האוניברסיטה הפתוחה

המחלקה למתמטיקה ומדעי המחשב

אלגוריתמי חלוקת עוגה הוגנים עם מידע מבוסס שטח ממשי

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תזה זו מוגשת כחלק מהדרישות לתואר שני במדעי המחשב

בהנחיית פרופ׳ ריקה גונן

תקציר

חלוקה הוגנת של שטח היא בעיה מעשית חשובה. לרוב, בכדי לפתור בעיה זו, נעשה שימוש בשירותיו של שמאי או על ידי מכירת השטח על פי ערך השוק שלו וחלוקת הרווחים. דרך נוספת לחלוקה הוגנת של שטח היא על ידי אלגוריתמים לחלוקה הוגנת של "עוגה". אלגוריתמים אלו כרוכים בעלויות שימוש נמוכות בהרבה מאשר שכירת שירותיו של שמאי, ומבחינה תיאורטית אלגוריתמים אלו הוגנים יותר כיוון שהם מבטיחים לכל סוכן חלק יחסי הוגן על פי פונקציית הערך של אותו סוכן, שעשויה להיות שונה מאשר זו של השמאי (או של השוק). למרות האמור לעיל, התיאוריה הנוכחית העוסקת בחלוקת עוגה הוגנת, אינה בשלה לשימוש מעשי במקרי חלוקת שטח ולכן כיום לא נעשה שימוש באלגוריתמים אלו.

אנו מנסים להקטין את הפער בין תיאוריה ופרקטיקה על ידי הצגת מספר דרכים להרחבת שני אלגוריתמים ידועים לחלוקה הוגנת של עוגה חד-ממדית על ידי היוריסטיקה, כך שיתאימו לחלוקת שטח דו-ממדי לחלקות מלבניות. כל היוריסטיקה נמדדת על ידי ביצוע סדרת סימולציות על גבי סט נתונים המבוסס על ערכים ממשיים של שטח עבור שלושה סטים שונים של מפות ומפה רביעית (בקרה) שמורכבת מערכים אקראיים. הסימולציות משוות את הביצועים של אלגוריתמי החלוקה לביצועי חלוקת השמאי ולמכירת השטח על ידי כמה מדדים, רווחה תועלתנית, רווחה שוויונית, קנאה וצורה גיאומטרית. אלגוריתמי החלוקה ההוגנת מוכיחים עצמם כעדיפים ברוב המדדים. יחד עם זאת, ניכר כי ביצועי אלגוריתמי החלוקה מושפעים רבות מפרטי המימוש הטכני ומפרטים מעשיים שלרוב לא מקבלים התייחסות בתיאוריה.

נוסף על הרחבת אלגוריתמי חלוקת עוגה, בעבודה זו מוצג פרוטוקול חדש לחלוקה מבוסס תכנון דינמי יחד עם בחינת שקלול התמורות בין סיבוכיות זמן הריצה לבין ביצועים. ביצועי הפרוטוקול המוצע טובים מאשר אלגוריתמי חלוקת העוגה הקלאסיים שנבחנו על פני רוב המדדים. יחד עם זאת, בוצעה הערכה של הרווח הצפוי לסוכן המדווח העדפות כוזבות באופן אסטרטגי בניסיון להגדיל את הרווח. בעוד שעל פי התאוריה, במקרה הגרוע ביותר אסטרטגיה שכזו עלולה לפגוע באופן מהותי בתוצאה, על פי הסימולציות ניתן לראות ששימוש באסטרטגיה בניסיון לשנות את תוצאות הריצה אינה מהווה בעיה מהותית בפועל במקרה הממוצע.

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תודות

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Fair Cake-Cutting Algorithms with Real Land-Value Data

by

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Abstract

Fair division of land is an important practical problem that is commonly handled either by hiring assessors or by selling and dividing the proceeds. A third way to divide land fairly is via algorithms for *fair cake-cutting*. Such un-intermediated methods are not only cheaper than an assessor but also theoretically fairer since they guarantee each agent a fair share according to his/her value function. However, the current theory of fair cake-cutting is not yet ready to optimally share a plot of land, and such algorithms are seldom used in practical land division.

We attempt to narrow the gap between theory and practice by presenting several heuristic adaptations of famous algorithms for one-dimensional cake-cutting to two-dimensional land-division. The heuristics are evaluated using extensive simulations on real land-value data maps from three different data sets and a fourth (control) map of random values. The simulations compare the performance of cake-cutting algorithms to sale and assessor division in various performance metrics, such as utilitarian welfare, egalitarian welfare, envy, and geometric shape. The cake-cutting algorithms perform better in most metrics. However, their performance is greatly influenced by technical implementation details and heuristics that are often overlooked by theorists.

We also propose a new protocol for practical cake-cutting using a dynamic programming approach and discuss its runtime complexity versus performance trade-off. Our new protocol performs better than the examined classic cake-cutting algorithms on most metrics. Experiments to assess the amount that a strategic agent can gain from reporting false preferences are also presented. The results show that the problem of strategic manipulation is much less severe than the worst-case predicted by theory.

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Dedicated to my beloved wife, for all the love and support

Preface

Fair division of land is an important practical problem. It occurs in inheritance cases, partnership resolutions, and public land allocations. It is commonly handled by hiring the services of expert human assessors, who calculate a division that is considered fair based on elaborate land-value assessments. If the partners do not want to trust an assessor, they often sell the land in the market and split the revenues.

But there is a different way to divide land fairly. In the last 70 years, economists and computer scientists have developed various algorithms for fair *cake-cutting* - a fair division of a heterogeneous resource, such as a cake, that is assumed to be divisible. The fair division of a cake is often used as an analogy for division of resources, such as advertisement space, broadcast time or land among agents with different preferences.

Theoretical research on fair cake cutting algorithms often ignores several practical concerns that may arise when using such algorithms for practical land division, such as choosing the two dimensional cuts directions, the implications real data and preferences by players has on the algorithms' results, cost-benefit of players acting strategically and more. The purpose of this research is to study the theoretical and practical implications of using fair cake cutting algorithms to fairly divide land using real land-value data.

Chapter 1

Introduction

Using automatic fair cake cutting algorithms for land division is not only cheaper than renting a human assessor — it is also theoretically fairer, since it guarantees to each agent a fair share by his/her personal value function, which may differ from the assessor's value function **(Section 3)**. As far as we know, these theoretically-superior algorithms are seldom used in practical land-division problems. Our goal is to bring theory and practice closer together, and this paper describes several steps we made in this direction.

First, we constructed instances of cake-cutting problems based on real land-value data. We constructed detailed two-dimensional land-value maps of New Zealand, most of Israel and a higher resolution map of Tel-Aviv city.

To simulate agents with different but correlated valuations, we constructed, from each land-value map, n different maps (one map per agent), where each map is based on the original map and some random noise. We experimented with various values of n between 4 and 128, two noise models, and various noise levels, ranging between 20% and 60% of the base value **(Section 4)**.

Second, we adapted the classic cake-cutting algorithms of Even and Paz (1984) and Steinhaus (1948) to divide a two-dimensional map into rectangular land-plots. This adaptation can be done in many ways, since each cut of the interval made by the original algorithm can be converted to a horizontal (east-west) cut or to a vertical (north-south) cut of the two-dimensional map. The cut direction can be fixed in advance, or it can be decided dynamically based on various heuristics such as cutting the longer side or choosing the cut that maximizes the social welfare. Most previous works that we

know of ignored these questions since they assumed that the cake is one-dimensional. However, when dividing a two-dimensional land, such decisions may affect the allocation quality. Therefore, we present a dynamic programming algorithm for finding the sequence of $n-1$ cut-directions that optimizes a given performance metric **(Section 5)**. The computation complexity of the algorithm is $O(n^2)$ and the query complexity is $O(n)$, this may be too much for large values of n. Therefore, we implemented over 10 different heuristics for choosing the cut-direction, which allow the algorithm to run with computational complexity of $O(n \log n)$ and query complexity of $O(\log n)$.

Third, we conducted extensive experiments with different numbers of agents and four different maps (the above three and a randomly-generated map). In each setting, we compared the cake-cutting algorithm in all its different variants to the two baseline methods commonly used today for dividing land: (1) *assessor division*, where the land is partitioned into pieces with the same base value (ignoring the "noise") and each agent receives one piece; (2) *market sale*, where the land is sold for its total market value and each agent receives $1/n$ of the proceeds. We compared the methods using several metrics: *utilitarian welfare* (the sum of agents' utilities), *egalitarian welfare* (the smallest agent utility), *envy* (the amount by which an agent considers another agent's share as better than his/her own share), *Nash social welfare*, and a new metric particularly important in land division — the *length/width ratio* of the resulting pieces.

We found substantial differences in performance among the different heuristics. For each performance metric, one or two heuristics were clearly superior to the others. This shows the importance of modelling land in two dimensions rather than reducing it to a 1-dimensional interval. The cake-cutting algorithms fared similar or better than assessor division in all metrics, and better than market sale in the two welfare metrics **(Section 6)**.

A potential problem with cake-cutting algorithms, that does not exist in market sale or assessor division, is that the agents may try to manipulate the algorithm by misrepresenting their preferences. We measured the *strategic gain* — the amount by which a strategic agent, who knows the actions of all other agents in the current round and plays a best response, can increase his/her utility relatively to a truthful agent. The average strategic gain of such an agent was less than 2.2% (same order of magnitude as the payment to a real-estate broker). In the more realistic situation, in which an agent does not know the actions of all other agents, strategic manipulations would probably be even less profitable, and may even result in losing value. This information

is helpful, as it allows us to advise participants in cake-cutting algorithms to report their valuations truthfully even though theoretically the algorithm is not truthful.

The advantage of the cake-cutting algorithms was much more pronounced in the two maps based on land values than in the random map. This illustrates the importance of using real value data for evaluating fair division algorithms. This is in contrast to previous work on simulation of fair division algorithms, which mostly used artificially-generated data.

The improvements provided by the cake-cutting algorithm are encouraging. They imply that the algorithmic "do it yourself" approach to fair division, which is cheaper than the common approaches of selling or employing an expert assessor, is also better in terms of social welfare and reducing envy. Still, we believe that there is a lot of room for future improvement by using sophisticated cake-cutting algorithms that are specifically tailored to perform well in a specific metric (**Section 7**).

Chapter 2

Related Work

Surveys on the theory of cake-cutting algorithms can be found in Brams and Taylor (1996); Robertson and Webb (1998); Brams (2007); Brânzei (2015); Lindner and Rothe (2016) and Procaccia (2016). Below we focus on the more practice-oriented works on fair division. These can be categorized into computerized simulations, case-studies and lab experiments.

2.1 Computerized Simulations

Fair division papers often use simulations to compare the performance of various algorithms. For example, Walsh (2011) used simulations to compare his new algorithms for *online cake-cutting* with existing algorithms for offline cake-cutting w.r.t. egalitarian and utilitarian welfare. Cavallo (2012) used simulations to compare his redistribution-based improvement of VCG to the original VCG w.r.t. fairness and welfare. Dickerson et al. (2014); Aziz et al. (2020) used simulations to check under what conditions a fair assignment of discrete objects exist with high probability.

In these works, the agents' valuations were drawn at random. Correlation between different valuations for the same item was modeled by assigning to each item a random *intrinsic value*, and adding a random noise for each agent.

Our work is similar to these works in that it, too, compares different methods for fair division using computerized simulations. It differs in that, in our model, the "intrinsic value" of each land-plot is not selected at random but rather based on real land values.

2.2 Case Studies

Flood (1958) studied a case of dividing gift parcels using the Knaster algorithm (Steinhaus, 1948). Pratt and Zeckhauser (1990) applied an auction-based division algorithm to a real-life problem of allocating silver heirlooms. They found that, although the algorithm was decentralized and most participants did not fully understand it or the preference information desired, it handled all major considerations well and was regarded as equitable.

Similarly, Tijs and Branzei (2004) describe some case studies of dividing the profits of cooperation between partners, in light of concepts from cooperative game theory, such as the Shapley value.

Several counter-factual studies checked the feasibility of using the Adjusted Winner fair-division protocol for resolving international disputes: Camp David Accords, Spratly Islands controversy and the Israeli-Palestinian conflict (Brams and Taylor, 1996; Brams and Togman, 1996; Brams and Denoon, 1997; Massoud, 2000).

Budish and Cantillon (2007, 2012) studied the course allocation mechanism in the Harvard business school. They found out that the mechanism is often manipulated, and this leads to Pareto-inefficiency and a significant loss of welfare.

Kurokawa et al. (2015) applied a *randomized leximin mechanism* for allocating public-school classrooms to charter-schools in California. Gal et al. (2017) test their algorithm for fair division of rooms and rent on data collected from users of the spliddit.org website (Goldman and Procaccia, 2015). Oluwasuji et al. (2018) test their heuristic algorithms for *fair load-shedding* on electricity-usage data, which they collected from a USA-based database and adapted to African consumption patterns.

Many authors test their algorithms on the PrefLib library (Mattei and Walsh, 2013) — a collection of real-world preference relations on discrete items.

All these works consider the allocation of discrete items (possibly with money). We contribute to this literature by studying the allocation of land, which is a *continuous good* that requires quite different division algorithms.

In a recent work, Aleskerov and Shvydun (2019) study the fair division of regions in the Arctic sea, which are not contained in the economic waters of any country, among neighboring countries. They divide the sea region into squares of area 50 km^2

each. For each square, they estimate the subjective value of each country based on the amount of resources of different types in the square, and its distance from the country border. They present a heuristic algorithm for computing an *equitable* allocation — an allocation in which all countries have the same level of subjective satisfaction. In contrast to our work, they treat the squares as discrete objects, and do not guarantee each country a connected piece.

2.3 Laboratory Experiments

In many experiments, agents are asked to divide items in the lab using several pre-specified procedures. The experimenters then compare different division procedures according to various criteria, such as: perceived fairness, economic efficiency, etc. (Schneider and Krämer, 2004; Dupuis-Roy and Gosselin, 2011). Other experiments test what desiderata are more important to the subjects when there is a conflict, e.g. between fairness and efficiency (Engelmann and Strobel, 2004; Fehr et al., 2006; Herreiner and Puppe, 2010). Some experiments check whether participants choose to play strategically, and how their manipulations affect the fairness of the final outcome (Parco and Rapoport, 2004; Daniel and Parco, 2005; Hortala-Vallve and Llorente-Saguer, 2010; Kyropoulou et al., 2019).

In lab experiments, due to practical reasons, the division instances are small (e.g. Kyropoulou et al. (2019) report a cake-cutting experiment with at most 4 participants per instance). In contrast, in our simulation experiments we could compare division procedures on much larger instances.

Chapter 3

Preliminaries

A land-estate C ("cake") has to be divided among n agents. Each agent i has a value-density function v_i, mapping each point of C to its monetary value for i. We define for each agent i its value-measure V_i, which maps each *subset* of C to its monetary value for i. The value-measure is the integral of the value-density, i.e., for each subset $X \subseteq C$, $V_i(X) = \int_{x \in X} v_i(x)dx$. Hence, each value-measure is an additive set function — the value of a union of two disjoint plots is the sum of the values of the plots. Additivity is crucial to the operation of most cake-cutting algorithms.

We assume that the agents' valuations are *piecewise-constant*: C is represented as a rectangular matrix, where each cell represents a rectangle for which the value-density of each agent is constant. Thus V_i can be represented by a matrix of the cell values, where V_i of a union of cells is the sum of the cells' values.

C should be partitioned into n disjoint contiguous pieces, $X_1, \ldots, X_n$, one piece per agent. C is assumed to be a rectangle, and each piece should be a rectangle too. Cut locations are not confined to matrix cells: a cut can break a cell and divide its value V_i between the two pieces proportionately to the cut location.

3.1 Evaluation Metrics

There are infinitely many land partitions, and there are various metrics by which a partition can be evaluated. Our metrics use the valuation of each agent relative to the agent's valuation of C.

The **Utilitarian Value** of a partition X is the sum of the agents' relative values:

$$\text{UV}(X) = \sum_{i=1}^{n} V_i(X_i)/V_i(C)$$

The **Egalitarian Value** of X is the smallest relative value of an agent (multiplied by n to make it comparable to UV):

$$\text{EV}(X) = n \min_{1 \le i \le n} V_i(X_i)/V_i(C)$$

A partition X is called *proportional* if $EV(X) \geq 1$.

For every two agents i and j, we calculate the *Envy* of agent i in agent j by comparing the value agent i got from his/her own land-plot to the value he/she could get from having agent j 's plot:

$$\textsc{Envy}(i,j) = \begin{cases} V_i(X_j)/V_i(X_i), & \text{if } V_i(X_j) > V_i(X_i) \\ 1 \quad , & \text{Otherwise} \end{cases}$$

The **Largest Envy** of a partition X is defined as:

$$\text{LE}(X) = \max_{1 \le i \le n} \max_{1 \le j \le n} \textsc{Envy}(i,j)$$

The **Nash Social welfare** of a partition X is defined as (multiplied by n to make it comparable to UV and EV):

$$\text{NS}(X) = n \left(\prod_{i=1}^{n} V_i(X_i)/V_i(C) \right)^{\frac{1}{n}}$$

As a baseline for comparison, note that if the land is sold and the money is divided among the agents, the relative value of each agent is exactly $1/n$, so $UV = EV = LE = NS = 1$ (we defined the metrics such that their baseline is 1).

When dividing land, the pieces should not only be valuable, but also have a usable geometric shape. A land-plot that is too thin cannot be efficiently used for building or

agriculture. Hence, besides the value-based metrics we introduce the *face-ratio* metrics. For each agent i we define:

$$\textsc{FaceRatio}(X_i) = \frac{\textsc{ShortDimension}(X_i)}{\textsc{LongDimension}(X_i)}$$

The maximum possible face-ratio is 1 (a square). Given two land-plots with the same value, we claim that a plot in which FaceRatio is too low, is harder to use (e.g., a 1-by-900 rectangle is harder to use than a 30-by-30 square). Therefore we prefer a large face-ratio.[1] The **Average Face Ratio** and **Smallest Face Ratio** of a partition X are defined by:

$$\text{AFR} = \sum_{i=1}^{n} \textsc{FaceRatio}(X_i)/n \; ; \; \text{SFR} = \min_{1 \le i \le n} \textsc{FaceRatio}(X_i)$$

[1] The face-ratio is not incorporated into the agents' value functions, since this would make the value functions non-additive, while most cake-cutting algorithms require additive valuations.

Chapter 4

Maps

We constructed four land-value maps.

The first map was of New Zealand. We used the Forest Profit Expectations Dataset (U10073) published by Motu — a New Zealand institute for Economic and Public Policy Research (http://motu.nz). It contains 3000-by-2000 matrix where each element represents a 500-by-500 meters land-block. The value describes the Net Present Value (NPV) of planting a forest in that land-block. The values were calculated by a certain model taking into account the yield of various kinds of trees, the export value of each kind of tree, the transportation costs to the nearest port, etc. The total matrix corresponds to a 1000-by-1500 kilometers rectangle which entirely contains New Zealand. We assume free disposal so blocks in the sea, as well as land blocks with negative NPV, were set to 0. In our experiment we reduced the dataset resolution due to cache memory limitations on our computational machines, so we summed each four adjacent matrix elements to create a single element representing a 1000-by-1000 meters land-block. All in all, we got a 1500-by-1000 matrix of non-negative values representing the estimated profit for an entrepreneur planting a forest in each spot.

The second map was of Israel. We found a database constructed by Madlan —a commercial Israeli website for classified real-estate ads (http://madlan.co.il). It contains a list of all major neighborhoods in Israel and their estimated PPM (price per square meter) calculated from a database of past apartment sales. We extracted each neighborhood latitude and longitude using a map of Israel. We created a 1150-by-1000 matrix representing the land of Israel —each element in the matrix corresponds to a 360-by-160 meters land-block and was initiated with 0. After calculating the approximate element representing each neighborhood location by its latitude and longitude we added

to this element and the 10-by-10 element block surrounding it (to represent a neighborhood area) the PPM value corresponding to the neighborhood resulting in a rough estimate of land value.

Arguably, it is not very realistic that an entire country will be divided. For that reason, the third map we constructed was of Tel-Aviv city. We used the same list of estimated neighborhoods' PPM and Tel-Aviv's city plans to construct a map where each neighborhood area received the appropriate PPM value resulting in a higher resolution real estate map of a relatively small area (52 squared kilometers).

For all three land-maps described, the values do not represent the actual market-price of each land-block. In New Zealand, many land-blocks already have planted forests, buildings or other improvements. In Israel and Tel-Aviv, our calculations did not take into account the neighborhood surrounding aspect. However, the land values in our map still show a *dependence on geographic location* and maintain interesting properties. For instance, the maps naturally contain regions of concentrated value where value gradually decreases in proportion to the distance from the high-value centers. For comparison, we generated a 1500-by-1000 land by choosing a value for each matrix element uniformly at random between 0 and 1,000,000.

Fig. 4.1 shows heat-maps illustrating the four maps. It shows that, while the random map is uniform, the rest are far from uniform: they have small regions with a high value and large regions with a small value.

Land-value maps represent an "objective" evaluation of the land. But, people's preferences have a subjective component too. For example, in the New Zealand map, different people may have different assumptions regarding the possible yield from each land-spot, different means of transporting woods to the port, etc. In general, different people may have different evaluation criteria. We capture this subjective component by a random noise that is added to the land-value map. To control the relative weight of the subjective versus the objective component, we introduce the noise-ratio parameter, denoted by r. From each land-value map we created two different datasets, each dataset containing n variants, one variant per agent. Each one of the two datasets was created using one of two noise patterns:

Uniform: For each variant, we selected the value for each land-block uniformly at random between $(1-r)V$ and $(1+r)V$, where V is taken from the land-value map. This noise pattern represents agents with different subjective valuations but no clear specific area of interest.

