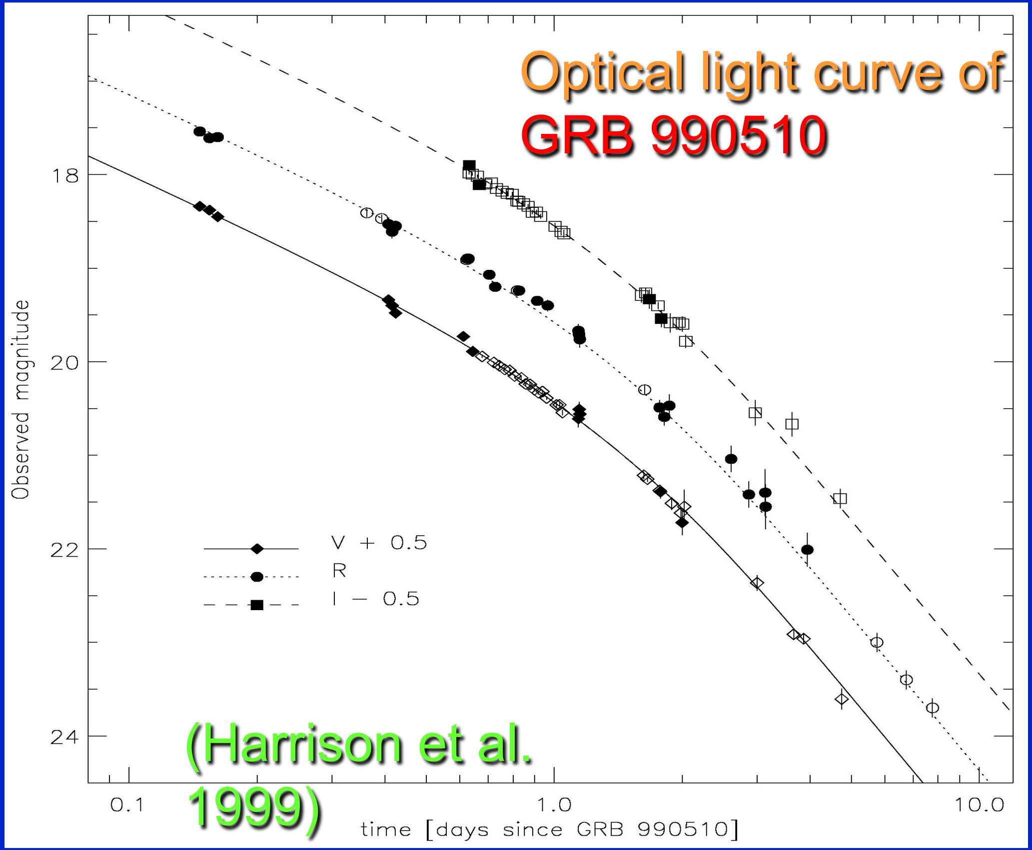
- The energy output in γ-rays assuming isotropic emission approaches or even exceeds $M_\odot c^2$
  - ⇒ difficult for a stellar mass progenitor
  - **True energy** is much smaller for a narrow **jet**
- Some long GRBs occur together with a SN

  ⇒ the outflow would contain $>M_\odot$ if spherical

  ⇒ only a small part of this mass can reach $\Gamma \gtrsim 100$

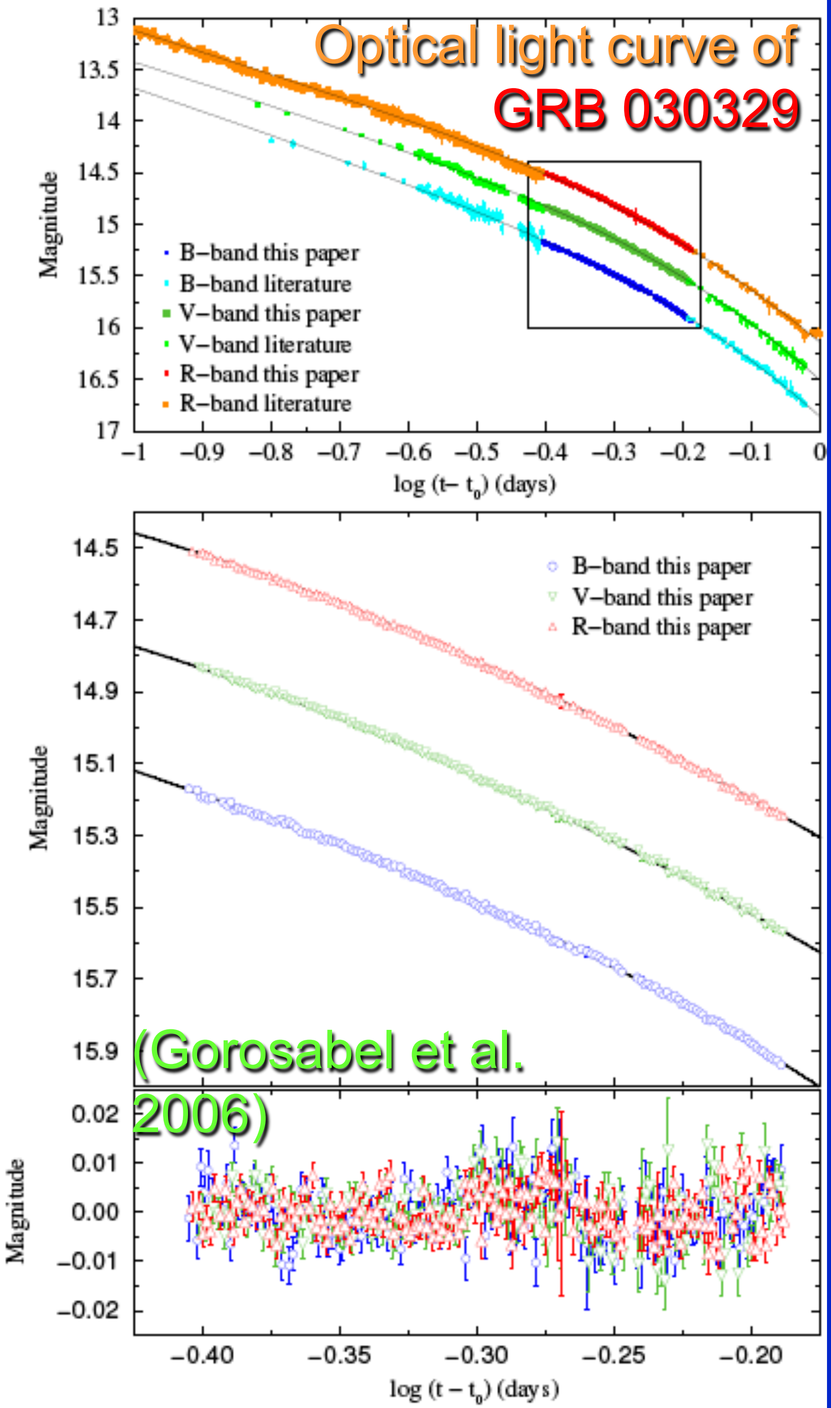
  & it would contain a small fraction of the energy
- Achromatic break or steepening of the afterglow light curves ("jet break")

Examples of Smooth & Achromatic Jet Breaks



Optical light curve of GRB 990510

(Harrison et al. 1999)

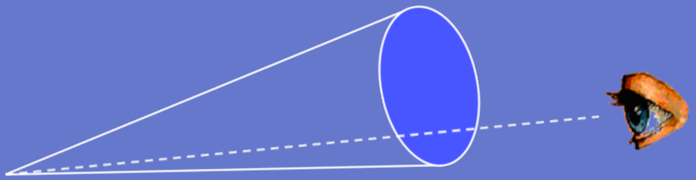


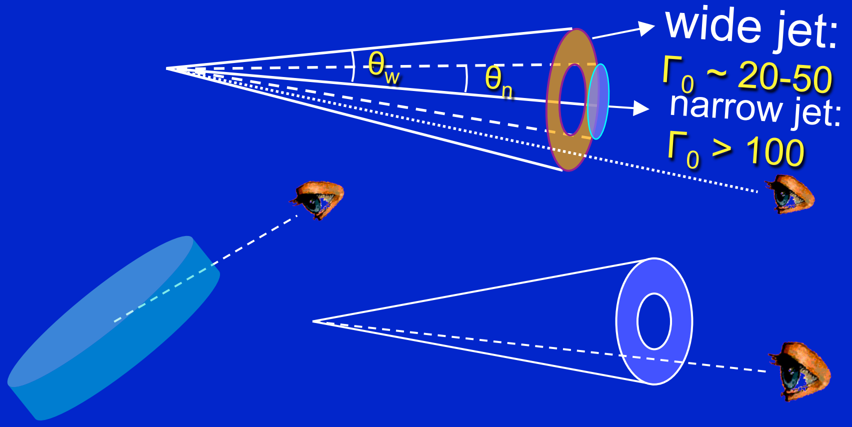
Optical light curve of GRB 030329

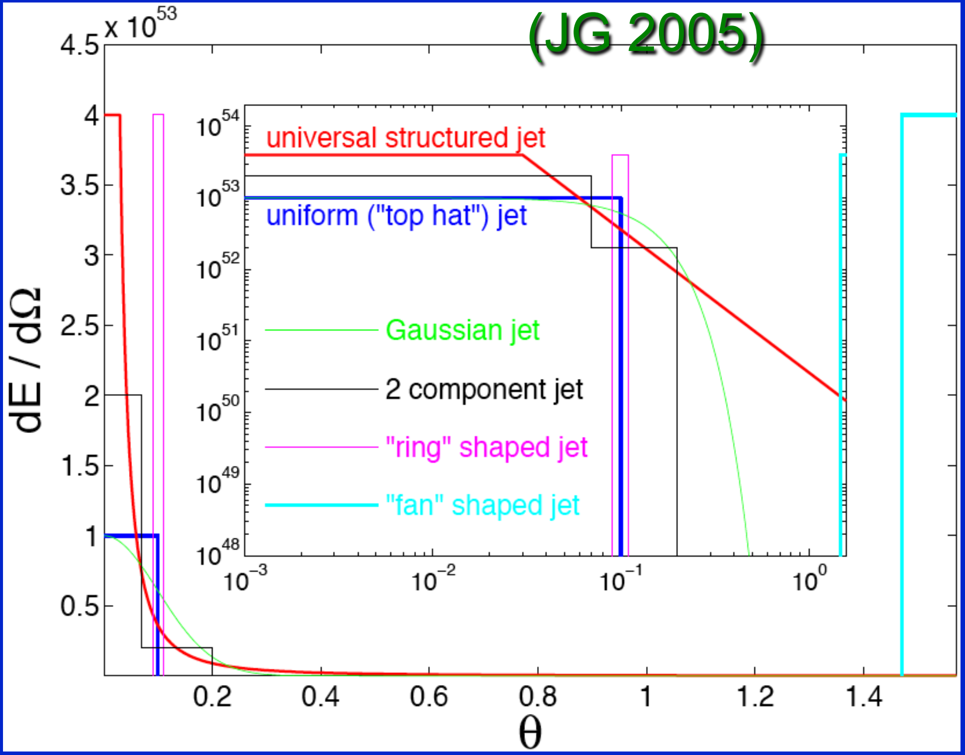
(Gorosabel et al. 2006)

The **Angular Structure** of GRB Jets:

