

Fast Johnson-Lindenstrauss: History, Recent Progress and Open Questions

Nir Ailon, Technion

Abstract

A Johnson-Lindenstrauss Transform is defined to be a distribution over k -by- n matrices with the following property: For any fixed unit vector x in \mathbb{R}^n , if A is drawn from the distribution then the estimator $\|Ax\|$ looks like a Gaussian centered at 1 with variance roughly $1/k$. A Fast Johnson-Lindenstrauss Transform (FJLT) has the additional property that Ax can be computed in time $O(n \log n)$. Such transformations are related to design of restricted isometry matrices, useful for universal sparse reconstruction.

FJLT constructions are known for k at most an order of \sqrt{n} . For k above \sqrt{n} , best constructions slightly compromise the distribution guarantees of $\|Ax\|$. I will survey the history of results and efforts for constructing FJLTs, including some recent progress.

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