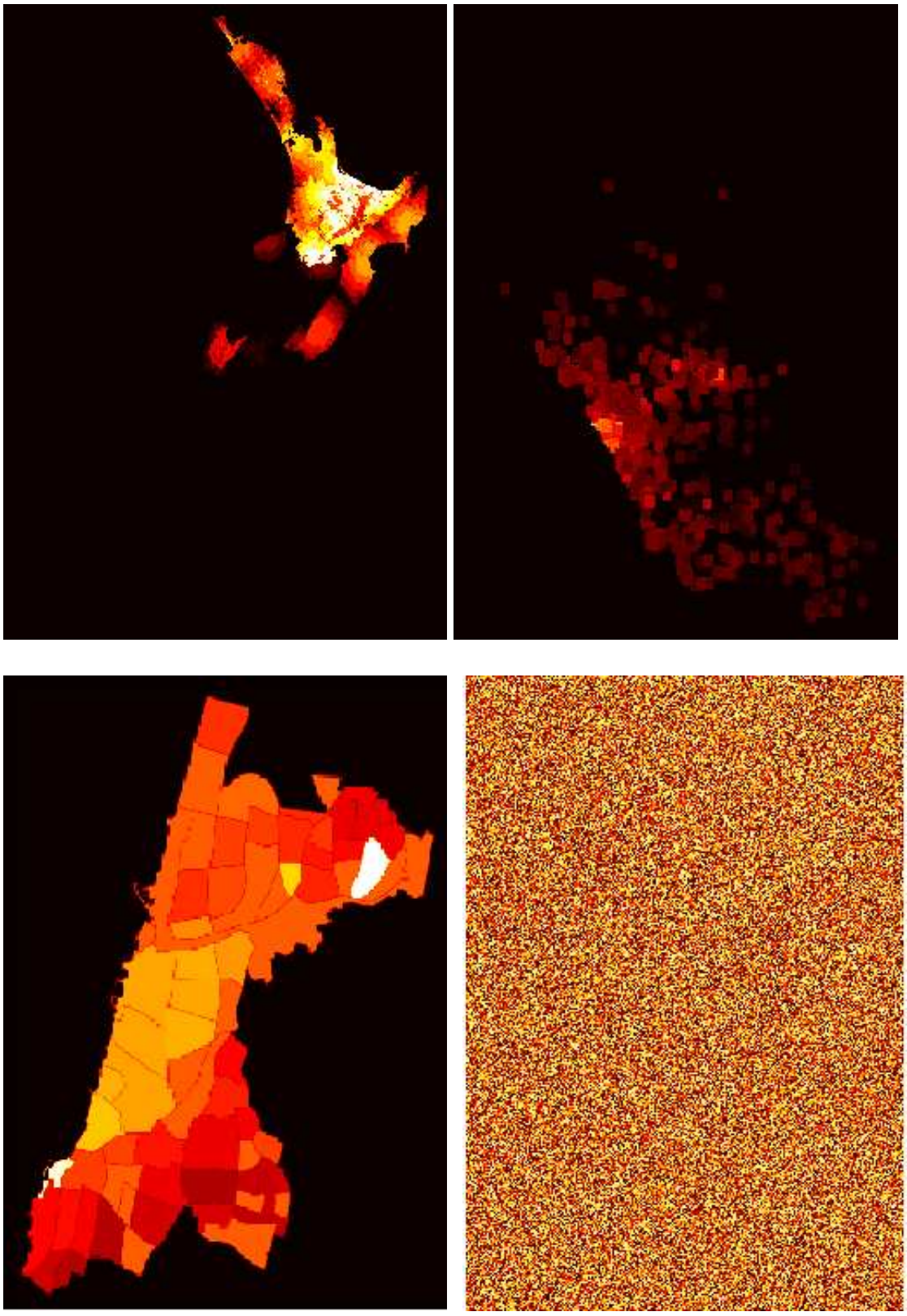


FIGURE 4.1: Heat maps based on the four land-value maps (top left- New Zealand; top right- Israel; bottom left- Tel-Aviv; bottom right- Random Noise)

Hotspot: For each variant, a map land-block coordinate (x_c, y_c) was selected uniformly at random. The goal was to represent a subjective preference for an agent specific area of interest. The point (x_c, y_c) represents a spot that the agent finds particularly valuable; the agent's valuation of other points is larger the closer they are to this "hot spot". The noise radius around (x_c, y_c) grows with r. Formally, we set the value for each land-block using a function for a right circular cone were the x and y axes are the width and height of the map and the z-axis is the agent's interest (added noise):

$$z_{cone}(x, y) = h + \sqrt{cot(\theta)} * \sqrt{tan^2(\theta) * ((x_c - x)^2 + (y_c - y)^2)}$$

Cone height h is chosen to be the shortest dimension of the map and $\theta := \arctan(1/4)$. We then set each land-block (x_j, y_j) value to $(1 + HSN(x_j, y_j, r))V$, where V is taken from the land-value map and HSN is the noise in range $[0, 1]$ calculated by using $z_{cone}(x_j, y_j)$ normalized by the top of the cone $z_{cone}(x_c, y_c)$ and truncated by r:

$$HSN(x_j, y_j, r) = \begin{cases} z_{cone}(x_j, y_j)/z_{cone}(x_c, y_c), & \text{if } HSN > 1 - r \\ 0 \quad , & \text{Otherwise} \end{cases}$$

Fig. 4.2 show heat-maps illustrating the hot spot noise pattern for different values of noise parameter r. The value of HSN is between 0 and 1, so for each land-block with original value V, its value after adding HSN is between V and $2V$.

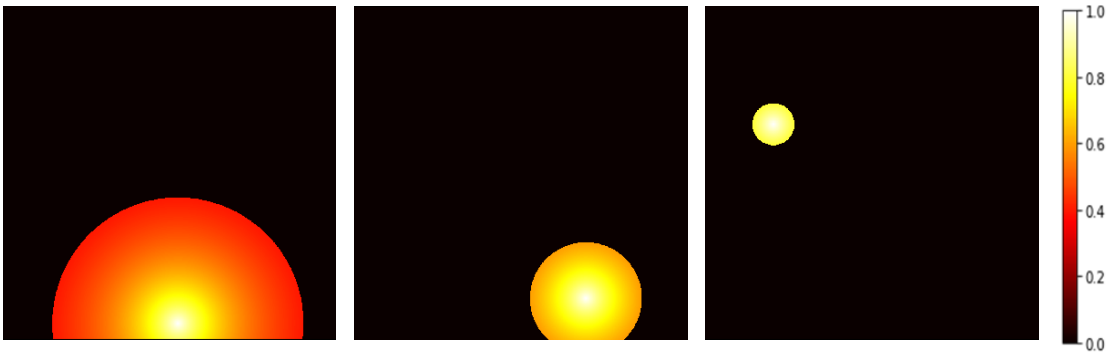


FIGURE 4.2: Heat maps illustrating the hot spot noise pattern (left to right - r=60%, r=40%, r=20%)

All in all, for each of the four base maps, we can evaluate the algorithm performance over either a dataset created using hotspot noise pattern or a dataset created using uniform noise pattern. Each dataset is made of n matrices of non-negative values, each representing a different piecewise-constant value measure.

Chapter 5

Algorithms and Heuristics

5.1 Basic Algorithms

There are many algorithms for finding a proportional cake-partition. In this experiment we focus on the *Even-Paz (EP)* algorithm (Even and Paz, 1984), since we believe it is the algorithm most likely to be used in a cake-cutting problem with a large number of agents. This is due to its simplicity, the low number of queries per agent ($\log_2 n$) and its optimal computational complexity: it runs in time $O(n \log n)$, which is provably the best possible (Woeginger and Sgall, 2007).

We first present the algorithm for the case that n is even. At the first iteration, each agent i is asked to make a *query-mark* x_i — a mark that divides C into two pieces of equal value in his/her eyes. The algorithm orders the marks and renumbers them such that $x_1 \leq \cdots \leq x_n$, then calculates the median $x_M := (x_{n/2} + x_{n/2+1})/2$. The algorithm cuts C at x_M, producing two sub-cakes, and recursively divides each sub-cake among the $n/2$ agents whose query-mark is in that part. When n is odd, the procedure is similar, except that the query-mark of each agent i should partition the cake into two parts with ratio $\lfloor \frac{n}{2} \rfloor : \lceil \frac{n}{2} \rceil$. The algorithm recursively divides one sub-cake among $\lfloor n/2 \rfloor$ agents and the other sub-cake among the other $\lceil n/2 \rceil$ agents.

When $n = 1$, the one agent receives all C.

Whether n is even or odd, each recursive call involves some k agents, and divides among them a subset of the land which they value at least k/n of the total land-value (where $k = n/2$ or $k = \lfloor n/2 \rfloor$ or $k - \lceil n/2 \rceil$). Therefore, when the algorithm concludes, each agent receives land-plot with a personal value of at least $1/n$ of the total.

We compare the results of Even-Paz to another classic proportional cake-cutting algorithm — *Last-Diminisher (LD)* (Steinhaus, 1948).[1] Similar to Even-Paz, at the first iteration, each agent i is asked to make a *query-mark* x_i, only that the mark is made to divide C into two pieces with value ratio $1 : n-1$ in his/her eyes. The algorithm orders the marks and renumbers them such that $x_1 \leq \cdots \leq x_n$, then cuts C at x_1, producing a piece for the agent that proposed query-mark x_1 and a sub-cake which is recursively divided among all other $n-1$ agents.

5.2 Adaptations to a 2-Dimensional Setting

Both Even-Paz and Last-Diminisher are well-defined for a one-dimensional cake. However, when C is two dimensional, in each iteration, the agents can make their query-marks in many different directions; the only requirement is that the query-marks of all agents at a given iteration do not intersect (so that they can be ordered). Even with the natural restriction that the cuts should be parallel to the axes (i.e., either horizontal or vertical), there are still 2^{n-1} options — two options per cut (for example, with $n = 4$ agents, there are two options for the first cut. This cut yields two sub-cakes, and in each of these sub-cakes there are two options. All in all there are 2^3 options). How can a practitioner decide which of these 2^{n-1} sequences to use?

One option is to decide on a certain metric to optimize, such as the utilitarian value. Given a specific instance, one can try all 2^{n-1} sequences and choose the one that yields the highest utilitarian value. We did this exhaustive search for $n = 4$ and $n = 8$ agents. However, for larger values of n this becomes impractical.[2] To avoid such exhaustive search we propose the use of a dynamic programming approach in order to find such optimal[3] cut sequence. For simplicity we present the *FindOptimalCutSequence (FOCS)* algorithm for an n which is a power of 2 (Algorithm 1). The proof that FOCS is optimal uses the following lemma.

Lemma 5.1. *Denote by X_C the division of cake C to n agents such that sub-cake C_1 is allocated to agents $a_1, ..., a_{n/2}$, and sub-cake C_2 to agents $a_{n/2+1}, ..., a_n$. Denote the*

[1] In Section 7 we discuss more algorithms we plan to experiment with in future work.

[2] Note that even this exhaustive search does not yield the maximum possible utilitarian value overall due to the parallel cuts assumption.

[3] It has been proven that even for 1-dimensional cake, neither Even-Paz nor Last-Diminisher yield the maximum utilitarian value; finding an allocation that maximizes the utilitarian value is NP-hard. The same is true for egalitarian value (Aumann et al., 2013). However, since the focus of the present work is the adaptation of one-dimensional algorithms to a two-dimensional cake, we consider the maximum utilitarian value attained by one of the 2^{n-1} possible cut-direction sequences as "optimal".

ALGORITHM 1: Find Optimal Cut Sequence

Input: A cake C, a set of $n = 2^k$ agents, and a preferred metric function m.
Output: The allocation of cake C to the n agents.
if n == *1* **then**
    return C as the agent's entire allocation;
else
    Each agent i makes a vertical half-mark x_i. Order marks from left to right and renumber the agents so $x_1 < \ldots < x_n$. Let $x_M := (x_{n/2} + x_{n/2+1})/2$. Let C_{left}, C_{right} be the halves of C to the left and right of x_M;
    Set $VerticalAllocation =$ $FOCS(C_{left},\ Agents\ 1 \ldots n/2,\ m) \cup FOCS(C_{right},\ Agents\ n/2+1 \ldots n,\ m)$;
    Each agent i makes a horizontal half-mark y_i. Order marks from top to bottom and renumber the agents so $y_1 < \ldots < y_n$. Let $y_M := (y_{n/2} + y_{n/2+1})/2$. Let C_{top}, C_{bottom} be the halves of C to the top and bottom of y_M;
    Set $HorizontalAllocation =$ $FOCS(C_{top},\ Agents\ 1 \ldots n/2,\ m) \cup FOCS(C_{bottom},\ Agents\ n/2+1 \ldots n,\ m)$;
    if $m(HorizontalAllocation) > m(VerticalAllocation)$ **then**
        return $HorizontalAllocation$;
    else
        return $VerticalAllocation$;
    end
end

rest of the division of C_1 and C_2 by X_{C_1} and X_{C_2} respectively. For every metric $m \in \{UV, EV, NS, AFR, SFR\}$ there exists a monotonic function f_m such that $m(X_C) = f_m(m(X_{C_1}), m(X_{C_2}))$.[4]

Proof. We show for every metric $m \in \{UV, EV, NS, AFR, SFR\}$ that such function exists:

1. $\text{UV}(X_C) = \sum_{i=1}^{n} \frac{V_i(X_i)}{V_i(C)} = \sum_{i=1}^{n/2} \frac{V_i(X_i)}{V_i(C)} + \sum_{i=n/2+1}^{n} \frac{V_i(X_i)}{V_i(C)} = \text{UV}(X_{C_1}) + \text{UV}(X_{C_2})$.
2. $\text{EV}(X_C) = n \min_{1 \le i \le n} \frac{V_i(X_i)}{V_i(C)} = n \min\left(\frac{1}{n/2}\text{EV}(X_{C_1}), \frac{1}{n/2}\text{EV}(X_{C_2})\right) =$ $2\min(\text{EV}(X_{C_1}), \text{EV}(X_{C_2}))$.
3. $\text{NS}(X_C) = n\left(\prod_{i=1}^{n} \frac{V_i(X_i)}{V_i(C)}\right)^{\frac{1}{n}} = 2\left(\frac{n}{2}\left(\prod_{i=1}^{n/2} \frac{V_i(X_i)}{V_i(C)}\right)^{\frac{2}{n}} \cdot \frac{n}{2}\left(\prod_{i=n/2+1}^{n} \frac{V_i(X_i)}{V_i(C)}\right)^{\frac{2}{n}}\right)^{\frac{1}{2}} =$ $2\left(\text{NS}(X_{C_1}) \cdot \text{NS}(X_{C_2})\right)^{\frac{1}{2}}$.
4. $\text{AFR}(X_C) = \left(\sum_{i=1}^{n} \textsc{FaceRatio}(X_i)\right)/n = \left(\text{AFR}(X_{C_1}) + \text{AFR}(X_{C_2})\right)/2$.

[4]Since LE computation depends on the relations between all pairs of agents, no such function exists for this metric.

5. $\text{SFR}(X_C) = \min_{1 \le i \le n} \textsc{FaceRatio}(X_i) = \min(\text{SFR}(X_{C_1}), \text{SFR}(X_{C_2}))$.

□

Theorem 5.2. *Given any cake C, a set of $n = 2^k$ agents (for some $k \in \mathbb{Z}_+$) and a metric function $m \in \{UV, EV, NS, AFR, SFR\}$, the cake division made by the optimal cut (Horizontal or Vertical) sequence for metric m amongst the 2^{n-1} options, is given by $FOCS(C, n, m)$.*

Proof. Let X_{mnC} be the division to n agents of cake C made by the optimal cut sequence for metric $m \in \{UV, EV, NS, AFR, SFR\}$. We will prove by induction that:

$$FOCS(C, n, m) = X_{mnC} \tag{5.1}$$

Base case: when $n = 2$, the algorithm tries all two possible cut sequences and chooses the best one, so (1) is true for $n = 2$.

Induction step: Let $k \in \mathbb{Z}_+$ be given and suppose (1) is true for $n = 2^k$. Then, for $n = 2^{k+1}$, consider two possible cut sequences:

1. $X_{C_{hor}}$: A sequence starting with a horizontal cut dividing C into C_{top} and C_{bottom}.
2. $X_{C_{ver}}$: A sequence starting with a vertical cut dividing C into C_{left} and C_{right}.

By the induction hypothesis, the optimal sequence for dividing each sub-cake $C' \in \{C_{top}, C_{bottom}, C_{left}, C_{right}\}$ among $n/2$ agents can be found using $FOCS(C', 2^k, m)$ – we denote those respective allocations by $X_{C_{top}}, X_{C_{bottom}}, X_{C_{left}}, X_{C_{right}}$.

Finally, from lemma 5.1 we can use the function f_m to measure $X_{C_{hor}}$ and $X_{C_{ver}}$ according to metric m and choose the best one:

$$m(X_{C_{hor}}) = f_m(X_{C_{top}}, X_{C_{bottom}}) \quad ; \quad m(X_{C_{ver}}) = f_m(X_{C_{left}}, X_{C_{right}})$$

Thus, (1) holds for $n = 2^{k+1}$ too. □

In each round, FOCS asks each agent 2 queries, and sends each agent to 2 recursive calls (vertical and horizontal). Therefore, the number of queries per agent is given by the recurrence $Q(n) = 2 + 2Q(n/2)$; $Q(1) = 0$, whose solution is $Q(n) = 2(n-1)$,

and the run-time is in $\Theta(n^2)$. This is much better than exhaustive search, but having to answer $2(n-1)$ queries may be quite bothersome for the agents. To allow for trade-off between query complexity and performance in the defined metrics, we suggest various heuristics for selecting the cut-direction in each round. With these heuristics, each agent is sent to only a single recursive call, as in Even-Paz. Therefore, the number of queries per agent is given by the recurrence $Q(n) = 2 + Q(n/2); Q(1) = 0$, whose solution is $2\log_2 n$, and the run-time is $\Theta(n \log n)$.

While the difference in run-time is not very significant in our setting, the difference in the number of queries per agent may be significant.

5.3 Cut-direction heuristics

We tested two sets of heuristics: predefined heuristics and greedy heuristics. The predefined heuristics are:

Hor: all $n-1$ cuts are horizontal (east to west).

Ver: all $n-1$ cuts are vertical (north to south).

HorVer: The first cut is horizontal; in each recursion level the direction switches.

VerHor: As above but the first cut is vertical. HorVer and VerHor heuristics were chosen in hope that they would have a better face-ratio than Hor and Ver.

For the *greedy heuristics*, each agent i makes two query-marks in each iteration: a vertical query-mark x_i and a horizontal query-mark y_i. We define the *proposed vertical cut* x_P and the *proposed horizontal cut* y_P according to the cut location selection of the examined algorithm. When using Even-Paz, $x_P := x_M$ and $y_P := y_M$. When using Last-Diminisher $x_P := x_1$ and $y_P := y_1$. We then select the cut by one of the following heuristics.

SmallestCut: Cut vertically at x_P if $x_P < y_P$; else cut horizontally at y_P.

SmallestPiece: Make a cut resulting in a smaller piece. Let x_A be the area to the left of x_P and y_A the area above y_P. Cut vertically at x_P if $x_A < y_A$; else cut at y_P.

HighestScatter: Define the vertical scatter as $x_S := x_n - x_1$ and the horizontal scatter as $y_S := y_n - y_1$. Cut vertically at x_P if $x_S > y_S$; else cut horizontally at y_P.

SquarePiece: Choose a cut direction resulting in a land-plot with a larger face ratio. Let x_F be the average of the face-ratios of the piece to the left and the piece to the right of x_P. Let y_F be the average of the face-ratios of the piece above and the piece below y_P. Cut vertically at x_P if $x_F > y_F$; else cut horizontally at y_P.

Two additional greedy heuristics we examined are based on the notion that local benefit of a specific iteration is influenced by the margin between the two query-cuts from both sides of the proposed cut. When using Even-Paz the vertical margin is the area between $x_{n/2}$ and $x_{n/2+1}$, and the horizontal margin is the area between $y_{n/2}$ and $y_{n/2+1}$. For Last-Diminisher the vertical margin is the area between x_1 and x_2, and the horizontal margin is the area between y_1 and y_2.

LargestMargin: Choose a cut direction resulting in a larger margin. Cut vertically at x_P if the length of the vertical margin is greater than the length of the horizontal margin; else cut horizontally at y_P.

MostValuableMargin: Choose a cut direction resulting in a more valuable margin. Cut vertically at x_P if the average value of the vertical margin (averaged over all n agents) is larger than the average value of the horizontal margin; else cut horizontally at y_P.

Post-processing heuristic. For each examined algorithm, after the allocation is found, we run a post-processing step where we allow for mutually-beneficial exchanges of allocated pieces to take place. The exchange is handled using the *Top Trading Cycle (TTC)* algorithm (Shapley and Scarf, 1974). After a division X is created by the tested division algorithm, we preform the iterative TTC algorithm as follows:

1. Initiate graph G with vertex a_i for each agent i and vertex p_j for each piece j in the X and with edges (p_j, a_i) for each allocation in X.
2. Ask each agent i to denote by Top_i his/her preferred piece.
3. Add edge (a_i, Top_i) to G for each agent i.
4. We find a cycle in G and implement the trade indicated by this cycle and remove all the agents and pieces involved from G.
5. If there are agent vertices remaining in G, go back to step (2).

Assessor division. We compare our algorithms' results to the performance of an assessor. To simulate an assessor division, the land is divided into n parts with the same base-land values, ignoring the random noise. The division is done using the following simple method. For some integers k_1, k_2 with $k_1 \cdot k_2 = n$, the assessor first partitions the land using vertical cuts into k_1 parallel strips of value $1/k_1$, and then partitions each such strip using horizontal cuts into k_2 plots of value $(1/k_1)/k_2 = 1/n$. We take $k_1 = 2^{\lceil (\log_2 n)/2 \rceil}$ and $k_2 = 2^{\lfloor (\log_2 n)/2 \rfloor}$, so that the pieces have a balanced aspect ratio. In particular, if n is a square number then $k_1 = k_2 = \sqrt{n}$.

Chapter 6

Experiments and Results

We ran experiments for each of the three maps, for n in $\{4, 8, 16, 32, 64, 128\}$. The value of n is chosen as a power of 2 for simplicity in assessor method implementation, we have also experimented with odd n values and found no significant difference in results. For each map and n, we ran 50 experiments with different randomly-generated noise. In most experiments the noise-ratio was $r = 0.6$. In each experiment, we ran each heuristic twice: once using Even-Paz algorithm and once using Last-Diminisher. All in all we tested 1,800 different experiment settings (3 dataset x 2 noise-patterns x 50 repetitions x 6 agent-numbers) and calculated 39,600 partitions (1,800 experiments x (2 algorithms x 10 heuristics + FOCS + Assessor)). For each partition, we calculated the evaluation metrics UV, EV, NS, LE, AFR, SFR and Run Duration (RD). For each setting and each metric, we calculated its average over the 50 runs and a 95% confidence interval. All reported p-values were calculated using a two-tailed t-test.

The results for the New Zealand, Israel and Tel-Aviv maps were very similar, but the results for the random map were substantially different. Below, we present graphs illustrating the results for New Zealand and selected results of Israel and random maps as well. The results for Tel-Aviv map as well as the full experiment result tables and code base for reproduction purposes are available at https://github.com/shtechman/fair-cakecutting-with-real-land-value-data.

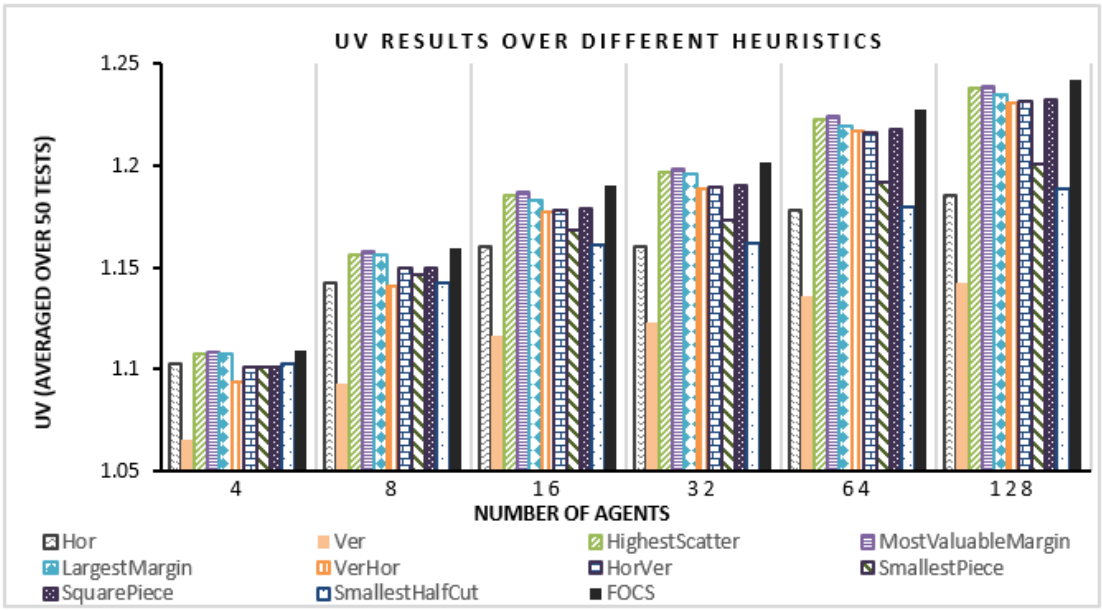


FIGURE 6.1: Utilitarian-value of Even-Paz with various heuristics for New Zealand dataset created by hotspot noise pattern (the graph for uniform noise pattern is similar).

TABLE 6.1: MostValuableMargin UV comparison to assessor, market-sale and FOCS, n=128

	New Zealand		Israel		Random	
	Uni.	HS	Uni.	HS	Uni.	HS
Assessor comparison	2.8%	23.9%	2.6%	26.6%	0.8%	56.6%
Market comparison	2.9%	24%	2.6%	26.6%	0.8%	57.1%
FOCS comparison	-0.1%	-0.2%	-0.1%	-0.4%	-0.02%	-0.6%

6.1 Social Welfare Metrics

Fig. 6.1 compares the UV of various heuristics. All heuristics score better than the baseline of $UV = 1$. Besides FOCS, which finds the optimal cut-sequence, the best-performing heuristic with both uniform and hotspot noise patterns is MostValuableMargin. Intuitively, the reason is that MostValuableMargin takes the most advantage of the differences between the agents' valuations.

MostValuableMargin is only slightly better than the other greedy heuristics ($p < 0.1$), but significantly better than the predefined heuristics ($p < 0.001$). The UV of Even-Paz + MostValuableMargin is significantly better than the market-sale ($p < 0.001$) and the advantage grows with n.