- **Jet structure: unclear** (uniform, structured, hollow cone,…)
  - Affects $E_{\gamma,\mathrm{iso}}$ → E_γ & observed GRB rate → true rate
  - Viewing-angle effects (afterglow & prompt - XRF)
  - Can also affect late time radio calorimetry
- Here I consider mainly a uniform “top hat” jet

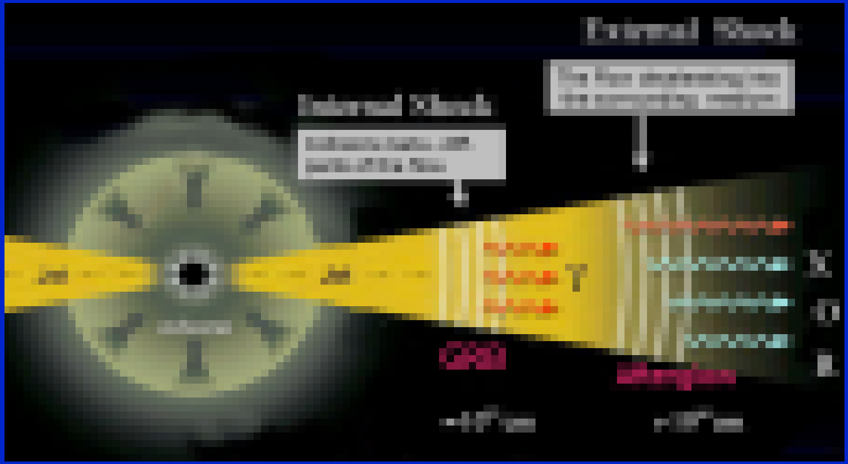






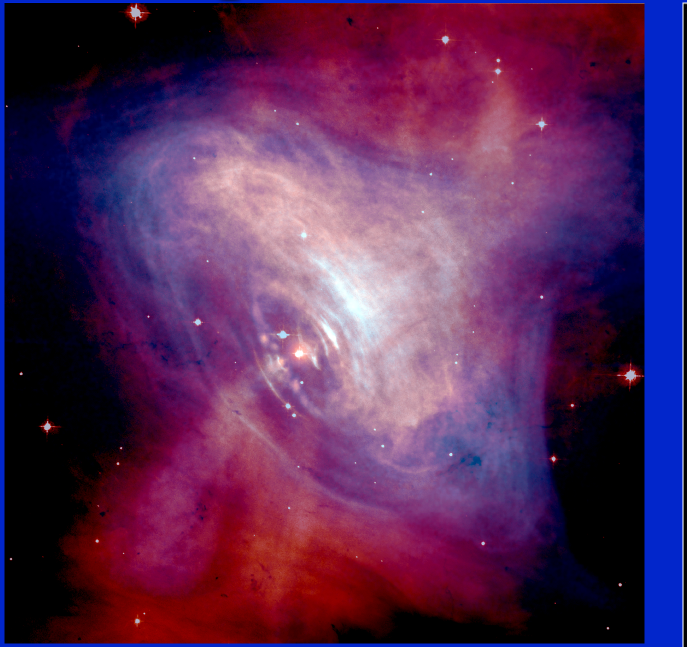
Stages in the **Dynamics** of GRB Jets:

- **Launching** of the jet: magnetic (B-Z?) neutrino annihilation?
- Acceleration: magnetic or thermal?
- For long GRBs: propagation inside progenitor star
- Collimation: stellar envelope, accretion disk wind, magnetic
- Coasting phase that ends at the deceleration radius R_{dec}
- At $R > R_{dec}$ most of the energy is in the shocked external medium: the composition & radial profile are forgotten, but the angular profile persists (locally: BM76 solution)
- Once $\Gamma < 1/\theta_0$ at $R > R_{jet}$ jet lateral expansion is possible
- Eventually the flow becomes Newtonian & spherical: self-similar Sedov-Taylor solution

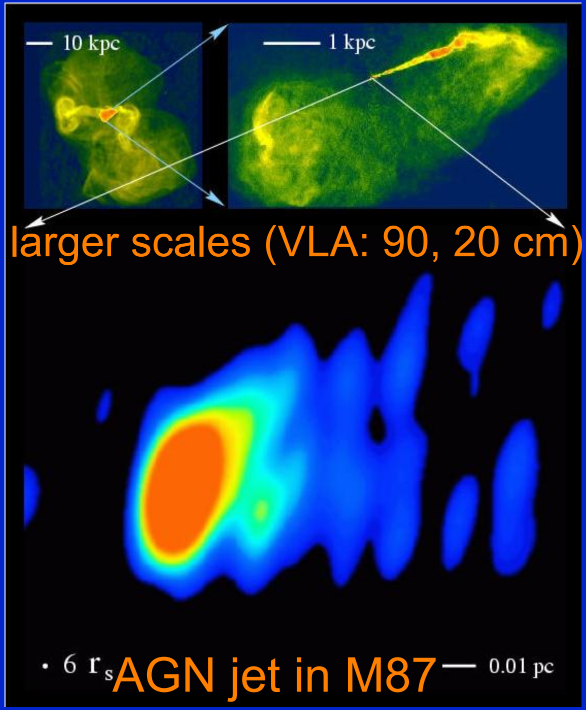


Relativistic Magnetic Acceleration:

- Relativistic ($v \approx c$) outflows/jets are very common in astrophysics & involve strong gravity at the source: PWN (NS), GRBs, AGN (SMBH), μ-quasars (BH/NS)
- Most models assume a steady flow for simplicity, despite observational evidence for time variability

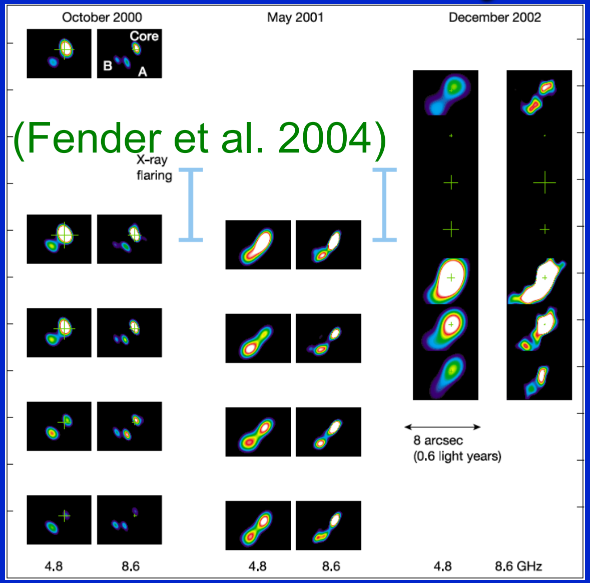


Crab Nebula: X-ray in blue, optical in red



larger scales (VLA: 90, 20 cm)

AGN jet in M87 (VLBA @ 43 GHz)



(Fender et al. 2004)

Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)

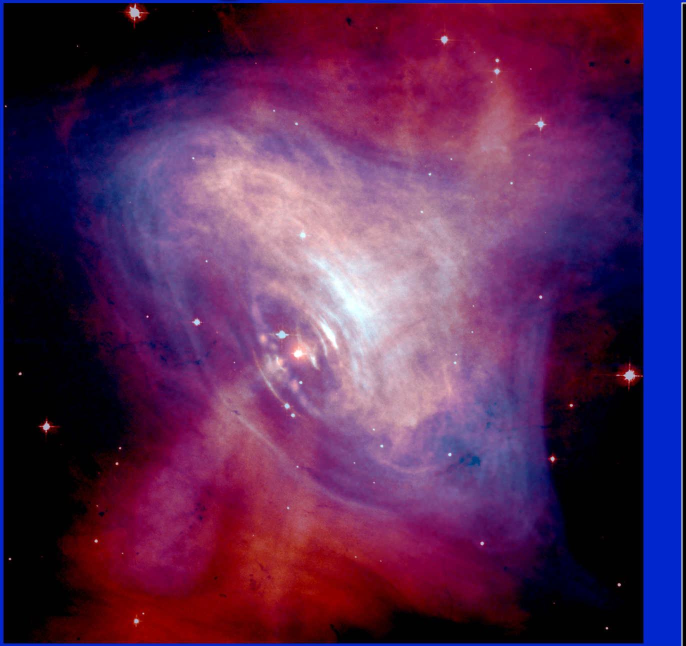
Relativistic Magnetic Acceleration:

Is the acceleration magnetic?

✔ ? ✔ ?

PWN (NS), GRBs, AGN (SMBH), μ-quasars (BH/NS)

- Most models assume a steady flow for simplicity, despite observational evidence for time variability



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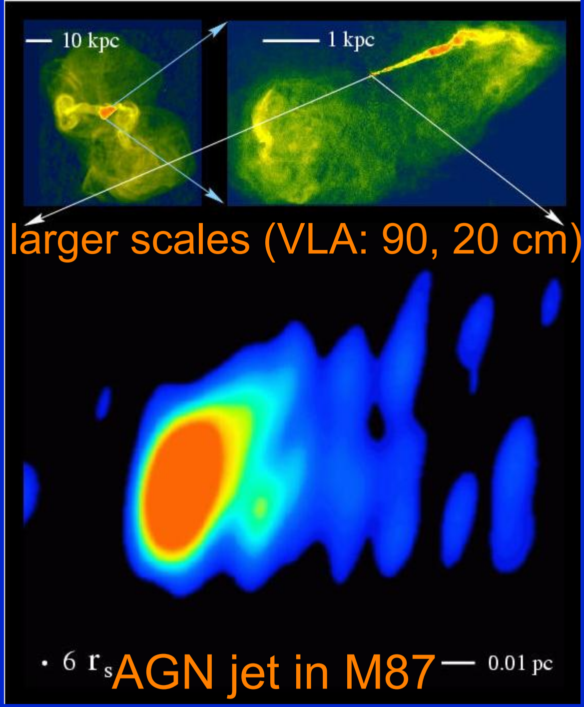

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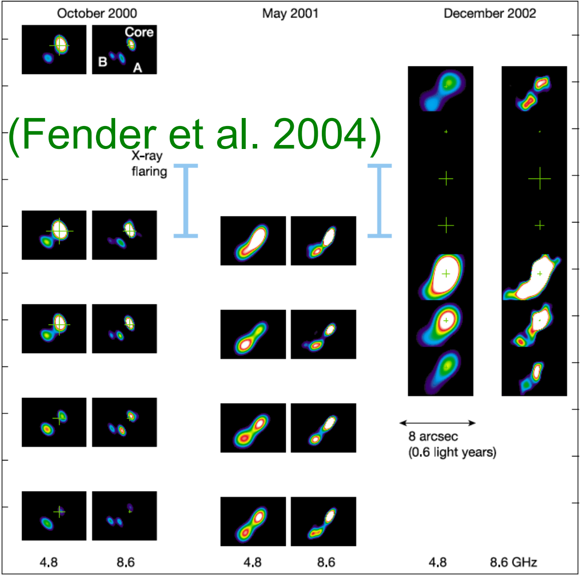


(Fender et al. 2004)

Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)

Thermal vs. Magnetic Acceleration:

- Most of the acceleration is in the supersonic regime

Key difference between thermal and magnetic steady state acceleration of relativistic supersonic flows:

- **Thermal**: fast, robust & efficient
- **Magnetic**: slow, delicate & less efficient