TABLE 6.2: MostValuableMargin NS comparison to assessor, market-sale and FOCS, n=128

	New Zealand		Israel		Random	
	Uni.	HS	Uni.	HS	Uni.	HS
Assessor comparison	2.9%	24.6%	2.6%	28%	0.8%	59%
Market comparison	2.9%	23%	2.6%	26.2%	0.8%	56%
FOCS comparison	-0.1%	-0.4%	-0.1%	-0.3%	-0.02%	-0.5%

TABLE 6.3: MostValuableMargin EV comparison to assessor, market-sale and FOCS, n=128

	New Zealand		Israel		Random	
	Uni.	HS	Uni.	HS	Uni.	HS
Assessor comparison	4.7%	71.3%	4.5%	66.3%	1.25%	48.9%
Market comparison	0.9%	6.4%	0.9%	6.3%	0.3%	28.7%
FOCS comparison	-0.6%	-1.3%	-0.4%	-3.8%	-0.2%	-4.5%

Table 6.1 contains the advantage of Even-Paz + MostValuableMargin over assessor and market-sale and the disadvantage compared to FOCS for the different maps and noise patterns with $n = 128$. The UV of an assessor division is very close to 1 – similar to market-sale (see Fig. 6.2).

The difference in the results between the hotspot noise and the uniform noise might be explained by the nature of both patterns. For example, if the land is being divided between two agents using hotspot noise, each agent would likely have a different area of interest as his/her hotspot, making it fairly simple to divide the land in such a way that each agent will receive their respective area of interest resulting in a high UV. On the other hand, when the noise is spread out uniformly at random it is harder to choose a similarly beneficial allocation.

The results for random map using hotspot noise are more then twice the results for the other maps, this illustrates the importance of using real land-value data to evaluate division algorithms in order to avoid over-optimistic results.

The graphs of EV and NS for the different heuristics are qualitatively similar to the graphs of UV. The best heuristic is again MostValuableMargin (see Fig. 6.3 and 6.4).

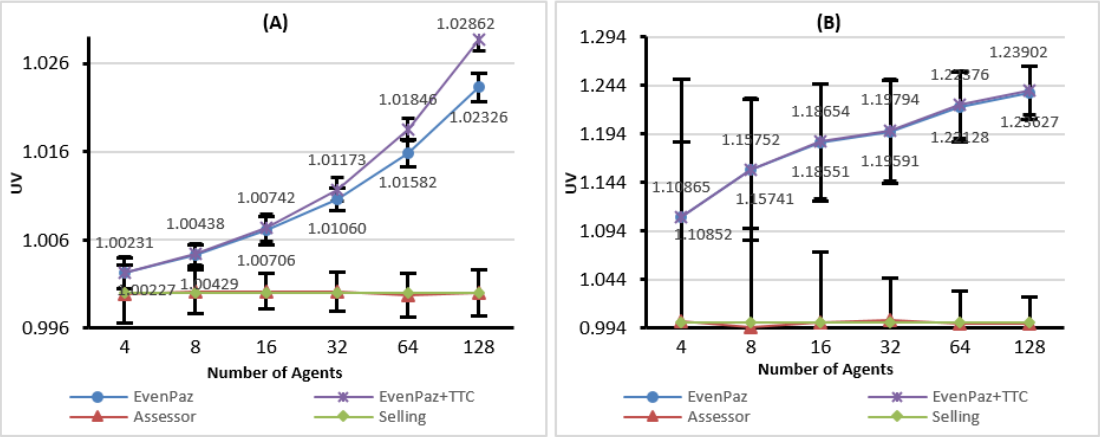


FIGURE 6.2: Utilitarian-value of Even-Paz with MostValuableMargin before and after TTC compared to UV of an assessor division and market sale. The vertical lines denote 95% confidence intervals.
(A) Uniform noise pattern (B) Hotspot noise pattern

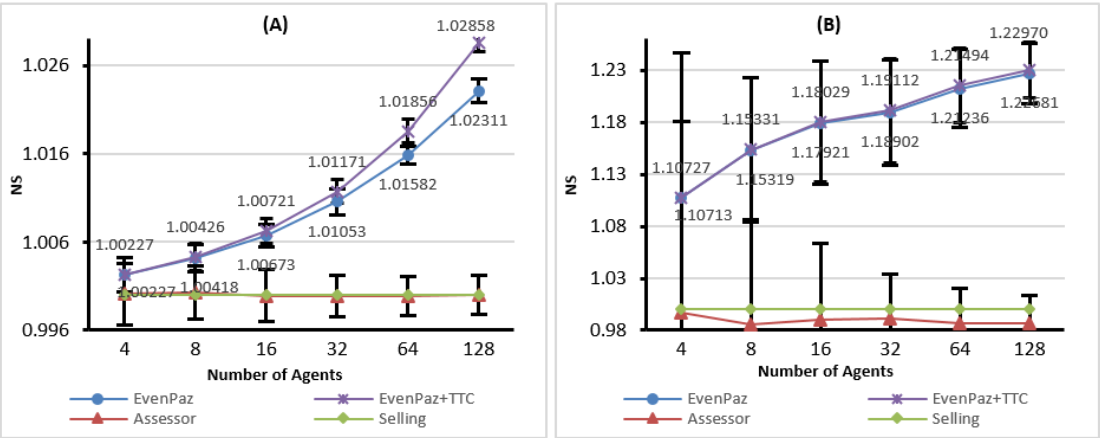


FIGURE 6.3: Nash social welfare of Even-Paz with MostValuableMargin before and after TTC compared to EV of an assessor division and market sale. The vertical lines denote 95% confidence intervals.
(A) Uniform noise pattern (B) Hotspot noise pattern

The NS of Even-Paz is very similar to the UV in value and in comparison to assessor and market sale performance. Besides FOCS, the best-performing heuristic with both noise patterns is MostValuableMargin.

Table 6.2 contains the advantage in NS of Even-Paz + MostValuableMargin over assessor and market-sale and the disadvantage compared to FOCS for the different maps and noise patterns with $n = 128$.

The EV of Even-Paz is always higher than the EV of selling the land and with MostValuableMargin it is significantly better ($p < 0.001$). For uniform noise, the advantage of Even-Paz grows with n but with hotspot noise, the advantage does not grow as n

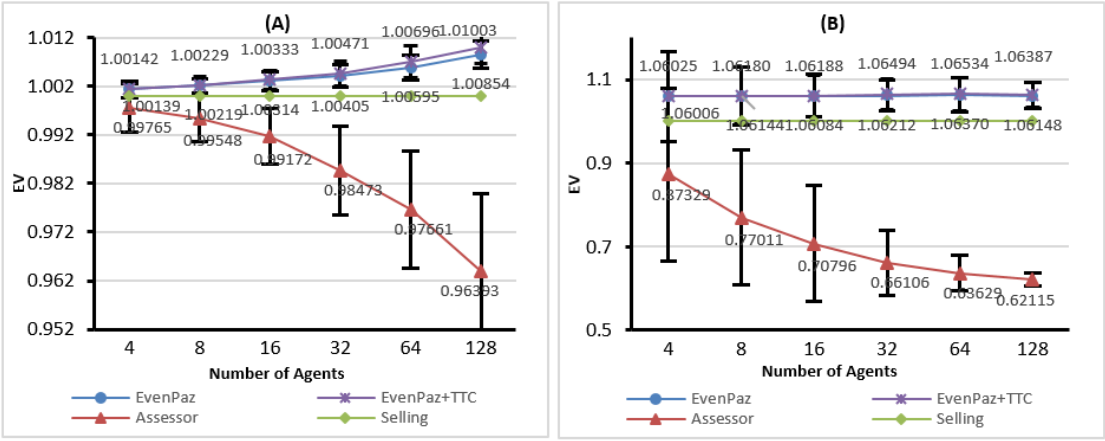


FIGURE 6.4: Egalitarian-value of Even-Paz with MostValuableMargin before and after TTC compared to EV of an assessor division and market sale. The vertical lines denote 95% confidence intervals.
(A) Uniform noise pattern (B) Hotspot noise pattern

grows; this trend can be explained by further examination of the hotspot noise pattern. Recall that we have created each agent's map by selecting a random coordinate and generated a cone-shaped noise pattern around it. As a result of this pattern, coordinates on the map that are far away from the noise center have little or no noise added to it creating areas on their "subjective" maps that are equivalent to the "objective" evaluation of the land, this means that two agents can have high correlation in their preferences for a subset of the land that mainly contains "objective" evaluations for them both, if those two agents were asked to divide that subset of land between them using the Even-Paz algorithm, both will propose a similar cutting location and the land will be divided accordingly resulting in each agent receiving roughly 50% of the land-plot resulting in a small EV.

Table 6.3 contains the advantage in EV of Even-Paz + MostValuableMargin over assessor and market-sale and the disadvantage compared to FOCS for the different maps and noise patterns with $n = 128$.

The EV of an assessor division is always *lower* than of selling the land. The disadvantage grows with n; with $n = 128$ it is about -37.8% for hotspot noise and -3.7% for uniform noise (see Fig. 6.4). This means that, in an assessor division, there is always at least one person who receives less than his/her fair share. Even receiving 3.7% below the fair share might make a person feel frustrated for being treated unfairly. This cannot happen when using the Even-Paz cake-cutting algorithm, since by design it always gives each agent at least a fair share.

Although both predefined heuristics Hor and Ver strictly allow cutting in a constant direction for the entire cut sequence, there is a noticeable difference in results between the two. Hor performs better than Ver across all measures. This difference is explained by the dimensions of the maps. Since the height of all the tested maps is longer than their width, Hor heuristics cuts along the longer dimension and thus performs better than Ver. When the same experiments were conducted with the maps rotated by 90 degrees, Ver performed better than Hor.

Applying the TTC algorithm to the outcome of Even-Paz + MostValuableMargin slightly improved UV and NS with uniform-noise, but had no noticeable effect on the UV and NS with hotspot-noise or on the EV.

6.2 Largest Envy

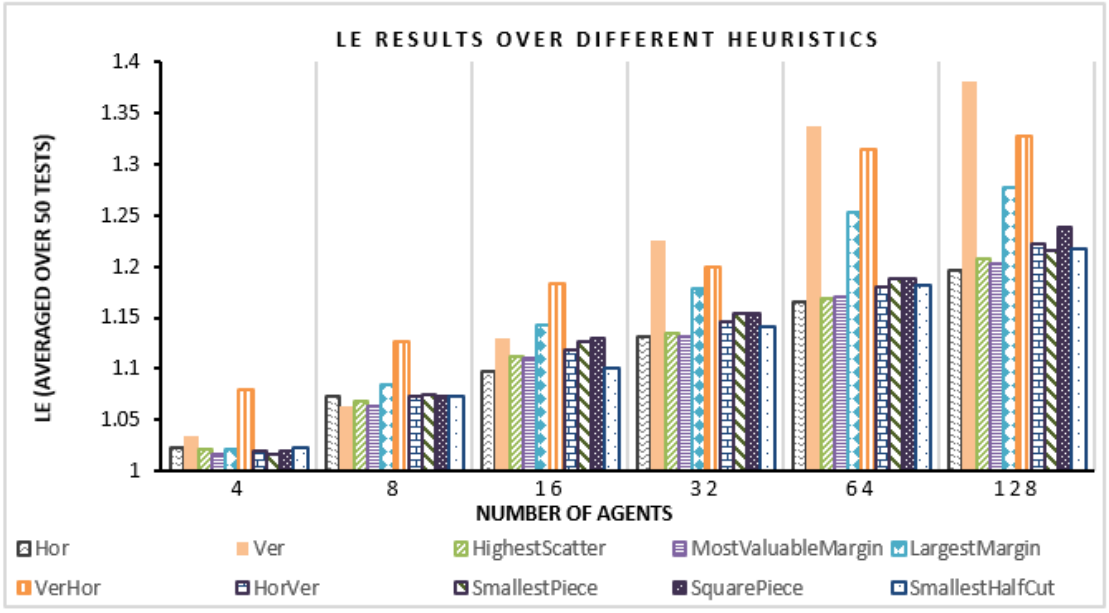


FIGURE 6.5: Largest-envy of Even-Paz with various heuristics. New Zealand map, hotspot noise.

Fig. 6.5 compares the LE of various heuristics.

Note that smaller LE is better, and market-sale always trivially attains the minimum LE which is 1. Besides market-sale, MostValuableMargin attains the smallest LE of the heuristics we tested, but the advantage is not statistically significant ($p = 0.2$).

TABLE 6.4: MostValuableMargin LE comparison to assessor and market-sale, n=128

	New Zealand		Israel		Random	
	Uni.	HS	Uni.	HS	Uni.	HS
Assessor comparison	3.5%	38.4%	3.35%	34.6%	1.0%	29.6%
Market comparison	-4.1%	-20.3%	-4.7%	-27.7%	-1.0%	-30.8%

The LE of both Even-Paz and assessor-division grows as n grows, but the LE of Even-Paz is better (smaller, see Fig. 6.6). For $n = 128$, Even-Paz + MostValuableMargin is significantly smaller than assessor division ($p < 0.001$).

Table 6.4 contains the advantage of Even-Paz + MostValuableMargin over assessor and the disadvantage over market-sale for the different maps and noise patterns with $n = 128$.

Since FOCS does not necessarily find the optimal cut-sequence for LE (see Theorem 5.2), we calculated the optimal cut-sequence by exhaustive search for $n = 4$ and $n = 8$ (for $n \geq 16$ the run-time is too large). For $n = 8$, the LE of the optimal cut-sequence (above the baseline of 1) was 2.5%, as opposed to 6.3% for MostValuableMargin. We also checked the LE of FOCS with the NS metric. Since an allocation that maximizes the Nash social welfare is envy-free (Segal-Halevi and Sziklai, 2019), one could expect that FOCS+NS would yield an allocation with a small envy. However, we found out that this is not the case: the LE of FOCS+NS is larger than that of MostValuableMargin.

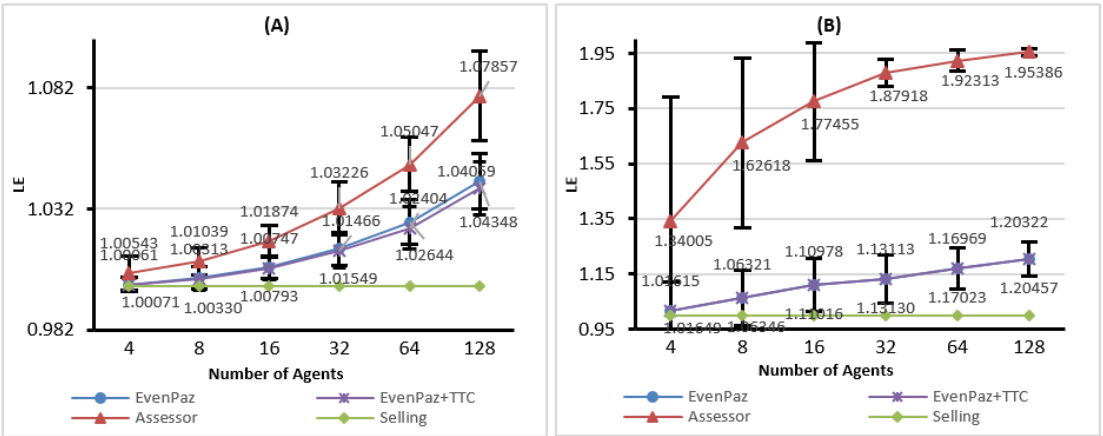


FIGURE 6.6: Largest-envy of Even-Paz with MostValuableMargin compared to LE of an assessor division and market sale. The vertical lines denote 95% confidence intervals. (A) Uniform noise pattern (B) Hotspot noise pattern

6.3 Smallest and Average Face Ratio

Fig. 6.7 compares the SFR and AFR of various heuristics. Besides FOCS, the best heuristic for both SFR and AFR, for both uniform and hotspot noise patterns, is SquarePiece ($p < 0.001$).

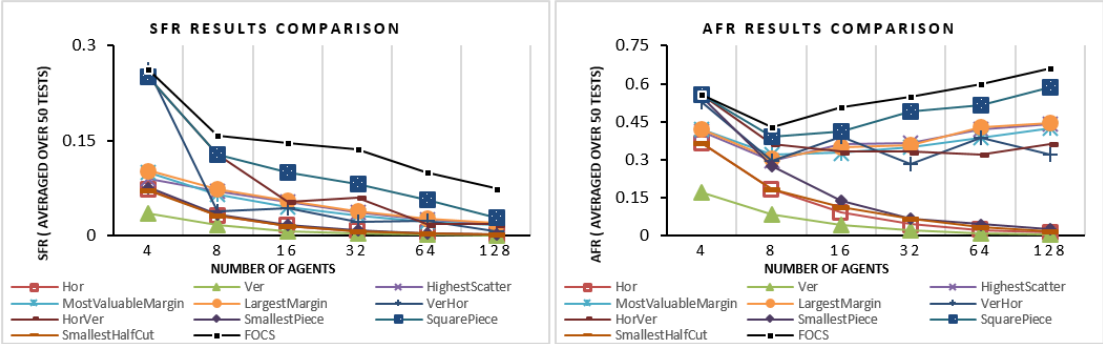


FIGURE 6.7: Smallest-face-ratio (left) and Average-face-ratio (right) for different heuristics for New Zealand dataset created by hotspot noise pattern.

In contrast to the previous metrics, we did not find a significant difference between Even-Paz + SquarePiece and an assessor division.

The AFR of Even-Paz with SquarePiece is always above 1/3, and sometimes above 1/2. However, the SFR (for $n = 128$) is very small — around 0.02. This means that at least one agent might get a very thin plot (with aspect ratio 1:50). The SFR of FOCS is better — around 0.1 — but it is still quite small. Handling this issue is an important problem for future work (see Section 7).

The results in the Israel map are similar. However, in the random map the SFR is always above $1/2$, even for large n. Intuitively, in the random map the value is distributed uniformly, so all cuts are made approximately in the center of the map, and the resulting pieces are square-like. In contrast, in real land-value maps, a large value is concentrated in small regions, so many cuts are made near the edge of the cake, and the resulting pieces are thin. This difference again illustrates the importance of using real land-value data to evaluate division algorithms. Evaluating the algorithms with only a randomly-generated map might lead one to be over-optimistic about the face-ratio and overlook the problem of thin pieces.

6.4 Comparison to Last-Diminisher

We ran the same set of experiments with the Last Diminisher algorithm instead of Even-Paz. Below we highlight the main similarities and differences.

- The UV, NS and LE were similar to Even-Paz. For UV and NS the best-performing heuristic was still MostValuableMargin and for LE it was LargestMargin.
- The EV was worse ($p < 0.001$). Because Last-Diminisher is designed to allocate the first agent exactly $1/n$ of C in his/her eyes, its EV always equals 1.
- The SFR and the AFR were even worse than that of the assessor ($p < 0.001$) and the best-performing heuristic was LargestMargin. Since Last-Diminisher partitions the agents into two unequal groups, the SquarePiece heuristic does not perform well.

All in all, Even-Paz is at least as good, and often better, than Last-Diminisher.

6.5 Complexity Comparison

Division algorithms that rely on a sequence of queries to the agents, require the agent to assess the land in question in order to provide a query-mark and a back and forth interaction between the algorithm and the agent. Hence, the query complexity might be a very significant measure for these algorithms.

The query complexity of Even-Paz depends on the heuristic — the number of queries per agent is $\log_2 n$ for a predefined heuristic and $2\log_2 n$ for a greedy heuristic. For both Last-Diminisher and FOCS, the number of queries per agent is $O(n)$. As n grows, this difference becomes substantial (e.g. for $n = 128$, FOCS requires 128 queries per agent while Even-Paz requires 14 at most).

In addition to query complexity, we also compare computational complexity. Recall that the computational complexity of Even-Paz grows like $O(n \log n)$; this means that the runtime would be feasible even for a larger number of agents.

On the other hand, for both Last-Diminisher and FOCS the computational complexity grows like $O(n^2)$; this becomes noticeable quickly as n grows.

Although this observation is interesting from a computational point of view, land division problems do not need to be solved repeatedly or even particularly quickly, therefore, the computational complexity measure is less significant compared to other measures we have tested.

6.6 Different Noise-Ratios

Recall that the noise-ratio r represents the weight of the agents' subjective preferences relative to the "objective" land-value data.

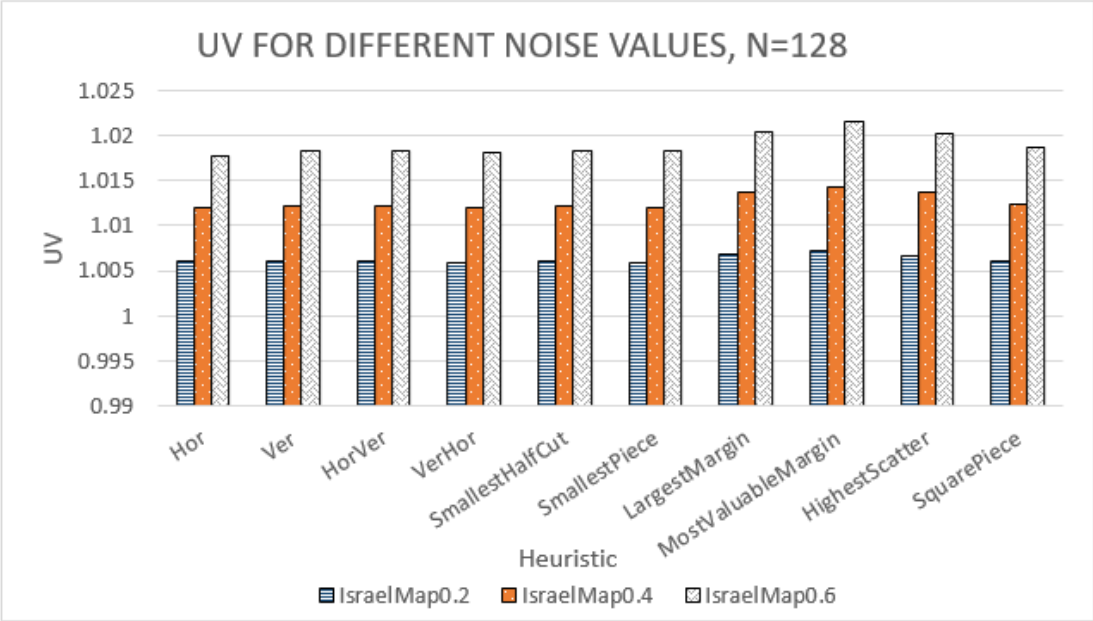


FIGURE 6.8: UV results of various heuristics for uniform noise using different noise-ratios, Israel map and $n = 128$.

To check the effect of this parameter, we ran experiments with lower noise-ratios — 0.4 and 0.2 — representing a lower weight to subjective preferences. We ran these experiments on the Israel map with $n = 128$ using uniform noise-pattern. We performed linear regression of UV using MostValuableMargin as a function of the noise-ratio r, and found the linear relation:

$$\text{UV} = 0.0359r + 1.00001$$

The correlation coefficient $R = 0.99$, coefficient of determination $R^2 = 0.99$ and the standard error is $5.9E - 05$. A similar relation exists for each one of the tested

TABLE 6.5: Strategic Gain with Even-Paz algorithm, New Zealand Map, Hotspot Noise, n=128

Cut-direction heuristic	Average strategic gain	Maximum strategic gain	Strategy risk
Ver	0.9%	7.2%	0.2%
Hor	0.9%	4.5%	0.1%
VerHor	1.9%	7.0%	0.4%
HorVer	1.7%	5.8%	0.2%
MostValuableMargin	1.9%	8.2%	4.3%
LargestMargin	2.2%	15.5%	28.6%
SmallestPiece	1.1%	4.9%	0.1%
HighestScatter	1.8%	6.0%	1.5%
SquarePiece	1.8%	8.3%	2.7%
SmallestCut	0.9%	4.6%	0.1%

heuristics. In other words, when using uniform noise, the advantage over the market-sale ($UV = 1$) grows linearly with r.

The situation with NS and EV is similar: the improvement over market-sale ($NS = 1, EV = 1$) is proportional to r. For other metrics (AFR, SFR, RD) the noise-ratio has no noticeable effect. The reason UV, NS and EV grow with r for uniform noise pattern is that, the noisier the valuations, the less correlated they become, making it easier to find a fair allocation with high social welfare. For example, if two agents prefer two disjoint regions, then it is possible to give each of them 100% of its value, while if two people are interested in the very same region, then each receives only 50%.

We ran the same experiment using the hotspot noise-pattern and found no significant linear relation between r and our metrics. This might be explained by the second variable introduced by the hotspot pattern — the hotspot center point. For two agents' maps, the center point location for each agent has a great effect on the correlation of the two maps, so the correlation does not depend on r alone as the case for the uniform noise pattern. When using hotspot noise pattern with $r = 0.6$ we have received the most statistically significant results and smallest variance from other r values we have tested. This illustrates the importance of calibrating the noise parameters when experimenting with randomly generated data.

6.7 Strategic Gain

It is known that most existing cake-cutting algorithms are not *strategyproof*, which means that agents may gain by strategically reporting false preferences (Brams and Taylor, 1996; Menon and Larson, 2017). In theory, the gain from such manipulation can be very high: there are instances in which a truthful agent receives only $1/n$ of the total value, while a strategic agent receives 100% of the value (Kyropoulou et al., 2019). However, such instances are highly artificial. In order to get a more realistic estimate on these effects, we ran experiments on our land-value based maps to check how much a single agent may gain by being strategic, assuming all other agents are truthful.

Calculating the strategic gain requires us to make some assumptions on how much information the agent has on the preferences of other agents, and how sophisticated the agent is calculating an optimal strategy. On one hand, a worst-case analysis would require us to assume that the strategic agent knows the value functions of all other agents, and uses a globally-optimal strategy. On the other hand, recent evidence from a laboratory experiment (Kyropoulou et al., 2019) shows that humans are far from optimal in manipulating cake-cutting algorithms. Even in simple algorithms like cut-and-choose on a 1-dimensional cake, they often make obvious mistakes such as cutting the cake to the right of their actual half-point when they know that their partner will choose the leftmost piece. As a middle ground, we decided to experiment with strategies that are *locally-optimal*, as described below.

Suppose the strategic agent is Alice, while the other $n - 1$ agents are truthful. We assume Alice is acting strategically in a local manner, meaning at each iteration Alice makes a greedy selection of her query-mark to maximize her value. The strategy of Alice for selecting her query-marks is calculated in two steps, at the first step Alice forces a better cut location in both possible cut directions using the following strategy:

1. Calculate the vertical and horizontal query-marks of all other n agents (this does not require to assume that Alice knows the entire valuation functions of the other agents — only that she knows their half-points).

2. If Alice's mark is among the leftmost (or bottom) $n/2$ agents, then she selects a mark slightly to the left (or below) of the mark number $n/2 + 1$ from the left (or bottom).

3. Otherwise, Alice's mark is among the rightmost (or top) $n/2$ agents; then she selects a mark slightly to the right (or above) of the mark number $n/2$ from the left (or bottom).

At the second step of her strategic selection, if the cutting heuristic is not a predefined one, Alice tries forcing the cut direction to further expand her local gain using the following strategy:

1. Calculate the selected cut direction D_{sel} according to the current cut heuristic and the newly selected query-marks. Additionally, calculate the potential cut locations for both horizontal and vertical cuts.

2. If Alice's gain from cutting in D_{sel} direction is less then her possible gain from cutting in the opposite direction D_{opp}, Alice searches for a different D_{sel} direction query-mark q that will force the current heuristic to select D_{opp} as the cut direction.

3. If such q was found, Alice selects her new query-mark for D_{sel} direction to be q, otherwise, Alice does not change her already selected strategic query-marks.

Using this strategy, Alice remains in the same group, while maximizing the piece given to her own group of $n/2$ agents. In some cases, depending on the cut heuristic and the other agents, Alice also influences the cut direction by intentionally selecting a bad query-mark in the direction she is less interested in.

The **Strategic Gain** of agent i is the increase in utility of agent i while playing strategic, vs. playing truthful. We calculate the increase in utility of agent i while playing strategic by conducting a standard truthful experiment followed by n additional strategic experiments, one for each agent i.

The **Average Strategic Gain** and **Maximum Strategic Gain** over all agents are defined by:

$$\text{ASG} = \sum_{i=1}^{n}(SG_i)/n \;\; ; \;\; \text{MSG} = \max_{1\leq i\leq n}(SG_i)$$

Note that, since our strategic agent acts greedily, she may fail to find the globally optimal manipulation. This means that she may gain less by acting strategically than by acting truthfully, because the greedy choice made at each iteration is designed to optimize the current iteration and does not take into account the effect on future choices

of other agents. We measure the **Strategy Risk**, which is the average percentage of agents who gained *less* by being strategic as opposed to being truthful.

Table 6.5 shows the results for 128 agents, New Zealand map and hotspot noise. The largest MSG in all our experiments was 15.5% (in New Zealand map, hotspot noise, 128 agents, LargestMargin heuristic). However, the ASG in all experiments was less than 2.2%.

For each one of our tested heuristics, there were agents who gained less by acting strategic than by being truthful. The LargestMargin heuristic had a significantly higher strategy risk then all other heuristics (28.6% in New Zealand map, hotspot noise, 128 agents), followed by MostValuableMargin, SquarePiece and HighestScatter. This is especially interesting because those heuristics has a generally higher MSG, suggesting a correlation between risk and reward in a strategic play. We believe that the high strategy risk in some of the tested heuristics can be explained by the magnitude by which the entire run of a tested heuristic is affected by each greedy selection of the strategic agent. Heuristics such as LargestMargin, MostValuableMargin, SquarePiece and HighestScatter are more sensitive to changes in cut locations and margin sizes than heuristics such as the predefined ones or even SmallestPiece that are less influenced by the strategic agent's greedy selection.

Our results suggests that, in general, strategic manipulations are not a big issue for land division algorithms — even when agents have complete knowledge of all other agents' preferences, they would be required to make a big effort to slightly increase their gain (in our experiments about 2.2%) and by doing so taking a risk of gaining less than they could have. In practice agents do not have complete knowledge of others' preferences, so it is even harder to gain from strategic manipulation.

6.8 Price of Fairness

In order to further examine the performance of our approach, we measured the *price of fairness*, which is the relative efficiency loss under a fair allocation compared to the one that maximizes UV, the sum of agent utilities (Aumann and Dombb, 2010; Bertsimas et al., 2011; Caragiannis et al., 2012). The price of fairness was calculated by running experiments using an algorithm we call **Highest Bidder**, which is a greedy algorithm that assigns each land-block to the agent who values it the most when comparing normalized valuations. At each iteration a land-block in C is put up for auction and

all agents are requested to place a bid, the land-block is then allocated to the highest bidder. Agent i's bid for land-block c is given by $V_i(c)/V_i(C)$. Using this algorithm we ensure the highest bidder for a land-block is the agent that values the land-block the most, thus maximizing UV. We define price of fairness POF as the increase in UV when using Highest-Bidder algorithm compared to using Even-Paz:

$$\text{POF} = \frac{UV_{HighestBidder} - UV_{EvenPaz}}{UV_{HighestBidder}}$$

Since POF is the price we pay for using a fair method, a smaller POF is better. Even-Paz + MostValuableMargin heuristic has shown the best performance for UV, hence, its corresponding POF is the smallest of all other heuristics. However, there is a substantial difference between the POF of the two different noise patterns. The POF of Even-Paz + MostValuableMargin for $n = 128$ is 35.34% when using uniform noise and 4.66% when using hotspot noise (the results are similar for the Israel map and the random map). Agents using the hotspot noise have higher correlated maps then the agents using the uniform noise which result in a higher UV. This high correlation is causing the agents to benefit less from an unfair algorithm based on out-bidding others. This difference illustrates the importance of creating agents valuations that represent real-life preferences when assessing division algorithms.

Chapter 7

Conclusions and Future Work

Our results have several practical implications for people dealing with fair division of land.

1. Cake-cutting algorithms may be a viable alternative to the common methods for land division, namely assessor division and market-sale. In particular, Even-Paz attains higher social welfare than both these methods, and lower envy than assessor division. The effect is larger when there are more agents and when there is more variation in the agents' valuations. Even-Paz performs better than the other cake-cutting algorithm we checked (Last Diminisher).

2. When adapting cake-cutting algorithms to two dimensions, FOCS performs well in most metrics, but has to be tailored to a specific performance measure for each different run, takes longer to execute than other tested methods and more importantly, requires many more queries per agent. Of the faster heuristics, MostValuableMargin is superior in terms of social welfare and envy, while SquarePiece is superior in terms of geometric shape. This shows a trilateral trade-off between query complexity, social welfare and geometric shape.

3. When assessing cake-cutting algorithms using artificially generated data, results are highly effected by the methods used to generate the data. Randomly created maps can give overly optimistic or pessimistic results compared to real land-value data. Furthermore, the performance of the algorithms are also highly influenced by the chosen method for adding randomly generated noise in order to represent different agents' valuations. Using a hotspot in our case to represent *desired point of interest*, the cake-cutting algorithms yielded much better results

than when using uniform random noise. In order to correctly assess how cake-cutting algorithms perform, we must model the preferences of our agents in as realistic way as possible.

4. Strategic manipulation may improve the welfare of an agent with complete information, but the improvement is minor. Greedy strategic manipulation can effect the entire run of the algorithm and result in smaller gain for the agent being strategic — the risk for this loss depends on the heuristic.

Our results also indicate that there is a lot of potential for future improvement.

7.1 Other Cake-Cutting Algorithms

Our present experiment focuses on the Even-Paz algorithm since it provides both good fairness guarantees and a reasonable computational and query complexity. However, there are other cake-cutting algorithms, which may fare better in some metrics.

Consider first the welfare-related metrics — UV and EV. It is known that finding a connected allocation maximizing either UV or EV is NP-hard even for a 1-dimensional cake, and even without additional fairness considerations. For a 1-dimensional cake, Aumann et al. (2013) provide a polynomial-time 8-factor approximation algorithm for the maximum UV. It guarantees connected pieces but does not guarantee fairness. They also present an FPTAS for the UV and EV, but its run-time is $\Omega(n \cdot 2^n)$, in contrast to the $O(n \log(n))$ runtime of Even-Paz. In future work, we are interested in checking what is the largest n for which these algorithms are practical, and how they perform in practice in terms of fairness.

Bei et al. (2012) present an algorithm for maximizing the UV subject to proportionality. However, the algorithm works only for utility functions of the form $ax + b$, where $x \in [0, 1]$ is the location on the 1-dimensional cake. For the more general case of piecewise-constant valuations, they prove that the problem is NP-hard to approximate within a factor of $O(\sqrt{n})$.

Consider now the LE metric. In theory, envy can be entirely eliminated by using *envy-free* cake-cutting algorithms (Su, 1999). However, no finite protocol can find a connected envy-free allocation of the entire cake for 3 or more agents (Stromquist, 2008).

There are various finite-time algorithms that guarantee envy-freeness when the valuations are piecewise-constant (Aziz and Ye, 2014; Kurokawa et al., 2013; Cohler et al., 2011), but they might allocate disconnected pieces, so they are less practical for our use-case of dividing land. Segal-Halevi et al. (2016) guarantee both envy-freeness and connectivity, but they may leave some cake unallocated, and (for $n > 3$) do not guarantee proportionality.

Two more recent works, by Arunachaleswaran et al. (2019) and Goldberg et al. (2019), guarantee connectivity but they are only approximately envy-free. A recent work by Barman and Rathi (2020), guarantee both envy-freeness and connectivity under the restriction that value densities of agents satisfy the monotone likelihood ratio property. In future work it would be beneficial to adapt this algorithm to a two-dimensional setting both for LE measure and for experimenting with additional types of noise patterns.

Welfare-related metrics (UV and EV) may be improved by cake-cutting algorithms that consider efficiency in addition to fairness, such as (Cohler et al., 2011; Bei et al., 2012).

Consider now the two face-ratio related metrics — SFR and AFR. Segal-Halevi et al. (2017, 2020) present several algorithms for dividing a two-dimensional land estate, such that the pieces are all squares, or (more generally) rectangles with a lower bound on their face ratio. These algorithms obviously do better than our heuristics in terms of SFR and AFR. However, they might do worse in terms of fairness and welfare. They do not guarantee that the entire land is divided, and they do not guarantee proportionality but only a multiplicative approximation of proportionality (e.g. each agent receives a square piece worth at least $1/(2n)$ of the total cake value, instead of the $1/n$ guaranteed by Even-Paz). In future work, it may be interesting to check what fairness and welfare these algorithms provide when applied to real land-value data.

Very recently, Legut (2020) presented an algorithm for proportional allocation of a square cake, which is similar to ours in that the cuts may be both horizontal and vertical, but different in that the cut directions are decided in advance. He mentions that choosing cut-directions may improve the geometric shape, but does not explain how to choose an optimal cut-direction.

Finally, consider the two strategy-related metrics — ASG and MSG. The gain from strategic manipulations can be entirely eliminated by using a *truthful* cake-cutting algorithm. Chen et al. (2013) and Bei et al. (2018) present truthful algorithms mainly for the special case of *piecewise-uniform* valuations, where each agent values each region

of the cake at either 0 or 1. They have algorithms for more general piecewise-constant valuations, but they assume a 1-dimensional cake and do not guarantee connectivity, so they are not applicable to our setting.

7.2 Different Noise Patterns

We have seen that different noise pattern significantly effect the results of the tested algorithms. Thus, an important future work project is to test which noise pattern best reflects the differing valuations of real people for land.

7.3 Different Manipulation Strategies

Our results suggests a relationship between the risk of losing from acting strategic and the prospect of a high reward. Studying the risk/reward ratio of different agent manipulation strategies in future work provides more insights on the vulnerability of different cake-cutting algorithms to strategy and the profitability of acting strategic.

Bibliography

Fuad Aleskerov and Sergey Shvydun. 2019. Allocation of Disputable Zones in the Arctic Region. *Group Decision and Negotiation* 28, 1 (2019), 11–42.

Eshwar Ram Arunachaleswaran, Siddharth Barman, Rachitesh Kumar, and Nidhi Rathi. 2019. Fair and Efficient Cake Division with Connected Pieces. *arXiv preprint arXiv:1907.11019* (2019).

Yonatan Aumann and Yair Dombb. 2010. The efficiency of fair division with connected pieces. In *International workshop on internet and network economics*. Springer, 26–37.

Yonatan Aumann, Yair Dombb, and Avinatan Hassidim. 2013. Computing socially-efficient cake divisions. In *Proceedings of the 2013 international conference on Autonomous agents and multi-agent systems*. International Foundation for Autonomous Agents and Multiagent Systems, 343–350.

Haris Aziz, Avinatan Hassidim, and Erel Segal-Halevi. 2020. Fair Allocation with Diminishing Differences. *Journal of Artificial Intelligence Research* 67 (2020), 471–507.

Haris Aziz and Chun Ye. 2014. Cake cutting algorithms for piecewise constant and piecewise uniform valuations. In *International Conference on Web and Internet Economics*. Springer, 1–14.

Siddharth Barman and Nidhi Rathi. 2020. Fair Cake Division Under Monotone Likelihood Ratios. *CoRR* abs/2006.00481 (2020). arXiv:2006.00481 https://arxiv.org/abs/2006.00481

Xiaohui Bei, Ning Chen, Xia Hua, Biaoshuai Tao, and Endong Yang. 2012. Optimal Proportional Cake Cutting with Connected Pieces. In *Proceedings of the 26th AAAI Conference on Artificial Intelligence (AAAI-12)*. 1263–1269.

Xiaohui Bei, Guangda Huzhang, and Warut Suksompong. 2018. Truthful Fair Division without Free Disposal, In Proceedings of IJCAI-18. *arXiv preprint arXiv:1804.06923*.

Dimitris Bertsimas, Vivek F. Farias, and Nikolaos Trichakis. 2011. The Price of Fairness. *Operations Research* 59, 1 (2011), 17–31.

Steven Brams and David Denoon. 1997. Fair division: A new approach to the Spratly Islands controversy. *International Negotiation* 2, 2 (1997), 303–329.

Steven Brams and Alan Taylor. 1996. *Fair division: From cake-cutting to dispute resolution.* Cambridge University Press.

Steven Brams and Jeffrey Togman. 1996. Camp David: Was the agreement fair? *Conflict Management and Peace Science* 15, 1 (1996), 99–112. https://doi.org/10.1177/073889429601500105

Steven J. Brams. 2007. *Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures* (first edition ed.). Princeton University Press. http://www.sciencedirect.com/science/article/pii/S089571770800

Simina Brânzei. 2015. *Computational Fair Division.* Ph.D. Dissertation. Faculty of Science and Technology in Aarhus university.

Eric Budish and Estelle Cantillon. 2007. Strategic Behavior in Multi-unit Assignment Problems: Theory and Evidence from Course Allocations. *Computational Social Systems and the Internet* 1, 6.7 (2007), 2007.

Eric Budish and Estelle Cantillon. 2012. The multi-unit assignment problem: Theory and evidence from course allocation at Harvard. *American Economic Review* 102, 5 (2012), 2237–71.

Ioannis Caragiannis, Christos Kaklamanis, Panagiotis Kanellopoulos, and Maria Kyropoulou. 2012. The efficiency of fair division. *Theory of Computing Systems* 50, 4 (2012), 589–610.

Ruggiero Cavallo. 2012. Fairness and welfare through redistribution when utility is transferable. In *Proceedings of the Conference on Artificial Intelligence (AAAI).*

Yiling Chen, John K. Lai, David C. Parkes, and Ariel D. Procaccia. 2013. Truth, justice, and cake cutting. *Games and Economic Behavior* 77, 1 (Jan. 2013), 284–297.

Yuga J. Cohler, John K. Lai, David C. Parkes, and Ariel D. Procaccia. 2011. Optimal Envy-Free Cake Cutting. In *Proceedings of the 25th AAAI Conference on Artificial Intelligence (AAAI-11).* 626–631.

Terry Daniel and James Parco. 2005. Fair, Efficient and Envy-Free Bargaining: An Experimental Test of the Brams-Taylor Adjusted Winner Mechanism. *Group Decision and Negotiation* 14, 3 (May 2005), 241–264.

John Dickerson, Jonathan Goldman, Jeremy Karp, Ariel Procaccia, and Tuomas Sandholm. 2014. The computational rise and fall of fairness.. In *Proceedings of the Conference on Artificial Intelligence (AAAI)*.

Nicolas Dupuis-Roy and Frédéric Gosselin. 2011. The Simpler, the Better: A New Challenge for Fair-Division Theory. In *Proceedings of the Annual Meeting of the Cognitive Science Society*. 3229–3234. http://csjarchive.cogsci.rpi.edu/Proceedings/2011/papers/0742/

Dirk Engelmann and Martin Strobel. 2004. Inequality aversion, efficiency, and maximin preferences in simple distribution experiments. *American Economic Review* 94, 4 (2004), 857–869.

Shimon Even and Azaria Paz. 1984. A Note on Cake Cutting. *Discrete Applied Mathematics* 7, 3 (March 1984), 285–296. https://doi.org/10.1016/0166-218x(84)90005-2

Ernst Fehr, Michael Naef, and Klaus Schmidt. 2006. Inequality aversion, efficiency, and maximin preferences in simple distribution experiments: Comment. *American Economic Review* 96, 5 (2006), 1912–1917.

Merrill Flood. 1958. Some experimental games. *Management Science* 5, 1 (1958), 5–26.

Yafiakov Kobi Gal, Moshe Mash, Ariel D Procaccia, and Yair Zick. 2017. Which is the fairest (rent division) of them all? *Journal of the ACM (JACM)* 64, 6 (2017), 39.

Paul W. Goldberg, Alexandros Hollender, and Warut Suksompong. 2019. Contiguous Cake Cutting: Hardness Results and Approximation Algorithms. (2019). arXiv:cs.GT/1911.05416

Jonathan Goldman and Ariel D Procaccia. 2015. Spliddit: Unleashing fair division algorithms. *ACM SIGecom Exchanges* 13, 2 (2015), 41–46.

Dorothea K Herreiner and Clemens Puppe. 2010. Inequality aversion and efficiency with ordinal and cardinal social preferencesfi?!An experimental study. *Journal of Economic Behavior & Organization* 76, 2 (2010), 238–253.

Rafael Hortala-Vallve and Aniol Llorente-Saguer. 2010. A simple mechanism for resolving conflict. *Games and Economic Behavior* 70, 2 (Nov. 2010), 375–391. https://doi.org/10.1016/j.geb.2010.02.005

David Kurokawa, John K. Lai, and Ariel D. Procaccia. 2013. How to Cut a Cake Before the Party Ends. In *Proceedings of the Twenty-Seventh AAAI Conference on Artificial Intelligence*. 555–561.

David Kurokawa, Ariel Procaccia, and Nisarg Shah. 2015. Leximin allocations in the real world. In *Proceedings of the Sixteenth ACM Conference on Economics and Computation*. ACM, 345–362.

Maria Kyropoulou, Josué Ortega, and Erel Segal-Halevi. 2019. Fair cake-cutting in practice. In *Proceedings of the 2019 ACM Conference on Economics and Computation*. ACM, 547–548.

Jerzy Legut. 2020. Simple fair division of a square. *Journal of Mathematical Economics* 86 (2020), 35–40.

Claudia Lindner and Jörg Rothe. 2016. Cake-Cutting: Fair Division of Divisible Goods. In *Economics and Computation: An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division*, Jörg Rothe (Ed.). Springer Berlin Heidelberg, Berlin, Heidelberg, 395–491.

Tansa Massoud. 2000. Fair division, adjusted winner procedure (AW), and the Israeli-Palestinian conflict. *Journal of Conflict Resolution* 44, 3 (June 2000), 333–358. https://doi.org/10.1177/0022002700044003003

Nicholas Mattei and Toby Walsh. 2013. Preflib: A library for preferences http://www.preflib. org. In *International Conference on Algorithmic DecisionTheory*. Springer, 259–270.

Vijay Menon and Kate Larson. 2017. Deterministic, strategyproof, and fair cake cutting. In *Proceedings of the Joint Conference on Artificial Intelligence (IJCAI)*.

Olabambo I Oluwasuji, Obaid Malik, Jie Zhang, Sarvapali D Ramchurn, et al. 2018. Algorithms for Fair Load Shedding in Developing Countries.. In *IJCAI*. 1590–1596.

James Parco and Amnon Rapoport. 2004. Enhancing honesty in bargaining under incomplete information: An experimental study of the bonus procedure. *Group Decision and Negotiation* 13, 6 (2004), 539–562.

John Pratt and Richard Zeckhauser. 1990. The fair and efficient division of the Winsor family silver. *Management Science* 36, 11 (Nov. 1990), 1293–1301. https://doi.org/10.1287/mnsc.36.11.1293

Ariel Procaccia. 2016. Cake cutting algorithms. In *Handbook of Computational Social Choice*, Felix Brandt, Vincent Conitzer, Ulle Endriss, Jerome Lang, and Ariel Procaccia (Eds.). Cambridge University Press, 311–330. https://doi.org/10.1017/CBO9781107446984.014

Jack Robertson and William Webb. 1998. *Cake-cutting algorithms: Be fair if you can.* AK Peters/CRC Press.

Gerald Schneider and Ulrike Krämer. 2004. The limitations of fair division. *Journal of Conflict Resolution* 48, 4 (2004), 506–524. https://doi.org/10.1177/0022002704266148

Erel Segal-Halevi, Avinatan Hassidim, and Yonatan Aumann. 2016. Waste makes haste: bounded time algorithms for envy-free cake cutting with free disposal. *ACM Transactions on Algorithms (TALG)* 13, 1 (2016), 12.

Erel Segal-Halevi, Shmuel Nitzan, Avinatan Hassidim, and Yonatan Aumann. 2017. Fair and square: Cake-cutting in two dimensions. *Journal of Mathematical Economics* 70 (Feb. 2017), 1–28. https://doi.org/10.1016/j.jmateco.2017.01.007

Erel Segal-Halevi, Shmuel Nitzan, Avinatan Hassidim, and Yonatan Aumann. 2020. Envy-Free Division of Land. *Mathematics of Operations Research* 45, 3 (2020), 896–922.

Erel Segal-Halevi and Balázs R Sziklai. 2019. Monotonicity and competitive equilibrium in cake-cutting. *Economic Theory* 68, 2 (2019), 363–401.

Lloyd Shapley and Herbert Scarf. 1974. On cores and indivisibility. *Journal of Mathematical Economics* 1, 1 (March 1974), 23–37. https://doi.org/10.1016/0304-4068(74)90033-0

Hugo Steinhaus. 1948. The problem of fair division. *Econometrica* 16, 1 (1948), 101–104.