The σ-problem: for a "standard" steady ideal MHD axisymmetric flow

- $\Gamma_\infty \sim \sigma_0^{1/3}$ & $\sigma_\infty \sim \sigma_0^{2/3} \gg 1$ for a spherical flow; $\sigma_0 = B_0^2/4\pi\rho_0 c^2$
- However, PWN observations (e.g. the Crab nebula) imply $\sigma \ll 1$ after the wind termination shock – the σ problem!!!
- A broadly similar problem persists in relativistic jet sources
- Jet **collimation** helps, but not enough: $\Gamma_\infty \sim \sigma_0^{1/3}\theta_{jet}^{-2/3}$, $\sigma_\infty \sim (\sigma_0\theta_{jet})^{2/3}$ & $\Gamma\theta_{jet} \lesssim \sigma^{1/2}$ (~ 1 for $\Gamma_\infty \sim \Gamma_{max} \sim \sigma_0$)
- Still $\sigma_\infty \gtrsim 1$ ⇒ inefficient internal shocks, $\Gamma_\infty\theta_{jet} \gg 1$ in GRBs
- Sudden drop in external pressure can give $\Gamma_\infty\theta_{jet} \gg 1$ but still $\sigma_\infty \gtrsim 1$ **(Tchekhovskoy et al. 2009)** ⇒ inefficient internal shocks

Alternatives to the "standard" model

- ~~Axisymmetry~~: non-axisymmetric instabilities (e.g. the current-driven kink instability) can tangle-up the magnetic field (Heinz & Begelman 2000)

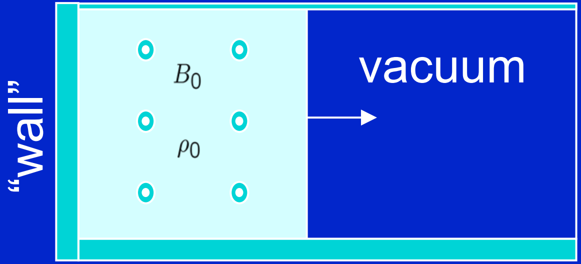
◆ If $\langle B_r^2 \rangle = \alpha \langle B_\phi^2 \rangle = \beta \langle B_z^2 \rangle$; $\alpha, \beta = \mathrm{const}$ then the magnetic field behaves as an ultra-relativistic gas: $p_{\mathrm{mag}} \propto V^{-4/3}$
$\Rightarrow$ magnetic acceleration as efficient as thermal

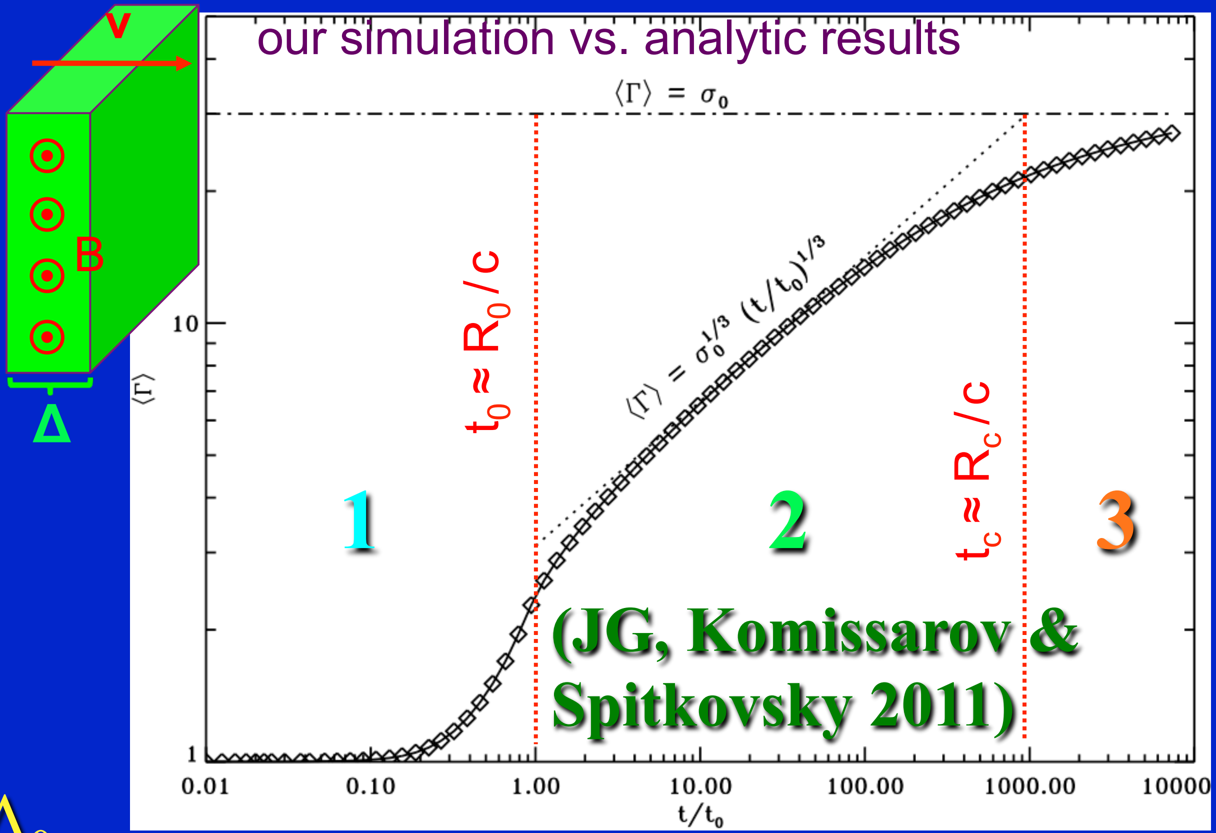
- ~~Ideal~~ MHD: a tangled magnetic field can reconnect (Drenkham & Spruit 2002; Lyubarsky 2010 - Kruskal-Schwarzschild instability (like R-T) in a "striped wind")
  magnetic energy → heat (+radiation) → kinetic energy
- ~~Steady state~~: **effects of strong time dependence** (JG, Komissarov & Spitkovsky 2011; JG 2012a, 2012b)

Impulsive Magnetic Acceleration: $\Gamma \propto R^{1/3}$

Useful **case study**:

Initial value of magnetization parameter: $\sigma_0 = \dfrac{B_0^2}{4\pi\rho_0 c^2} >> 1$




our simulation vs. analytic results

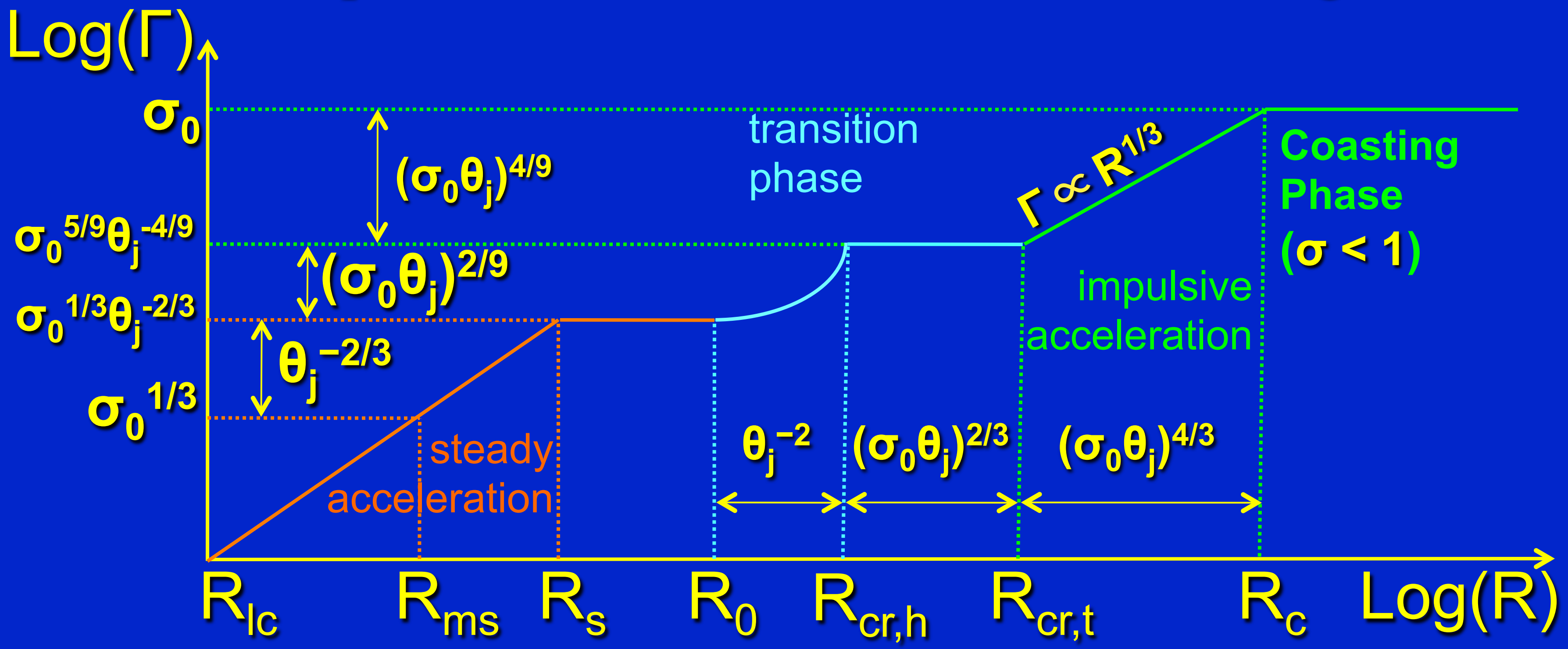
(JG, Komissarov & Spitkovsky 2011)

1. $\langle\Gamma\rangle_E \approx \sigma_0^{1/3}$ by $R_0 \sim \Delta_0$
2. $\langle\Gamma\rangle_E \propto R^{1/3}$ between $R_0 \sim \Delta_0$ & $R_c \sim \sigma_0^2 R_0$ and then $\langle\Gamma\rangle_E \approx \sigma_0$
3. At $R > R_c$ the sell spreads as $\Delta \propto R$ & $\sigma \sim R_c/R$ rapidly drops

- **Complete conversion of magnetic to kinetic energy!**
- **This allows efficient dissipation by shocks at large radii**