Walter Stromquist. 2008. Envy-free cake divisions cannot be found by finite protocols. *the electronic journal of combinatorics* 15, 1 (2008), 11.

Francis E. Su. 1999. Rental Harmony: Sperner's Lemma in Fair Division. *The American Mathematical Monthly* 106, 10 (Dec. 1999), 930–942.

Stef Tijs and Rodica Branzei. 2004. Cases in cooperation and cutting the cake. *SSRN preprint* (Dec. 2004). `http://ssrn.com/abstract=627424`

Toby Walsh. 2011. Online Cake Cutting. *Algorithmic Decision Theory* 6992 (2011), 292–305.

Gerhard J. Woeginger and Jiří Sgall. 2007. On the complexity of cake cutting. *Discrete Optimization* 4, 2 (June 2007), 213–220.

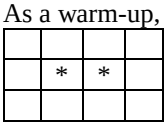
Appendix

A All Cut-Patterns Are Needed for Maximizing Utilitarian Gain

This appendix shows that for some arbitrarily large Resultss of n, there are 2^{n-1} different scenarios (i.e., sets of n maps), where in each scenario, a different sequence of cut-directions is needed in order to get the maximum possible utilitarian Results.

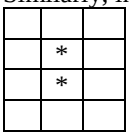
In all our scenarios, all agents have piecewise-uniform valuations. For every agent i, there is a unique cell on the map that the agent Resultss at 1, and the agent Resultss all other cells at 0. We present the scenarios concisely by showing a single map with the cells wanted by the agents marked by *.

As a warm-up, consider the following map, which we denote by **V** (for a reason that will soon be clear):



This map represents a scenario with n=2 agents, in which one agent Resultss at 1 only the cell at row 2 column 2, and the other agent Resultss at 1 only the cell at row 2 column 3. In this scenario, running Even-Paz with a vertical cut gives each agent a Results of 1, so the utilitarian Results is 2; running Even-Paz with a horizontal cut gives each agent a Results of ½, so the utilitarian Results is 1. Hence, the unique cut-direction that maximizes the utilitarian Results is vertical; therefore we denote this pattern by V.

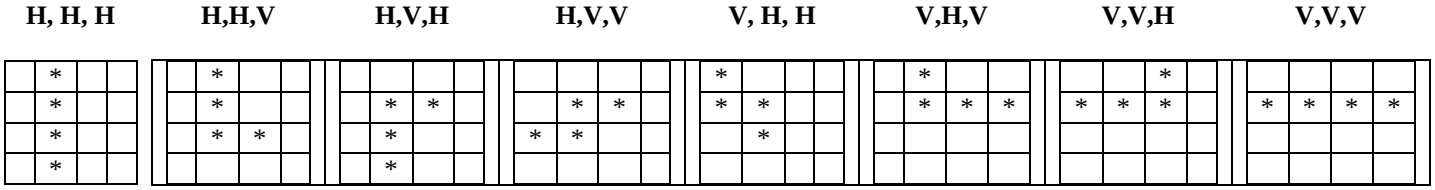
Similarly, in the following map, which we denote by **H**, the unique direction maximizing the utilitarian Results is horizontal:



Now, we show how to construct such maps for larger Resultss of n. For simplicity, we focus on Resultss of n that are powers of two. Suppose we already have 2^{n-1} different patterns for n agents. We generate 2^{2n-1} different patterns for $2n$ agents in the following way:

- Pick a direction for the initial cut (2 options -- either Horizontal or Vertical);
- Pick a pattern for one side of the cut (2^{n-1} options);
- Pick a pattern for the other side of the cut (2^{n-1} options).
- Arrange the two patterns based on the direction of the initial cut:
    - If the initial cut is horizontal, then arrange the two patterns one above the other, with an overlap of one cell in their horizontal projection;
    - Otherwise (the initial cut is vertical), arrange the two patterns one alongside the other, with an overlap of one cell in their vertical projection.

For n=4 agents, this procedure gives the following 8 patterns. The letters above each pattern correspond to the selections made in the above construction procedure. They also correspond to the unique sequence of 3 cuts with the maximum utilitarian Results in this scenario:



Consider, as an example, the pattern V,H,H:

- If the initial cut is vertical (V) and the two cuts in the sub-cakes are horizontal (H,H), then clearly each agent's Results is 1 and the utilitarian Results is 4.
- If the initial cut is vertical (V) and one or both cuts in the sub-cakes is vertical (V,H or H,V or V,V), then the agents in that sub-cake get a Results of ½ and the utilitarian Results is at most 3.
- If the initial cut is horizontal (H), then it cuts at the middle of the second row, so the two agents whose wanted cell is at that row get a Results of at most ½, and again the utilitarian Results is at most 3.

We emphasize that this set of examples does not imply an exponential lower bound on the run-time complexity of finding the optimal sequence of cuts. In fact, in the examples described above, the greedy heuristic that we called "Most Valuable Margin" always attains the maximum utilitarian welfare. Thus, it remains an open question whether the sequence of cuts that maximizes the utilitarian welfare (from among the 2^{n-1} possible sequences) can be found in polynomial time.

B Results – Strategy Experiments

This appendix shows the results for our strategy experiments, which was held as follows:
We ran the standard truthful experiment using a group of 128 agent, then we ran 128 more experiments, for each agent i, we ran an experiment were we assumed all agents are truthful except agent i that knows all of the other agents' preferences and plays the best move at each iteration of the algorithm. The agent's best move is to choose to be on one side or the other of the predicted cut in the current iteration by choosing his/her own "half-cut" to be on that chosen side.
Using the truthful experiment as the truthful gain of all agents and the 128 experiments as the untruthful gain of each agent respectively we then calculated the SG_i of agent i by subtracting the truthful gain from the untruthful gain of agent i.
Finally, we calculated ASG to be the average of SG_i over all agents and MSG to be the maximum SG_i.
To ensure significant results we repeated this "experiment set" 50 times for different groups of agents and averaged all "experiment sets" .

Strategy Experiment Results – New-Zealand Map with uniform noise 0.6

Cut Pattern	ASG	ASG Improvement (%)	ASG stdev	MSG	MSG Improvement (%)	MSG stdev
Ver	8.00E-05	1.03%	4.95E-05	0.000344	4.44%	0.000184
VerHor	9.65E-05	1.24%	5.29E-05	0.000379	4.90%	0.000196
SmallestHalfCut	0.000112	1.43%	6.00E-05	0.000407	5.29%	0.000205
Hor	0.000109	1.40%	5.90E-05	0.000387	5.02%	0.000198
MostValuableMargin	0.000107	1.36%	6.47E-05	0.000452	5.86%	0.000163
LargestMargin	0.000101	1.30%	6.16E-05	0.000495	6.47%	8.13E-05
SmallestPiece	8.58E-05	1.10%	5.48E-05	0.00034	4.39%	0.000206
HorVer	0.000104	1.34%	6.28E-05	0.000404	5.22%	0.000215
HighestScatter	0.000103	1.32%	6.38E-05	0.000404	5.24%	0.000193
SquarePiece	9.92E-05	1.27%	6.03E-05	0.000406	5.25%	0.000153

Strategy Experiment Results – New-Zealand Map with hotspot noise 0.6

Cut Pattern	ASG	ASG Improvement (%)	ASG stdev	MSG	MSG Improvement (%)	MSG stdev
Ver	9.22E-05	0.89%	4.38E-05	0.000981	7.18%	0.000816
VerHor	0.000188	1.86%	4.96E-05	0.000804	7.00%	0.000266
SmallestHalfCut	9.61E-05	0.91%	2.28E-05	0.000624	4.60%	0.000217
Hor	9.17E-05	0.86%	2.23E-05	0.000623	4.54%	0.000218
MostValuableMargin	0.000193	1.90%	3.41E-05	0.000936	8.15%	0.000251
LargestMargin	0.000213	2.21%	7.36E-05	0.00143	15.51%	0.000443
SmallestPiece	0.000115	1.11%	2.49E-05	0.000648	4.85%	0.000211
HorVer	0.000175	1.73%	3.31E-05	0.000734	5.79%	0.000174
HighestScatter	0.000181	1.78%	3.01E-05	0.000745	6.01%	0.000186
SquarePiece	0.000182	1.82%	3.86E-05	0.000918	8.28%	0.000271

C Results – New-Zealand Map with hotspot noise parameter 0.6

Egalitarian Results

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.058501	0.87329	1.058689	1	0.109887	0.207089	0.109486
8	1.060005	0.770108	1.060713	1	0.065329	0.161406	0.064276
16	1.059516	0.707957	1.062756	1	0.05225	0.139346	0.050105
32	1.060802	0.661056	1.063107	1	0.041615	0.077031	0.040591
64	1.06257	0.636293	1.06515	1	0.043243	0.042059	0.043842
128	1.063951	0.621151	1.065336	1	0.03039	0.015662	0.029986

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.054066	0.87329	1.054246	1	0.112263	0.207089	0.112176
8	1.046263	0.770108	1.046268	1	0.065434	0.161406	0.065416
16	1.037081	0.707957	1.037081	1	0.037999	0.139346	0.037999
32	1.031203	0.661056	1.031379	1	0.02945	0.077031	0.029332
64	1.023531	0.636293	1.023624	1	0.022468	0.042059	0.022613
128	1.020554	0.621151	1.020811	1	0.01596	0.015662	0.015915

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.051092	0.87329	1.051239	1	0.112285	0.207089	0.112053
8	1.051931	0.770108	1.053187	1	0.056115	0.161406	0.056976
16	1.051669	0.707957	1.054169	1	0.045749	0.139346	0.044688
32	1.052321	0.661056	1.054987	1	0.0389	0.077031	0.039841
64	1.04907	0.636293	1.056219	1	0.034215	0.042059	0.043587
128	1.044993	0.621151	1.050262	1	0.022858	0.015662	0.024753

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.053999	0.87329	1.054889	1	0.11312	0.207089	0.111372
8	1.04742	0.770108	1.047856	1	0.064692	0.161406	0.064608
16	1.040292	0.707957	1.041147	1	0.03452	0.139346	0.033613
32	1.037298	0.661056	1.038329	1	0.028513	0.077031	0.028903
64	1.029389	0.636293	1.031074	1	0.02675	0.042059	0.028565
128	1.026424	0.621151	1.02754	1	0.016757	0.015662	0.01824

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.035331	0.87329	1.035527	1	0.07593	0.207089	0.075718
8	1.03808	0.770108	1.040204	1	0.054627	0.161406	0.055756
16	1.049037	0.707957	1.049856	1	0.041955	0.139346	0.041308
32	1.044895	0.661056	1.047212	1	0.04199	0.077031	0.04315
64	1.044294	0.636293	1.045872	1	0.028131	0.042059	0.029008
128	1.0398	0.621151	1.04201	1	0.022627	0.015662	0.023218

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.051092	0.87329	1.051239	1	0.112285	0.207089	0.112053
8	1.051858	0.770108	1.053032	1	0.056141	0.161406	0.056734
16	1.051921	0.707957	1.054058	1	0.04611	0.139346	0.04695
32	1.051572	0.661056	1.056365	1	0.040223	0.077031	0.039851
64	1.050105	0.636293	1.05848	1	0.0349	0.042059	0.041716
128	1.045941	0.621151	1.051253	1	0.022089	0.015662	0.024621

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.060064	0.87329	1.060252	1	0.108141	0.207089	0.107705
8	1.061444	0.770108	1.061795	1	0.069415	0.161406	0.068882
16	1.06084	0.707957	1.06188	1	0.05093	0.139346	0.049584
32	1.062117	0.661056	1.064936	1	0.038001	0.077031	0.035888
64	1.063697	0.636293	1.065338	1	0.040686	0.042059	0.039833
128	1.06148	0.621151	1.063867	1	0.030572	0.015662	0.02965

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.054066	0.87329	1.054246	1	0.112263	0.207089	0.112176
8	1.046263	0.770108	1.046268	1	0.065434	0.161406	0.065416
16	1.037081	0.707957	1.037081	1	0.037999	0.139346	0.037999
32	1.031203	0.661056	1.031379	1	0.02945	0.077031	0.029332
64	1.023531	0.636293	1.023624	1	0.022468	0.042059	0.022613
128	1.020554	0.621151	1.020811	1	0.01596	0.015662	0.015915

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.058811	0.87329	1.058811	1	0.107639	0.207089	0.107639
8	1.057889	0.770108	1.058344	1	0.064671	0.161406	0.06418
16	1.058419	0.707957	1.059389	1	0.049469	0.139346	0.049838
32	1.05938	0.661056	1.060934	1	0.041597	0.077031	0.040885
64	1.051082	0.636293	1.052921	1	0.033487	0.042059	0.032543
128	1.047971	0.621151	1.049702	1	0.029232	0.015662	0.030434

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.0266	0.87329	1.026701	1	0.053036	0.207089	0.052903
8	1.026162	0.770108	1.026439	1	0.031865	0.161406	0.031759
16	1.029611	0.707957	1.029834	1	0.026163	0.139346	0.026518
32	1.026545	0.661056	1.027047	1	0.01679	0.077031	0.018344
64	1.026341	0.636293	1.026361	1	0.015512	0.042059	0.015535
128	1.025201	0.621151	1.025838	1	0.012281	0.015662	0.011715

Utilitarian Results

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.106874904	1.001686613	1.107521162	1	0.145674671	0.184595762	0.143647306
8	1.155460769	0.995416807	1.156526932	1	0.072466283	0.101851059	0.071790088
16	1.182590832	1.000659051	1.185133742	1	0.063360643	0.072264932	0.060434518
32	1.193544834	1.002734599	1.196639375	1	0.053871552	0.042395695	0.053057999
64	1.219643906	0.998925221	1.222718282	1	0.036051065	0.03362939	0.034174172
128	1.23467776	0.99938386	1.238218557	1	0.027084567	0.02692978	0.025229038

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.102521891	1.001686613	1.102820843	1	0.150511943	0.184595762	0.150170737
8	1.14257139	0.995416807	1.142726863	1	0.073121527	0.101851059	0.072895866
16	1.160169794	1.000659051	1.160315395	1	0.055413091	0.072264932	0.055404386
32	1.1598608	1.002734599	1.160494006	1	0.050803546	0.042395695	0.049829479
64	1.177223805	0.998925221	1.177741523	1	0.03070443	0.03362939	0.029956371
128	1.185177659	0.99938386	1.185474391	1	0.021608222	0.02692978	0.021119912

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.100839511	1.001686613	1.101165495	1	0.139849338	0.184595762	0.139213189
8	1.149614472	0.995416807	1.150034313	1	0.074218602	0.101851059	0.07398664
16	1.175642796	1.000659051	1.178240161	1	0.058151163	0.072264932	0.055440038
32	1.185309812	1.002734599	1.189523115	1	0.051104967	0.042395695	0.051841485
64	1.209374417	0.998925221	1.215661552	1	0.030651066	0.03362939	0.030991074
128	1.222146851	0.99938386	1.231404383	1	0.02288069	0.02692978	0.02268051

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.101057246	1.001686613	1.101573652	1	0.142796538	0.184595762	0.142603131
8	1.146238457	0.995416807	1.146497453	1	0.072965783	0.101851059	0.072416511
16	1.167806248	1.000659051	1.168659489	1	0.05486745	0.072264932	0.054587142
32	1.171936129	1.002734599	1.172971944	1	0.049107581	0.042395695	0.047711763
64	1.190038191	0.998925221	1.191560771	1	0.029882995	0.03362939	0.028197576
128	1.1997943	0.99938386	1.200786613	1	0.020136777	0.02692978	0.019731938

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.092381905	1.001686613	1.094269593	1	0.138562054	0.184595762	0.13508897
8	1.135325015	0.995416807	1.14076466	1	0.080319761	0.101851059	0.075934508
16	1.173117968	1.000659051	1.177170119	1	0.055188004	0.072264932	0.053598068
32	1.182722632	1.002734599	1.188347674	1	0.048904237	0.042395695	0.049556471
64	1.208998632	0.998925221	1.21646876	1	0.02861008	0.03362939	0.028832741
128	1.223270636	0.99938386	1.230994856	1	0.020485095	0.02692978	0.020584875

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.100839511	1.001686613	1.101165495	1	0.139849338	0.184595762	0.139213189
8	1.149195445	0.995416807	1.149591601	1	0.074393733	0.101851059	0.074186259
16	1.176132603	1.000659051	1.178631422	1	0.059274594	0.072264932	0.056503365
32	1.186295194	1.002734599	1.190549312	1	0.051181452	0.042395695	0.051430645
64	1.209321655	0.998925221	1.217377334	1	0.03335495	0.03362939	0.032964393
128	1.221471538	0.99938386	1.232106229	1	0.023690255	0.02692978	0.023123243

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.108515959	1.001686613	1.108653021	1	0.142314199	0.184595762	0.141891552
8	1.157411167	0.995416807	1.157522872	1	0.072565718	0.101851059	0.072704453
16	1.185507205	1.000659051	1.186541369	1	0.060080203	0.072264932	0.058921147
32	1.195906885	1.002734599	1.19793689	1	0.052654883	0.042395695	0.05192772
64	1.221277422	0.998925221	1.223763725	1	0.035892734	0.03362939	0.034469714
128	1.236272722	0.99938386	1.239023545	1	0.027489406	0.02692978	0.025229258

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.102521891	1.001686613	1.102820843	1	0.150511943	0.184595762	0.150170737
8	1.14257139	0.995416807	1.142726863	1	0.073121527	0.101851059	0.072895866
16	1.160588138	1.000659051	1.160820214	1	0.055825893	0.072264932	0.055901456
32	1.16153942	1.002734599	1.162247155	1	0.050878668	0.042395695	0.050124127
64	1.179315165	0.998925221	1.179969387	1	0.031067837	0.03362939	0.03002183
128	1.187727354	0.99938386	1.188320539	1	0.021714837	0.02692978	0.021231183

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.107566486	1.001686613	1.107566486	1	0.142939694	0.184595762	0.142939694
8	1.155013419	0.995416807	1.155796713	1	0.076251138	0.101851059	0.074611478
16	1.181812389	1.000659051	1.182956619	1	0.059530839	0.072264932	0.057987508
32	1.192931334	1.002734599	1.195618809	1	0.053922078	0.042395695	0.053116897
64	1.215531294	0.998925221	1.219596962	1	0.036330054	0.03362939	0.034254536
128	1.22993535	0.99938386	1.234719534	1	0.024637061	0.02692978	0.023469435

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.065286291	1.001686613	1.065362374	1	0.088595908	0.184595762	0.088347649
8	1.093042816	0.995416807	1.093234209	1	0.057401795	0.101851059	0.057412401
16	1.115745399	1.000659051	1.116674275	1	0.036336716	0.072264932	0.035585717
32	1.122061141	1.002734599	1.122791724	1	0.03630839	0.042395695	0.035122609
64	1.135578359	0.998925221	1.136228768	1	0.023007324	0.03362939	0.022107093
128	1.142174542	0.99938386	1.14272215	1	0.018218264	0.02692978	0.017031371

Nash Social Results

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.105440947	0.996527513	1.10609043	1	0.142955848	0.183671588	0.143647306
8	1.151307237	0.986004702	1.15242314	1	0.069723694	0.100385639	0.071790088
16	1.176436796	0.98992269	1.17907476	1	0.06277421	0.073369425	0.060434518
32	1.186702622	0.990884888	1.189916488	1	0.052323459	0.043412911	0.053057999
64	1.210797464	0.986733167	1.213985506	1	0.037988054	0.033826843	0.034174172
128	1.225374607	0.986652518	1.229027324	1	0.028450249	0.027274878	0.025229038

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.101044556	0.996527513	1.101351577	1	0.147547041	0.183671588	0.150170737
8	1.138048449	0.986004702	1.138214883	1	0.070073297	0.100385639	0.072895866
16	1.153273885	0.98992269	1.153420327	1	0.053890384	0.073369425	0.055404386
32	1.152050625	0.990884888	1.152677627	1	0.04848428	0.043412911	0.049829479
64	1.166986146	0.986733167	1.167503058	1	0.031131693	0.033826843	0.029956371
128	1.174207977	0.986652518	1.174505095	1	0.02211852	0.027274878	0.021119912

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.099393678	0.996527513	1.099725508	1	0.137189294	0.183671588	0.139213189
8	1.14538393	0.986004702	1.145822983	1	0.070607129	0.100385639	0.07398664
16	1.169637204	0.98992269	1.17230131	1	0.057147109	0.073369425	0.055440038
32	1.178817136	0.990884888	1.183148508	1	0.049428357	0.043412911	0.051841485
64	1.201158444	0.986733167	1.207661955	1	0.032433912	0.033826843	0.030991074
128	1.213553078	0.986652518	1.222994485	1	0.023832003	0.027274878	0.02268051

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.099633926	0.996527513	1.100169219	1	0.14012311	0.183671588	0.142603131
8	1.141829538	0.986004702	1.14209658	1	0.069702367	0.100385639	0.072416511
16	1.161145014	0.98992269	1.162033174	1	0.052797408	0.073369425	0.054587142
32	1.16471569	0.990884888	1.165771528	1	0.046633362	0.043412911	0.047711763
64	1.180673275	0.986733167	1.182232038	1	0.030242872	0.033826843	0.028197576
128	1.189959772	0.986652518	1.190979247	1	0.020166186	0.027274878	0.019731938

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.091009144	0.996527513	1.092845742	1	0.135251	0.183671588	0.13508897
8	1.130851057	0.986004702	1.136358067	1	0.075158457	0.100385639	0.075934508
16	1.166140057	0.98992269	1.17002458	1	0.052992189	0.073369425	0.053598068
32	1.17543194	0.990884888	1.181055955	1	0.047284123	0.043412911	0.049556471
64	1.199550871	0.986733167	1.207001784	1	0.029017946	0.033826843	0.028832741
128	1.213555886	0.986652518	1.221235325	1	0.021169236	0.027274878	0.020584875

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.099393678	0.996527513	1.099725508	1	0.137189294	0.183671588	0.139213189
8	1.145020217	0.986004702	1.145434368	1	0.070895426	0.100385639	0.074186259
16	1.170172137	0.98992269	1.172711723	1	0.058346617	0.073369425	0.056503365
32	1.179938836	0.990884888	1.184320205	1	0.049567162	0.043412911	0.051430645
64	1.201221017	0.986733167	1.209405878	1	0.034673593	0.033826843	0.032964393
128	1.212900263	0.986652518	1.223755834	1	0.024468284	0.027274878	0.023123243

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.107126884	0.996527513	1.107265849	1	0.139664878	0.183671588	0.141891552
8	1.153189004	0.986004702	1.153309619	1	0.069624341	0.100385639	0.072704453
16	1.179210404	0.98992269	1.18029392	1	0.05902637	0.073369425	0.058921147
32	1.189021713	0.990884888	1.191123619	1	0.050965281	0.043412911	0.05192772
64	1.212359903	0.986733167	1.214937026	1	0.03755889	0.033826843	0.034469714
128	1.226807792	0.986652518	1.229703011	1	0.02910193	0.027274878	0.025229258

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.101044556	0.996527513	1.101351577	1	0.147547041	0.183671588	0.150170737
8	1.138048449	0.986004702	1.138214883	1	0.070073297	0.100385639	0.072895866
16	1.153674268	0.98992269	1.153911646	1	0.05418694	0.073369425	0.055901456
32	1.153547579	0.990884888	1.154245609	1	0.048463792	0.043412911	0.050124127
64	1.168902544	0.986733167	1.16953981	1	0.031429526	0.033826843	0.03002183
128	1.176595617	0.986652518	1.177165109	1	0.022199475	0.027274878	0.021231183

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.106190634	0.996527513	1.106190634	1	0.140120121	0.183671588	0.142939694
8	1.150585936	0.986004702	1.151401544	1	0.072695467	0.100385639	0.074611478
16	1.175360134	0.98992269	1.17654743	1	0.058315208	0.073369425	0.057987508
32	1.185886752	0.990884888	1.18863319	1	0.052011748	0.043412911	0.053116897
64	1.206040033	0.986733167	1.210184391	1	0.036796482	0.033826843	0.034254536
128	1.220314013	0.986652518	1.225241023	1	0.025650078	0.027274878	0.023469435