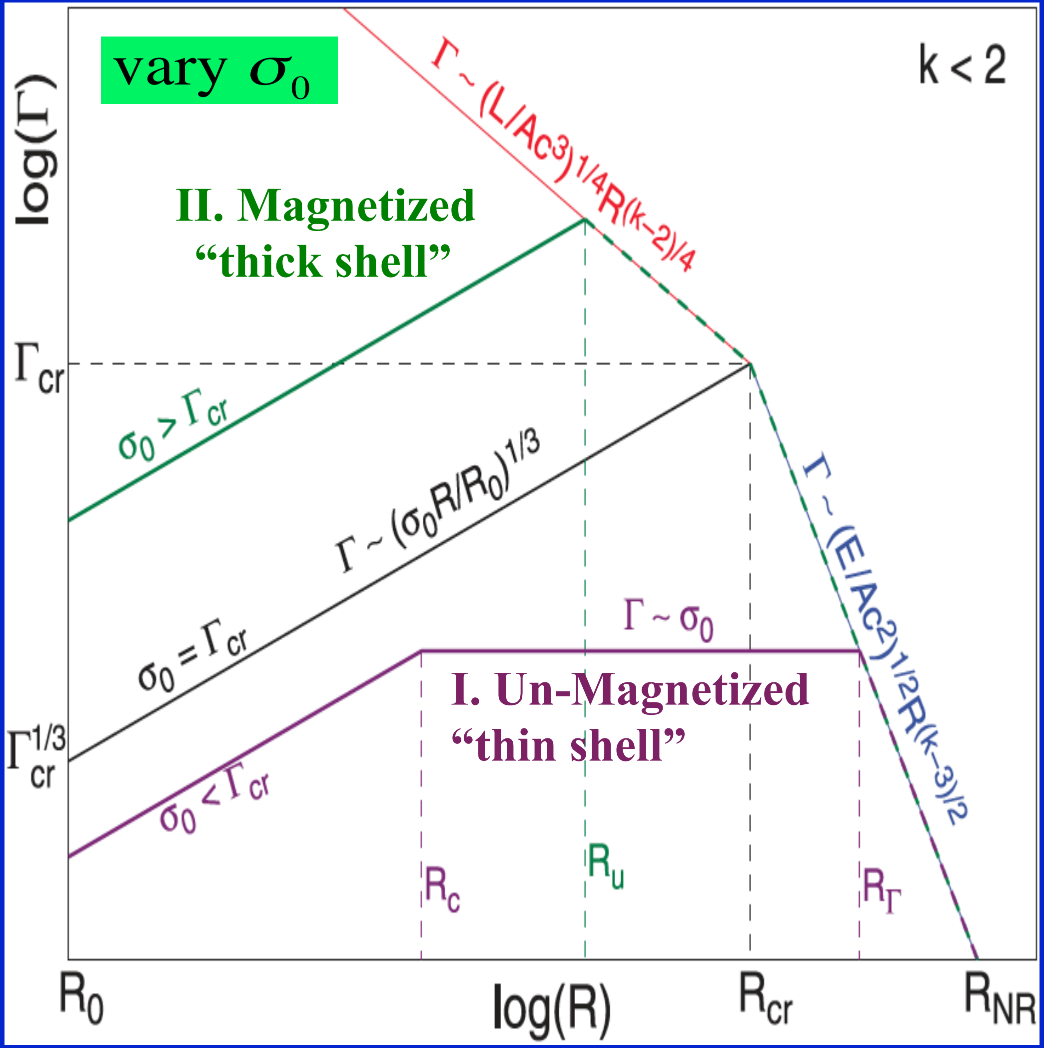
1st Steady then Impulsive Acceleration

- Our test case problem may be directly relevant for giant flares in SGRs (active magnetars); however:
- In most astrophysical relativistic (jet) sources (GRBs, AGN, μ-quasars) the variability timescale $(t_v \approx R_0/c)$ is long enough $(>R_{ms}/c)$ that steady acceleration operates & saturates (at R_s)
- Then the impulsive acceleration kicks in, resulting in $\sigma < 1$



Impulsive Magnetic Acceleration: single shell propagating in an external medium

acceleration & deceleration are tightly coupled (JG 2012)



$$\rho_{\rm ext} = AR^{-k}$$

$$R_{\rm cr} \sim R_0\Gamma_{\rm cr}^2 \sim \left(\frac{ER_0}{Ac^2}\right)^{\frac{1}{4-k}}$$

I. "Thin shell", low-σ : strong reverse shock, peaks at ≫ $T_{\rm GRB}$

II. "Thick shell", high-σ: weak or no reverse shock, $T_{\rm dec} \sim T_{\rm GRB}$

III. like II, but the flow becomes independent of σ_0

IV. a Newtonian flow (if $\rho_{\rm ext}$ is very high, e.g. inside a star)

II*. if $\rho_{\rm ext}$ drops very sharply

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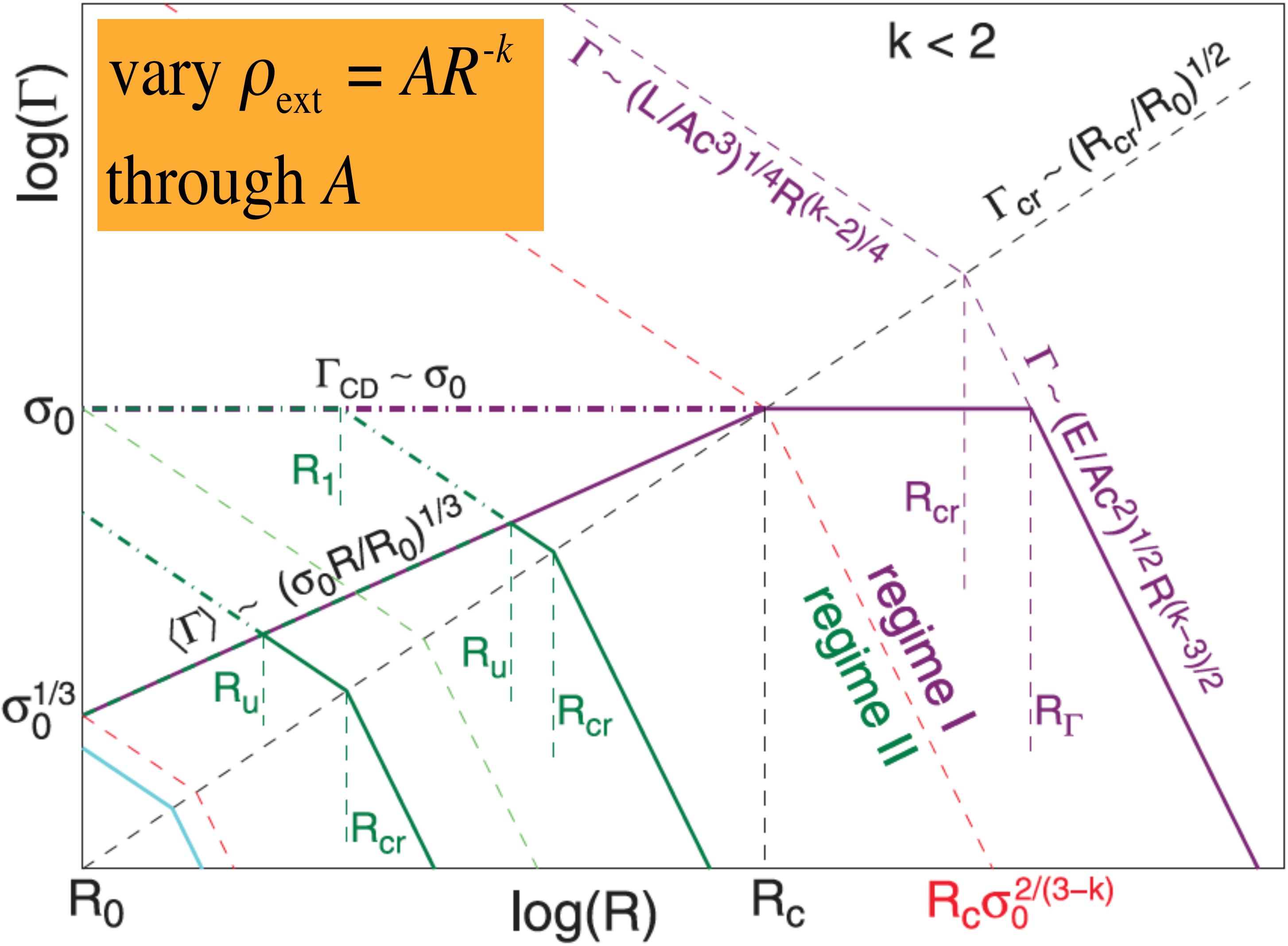

vary $\rho_{ext} = AR^{-k}$ through A

Dynamical Regimes:

I. "Thin shell", low-σ: strong reverse shock, peaks at $\gg T_{\rm GRB}$

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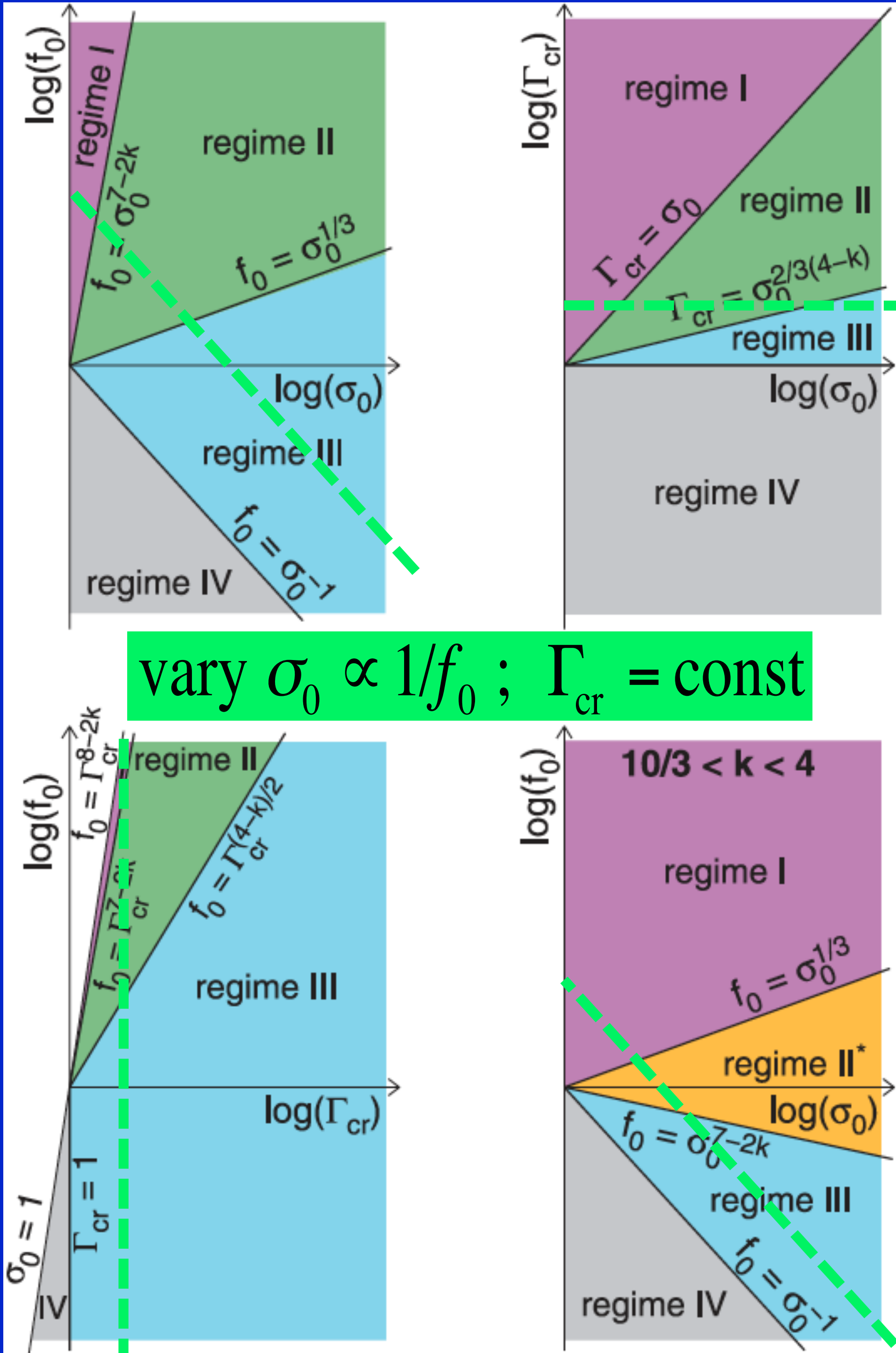
II*. if $\rho_{\rm ext}$ drops very sharply

$$\sigma_0 = B_0^2/4\pi\rho_0 c^2$$

$$f_0 = \rho_0/\rho_{\rm ext}(R_0)\ ,\quad \rho_{\rm ext} = AR^{-k}$$

$$\Gamma_{\rm cr} \sim (f_0\sigma_0)^{1/(8-2k)}$$



vary $\sigma_0 \propto 1/f_0\ ;\ \Gamma_{\rm cr} = {\rm const}$

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I. "Thin shell", low-σ: strong reverse shock, peaks at $\gg T_{GRB}$