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.064453855	0.996527513	1.064531067	1	0.086610646	0.183671588	0.088347649
8	1.090626389	0.986004702	1.090826252	1	0.054982537	0.100385639	0.057412401
16	1.111374283	0.98992269	1.11222051	1	0.034930034	0.073369425	0.035585717
32	1.115996299	0.990884888	1.116711175	1	0.033160214	0.043412911	0.035122609
64	1.128021208	0.986733167	1.128635209	1	0.021933277	0.033826843	0.022107093
128	1.134010202	0.986652518	1.134512341	1	0.016607982	0.027274878	0.017031371

Largest Envy Results

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.022517192	1.340050707	1.02184733	1	0.101775442	0.452322282	0.143647306
8	1.069678297	1.626175769	1.068110442	1	0.118488548	0.30793347	0.071790088
16	1.114937386	1.774549155	1.112751291	1	0.098354578	0.214890882	0.060434518
32	1.136394605	1.879184984	1.134379125	1	0.091017035	0.050581946	0.053057999
64	1.171368862	1.923131987	1.169222523	1	0.074437466	0.038505936	0.034174172
128	1.207719154	1.953859326	1.207089519	1	0.069474982	0.013993133	0.025229038

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.024095849	1.340050707	1.023825723	1	0.100299186	0.452322282	0.150170737
8	1.074027476	1.626175769	1.074021435	1	0.077290294	0.30793347	0.072895866
16	1.096865063	1.774549155	1.096865063	1	0.073965049	0.214890882	0.055404386
32	1.131641885	1.879184984	1.131481142	1	0.0700708	0.050581946	0.049829479
64	1.16524827	1.923131987	1.165224507	1	0.05648544	0.038505936	0.029956371
128	1.19603312	1.953859326	1.195854341	1	0.049367571	0.013993133	0.021119912

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.019794991	1.340050707	1.019794991	1	0.095950713	0.452322282	0.139213189
8	1.074859644	1.626175769	1.073629912	1	0.093701397	0.30793347	0.07398664
16	1.120088479	1.774549155	1.118572514	1	0.086980712	0.214890882	0.055440038
32	1.147546136	1.879184984	1.145582008	1	0.091225044	0.050581946	0.051841485
64	1.183883613	1.923131987	1.179237136	1	0.064827207	0.038505936	0.030991074
128	1.227099636	1.953859326	1.221928408	1	0.070253554	0.013993133	0.02268051

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.017938596	1.340050707	1.017089853	1	0.083170306	0.452322282	0.142603131
8	1.076032016	1.626175769	1.075598395	1	0.088773803	0.30793347	0.072416511
16	1.128650534	1.774549155	1.126284867	1	0.115189816	0.214890882	0.054587142
32	1.154113692	1.879184984	1.153424857	1	0.115400808	0.050581946	0.047711763
64	1.188604649	1.923131987	1.188331582	1	0.095113225	0.038505936	0.028197576
128	1.21624351	1.953859326	1.215918573	1	0.079617714	0.013993133	0.019731938

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.079354978	1.340050707	1.079354978	1	0.226896674	0.452322282	0.13508897
8	1.129649063	1.626175769	1.12707139	1	0.150670464	0.30793347	0.075934508
16	1.183346337	1.774549155	1.182821179	1	0.223592442	0.214890882	0.053598068
32	1.200543601	1.879184984	1.198627138	1	0.173618827	0.050581946	0.049556471
64	1.318408037	1.923131987	1.314207903	1	0.225784439	0.038505936	0.028832741
128	1.333206156	1.953859326	1.327805174	1	0.164410982	0.013993133	0.020584875

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.019794991	1.340050707	1.019794991	1	0.095950713	0.452322282	0.139213189
8	1.073513764	1.626175769	1.072512107	1	0.09631223	0.30793347	0.074186259
16	1.130316786	1.774549155	1.129833192	1	0.094815381	0.214890882	0.056503365
32	1.157093456	1.879184984	1.154526563	1	0.112877676	0.050581946	0.051430645
64	1.19558573	1.923131987	1.187644676	1	0.084706927	0.038505936	0.032964393
128	1.241563285	1.953859326	1.237544682	1	0.071991204	0.013993133	0.023123243

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.016488772	1.340050707	1.016145234	1	0.105043946	0.452322282	0.141891552
8	1.063460653	1.626175769	1.063209863	1	0.100393691	0.30793347	0.072704453
16	1.110156173	1.774549155	1.109777565	1	0.097322102	0.214890882	0.058921147
32	1.131300495	1.879184984	1.131125201	1	0.088805447	0.050581946	0.05192772
64	1.170231329	1.923131987	1.169689587	1	0.073511587	0.038505936	0.034469714
128	1.204565	1.953859326	1.203221905	1	0.063283179	0.013993133	0.025229258

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.024095849	1.340050707	1.023825723	1	0.100299186	0.452322282	0.150170737
8	1.074027476	1.626175769	1.074021435	1	0.077290294	0.30793347	0.072895866
16	1.100075242	1.774549155	1.100075242	1	0.071740793	0.214890882	0.055901456
32	1.140961853	1.879184984	1.140836341	1	0.068172001	0.050581946	0.050124127
64	1.181806104	1.923131987	1.181490824	1	0.05466883	0.038505936	0.03002183
128	1.21781543	1.953859326	1.21763484	1	0.047677006	0.013993133	0.021231183

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.021856666	1.340050707	1.021856666	1	0.119736977	0.452322282	0.142939694
8	1.085769443	1.626175769	1.08428824	1	0.168764364	0.30793347	0.074611478
16	1.143838087	1.774549155	1.143145894	1	0.198055118	0.214890882	0.057987508
32	1.178920367	1.879184984	1.177727298	1	0.211800821	0.050581946	0.053116897
64	1.255012742	1.923131987	1.253369349	1	0.260505917	0.038505936	0.034254536
128	1.283167697	1.953859326	1.277606106	1	0.201499256	0.013993133	0.023469435

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.034467751	1.340050707	1.034254324	1	0.113169688	0.452322282	0.088347649
8	1.063073958	1.626175769	1.06296261	1	0.075633446	0.30793347	0.057412401
16	1.130188234	1.774549155	1.130115339	1	0.088265147	0.214890882	0.035585717
32	1.226806425	1.879184984	1.2257104	1	0.237392409	0.050581946	0.035122609
64	1.337603499	1.923131987	1.337536335	1	0.365681524	0.038505936	0.022107093
128	1.380761878	1.953859326	1.380761878	1	0.218993168	0.013993133	0.017031371

Average Face Ratio Results

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.412924785	0.558331937	1	0.244581389	1.16E-16
8	0.296494683	0.431197869	1	0.122101233	3.60E-17
16	0.359847098	0.452647497	1	0.092251625	3.35E-17
32	0.365029107	0.383458149	1	0.076084866	7.34E-17
64	0.418461201	0.417685542	1	0.064606309	7.24E-17
128	0.441851526	0.365192052	1	0.040635565	6.62E-17

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.365196078	0.558331937	1	9.60E-17	1.16E-16
8	0.182598039	0.431197869	1	3.72E-17	3.60E-17
16	0.09129902	0.452647497	1	2.00E-17	3.35E-17
32	0.04564951	0.383458149	1	8.70E-18	7.34E-17
64	0.022824755	0.417685542	1	4.26E-18	7.24E-17
128	0.011412377	0.365192052	1	3.10E-18	6.62E-17

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.55822818	0.558331937	1	0.016149768	1.16E-16
8	0.361818583	0.431197869	1	0.007069373	3.60E-17
16	0.334616222	0.452647497	1	0.008736068	3.35E-17
32	0.332001409	0.383458149	1	0.008051949	7.34E-17
64	0.320089193	0.417685542	1	0.00633465	7.24E-17
128	0.361337223	0.365192052	1	0.004957348	6.62E-17

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.55057395	0.558331937	1	0.022614324	1.16E-16
8	0.275562289	0.431197869	1	0.007673043	3.60E-17
16	0.137168391	0.452647497	1	0.00219436	3.35E-17
32	0.068468922	0.383458149	1	0.000529747	7.34E-17
64	0.046655096	0.417685542	1	0.00175283	7.24E-17
128	0.025303074	0.365192052	1	0.000490248	6.62E-17

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.526153809	0.558331937	1	0.037931709	1.16E-16
8	0.29646511	0.431197869	1	0.005993356	3.60E-17
16	0.390299465	0.452647497	1	0.016104169	3.35E-17
32	0.283798036	0.383458149	1	0.010127647	7.34E-17
64	0.385065274	0.417685542	1	0.016904884	7.24E-17
128	0.321065223	0.365192052	1	0.009366207	6.62E-17

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.55822818	0.558331937	1	0.016149768	1.16E-16
8	0.38934417	0.431197869	1	0.012753856	3.60E-17
16	0.410489553	0.452647497	1	0.010768933	3.35E-17
32	0.491524594	0.383458149	1	0.007345942	7.34E-17
64	0.515439685	0.417685542	1	0.010710535	7.24E-17
128	0.587346864	0.365192052	1	0.007625801	6.62E-17

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.418725745	0.558331937	1	0.324876484	1.16E-16
8	0.322201257	0.431197869	1	0.160463009	3.60E-17
16	0.330511219	0.452647497	1	0.138627372	3.35E-17
32	0.351226805	0.383458149	1	0.091035902	7.34E-17
64	0.38843711	0.417685542	1	0.072013219	7.24E-17
128	0.424228047	0.365192052	1	0.038185159	6.62E-17

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.365196078	0.558331937	1	9.60E-17	1.16E-16
8	0.182598039	0.431197869	1	3.72E-17	3.60E-17
16	0.111691207	0.452647497	1	0.011384665	3.35E-17
32	0.06672337	0.383458149	1	0.004507332	7.34E-17
64	0.034522398	0.417685542	1	0.001860167	7.24E-17
128	0.01805948	0.365192052	1	5.07E-05	6.62E-17

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.419425445	0.558331937	1	0.252352547	1.16E-16
8	0.30527556	0.431197869	1	0.107017758	3.60E-17
16	0.347238102	0.452647497	1	0.134871308	3.35E-17
32	0.359576851	0.383458149	1	0.113281386	7.34E-17
64	0.427865739	0.417685542	1	0.082670052	7.24E-17
128	0.446114735	0.365192052	1	0.047203264	6.62E-17

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.17114094	0.558331937	1	5.03E-17	1.16E-16
8	0.08557047	0.431197869	1	1.70E-17	3.60E-17
16	0.042785235	0.452647497	1	8.11E-18	3.35E-17
32	0.021392617	0.383458149	1	4.65E-18	7.34E-17
64	0.010696309	0.417685542	1	2.30E-18	7.24E-17
128	0.005348154	0.365192052	1	1.12E-18	6.62E-17

Smallest Face Ratio Results

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.089548404	0.246301227	1	0.157494727	0
8	0.069733857	0.09338135	1	0.051058807	0
16	0.053331216	0.040181471	1	0.039261721	0
32	0.036803582	0.042319339	1	0.020127846	0
64	0.025263957	0.02228519	1	0.012401853	0
128	0.019096646	0.019324674	1	0.009545919	0

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.07125096	0.246301227	1	0.019216557	0
8	0.032134406	0.09338135	1	0.00358764	0
16	0.015177063	0.040181471	1	0.001554242	0
32	0.007467599	0.042319339	1	0.000502208	0
64	0.00368879	0.02228519	1	0.000165241	0
128	0.001819661	0.019324674	1	6.12E-05	0

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.250159103	0.246301227	1	0.019435602	0
8	0.128212932	0.09338135	1	0.01157882	0
16	0.052870124	0.040181471	1	0.004644314	0
32	0.059359429	0.042319339	1	0.002619588	0
64	0.017951174	0.02228519	1	0.00083337	0
128	0.018720924	0.019324674	1	0.000642424	0

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.076022369	0.246301227	1	0.00987042	0
8	0.034223831	0.09338135	1	0.003975929	0
16	0.016100243	0.040181471	1	0.000896898	0
32	0.007910011	0.042319339	1	0.000285897	0
64	0.003866293	0.02228519	1	0.000100198	0
128	0.001909269	0.019324674	1	3.00E-05	0

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.262017055	0.246301227	1	0.009717575	0
8	0.037680168	0.09338135	1	0.005450371	0
16	0.043626154	0.040181471	1	0.004708211	0
32	0.021694186	0.042319339	1	0.001381535	0
64	0.023105691	0.02228519	1	0.002109086	0
128	0.007629214	0.019324674	1	0.000185524	0

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.250159103	0.246301227	1	0.019435602	0
8	0.128024865	0.09338135	1	0.011572099	0
16	0.099963333	0.040181471	1	0.01434114	0
32	0.081717772	0.042319339	1	0.006225975	0
64	0.056324323	0.02228519	1	0.008266268	0
128	0.028710745	0.019324674	1	0.001518638	0

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.100303091	0.246301227	1	0.231980622	0
8	0.065248787	0.09338135	1	0.074609081	0

16	0.045723357	0.040181471	1	0.036413832	0
32	0.032058591	0.042319339	1	0.027114752	0
64	0.021719944	0.02228519	1	0.014115965	0
128	0.016358671	0.019324674	1	0.009139158	0

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.07125096	0.246301227	1	0.019216557	0
8	0.032134406	0.09338135	1	0.00358764	0
16	0.015177063	0.040181471	1	0.001554242	0
32	0.007467599	0.042319339	1	0.000502208	0
64	0.00368879	0.02228519	1	0.000165241	0
128	0.001819661	0.019324674	1	6.12E-05	0

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.102397165	0.246301227	1	0.221476954	0
8	0.072896569	0.09338135	1	0.0645962	0
16	0.055642591	0.040181471	1	0.045007752	0
32	0.038387676	0.042319339	1	0.026959298	0
64	0.027029809	0.02228519	1	0.016005034	0
128	0.020839206	0.019324674	1	0.009145062	0

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.03566097	0.246301227	1	0.006745564	0
8	0.016444708	0.09338135	1	0.00188655	0
16	0.007673022	0.040181471	1	0.000390997	0
32	0.003757485	0.042319339	1	8.07E-05	0
64	0.001877199	0.02228519	1	3.73E-05	0
128	0.000937058	0.019324674	1	1.18E-05	0

FOCS results

	N=128	N=64	N=32	N=16	N=8	N=4
EV	1.078119	1.077706	1.079582	1.074413	1.069457	1.061976
UV	1.24188	1.227534	1.201505	1.190031	1.159045	1.109376
AFR	0.660129	0.596604	0.547481	0.508084	0.429604	0.558228
SFR	0.075507	0.100157	0.136544	0.146652	0.158607	0.26203

D Results – Israel Map with hotspot noise parameter 0.6

Egalitarian Results

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.047171454	0.867909769	1.04740747	1	0.10797366	0.207089	0.109486
8	1.050337931	0.762026919	1.052163436	1	0.055327922	0.161406	0.064276
16	1.050992933	0.713384134	1.053799669	1	0.050409485	0.139346	0.050105
32	1.046418729	0.681156111	1.04940038	1	0.041657254	0.077031	0.040591
64	1.040787576	0.652985937	1.041699097	1	0.032226271	0.042059	0.043842
128	1.033264428	0.638963218	1.033703657	1	0.027346122	0.015662	0.029986

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.072390433	0.867909769	1.072390433	1	0.136791462	0.207089	0.112176
8	1.066450078	0.762026919	1.066892974	1	0.061010188	0.161406	0.065416
16	1.078195774	0.713384134	1.080353511	1	0.055615879	0.139346	0.037999
32	1.069837447	0.681156111	1.071042376	1	0.053850298	0.077031	0.029332
64	1.05943015	0.652985937	1.064151521	1	0.052426909	0.042059	0.022613
128	1.063912776	0.638963218	1.067902335	1	0.042766096	0.015662	0.015915

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.05549782	0.867909769	1.056489311	1	0.112566026	0.207089	0.112053
8	1.058643867	0.762026919	1.061151763	1	0.056769285	0.161406	0.056976
16	1.062092676	0.713384134	1.069434894	1	0.055444137	0.139346	0.044688
32	1.052785377	0.681156111	1.056315326	1	0.046465715	0.077031	0.039841
64	1.043471108	0.652985937	1.049200748	1	0.036038063	0.042059	0.043587
128	1.037881678	0.638963218	1.042996468	1	0.027456891	0.015662	0.024753

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.051677876	0.867909769	1.051677876	1	0.112899501	0.207089	0.111372
8	1.045818523	0.762026919	1.04694159	1	0.050364645	0.161406	0.064608
16	1.053205761	0.713384134	1.053978844	1	0.049091197	0.139346	0.033613
32	1.049560178	0.681156111	1.050092738	1	0.031632249	0.077031	0.028903
64	1.047351256	0.652985937	1.047609191	1	0.024298059	0.042059	0.028565
128	1.04405625	0.638963218	1.044903207	1	0.020653381	0.015662	0.01824

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.067786109	0.867909769	1.067786109	1	0.126059121	0.207089	0.075718
8	1.067009986	0.762026919	1.067659734	1	0.064429511	0.161406	0.055756
16	1.074593143	0.713384134	1.078795272	1	0.060669223	0.139346	0.041308
32	1.066320313	0.681156111	1.068380147	1	0.053372788	0.077031	0.04315
64	1.064239404	0.652985937	1.064884214	1	0.044672561	0.042059	0.029008
128	1.061489663	0.638963218	1.062452556	1	0.048127644	0.015662	0.023218

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.064325855	0.867909769	1.065791093	1	0.1355989	0.207089	0.112053
8	1.056036606	0.762026919	1.05754502	1	0.057001043	0.161406	0.056734
16	1.073659807	0.713384134	1.080814968	1	0.069715289	0.139346	0.04695
32	1.071243015	0.681156111	1.076778547	1	0.058728407	0.077031	0.039851
64	1.067010734	0.652985937	1.067270739	1	0.04465731	0.042059	0.041716
128	1.074694225	0.638963218	1.075665035	1	0.031394506	0.015662	0.024621

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.050569536	0.867909769	1.055037462	1	0.115820482	0.207089	0.107705
8	1.053489459	0.762026919	1.056171178	1	0.055483194	0.161406	0.068882
16	1.058644254	0.713384134	1.064740541	1	0.054885361	0.139346	0.049584
32	1.05189861	0.681156111	1.055191335	1	0.048918562	0.077031	0.035888
64	1.044954183	0.652985937	1.050381281	1	0.03902001	0.042059	0.039833
128	1.03772621	0.638963218	1.040816209	1	0.025998893	0.015662	0.02965

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.045702139	0.867909769	1.046407352	1	0.102125285	0.207089	0.112176
8	1.041740916	0.762026919	1.042096947	1	0.051634228	0.161406	0.065416
16	1.050450863	0.713384134	1.052874219	1	0.042444515	0.139346	0.037999
32	1.042103054	0.681156111	1.0429226	1	0.0435791	0.077031	0.029332
64	1.032839177	0.652985937	1.033068428	1	0.029665965	0.042059	0.022613
128	1.029849206	0.638963218	1.030270449	1	0.026000414	0.015662	0.015915

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.045702139	0.867909769	1.046407352	1	0.102125285	0.207089	0.107639
8	1.041740916	0.762026919	1.042096947	1	0.051634228	0.161406	0.06418
16	1.050450863	0.713384134	1.052874219	1	0.042444515	0.139346	0.049838
32	1.042103054	0.681156111	1.0429226	1	0.0435791	0.077031	0.040885
64	1.032839177	0.652985937	1.033068428	1	0.029665965	0.042059	0.032543
128	1.029849206	0.638963218	1.030270449	1	0.026000414	0.015662	0.030434

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.064325855	0.867909769	1.065791093	1	0.1355989	0.207089	0.052903
8	1.056335454	0.762026919	1.057962171	1	0.054846116	0.161406	0.031759
16	1.068682248	0.713384134	1.074354715	1	0.066440186	0.139346	0.026518
32	1.068898085	0.681156111	1.073011522	1	0.059214195	0.077031	0.018344
64	1.062191443	0.652985937	1.063774489	1	0.041304847	0.042059	0.015535
128	1.064794754	0.638963218	1.065506124	1	0.026907699	0.015662	0.011715

Utilitarian Results

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.100216726	1.005510464	1.100463582	1	0.117505673	0.184595762	0.143647306
8	1.139288816	0.99804579	1.14061028	1	0.067006358	0.101851059	0.071790088
16	1.168282241	0.999276963	1.170845613	1	0.041221786	0.072264932	0.060434518
32	1.187320046	1.000006204	1.189676602	1	0.027633384	0.042395695	0.053057999
64	1.196447386	1.002655579	1.198337511	1	0.019207869	0.03362939	0.034174172
128	1.206235032	0.99992717	1.207500645	1	0.010774357	0.02692978	0.025229038

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.122654312	1.005510464	1.122654312	1	0.132633244	0.184595762	0.150170737
8	1.163849501	0.99804579	1.164434637	1	0.078198302	0.101851059	0.072895866
16	1.199280563	0.999276963	1.202179906	1	0.048052896	0.072264932	0.055404386
32	1.224263973	1.000006204	1.227449391	1	0.031196883	0.042395695	0.049829479
64	1.239409416	1.002655579	1.244267539	1	0.025686338	0.03362939	0.029956371
128	1.255666	0.99992717	1.261252836	1	0.016671326	0.02692978	0.021119912

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.113413858	1.005510464	1.114157927	1	0.117795345	0.184595762	0.139213189
8	1.158625419	0.99804579	1.162148969	1	0.073669631	0.101851059	0.07398664
16	1.195069898	0.999276963	1.20064001	1	0.047151727	0.072264932	0.055440038
32	1.221474184	1.000006204	1.22650533	1	0.03252375	0.042395695	0.051841485
64	1.237174541	1.002655579	1.243154346	1	0.022163483	0.03362939	0.030991074
128	1.255328299	0.99992717	1.260852595	1	0.017980317	0.02692978	0.02268051

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.100766754	1.005510464	1.100766754	1	0.126695325	0.184595762	0.142603131
8	1.130216852	0.99804579	1.130720147	1	0.074181299	0.101851059	0.072416511
16	1.150900483	0.999276963	1.151863893	1	0.041002775	0.072264932	0.054587142
32	1.168827871	1.000006204	1.169480259	1	0.022920501	0.042395695	0.047711763
64	1.175619576	1.002655579	1.176186826	1	0.015842681	0.03362939	0.028197576
128	1.180289103	0.99992717	1.181038209	1	0.009977031	0.02692978	0.019731938

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.122223461	1.005510464	1.122223461	1	0.130278947	0.184595762	0.13508897
8	1.166467784	0.99804579	1.167035718	1	0.078381972	0.101851059	0.075934508
16	1.20173892	0.999276963	1.2046406	1	0.046238512	0.072264932	0.053598068
32	1.226976889	1.000006204	1.229732405	1	0.029992439	0.042395695	0.049556471
64	1.246162441	1.002655579	1.248873712	1	0.02246223	0.03362939	0.028832741
128	1.263427481	0.99992717	1.265933483	1	0.016476566	0.02692978	0.020584875

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.116965688	1.005510464	1.119186748	1	0.13871314	0.184595762	0.139213189
8	1.153317177	0.99804579	1.155815547	1	0.080317096	0.101851059	0.074186259
16	1.193145602	0.999276963	1.199286072	1	0.047547368	0.072264932	0.056503365
32	1.221138979	1.000006204	1.22597828	1	0.029477959	0.042395695	0.051430645
64	1.240327887	1.002655579	1.244629691	1	0.020928758	0.03362939	0.032964393
128	1.257773688	0.99992717	1.263559342	1	0.01467776	0.02692978	0.023123243

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.106356361	1.005510464	1.109869969	1	0.116823906	0.184595762	0.141891552
8	1.153172065	0.99804579	1.157059351	1	0.070421551	0.101851059	0.072704453
16	1.18928305	0.999276963	1.19598677	1	0.04803329	0.072264932	0.058921147
32	1.215643823	1.000006204	1.221361568	1	0.030943634	0.042395695	0.05192772
64	1.232639548	1.002655579	1.239766283	1	0.022113497	0.03362939	0.034469714
128	1.25089943	0.99992717	1.256901077	1	0.016580786	0.02692978	0.025229258

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.098308053	1.005510464	1.098740105	1	0.117688257	0.184595762	0.150170737
8	1.132818062	0.99804579	1.132938008	1	0.06769275	0.101851059	0.072895866
16	1.158711068	0.999276963	1.159595392	1	0.038179679	0.072264932	0.055901456
32	1.171767792	1.000006204	1.172687245	1	0.02822875	0.042395695	0.050124127
64	1.177089527	1.002655579	1.177984545	1	0.017149343	0.03362939	0.03002183
128	1.184375079	0.99992717	1.185092249	1	0.008673832	0.02692978	0.021231183

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.098308053	1.005510464	1.098740105	1	0.117688257	0.184595762	0.142939694
8	1.132818062	0.99804579	1.132938008	1	0.06769275	0.101851059	0.074611478
16	1.158711068	0.999276963	1.159595392	1	0.038179679	0.072264932	0.057987508
32	1.171767792	1.000006204	1.172687245	1	0.02822875	0.042395695	0.053116897
64	1.177089527	1.002655579	1.177984545	1	0.017149343	0.03362939	0.034254536
128	1.184375079	0.99992717	1.185092249	1	0.008673832	0.02692978	0.023469435

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.116965688	1.005510464	1.119186748	1	0.13871314	0.184595762	0.088347649
8	1.155583781	0.99804579	1.158345997	1	0.083224713	0.101851059	0.057412401
16	1.194296134	0.999276963	1.199573534	1	0.0481193	0.072264932	0.035585717
32	1.22172589	1.000006204	1.226700069	1	0.029314486	0.042395695	0.035122609
64	1.241487768	1.002655579	1.246253357	1	0.021417491	0.03362939	0.022107093
128	1.259444174	0.99992717	1.265765725	1	0.015923529	0.02692978	0.017031371

Nash Social Results

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.098815881	0.999290372	1.099066039	1	0.116238959	0.183671588	0.143647306
8	1.13607867	0.98795856	1.137448385	1	0.064999022	0.100385639	0.071790088
16	1.163997997	0.987344317	1.166668729	1	0.041806618	0.073369425	0.060434518
32	1.182865172	0.986826634	1.185308121	1	0.027918752	0.043412911	0.053057999
64	1.191597659	0.988989835	1.193534742	1	0.019968534	0.033826843	0.034174172
128	1.201175619	0.985407327	1.202463071	1	0.01080004	0.027274878	0.025229038

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.121046549	0.999290372	1.121046549	1	0.131576139	0.183671588	0.150170737
8	1.160414507	0.98795856	1.16102073	1	0.075992833	0.100385639	0.072895866
16	1.195298753	0.987344317	1.198348651	1	0.048209622	0.073369425	0.055404386
32	1.220181475	0.986826634	1.223457808	1	0.031529557	0.043412911	0.049829479
64	1.235092304	0.988989835	1.240045901	1	0.025923939	0.033826843	0.029956371
128	1.251198574	0.985407327	1.25688091	1	0.016747631	0.027274878	0.021119912

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.111383703	0.999290372	1.112134546	1	0.115950704	0.183671588	0.139213189
8	1.154833407	0.98795856	1.158464346	1	0.070135096	0.100385639	0.07398664
16	1.190918596	0.987344317	1.196775635	1	0.047256889	0.073369425	0.055440038
32	1.217094683	0.986826634	1.222298325	1	0.032348301	0.043412911	0.051841485
64	1.232557318	0.988989835	1.238811083	1	0.022273691	0.033826843	0.030991074
128	1.250591819	0.985407327	1.256329062	1	0.01774976	0.027274878	0.02268051

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.099337228	0.999290372	1.099337228	1	0.1252772	0.183671588	0.142603131
8	1.127331227	0.98795856	1.12785221	1	0.071693139	0.100385639	0.072416511
16	1.147947204	0.987344317	1.148958131	1	0.040949239	0.073369425	0.054587142
32	1.165502297	0.986826634	1.166173439	1	0.02297922	0.043412911	0.047711763
64	1.172177309	0.988989835	1.172756714	1	0.016513191	0.033826843	0.028197576
128	1.17687372	0.985407327	1.177616132	1	0.010211316	0.027274878	0.019731938

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.120395739	0.999290372	1.120395739	1	0.129053598	0.183671588	0.13508897
8	1.162725709	0.98795856	1.163311952	1	0.075836267	0.100385639	0.075934508
16	1.197553944	0.987344317	1.200631971	1	0.04661793	0.073369425	0.053598068
32	1.222740633	0.986826634	1.225602002	1	0.029872189	0.043412911	0.049556471
64	1.241663992	0.988989835	1.244453661	1	0.023080453	0.033826843	0.028832741
128	1.259078066	0.985407327	1.261684589	1	0.016623516	0.027274878	0.020584875

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.115466731	0.999290372	1.117719167	1	0.137356931	0.183671588	0.139213189
8	1.150108564	0.98795856	1.152694088	1	0.078177517	0.100385639	0.074186259
16	1.189117339	0.987344317	1.195346567	1	0.04844118	0.073369425	0.056503365
32	1.217130537	0.986826634	1.222125611	1	0.030217568	0.043412911	0.051430645
64	1.236092362	0.988989835	1.240383637	1	0.021842852	0.033826843	0.032964393
128	1.253678615	0.985407327	1.259482607	1	0.015029797	0.027274878	0.023123243

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.104366331	0.999290372	1.107928951	1	0.114377127	0.183671588	0.141891552
8	1.14937607	0.98795856	1.153362645	1	0.066764998	0.100385639	0.072704453
16	1.185454501	0.987344317	1.192398525	1	0.048064725	0.073369425	0.058921147
32	1.211626516	0.986826634	1.217437245	1	0.03064118	0.043412911	0.05192772
64	1.228455045	0.988989835	1.235775897	1	0.022484536	0.033826843	0.034469714
128	1.24659101	0.985407327	1.252756519	1	0.016546092	0.027274878	0.025229258

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.096831484	0.999290372	1.097273951	1	0.116205683	0.183671588	0.150170737
8	1.129425185	0.98795856	1.129555629	1	0.065162634	0.100385639	0.072895866
16	1.15425465	0.987344317	1.155198372	1	0.0383712	0.073369425	0.055901456
32	1.167160316	0.986826634	1.168126428	1	0.028503416	0.043412911	0.050124127
64	1.172040899	0.988989835	1.172975407	1	0.018287777	0.033826843	0.03002183
128	1.179154578	0.985407327	1.179881326	1	0.009243022	0.027274878	0.021231183

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.096831484	0.999290372	1.097273951	1	0.116205683	0.183671588	0.142939694
8	1.129425185	0.98795856	1.129555629	1	0.065162634	0.100385639	0.074611478
16	1.15425465	0.987344317	1.155198372	1	0.0383712	0.073369425	0.057987508
32	1.167160316	0.986826634	1.168126428	1	0.028503416	0.043412911	0.053116897
64	1.172040899	0.988989835	1.172975407	1	0.018287777	0.033826843	0.034254536
128	1.179154578	0.985407327	1.179881326	1	0.009243022	0.027274878	0.023469435

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.115466731	0.999290372	1.117719167	1	0.137356931	0.183671588	0.088347649
8	1.152146454	0.98795856	1.154999685	1	0.080542541	0.100385639	0.057412401
16	1.190190779	0.987344317	1.195690667	1	0.048520308	0.073369425	0.035585717
32	1.21755311	0.986826634	1.222671286	1	0.029831358	0.043412911	0.035122609
64	1.23702977	0.988989835	1.241859935	1	0.022185123	0.033826843	0.022107093
128	1.254955077	0.985407327	1.261369342	1	0.016076421	0.027274878	0.017031371

Largest Envy Results

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.033721247	1.375403855	1.032980774	1	0.127184388	0.452322282	0.143647306
8	1.069678505	1.62629901	1.069143325	1	0.103670286	0.30793347	0.071790088
16	1.134759791	1.777080054	1.132718676	1	0.10304375	0.214890882	0.060434518
32	1.213987214	1.86980781	1.211350532	1	0.134317096	0.050581946	0.053057999
64	1.332292227	1.917497967	1.330820703	1	0.132062656	0.038505936	0.034174172
128	1.363531218	1.953914662	1.356143812	1	0.129150049	0.013993133	0.025229038

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.017811597	1.375403855	1.017811597	1	0.092858565	0.452322282	0.150170737
8	1.082327362	1.62629901	1.081370922	1	0.145201959	0.30793347	0.072895866
16	1.152300236	1.777080054	1.146416909	1	0.132562485	0.214890882	0.055404386
32	1.231975345	1.86980781	1.230377446	1	0.155633345	0.050581946	0.049829479
64	1.274306436	1.917497967	1.26895108	1	0.131201196	0.038505936	0.029956371
128	1.341758185	1.953914662	1.332451646	1	0.171534533	0.013993133	0.021119912

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.03468656	1.375403855	1.032238765	1	0.152368474	0.452322282	0.139213189
8	1.091227245	1.62629901	1.086019601	1	0.141018071	0.30793347	0.07398664
16	1.175765024	1.777080054	1.162960312	1	0.149427618	0.214890882	0.055440038
32	1.261615142	1.86980781	1.256039451	1	0.11070413	0.050581946	0.051841485
64	1.311919819	1.917497967	1.302569154	1	0.094075049	0.038505936	0.030991074
128	1.369283112	1.953914662	1.363970667	1	0.102462581	0.013993133	0.02268051

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.030405801	1.375403855	1.030405801	1	0.125082135	0.452322282	0.142603131
8	1.079440905	1.62629901	1.078994515	1	0.125292882	0.30793347	0.072416511
16	1.097822101	1.777080054	1.097322807	1	0.090936574	0.214890882	0.054587142
32	1.128980671	1.86980781	1.128668304	1	0.086396654	0.050581946	0.047711763
64	1.137949394	1.917497967	1.13780536	1	0.072714877	0.038505936	0.028197576
128	1.183852417	1.953914662	1.18331357	1	0.146742182	0.013993133	0.019731938

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.017253477	1.375403855	1.017253477	1	0.086556676	0.452322282	0.13508897
8	1.071906931	1.62629901	1.071252649	1	0.120068665	0.30793347	0.075934508
16	1.134108807	1.777080054	1.126864639	1	0.125960137	0.214890882	0.053598068
32	1.218384477	1.86980781	1.214597713	1	0.135040886	0.050581946	0.049556471
64	1.235784781	1.917497967	1.232790684	1	0.103809545	0.038505936	0.028832741
128	1.278363116	1.953914662	1.277124173	1	0.086630627	0.013993133	0.020584875

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.035899317	1.375403855	1.033318489	1	0.140054522	0.452322282	0.139213189
8	1.088577187	1.62629901	1.086131324	1	0.133405972	0.30793347	0.074186259
16	1.143995947	1.777080054	1.133876653	1	0.130497549	0.214890882	0.056503365
32	1.182773129	1.86980781	1.17627196	1	0.110227832	0.050581946	0.051430645
64	1.22969332	1.917497967	1.228939808	1	0.080367008	0.038505936	0.032964393
128	1.248473338	1.953914662	1.245091519	1	0.054856091	0.013993133	0.023123243

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.054134091	1.375403855	1.047032827	1	0.178750653	0.452322282	0.141891552
8	1.097576227	1.62629901	1.092259553	1	0.121331734	0.30793347	0.072704453
16	1.185909879	1.777080054	1.172087396	1	0.148778204	0.214890882	0.058921147
32	1.256265863	1.86980781	1.252164012	1	0.113819507	0.050581946	0.05192772
64	1.299196941	1.917497967	1.292194788	1	0.096553452	0.038505936	0.034469714
128	1.323839893	1.953914662	1.320087004	1	0.092081976	0.013993133	0.025229258

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.030345684	1.375403855	1.028951857	1	0.125116971	0.452322282	0.150170737
8	1.069138172	1.62629901	1.068774329	1	0.086316795	0.30793347	0.072895866
16	1.115395341	1.777080054	1.113460942	1	0.107939775	0.214890882	0.055901456
32	1.175808977	1.86980781	1.174890741	1	0.112822401	0.050581946	0.050124127
64	1.21745374	1.917497967	1.21649023	1	0.095311601	0.038505936	0.03002183
128	1.231202691	1.953914662	1.230499561	1	0.087452873	0.013993133	0.021231183

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.030345684	1.375403855	1.028951857	1	0.125116971	0.452322282	0.142939694
8	1.069138172	1.62629901	1.068774329	1	0.086316795	0.30793347	0.074611478
16	1.115395341	1.777080054	1.113460942	1	0.107939775	0.214890882	0.057987508
32	1.175808977	1.86980781	1.174890741	1	0.112822401	0.050581946	0.053116897
64	1.21745374	1.917497967	1.21649023	1	0.095311601	0.038505936	0.034254536
128	1.231202691	1.953914662	1.230499561	1	0.087452873	0.013993133	0.023469435

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.035899317	1.375403855	1.033318489	1	0.140054522	0.452322282	0.088347649
8	1.092890768	1.62629901	1.090499545	1	0.130809266	0.30793347	0.057412401
16	1.141478267	1.777080054	1.136764386	1	0.120251661	0.214890882	0.035585717
32	1.202828697	1.86980781	1.190886495	1	0.114554592	0.050581946	0.035122609
64	1.240457191	1.917497967	1.238094741	1	0.090160296	0.038505936	0.022107093
128	1.296575716	1.953914662	1.294335523	1	0.102235513	0.013993133	0.017031371

Average Face Ratio Results

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.49311459	0.710369178	1	0.025391296	0
8	0.247366319	0.430989821	1	0.008780507	5.74E-17
16	0.123133303	0.452422309	1	0.003013886	3.71E-17
32	0.078216788	0.416180016	1	0.00373289	5.49E-17
64	0.041455667	0.452487257	1	0.005469185	6.95E-17
128	0.025473436	0.437569916	1	0.011652393	2.71E-17

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.620359452	0.710369178	1	0.434010789	0
8	0.444044518	0.430989821	1	0.165142043	5.74E-17
16	0.395378593	0.452422309	1	0.107198109	3.71E-17
32	0.412689381	0.416180016	1	0.095412791	5.49E-17
64	0.433918233	0.452487257	1	0.069134913	6.95E-17
128	0.48023792	0.437569916	1	0.055635562	2.71E-17

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.547380398	0.710369178	1	0.517697969	0
8	0.476597346	0.430989821	1	0.134028205	5.74E-17
16	0.41381753	0.452422309	1	0.090647017	3.71E-17
32	0.455264542	0.416180016	1	0.093790921	5.49E-17
64	0.447198577	0.452487257	1	0.054304969	6.95E-17
128	0.512328514	0.437569916	1	0.038616024	2.71E-17

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.2875	0.710369178	1	6.19E-17	0
8	0.14375	0.430989821	1	2.55E-17	5.74E-17
16	0.071875	0.452422309	1	1.28E-17	3.71E-17
32	0.0359375	0.416180016	1	6.42E-18	5.49E-17
64	0.01796875	0.452487257	1	3.50E-18	6.95E-17
128	0.008984375	0.437569916	1	1.84E-18	2.71E-17

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.548631082	0.710369178	1	0.531040094	0
8	0.44098498	0.430989821	1	0.171107016	5.74E-17
16	0.397675221	0.452422309	1	0.129389478	3.71E-17
32	0.398048173	0.416180016	1	0.101587192	5.49E-17
64	0.400629233	0.452487257	1	0.077898516	6.95E-17
128	0.440630792	0.437569916	1	0.059316939	2.71E-17

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.70897214	0.710369178	1	0.033798968	0
8	0.421980149	0.430989821	1	0.008116294	5.74E-17
16	0.485733787	0.452422309	1	0.019908289	3.71E-17
32	0.345829421	0.416180016	1	0.01343597	5.49E-17
64	0.445011967	0.452487257	1	0.007520857	6.95E-17
128	0.381676692	0.437569916	1	0.008380119	2.71E-17

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.657403343	0.710369178	1	0.081289754	0
8	0.4731993	0.430989821	1	0.027578701	5.74E-17
16	0.447456567	0.452422309	1	0.013160032	3.71E-17
32	0.431009172	0.416180016	1	0.01415485	5.49E-17
64	0.415627678	0.452487257	1	0.008754382	6.95E-17
128	0.432232198	0.437569916	1	0.007405058	2.71E-17

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.217391304	0.710369178	1	5.89E-17	0
8	0.108695652	0.430989821	1	2.06E-17	5.74E-17
16	0.054347826	0.452422309	1	1.02E-17	3.71E-17
32	0.027173913	0.416180016	1	4.69E-18	5.49E-17
64	0.013586957	0.452487257	1	2.39E-18	6.95E-17
128	0.006793478	0.437569916	1	1.25E-18	2.71E-17

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.217391304	0.710369178	1	5.89E-17	0
8	0.108695652	0.430989821	1	2.06E-17	5.74E-17
16	0.054347826	0.452422309	1	1.02E-17	3.71E-17
32	0.027173913	0.416180016	1	4.69E-18	5.49E-17
64	0.013586957	0.452487257	1	2.39E-18	6.95E-17
128	0.006793478	0.437569916	1	1.25E-18	2.71E-17

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.70897214	0.710369178	1	0.033798968	0
8	0.528658572	0.430989821	1	0.011533309	5.74E-17
16	0.5400947	0.452422309	1	0.034335896	3.71E-17
32	0.526794767	0.416180016	1	0.019936627	5.49E-17
64	0.542273308	0.452487257	1	0.007955749	6.95E-17
128	0.577943794	0.437569916	1	0.008213042	2.71E-17

Smallest Face Ratio Results

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.101090971	0.535998851	1	0.02092075	0
8	0.047160109	0.149361388	1	0.006598296	0
16	0.021879371	0.168974985	1	0.002726005	0
32	0.010412482	0.085870271	1	0.000698483	0
64	0.005033845	0.044461304	1	0.000198875	0
128	0.002485256	0.037598358	1	6.54E-05	0

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.421310643	0.535998851	1	0.634070506	0
8	0.111997215	0.149361388	1	0.061406084	0
16	0.0824523	0.168974985	1	0.048993777	0
32	0.044674321	0.085870271	1	0.026180695	0
64	0.033110268	0.044461304	1	0.01551822	0
128	0.02462357	0.037598358	1	0.008118926	0

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.329866728	0.535998851	1	0.72481219	0
8	0.109677037	0.149361388	1	0.047752696	0
16	0.074672079	0.168974985	1	0.042291168	0
32	0.050948089	0.085870271	1	0.019701098	0
64	0.032657842	0.044461304	1	0.014802571	0
128	0.026354267	0.037598358	1	0.010072393	0

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.087904847	0.535998851	1	0.011653019	0
8	0.037705233	0.149361388	1	0.004605878	0
16	0.016456806	0.168974985	1	0.001877751	0
32	0.007697522	0.085870271	1	0.000544471	0
64	0.003683846	0.044461304	1	0.000198259	0
128	0.001777502	0.037598358	1	6.61E-05	0

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.305788618	0.535998851	1	0.711663251	0
8	0.10493008	0.149361388	1	0.081694442	0
16	0.06938166	0.168974985	1	0.051523446	0
32	0.042433691	0.085870271	1	0.023940844	0
64	0.028734939	0.044461304	1	0.015460984	0
128	0.020033188	0.037598358	1	0.009724247	0

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.53510954	0.535998851	1	0.046215558	0
8	0.120037106	0.149361388	1	0.029529945	0
16	0.160942866	0.168974985	1	0.020204301	0
32	0.038618839	0.085870271	1	0.006158118	0
64	0.045805795	0.044461304	1	0.00448916	0
128	0.020724693	0.037598358	1	0.000872373	0

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.508769176	0.535998851	1	0.175080337	0
8	0.09608054	0.149361388	1	0.012314194	0
16	0.089275057	0.168974985	1	0.007439913	0
32	0.053656322	0.085870271	1	0.004168455	0
64	0.043478529	0.044461304	1	0.002306118	0
128	0.020677962	0.037598358	1	0.001276875	0

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.080236092	0.535998851	1	0.012515535	0
8	0.036325847	0.149361388	1	0.003869921	0
16	0.017128368	0.168974985	1	0.001562136	0
32	0.008122387	0.085870271	1	0.000425488	0
64	0.003908587	0.044461304	1	0.000144355	0
128	0.001919777	0.037598358	1	4.89E-05	0

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.080236092	0.535998851	1	0.012515535	0
8	0.036325847	0.149361388	1	0.003869921	0
16	0.017128368	0.168974985	1	0.001562136	0
32	0.008122387	0.085870271	1	0.000425488	0
64	0.003908587	0.044461304	1	0.000144355	0
128	0.001919777	0.037598358	1	4.89E-05	0

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.53510954	0.535998851	1	0.046215558	0
8	0.148294262	0.149361388	1	0.021119146	0
16	0.047692262	0.168974985	1	0.002971111	0
32	0.043290964	0.085870271	1	0.010725194	0
64	0.046297149	0.044461304	1	0.004121397	0
128	0.028493426	0.037598358	1	0.001817923	0

FOCS results

	N=128	N=64	N=32	N=16	N=8	N=4
EV	1.104844745	1.102182038	1.100750626	1.101590317	1.07836701	1.074754064
UV	1.270886029	1.252568696	1.235147724	1.208665292	1.170101594	1.123994012
AFR	0.649518204	0.588173021	0.581723495	0.559946568	0.566060559	0.70897214
SFR	0.110003436	0.099760852	0.105724072	0.170326446	0.155341862	0.54637376

E Results – Random Map with hotspot noise parameter 0.6

Egalitarian Results

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.092391614	0.905788271	1.092473173	1	0.135906616	0.118475699	0.135368004
8	1.121840767	0.875756194	1.121840767	1	0.104645156	0.031106045	0.104645156
16	1.137264015	0.867342753	1.138430613	1	0.137533114	0.012128696	0.137646807
32	1.175609971	0.864587828	1.176670146	1	0.137240577	0.001545496	0.13671425
64	1.221236608	0.86436122	1.225648358	1	0.134454964	5.98E-05	0.133328011
128	1.281843454	0.864346622	1.286987942	1	0.117760443	4.37E-05	0.113981528

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.088427022	0.905788271	1.088807238	1	0.132755526	0.118475699	0.131463393
8	1.116963326	0.875756194	1.118508138	1	0.113354693	0.031106045	0.108979285
16	1.133596869	0.867342753	1.138206509	1	0.134329434	0.012128696	0.136299695
32	1.165390897	0.864587828	1.166622353	1	0.132014262	0.001545496	0.130677257
64	1.22093063	0.86436122	1.222180855	1	0.135616552	5.98E-05	0.137723135
128	1.279118388	0.864346622	1.28634904	1	0.122530744	4.37E-05	0.109464273

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.062577173	0.905788271	1.063592039	1	0.12239929	0.118475699	0.121127922
8	1.063485723	0.875756194	1.065103434	1	0.092127998	0.031106045	0.088300284
16	1.059771059	0.867342753	1.061041055	1	0.067195337	0.012128696	0.065011067
32	1.074082685	0.864587828	1.074654846	1	0.0769143	0.001545496	0.075839254
64	1.079634548	0.86436122	1.079746137	1	0.060223442	5.98E-05	0.060096951
128	1.0862731	0.864346622	1.086309274	1	0.057626913	4.37E-05	0.057650282

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.046914816	0.905788271	1.046914816	1	0.108945218	0.118475699	0.108945218
8	1.040772212	0.875756194	1.041272071	1	0.057667933	0.031106045	0.056735478
16	1.04756749	0.867342753	1.04756749	1	0.049408105	0.012128696	0.049408105
32	1.054017374	0.864587828	1.054017374	1	0.041305758	0.001545496	0.041305758
64	1.053682993	0.86436122	1.053700914	1	0.036118423	5.98E-05	0.036163786
128	1.054451814	0.864346622	1.054592496	1	0.032016554	4.37E-05	0.031694896

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.071990607	0.905788271	1.073644476	1	0.120909757	0.118475699	0.119907956
8	1.099828604	0.875756194	1.105366877	1	0.105510309	0.031106045	0.110242585
16	1.130537658	0.867342753	1.135286228	1	0.132317452	0.012128696	0.129581212
32	1.162230126	0.864587828	1.16623995	1	0.12420954	0.001545496	0.116701847
64	1.219093239	0.86436122	1.222279666	1	0.137257006	5.98E-05	0.138816533
128	1.278323247	0.864346622	1.286748924	1	0.117512384	4.37E-05	0.109445841