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$$\sigma_0 = B_0^2 / 4\pi\rho_0 c^2$$

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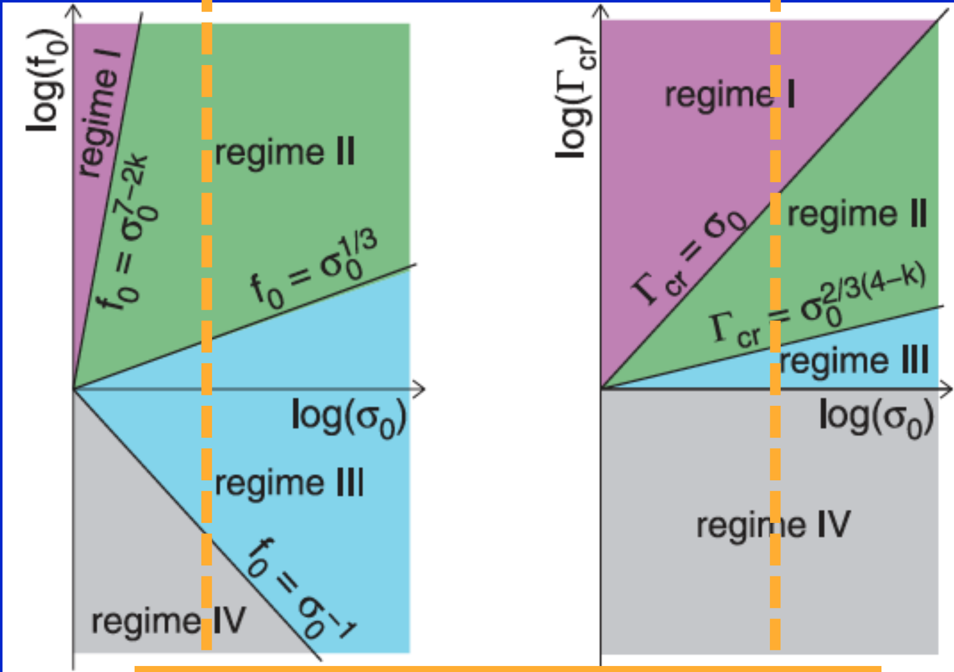
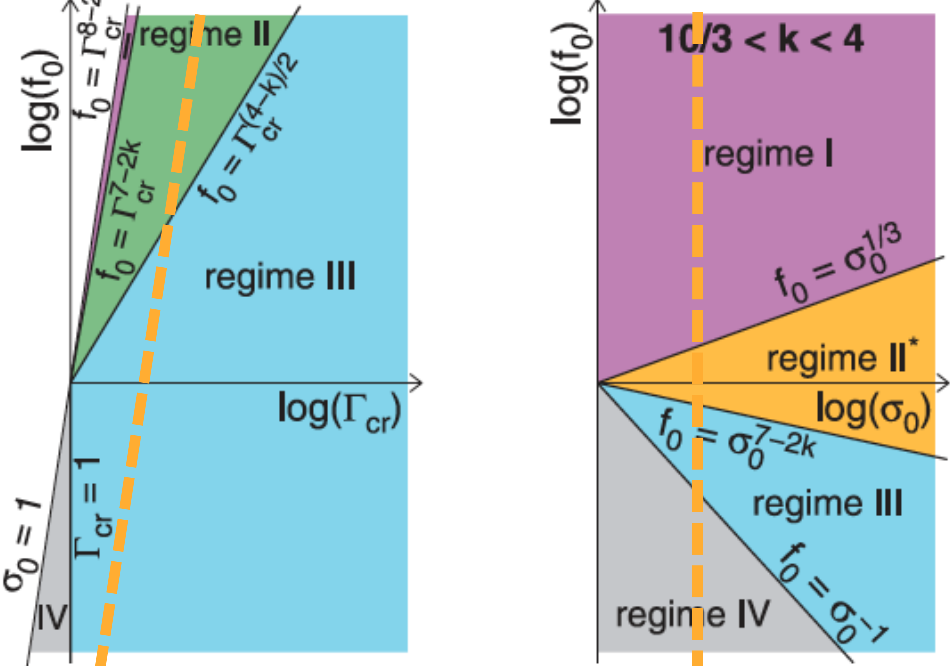


vary f_0 , $\Gamma_{cr} \propto f_0^{1/(8-2k)}$; σ_0 = const



Many sub-shells: acceleration, collisions

(JG 2012b)

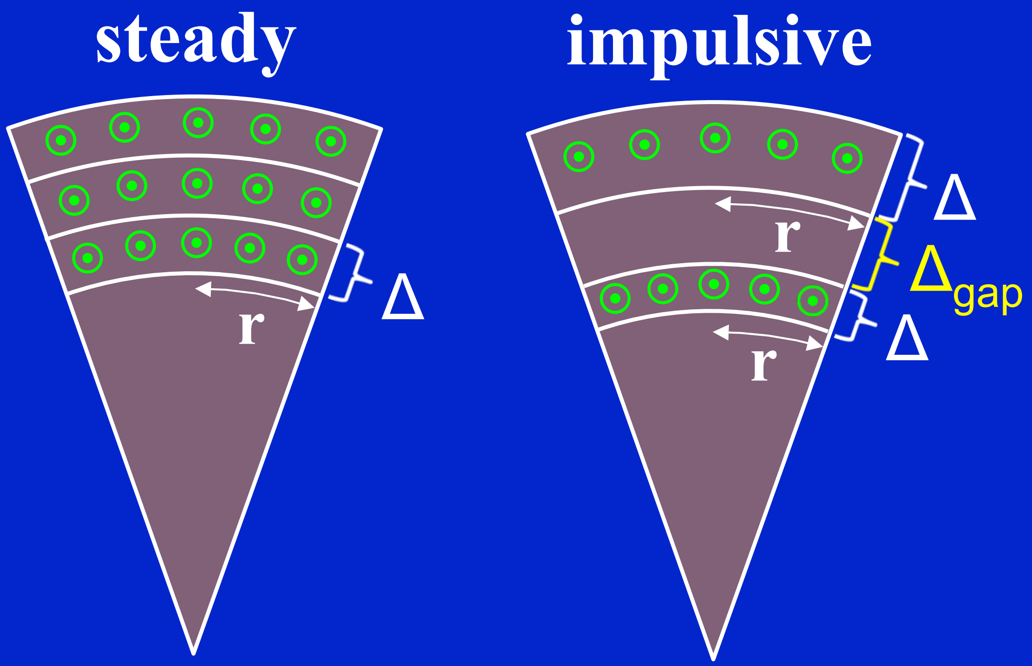
Flux freezing (ideal MHD):

$\Phi \sim B\,r\,\Delta = \textbf{constant}$

$E_{EM} \sim B^2\,r^2\,\Delta \propto 1/\Delta$

$$\frac{\text{total energy}}{\text{rest energy}} = (1+\sigma)\Gamma$$

acceleration $(\Gamma\uparrow) \Leftrightarrow \sigma\downarrow$



steady

impulsive

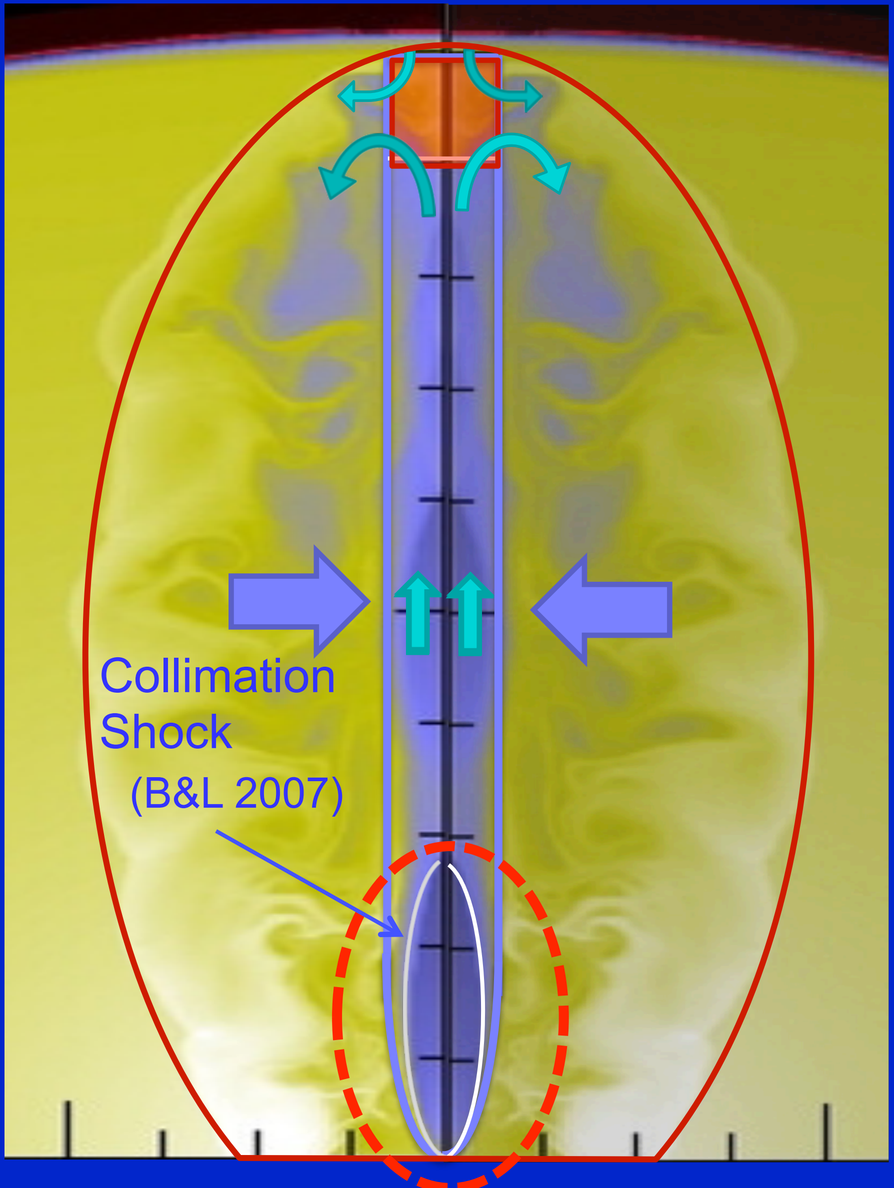
constant shell width Δ

shell width Δ grows

- For a long lived variable source (e.g. AGN), each sub shell can expand by $1+\Delta_{gap}/\Delta_0 \Rightarrow \sigma_\infty = (E_{total}/E_{EM,\infty} - 1)^{-1} \sim \Delta_0/\Delta_{gap}$
- For a finite # of sub-shells the merged shell can still expand
- Sub-shells can lead to a low-magnetization thick shell & enable the outflow to reach higher Lorentz factors

Jet propagation inside the progenitor star

- The Jet develops a slowly moving ‘head’
- At the head matter decelerates by a reverse shock & flows sideways forming a high-pressure cocoon that collimates the jet
- At the head there is a pressure balance between the shocked jet material & external medium
- The engine must continuously work until the jet breaks out
- After the jet breaks out it can freely accelerate & form the GRB



Jet propagation inside the progenitor star

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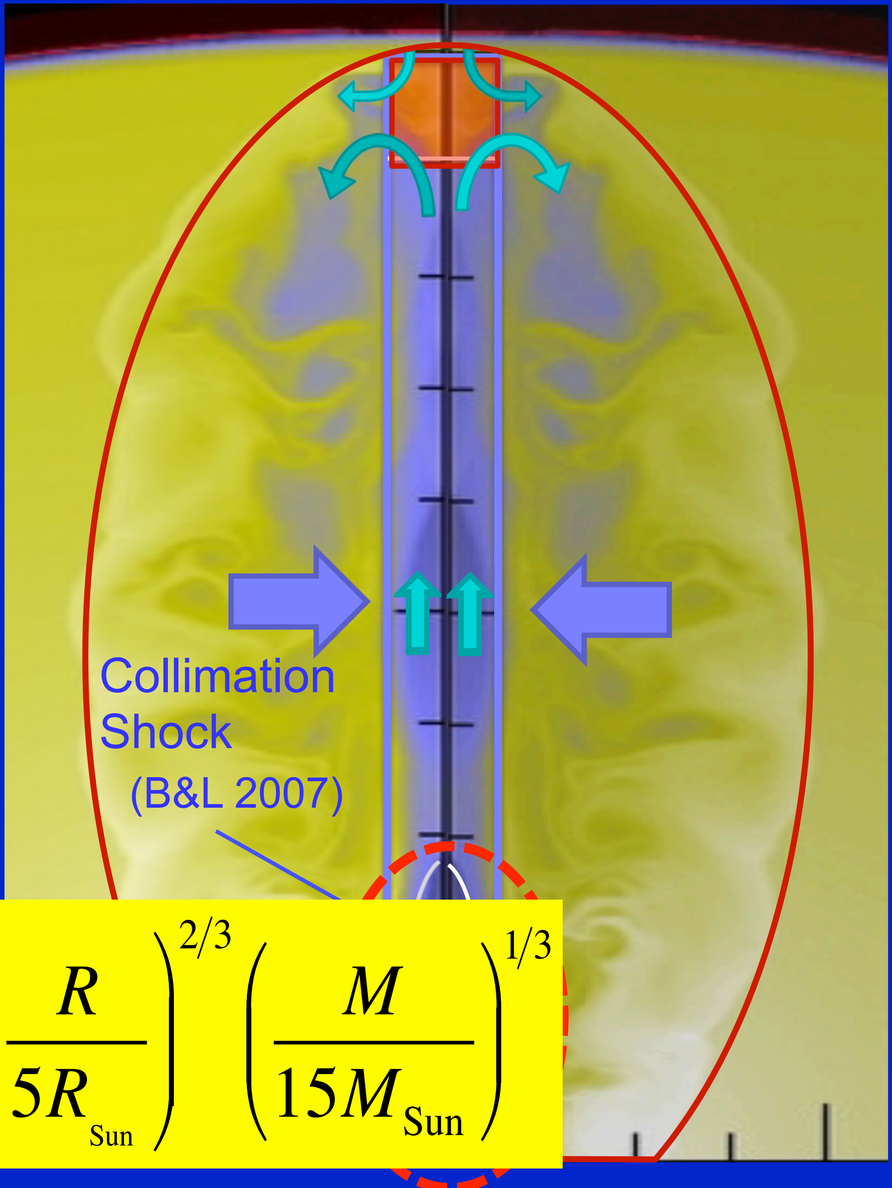
- At the head matter decelerates by a reverse shock & flows sideways forming a high-pressure cocoon that collimates the jet
- At the head there is a pressure balance between the shocked jet material & external medium
- Breakout time (Bromberg et al. 2011)

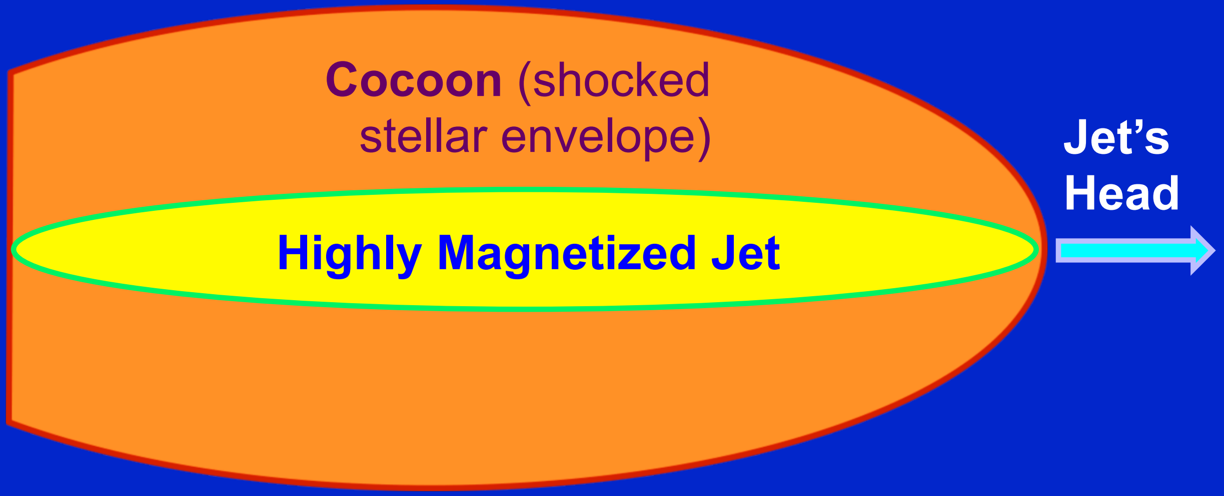
$$t_b \cong 15\,\mathrm{sec}\left(\frac{L_{iso}}{10^{51}\,erg/s}\right)^{-1/3}\left(\frac{\theta_0}{10^{\circ}}\right)^{2/3}\left(\frac{R}{5R_{Sun}}\right)^{2/3}\left(\frac{M}{15M_{Sun}}\right)^{1/3}$$



Jet propagation inside the progenitor star

highly magnetized vs. hydrodynamics jets

- The flow must decelerate to match it's head velocity, but for high-σ a shock can't do it ⇒ the jet converges near its head
- Narrower head ⇒ larger head velocity ⇒ faster jet breakout
- Relativistic head ⇒ less energy into cocoon & supernova
- The head velocity is independent of the detailed jet structure ⇒ simplifies the model & allows (semi-) analytic solutions



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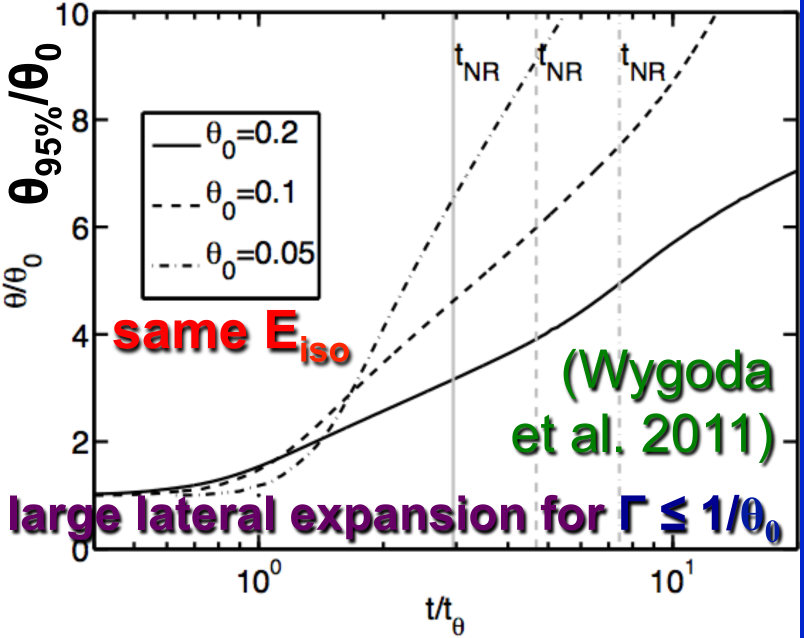
- The head velocity is independent of the detailed jet structure ⇒ simplifies the model & allows (semi-) analytic solutions
- Levinson & Begelman (2013): current-driven instabilities dissipate most of the magnetic field → a hydrodynamic jet
- This is still unclear & strongly effects the jet dynamics

Afterglow Jet Dynamics: Analytic vs. Numerical

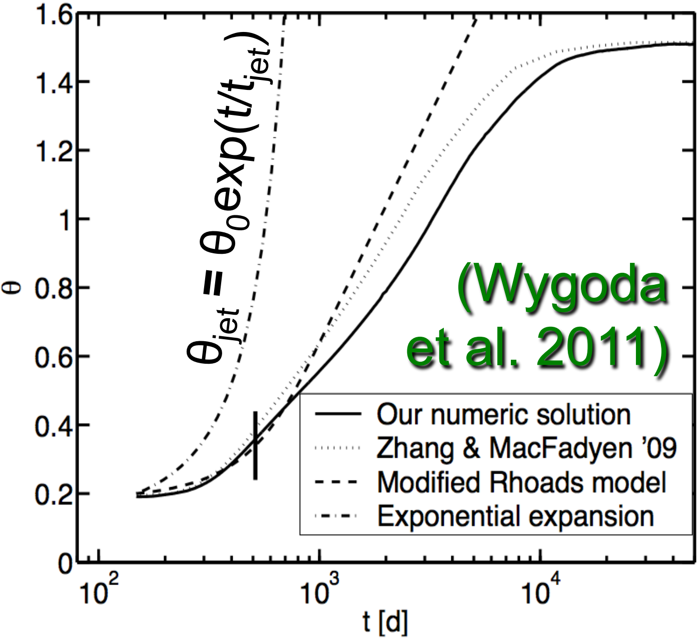
- **Analytic** results (Rhoads 1997, 99; Sari, Piran & Halpern 99): **exponential lateral expansion** at $R > R_{jet}$ e.g. $\Gamma \sim (c_s/c\theta_0)\exp(-R/R_{jet})$, $\theta_{jet} \sim \theta_0(R_{jet}/R)\exp(R/R_{jet})$
  - Supported by a self-similar solution (Gruvinov 2007)
- **Hydro-simulations**: very **mild** (logarithmic) lateral expansion while jet is relativistic (JG et al. 2001)



same E_{iso}

(Wygoda et al. 2011)

large lateral expansion for $\Gamma \leq 1/\theta_0$



$\theta_{jet} = \theta_0\exp(t/t_{jet})$

(Wygoda et al. 2011)

Modest θ_0 ⇒ small region of validity

Generalized Analytic model (JG & Piran 2012)

- **Lateral expansion:**

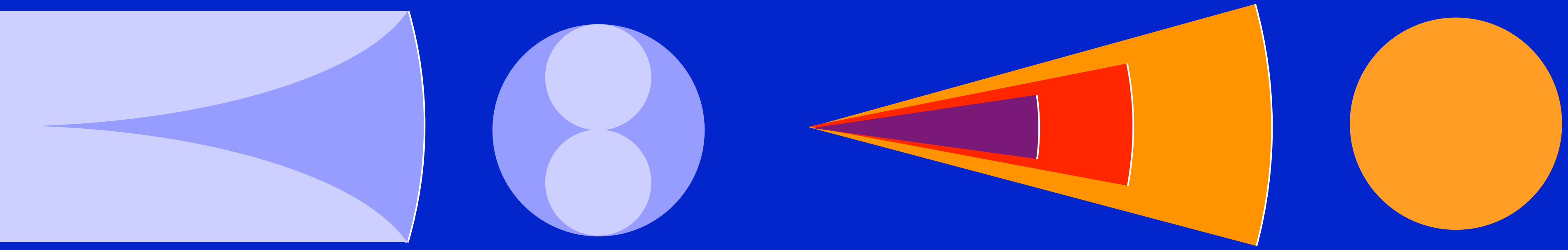
1. new recipe: $\beta_\theta/\beta_r \sim 1/(\Gamma^2\Delta\theta) \sim 1/(\Gamma^2\theta_j)$ (based on $\hat{\beta} = \hat{\boldsymbol{n}}$)
2. old recipe: $\beta_\theta = u_\theta/\Gamma = u'_\theta/\Gamma \sim \beta_r/\Gamma$ (assumes $u'_\theta \sim \beta_r \sim c_s$)

Generalized recipe:

$$\frac{d\theta_j}{d\ln R} = \frac{\beta_\theta}{\beta_r} \approx \frac{1}{\Gamma^{1+a}\theta_j^a}\ , \quad a = \begin{cases} 1 & (\hat{\beta} = \hat{\boldsymbol{n}}) \\ 0 & (u'_\theta \sim 1) \end{cases}$$

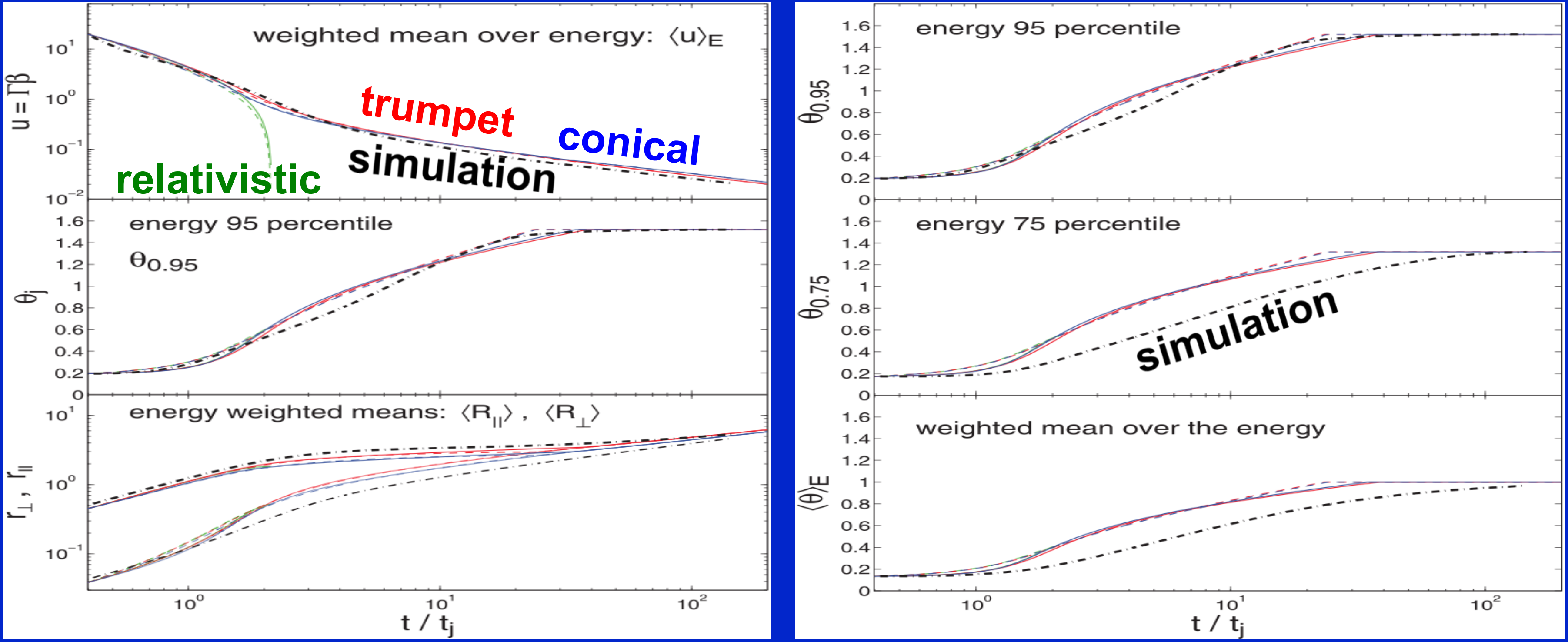
◆ New recipe: lower β_θ for $\Gamma > 1/\theta_0$ but higher β_θ for $\Gamma < 1/\theta_0$

- Does not assume $\Gamma \gg 1$ or $\theta_j \ll 1$ (& variable: $\Gamma \rightarrow u = \Gamma\beta$)
- Sweeping-up external medium: trumpet vs. conical models



Comparison to Simulations (JG & Piran 2012)

- Main effect of relaxing the $\Gamma \gg 1$, $\theta_j \ll 1$ approximation: quasi-logarithmic (~~exponential~~) lateral expansion for $\theta_0 \gtrsim 0.05$
- There is a reasonable overall **agreement** between the **analytic** generalized models and the hydro-**simulations**
- Analytic models: over-simplified, but capture the essence

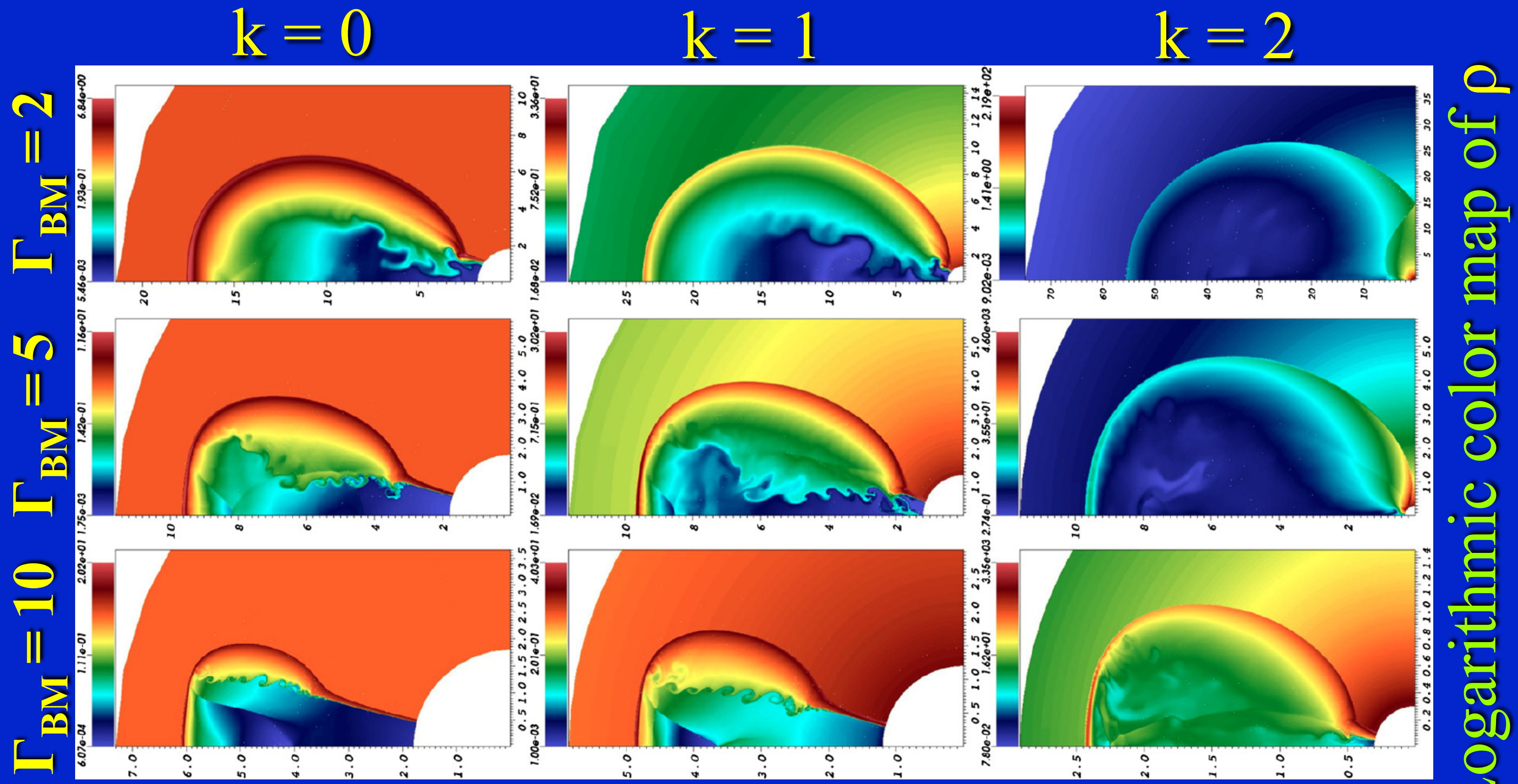


2D hydro-simulation by F. De Colle et al. 2012, with $\theta_0 = 0.2$, $k = 0$

Afterglow jet in stratified external media

(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

- Previous simulations were all for $k = 0$ where $\rho_{\rm ext} \propto R^{-k}$
- Larger k (e.g. $k = 1, 2$) are motivated by the stellar wind of a massive star progenitor for long GRBs



$\theta_0 = 0.2$, $E_{\rm iso} = 10^{53}$ erg, $n_{\rm ext}(R_{\rm jet}) \sim 1\ {\rm cm}^{-3}$

Afterglow jet in stratified external media

(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

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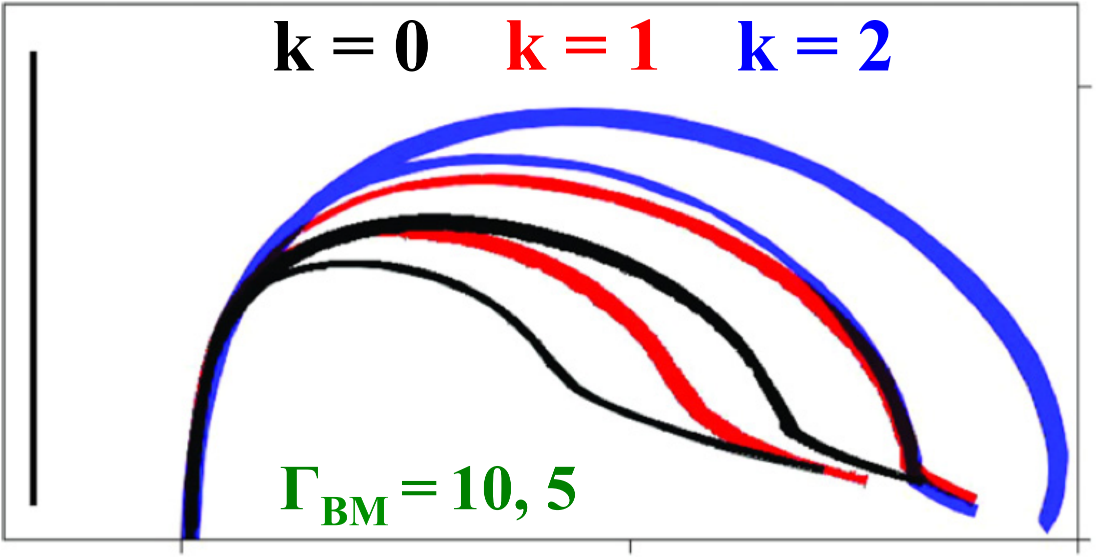
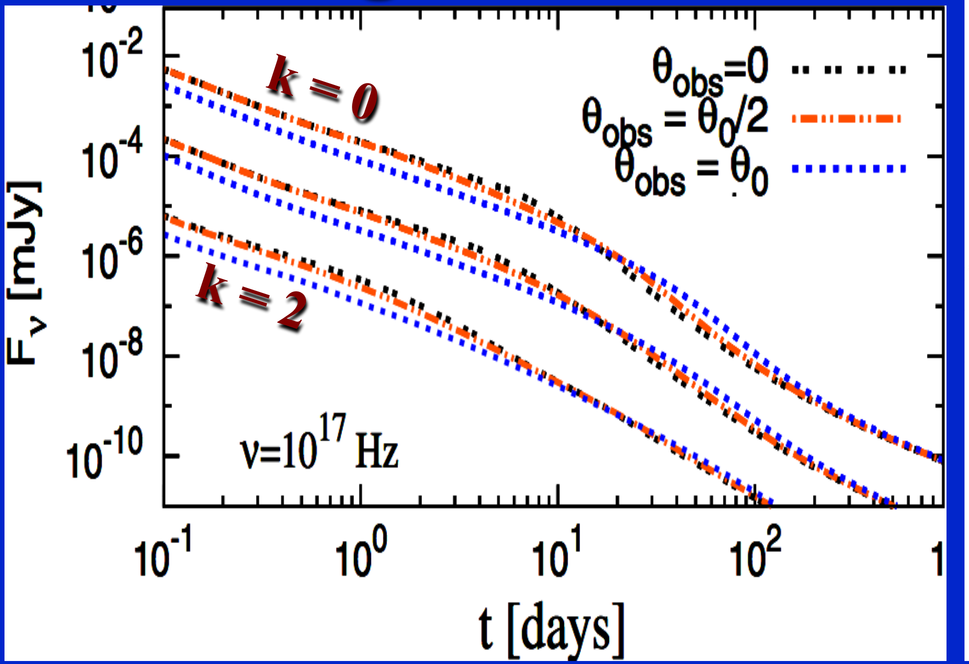


- At the same Lorentz factor larger k show larger sideways expansion since they sweep up mass and decelerate more slowly (e.g. $M \propto R^{3-k}$, $\Gamma \propto R^{(3-k)/2}$ in the spherical case) and spend more time at lower Γ (and β_θ decreases with Γ)

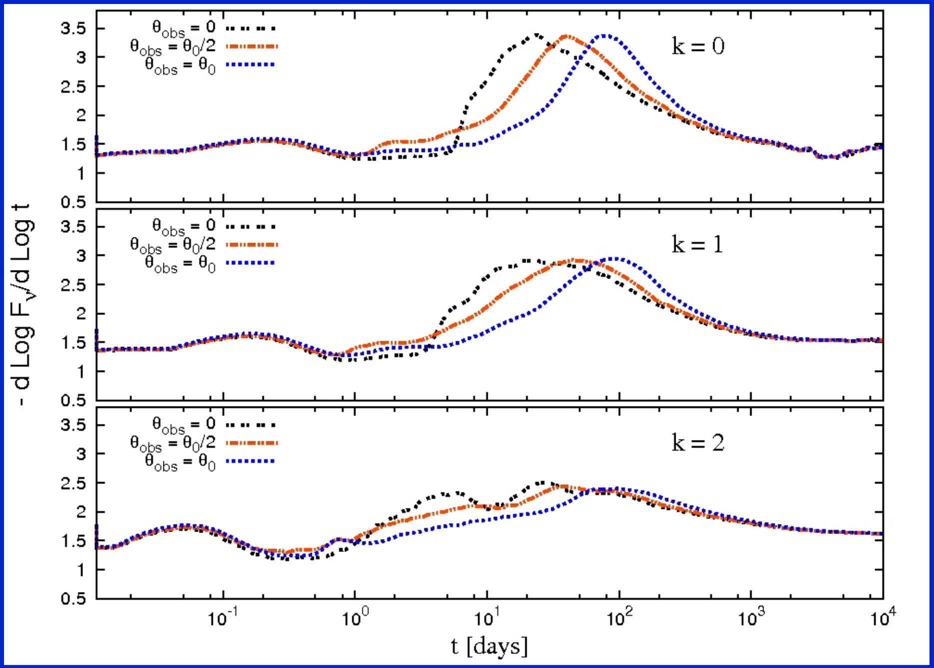
The shape of the jet break

- Jet break becomes smoother with increasing k (as expected analytically; Kumar & Panaitescu 2000 – KP00)
- However, the jet break is significantly sharper than found by KP00 $\Rightarrow$ better prospects for detection
- Varying $\theta_{obs} < \theta_0$ dominates over varying $k \lesssim 2$

Lightcurves



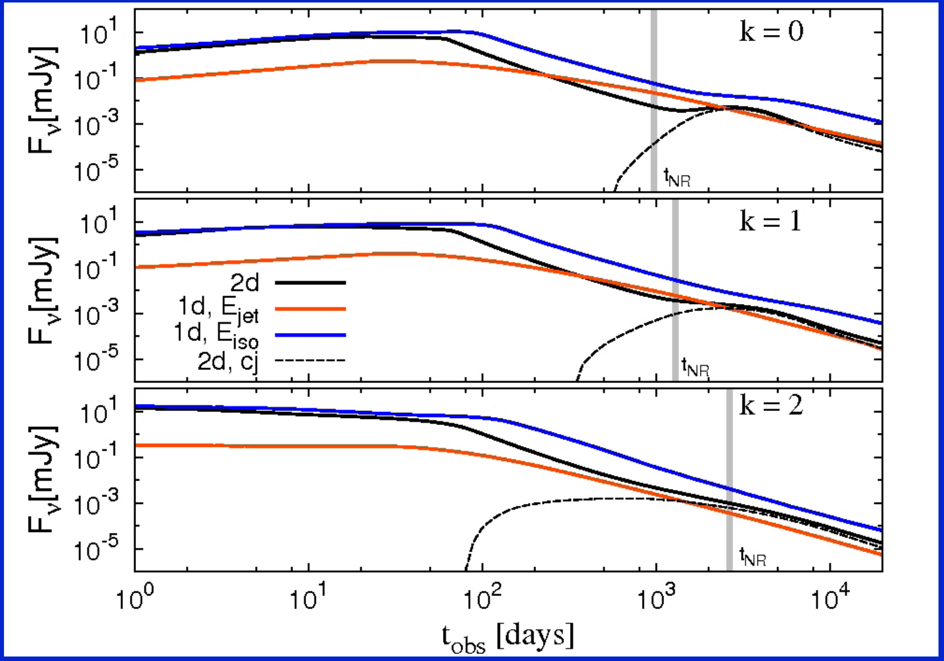
Temporal index



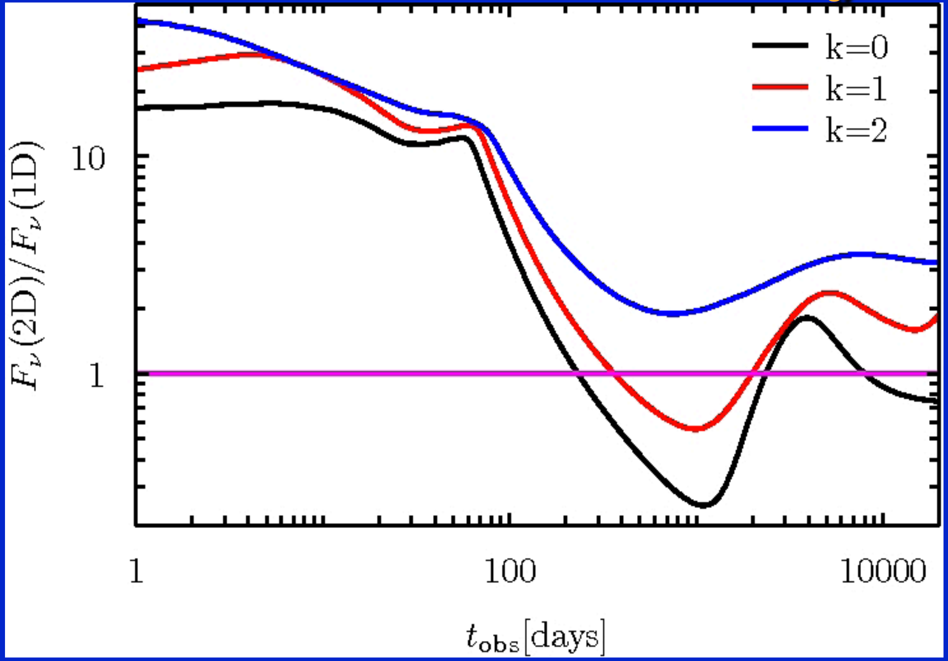
Late time Radio emission & Calorimetry

- The bump in the lightcurve from the counter jet is much less pronounced for larger k (as the counter jet decelerates & becomes visible more slowly) ⇒ hard to detect
- The error in the estimated energy assuming a spherical flow depends on the observation time t_{obs} & on k

Radio Lightcurves



Flux Ratio: 2D/1D(E_{jet})



Conclusions:

- Magnetic acceleration: likely option worth further study
- Jet propagation in star: can help probe jet magnetization
- Jet lateral expansion: analytic models & simulations agree
  - For $\theta_0 \gtrsim 0.05$: quasi-logarithmic (~~exponential~~) lateral expansion
  - For $\theta_0 \ll 0.05$: an early exponential lateral expansion phase (but such narrow GRB jets appear to be rare)
  - Jet becomes first sub-relativistic, then (slowly) spherical
- Jet in a stratified external medium: $\rho_{\rm ext} \propto R^{-k}$ for $k = 0, 1, 2$
  - larger k jets sweep-up mass & slow down more slowly
    $\Rightarrow$ sideways expansion is faster at $t < t_j$ & slower at $t > t_j$
    $\Rightarrow$ become spherical slower; harder to see counter jet
  - Jet break is smoother for larger k but possibly detectable
  - Jet break sharpness affected more by $\theta_{\rm obs} < \theta_0$ than $k \lesssim 2$
  - Radio calorimetry accuracy affected both by $t_{\rm obs}$ & k