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.07678656	0.905788271	1.07742297	1	0.140754444	0.118475699	0.139947087
8	1.093977621	0.875756194	1.096442538	1	0.111580293	0.031106045	0.111129492
16	1.107364219	0.867342753	1.110799316	1	0.119934978	0.012128696	0.12012153
32	1.157616066	0.864587828	1.160197278	1	0.132196032	0.001545496	0.130022112
64	1.203424956	0.86436122	1.207193651	1	0.141245435	5.98E-05	0.141485394
128	1.268973617	0.864346622	1.27577298	1	0.119741329	4.37E-05	0.110567651

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.071990607	0.905788271	1.073644476	1	0.120909757	0.118475699	0.119907956
8	1.099828604	0.875756194	1.105366877	1	0.105510309	0.031106045	0.110242585
16	1.130537658	0.867342753	1.135286228	1	0.132317452	0.012128696	0.129581212
32	1.162230126	0.864587828	1.16623995	1	0.12420954	0.001545496	0.116701847
64	1.219093239	0.86436122	1.222349502	1	0.137257006	5.98E-05	0.13857582
128	1.278323247	0.864346622	1.286748924	1	0.117512384	4.37E-05	0.109445841

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.090573145	0.905788271	1.090573145	1	0.127264794	0.118475699	0.127264794
8	1.121862181	0.875756194	1.12213089	1	0.115210618	0.031106045	0.114548281
16	1.144283326	0.867342753	1.144355278	1	0.115479251	0.012128696	0.115182102
32	1.185039241	0.864587828	1.185583921	1	0.115992368	0.001545496	0.114831892
64	1.22066773	0.86436122	1.22066773	1	0.143428434	5.98E-05	0.143428434
128	1.296804397	0.864346622	1.297986891	1	0.109612583	4.37E-05	0.10901314

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.049257221	0.905788271	1.049257221	1	0.117504924	0.118475699	0.117504924
8	1.047088456	0.875756194	1.047701338	1	0.062594405	0.031106045	0.061414913
16	1.056055577	0.867342753	1.059511807	1	0.057190362	0.012128696	0.060165035
32	1.060858557	0.864587828	1.061947819	1	0.051193163	0.001545496	0.05401433
64	1.062145411	0.86436122	1.064051009	1	0.0444119	5.98E-05	0.045977086
128	1.060071851	0.864346622	1.060890027	1	0.038514481	4.37E-05	0.037517566

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.065608655	0.905788271	1.066108057	1	0.125932764	0.118475699	0.124689248
8	1.079336825	0.875756194	1.080091969	1	0.097871188	0.031106045	0.098515645
16	1.117925548	0.867342753	1.125259907	1	0.09618327	0.012128696	0.10161368
32	1.16163669	0.864587828	1.164323333	1	0.113295187	0.001545496	0.109362067
64	1.201865655	0.86436122	1.202290233	1	0.133084516	5.98E-05	0.132678764
128	1.25629186	0.864346622	1.261826708	1	0.123380672	4.37E-05	0.12017635

Utilitarian Results

MostResultsableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.148829233	1.006338564	1.15005188	1	0.118602999	0.2371521	0.114753845
8	1.253185262	0.997960364	1.253343033	1	0.097582292	0.114129574	0.097813279
16	1.35719311	1.010631558	1.358916898	1	0.07268794	0.09620778	0.069656273
32	1.447067057	1.00093302	1.448198375	1	0.05549952	0.069928652	0.053040401
64	1.509527865	0.998050924	1.510826747	1	0.03653134	0.042570732	0.035259809
128	1.564438472	0.996163241	1.565699637	1	0.020367311	0.033148873	0.019838898

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.145045423	1.006338564	1.147097745	1	0.120587555	0.2371521	0.115445553
8	1.249783376	0.997960364	1.25105827	1	0.097063368	0.114129574	0.094981678
16	1.354047939	1.010631558	1.356966349	1	0.080717204	0.09620778	0.075643444
32	1.443666666	1.00093302	1.445069462	1	0.056289188	0.069928652	0.055293682
64	1.507960697	0.998050924	1.509777876	1	0.036162696	0.042570732	0.036195478
128	1.562797778	0.996163241	1.564394386	1	0.020586349	0.033148873	0.020169446

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.116744136	1.006338564	1.11854456	1	0.125562318	0.2371521	0.12107353
8	1.164131225	0.997960364	1.165979529	1	0.083570954	0.114129574	0.076826988
16	1.189962558	1.010631558	1.190496552	1	0.034683806	0.09620778	0.032675739
32	1.21016515	1.00093302	1.210416866	1	0.029482102	0.069928652	0.028962168
64	1.217694706	0.998050924	1.217805095	1	0.020864258	0.042570732	0.02042372
128	1.225389407	0.996163241	1.225431997	1	0.011739859	0.033148873	0.011741196

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.080307255	1.006338564	1.080307255	1	0.108029125	0.2371521	0.108029125
8	1.098828933	0.997960364	1.099787231	1	0.047584513	0.114129574	0.044813794
16	1.123084955	1.010631558	1.123084955	1	0.029472874	0.09620778	0.029472874
32	1.1310096	1.00093302	1.131022614	1	0.015961004	0.069928652	0.01596404
64	1.13819394	0.998050924	1.138216156	1	0.008622171	0.042570732	0.008621548
128	1.139852603	0.996163241	1.139942533	1	0.005695317	0.033148873	0.005664643

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.126331376	1.006338564	1.129637083	1	0.13657847	0.2371521	0.131756395
8	1.231882223	0.997960364	1.236050744	1	0.103815798	0.114129574	0.106079156
16	1.343490021	1.010631558	1.350888644	1	0.080848442	0.09620778	0.071443758
32	1.433841915	1.00093302	1.43676814	1	0.056312951	0.069928652	0.053138497
64	1.500399883	0.998050924	1.503180464	1	0.034491189	0.042570732	0.036141337
128	1.557169752	0.996163241	1.560119553	1	0.021151744	0.033148873	0.02049314

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.130169962	1.006338564	1.130942869	1	0.142964527	0.2371521	0.142257187
8	1.226941602	0.997960364	1.228269586	1	0.117182535	0.114129574	0.113448488
16	1.335508156	1.010631558	1.339274601	1	0.085742601	0.09620778	0.081335815
32	1.429046108	1.00093302	1.430355155	1	0.062685293	0.069928652	0.062886058
64	1.49815216	0.998050924	1.500420314	1	0.038743041	0.042570732	0.037970005
128	1.554992993	0.996163241	1.5564647	1	0.021913907	0.033148873	0.021589805

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.126331376	1.006338564	1.129637083	1	0.13657847	0.2371521	0.131756395
8	1.231882223	0.997960364	1.236050744	1	0.103815798	0.114129574	0.106079156
16	1.343490021	1.010631558	1.350888644	1	0.080848442	0.09620778	0.071443758
32	1.433841915	1.00093302	1.43676814	1	0.056312951	0.069928652	0.053138497
64	1.50020999	0.998050924	1.503000464	1	0.034598597	0.042570732	0.036288038
128	1.557169752	0.996163241	1.560119553	1	0.021151744	0.033148873	0.02049314

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.148256437	1.006338564	1.148256437	1	0.118557834	0.2371521	0.118557834
8	1.252170095	0.997960364	1.25228169	1	0.099252224	0.114129574	0.099136305
16	1.353940152	1.010631558	1.356799	1	0.076966971	0.09620778	0.072124947
32	1.443496353	1.00093302	1.446786952	1	0.070095617	0.069928652	0.059669358
64	1.507480811	0.998050924	1.508374033	1	0.037912994	0.042570732	0.037712147
128	1.562820202	0.996163241	1.564282335	1	0.02192526	0.033148873	0.021429604

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.099883102	1.006338564	1.099883102	1	0.132566047	0.2371521	0.132566047
8	1.151373402	0.997960364	1.152821149	1	0.088264487	0.114129574	0.087052404
16	1.202552312	1.010631558	1.204623889	1	0.063173229	0.09620778	0.061439359
32	1.229644565	1.00093302	1.230627131	1	0.0366983	0.069928652	0.035532578
64	1.244352582	0.998050924	1.245419807	1	0.025787197	0.042570732	0.024456168
128	1.253648592	0.996163241	1.254025429	1	0.02212411	0.033148873	0.021695711

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.121657172	1.006338564	1.126515523	1	0.137949747	0.2371521	0.134094464
8	1.196844137	0.997960364	1.199367678	1	0.08832118	0.114129574	0.08867336
16	1.333135194	1.010631558	1.342426219	1	0.080449633	0.09620778	0.072017093
32	1.407304872	1.00093302	1.415078638	1	0.056542756	0.069928652	0.05125849
64	1.493546497	0.998050924	1.500412413	1	0.040758948	0.042570732	0.038104812
128	1.542169186	0.996163241	1.546344338	1	0.023155254	0.033148873	0.021337442

Nash Social Results

MostResultsableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.147459881	1.000842734	1.148668607	1	0.119406254	0.229000687	0.115257993
8	1.248757867	0.988573829	1.248926824	1	0.097099228	0.106056497	0.09731355
16	1.348284347	0.997207162	1.350194704	1	0.075118571	0.084371023	0.071620476
32	1.437635054	0.9864868	1.438849688	1	0.060494918	0.060407326	0.05776311
64	1.501506277	0.983122649	1.502875678	1	0.039585005	0.036792844	0.038144508
128	1.558220535	0.980856148	1.559509063	1	0.023433895	0.02839544	0.022900655

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.143698836	1.000842734	1.145730878	1	0.121132151	0.229000687	0.115656339
8	1.245333707	0.988573829	1.246612623	1	0.0973992	0.106056497	0.095280325
16	1.345620239	0.997207162	1.348730046	1	0.083040791	0.084371023	0.077728011
32	1.434134145	0.9864868	1.435642075	1	0.061263072	0.060407326	0.060034449
64	1.4999232	0.983122649	1.501780459	1	0.039393137	0.036792844	0.03943331
128	1.556590592	0.980856148	1.558222916	1	0.023694586	0.02839544	0.023183601

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.115308145	1.000842734	1.117112522	1	0.125420616	0.229000687	0.120795751
8	1.161052174	0.988573829	1.162853081	1	0.083487602	0.106056497	0.076945646
16	1.186144155	0.997207162	1.186709905	1	0.03620637	0.084371023	0.033959363
32	1.206996375	0.9864868	1.207270482	1	0.031580187	0.060407326	0.0309912
64	1.215226707	0.983122649	1.215344691	1	0.02255319	0.036792844	0.022082643
128	1.223410885	0.980856148	1.223453685	1	0.013207002	0.02839544	0.01320891

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.079542173	1.000842734	1.079542173	1	0.107885002	0.229000687	0.107885002
8	1.097313644	0.988573829	1.098285276	1	0.047880363	0.106056497	0.045044518
16	1.121574914	0.997207162	1.121574914	1	0.030331835	0.084371023	0.030331835
32	1.129933665	0.9864868	1.129946713	1	0.016965972	0.060407326	0.016968884
64	1.137313679	0.983122649	1.1373361	1	0.009250495	0.036792844	0.009249403
128	1.139056675	0.980856148	1.139147062	1	0.006130081	0.02839544	0.006098326

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.125133779	1.000842734	1.128417059	1	0.136229146	0.229000687	0.131139136
8	1.227349786	0.988573829	1.231565222	1	0.10293935	0.106056497	0.105441845
16	1.3353302	0.997207162	1.342776737	1	0.082980905	0.084371023	0.072908319
32	1.424494248	0.9864868	1.427481463	1	0.060830989	0.060407326	0.05754098
64	1.492451861	0.983122649	1.495271774	1	0.037677463	0.036792844	0.03951497
128	1.551039197	0.980856148	1.554006398	1	0.024133975	0.02839544	0.023445284

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.128892348	1.000842734	1.129681596	1	0.142730493	0.229000687	0.14202327
8	1.222235402	0.988573829	1.223570203	1	0.116457205	0.106056497	0.112711916
16	1.326712791	0.997207162	1.330527765	1	0.087156922	0.084371023	0.082463526
32	1.419552565	0.9864868	1.420940709	1	0.067343409	0.060407326	0.067489873
64	1.490048678	0.983122649	1.492390463	1	0.041829327	0.036792844	0.040824128
128	1.54875118	0.980856148	1.550241357	1	0.024924658	0.02839544	0.024589278

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.125133779	1.000842734	1.128417059	1	0.136229146	0.229000687	0.131139136
8	1.227349786	0.988573829	1.231565222	1	0.10293935	0.106056497	0.105441845
16	1.3353302	0.997207162	1.342776737	1	0.082980905	0.084371023	0.072908319
32	1.424494248	0.9864868	1.427481463	1	0.060830989	0.060407326	0.05754098
64	1.49229007	0.983122649	1.495120629	1	0.037826084	0.036792844	0.03968532
128	1.551039197	0.980856148	1.554006398	1	0.024133975	0.02839544	0.023445284

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.146998659	1.000842734	1.146998659	1	0.118557733	0.229000687	0.118557733
8	1.248031381	0.988573829	1.24815516	1	0.099222414	0.106056497	0.099086685
16	1.345628854	0.997207162	1.348430953	1	0.078783006	0.084371023	0.073750592
32	1.434127342	0.9864868	1.437309846	1	0.073917801	0.060407326	0.064148019
64	1.498305684	0.983122649	1.499215338	1	0.042090268	0.036792844	0.041852143
128	1.556516517	0.980856148	1.557985021	1	0.025378222	0.02839544	0.024828752

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.098691054	1.000842734	1.098691054	1	0.131767405	0.229000687	0.131767405
8	1.146613337	0.988573829	1.148101349	1	0.085296662	0.106056497	0.084006019
16	1.194927652	0.997207162	1.197000388	1	0.060859442	0.084371023	0.059118387
32	1.221605499	0.9864868	1.222611979	1	0.035385964	0.060407326	0.034111322
64	1.235860722	0.983122649	1.23694394	1	0.024103972	0.036792844	0.022781949
128	1.244614927	0.980856148	1.245017407	1	0.021135532	0.02839544	0.020649185

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.120337149	1.000842734	1.125031365	1	0.137524335	0.229000687	0.133003299
8	1.193125599	0.988573829	1.195730266	1	0.088541615	0.106056497	0.088748731
16	1.32476776	0.997207162	1.33403967	1	0.081916411	0.084371023	0.072918963
32	1.398831183	0.9864868	1.406746098	1	0.06063862	0.060407326	0.054572787
64	1.484596046	0.983122649	1.491136001	1	0.044659416	0.036792844	0.042147084
128	1.534936229	0.980856148	1.539065471	1	0.026608074	0.02839544	0.025012675

Largest Envy Results

MostResultsableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.012304289	1.375042598	1.010110192	1	0.125110538	0.287555918	0.105270735
8	1.066384701	1.57742097	1.066384701	1	0.15584096	0.070716861	0.15584096
16	1.243768728	1.678975255	1.242789401	1	0.315003133	0.020904999	0.314228841
32	1.326016517	1.751987776	1.324992267	1	0.365314682	0.009580305	0.364065947
64	1.353750289	1.809907406	1.350053955	1	0.256171045	0.005850012	0.252263737
128	1.310338139	1.85871503	1.307830307	1	0.200502912	0.001665898	0.197740679

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.019562344	1.375042598	1.015057682	1	0.128435349	0.287555918	0.105942155
8	1.093940929	1.57742097	1.089815607	1	0.180067256	0.070716861	0.178586376
16	1.260939784	1.678975255	1.253500021	1	0.329056888	0.020904999	0.327780672
32	1.342517446	1.751987776	1.339219157	1	0.346742566	0.009580305	0.342310667
64	1.358577828	1.809907406	1.356261775	1	0.282129607	0.005850012	0.280613818
128	1.315160007	1.85871503	1.309661872	1	0.217171641	0.001665898	0.198250698

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.024176738	1.375042598	1.020996462	1	0.120910819	0.287555918	0.115355292
8	1.074887129	1.57742097	1.066939963	1	0.197703982	0.070716861	0.164161413
16	1.082886542	1.678975255	1.080033723	1	0.116920405	0.020904999	0.110746015
32	1.093818359	1.751987776	1.093336964	1	0.109398291	0.009580305	0.109531688
64	1.087951622	1.809907406	1.087951622	1	0.11839554	0.005850012	0.11839554
128	1.079919368	1.85871503	1.079884506	1	0.090300764	0.001665898	0.090351214

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.009592664	1.375042598	1.009592664	1	0.050719164	0.287555918	0.050719164
8	1.037576421	1.57742097	1.03690957	1	0.079572608	0.070716861	0.078387657
16	1.04231999	1.678975255	1.04231999	1	0.046340597	0.020904999	0.046340597
32	1.037565327	1.751987776	1.037565327	1	0.041120753	0.009580305	0.041120753
64	1.031628129	1.809907406	1.031628129	1	0.032041647	0.005850012	0.032041647
128	1.027708626	1.85871503	1.027668361	1	0.024577031	0.001665898	0.024485306

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.039196378	1.375042598	1.032190807	1	0.194975516	0.287555918	0.175992222
8	1.161008417	1.57742097	1.144545458	1	0.230555134	0.070716861	0.247130323
16	1.273527738	1.678975255	1.253138758	1	0.312714787	0.020904999	0.278222418
32	1.364172119	1.751987776	1.359481179	1	0.369815077	0.009580305	0.363602547
64	1.370665946	1.809907406	1.366691546	1	0.273386297	0.005850012	0.274289621
128	1.327372801	1.85871503	1.324611451	1	0.213307209	0.001665898	0.208776508

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.034210037	1.375042598	1.032371835	1	0.171422747	0.287555918	0.170183216
8	1.118816217	1.57742097	1.114519343	1	0.297831114	0.070716861	0.290846773
16	1.254243426	1.678975255	1.244925145	1	0.285085442	0.020904999	0.28131422
32	1.31163974	1.751987776	1.309478051	1	0.311119893	0.009580305	0.309407111
64	1.355052562	1.809907406	1.348199476	1	0.242220794	0.005850012	0.232101923
128	1.313748698	1.85871503	1.307665708	1	0.211733245	0.001665898	0.193738474

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.039196378	1.375042598	1.032190807	1	0.194975516	0.287555918	0.175992222
8	1.161008417	1.57742097	1.144545458	1	0.230555134	0.070716861	0.247130323
16	1.273527738	1.678975255	1.253138758	1	0.312714787	0.020904999	0.278222418
32	1.364172119	1.751987776	1.359481179	1	0.369815077	0.009580305	0.363602547
64	1.370665946	1.809907406	1.366600384	1	0.273386297	0.005850012	0.274186974
128	1.327372801	1.85871503	1.324611451	1	0.213307209	0.001665898	0.208776508

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.009987651	1.375042598	1.009987651	1	0.092761631	0.287555918	0.092761631
8	1.071909972	1.57742097	1.071634951	1	0.151044684	0.070716861	0.150776593
16	1.258525597	1.678975255	1.255100739	1	0.292155606	0.020904999	0.291792282
32	1.32166751	1.751987776	1.317663172	1	0.356915257	0.009580305	0.349651791
64	1.38716543	1.809907406	1.38716543	1	0.259153342	0.005850012	0.259153342
128	1.332221165	1.85871503	1.332063549	1	0.230230027	0.001665898	0.230146367

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.037230278	1.375042598	1.037230278	1	0.196290431	0.287555918	0.196290431
8	1.124834808	1.57742097	1.122347866	1	0.222414169	0.070716861	0.22675056
16	1.230260053	1.678975255	1.218184234	1	0.250849295	0.020904999	0.255509923
32	1.263135719	1.751987776	1.257769256	1	0.216052396	0.009580305	0.222192951
64	1.290700712	1.809907406	1.287062183	1	0.235461669	0.005850012	0.238871932
128	1.304094851	1.85871503	1.30404141	1	0.186459262	0.001665898	0.186168245

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.063403826	1.375042598	1.056370924	1	0.250801581	0.287555918	0.227593594
8	1.12011993	1.57742097	1.117142799	1	0.252442463	0.070716861	0.253256253
16	1.303906967	1.678975255	1.29408793	1	0.253494534	0.020904999	0.263093698
32	1.293374379	1.751987776	1.283858706	1	0.229982685	0.009580305	0.218108746
64	1.408576433	1.809907406	1.405087247	1	0.246487701	0.005850012	0.241918577
128	1.389292906	1.85871503	1.380343028	1	0.20277534	0.001665898	0.202348392

Average Face Ratio Results

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.513647376	0.684563776	1	0.4514019	0
8	0.571051014	0.730392711	1	0.296749024	1.13E-16
16	0.581266271	0.684564094	1	0.124809714	1.20E-16
32	0.591689081	0.730394497	1	0.101475484	0
64	0.558912167	0.6845645	1	0.06482043	7.28E-17
128	0.573158539	0.730406013	1	0.047559852	1.32E-16

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.563860864	0.684563776	1	0.402526666	0
8	0.624360537	0.730392711	1	0.225847827	1.13E-16
16	0.627567377	0.684564094	1	0.095341563	1.20E-16
32	0.627473514	0.730394497	1	0.078487797	0
64	0.592641511	0.6845645	1	0.052527046	7.28E-17
128	0.62411035	0.730406013	1	0.049371371	1.32E-16

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.365196078	0.684563776	1	8.74E-17	0
8	0.182598039	0.730392711	1	3.41E-17	1.13E-16
16	0.09129902	0.684564094	1	1.37E-17	1.20E-16
32	0.04564951	0.730394497	1	8.17E-18	0
64	0.022824755	0.6845645	1	3.40E-18	7.28E-17
128	0.011412377	0.730406013	1	1.86E-18	1.32E-16

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.17114094	0.684563776	1	4.72E-17	0
8	0.08557047	0.730392711	1	1.80E-17	1.13E-16
16	0.042785235	0.684564094	1	8.46E-18	1.20E-16
32	0.021392617	0.730394497	1	3.50E-18	0
64	0.010696309	0.6845645	1	2.49E-18	7.28E-17
128	0.005348154	0.730406013	1	1.16E-18	1.32E-16

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.686852994	0.684563776	1	0.007012653	0
8	0.734626263	0.730392711	1	0.008076881	1.13E-16
16	0.690475732	0.684564094	1	0.005534339	1.20E-16
32	0.734853131	0.730394497	1	0.006293634	0
64	0.689177424	0.6845645	1	0.00423916	7.28E-17
128	0.733219241	0.730406013	1	0.002703484	1.32E-16

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.478413289	0.684563776	1	0.488147562	0
8	0.482474953	0.730392711	1	0.324056736	1.13E-16
16	0.503408088	0.684564094	1	0.149361722	1.20E-16
32	0.498813526	0.730394497	1	0.148995472	0
64	0.494211175	0.6845645	1	0.081667779	7.28E-17
128	0.477604119	0.730406013	1	0.099324718	1.32E-16

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.686852994	0.684563776	1	0.007012653	0
8	0.734626263	0.730392711	1	0.008076881	1.13E-16
16	0.690475732	0.684564094	1	0.005534339	1.20E-16
32	0.734853131	0.730394497	1	0.006293634	0
64	0.689154669	0.6845645	1	0.004206601	7.28E-17
128	0.733219241	0.730406013	1	0.002703484	1.32E-16

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.587869989	0.684563776	1	0.365981029	0
8	0.632631984	0.730392711	1	0.228389002	1.13E-16
16	0.636441175	0.684564094	1	0.097649526	1.20E-16
32	0.635972451	0.730394497	1	0.098197145	0
64	0.601814875	0.6845645	1	0.063793269	7.28E-17
128	0.627119975	0.730406013	1	0.059261718	1.32E-16

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.356287797	0.684563776	1	0.372956256	0
8	0.286788891	0.730392711	1	0.155250983	1.13E-16
16	0.154974972	0.684564094	1	0.025385237	1.20E-16
32	0.077273746	0.730394497	1	0.010235988	0
64	0.040587589	0.6845645	1	0.013299748	7.28E-17
128	0.021045775	0.730406013	1	0.009083544	1.32E-16

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.690577347	0.684563776	1	0.015759388	0
8	0.345206869	0.730392711	1	0.004757087	1.13E-16
16	0.695900901	0.684564094	1	0.008480689	1.20E-16
32	0.347192719	0.730394497	1	0.003354496	0
64	0.693768952	0.6845645	1	0.006772766	7.28E-17
128	0.345413201	0.730406013	1	0.00217239	1.32E-16

Smallest Face Ratio Results

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.371323811	0.6843255	1	0.518955166	0
8	0.321914895	0.729225791	1	0.348192522	0
16	0.301119031	0.682541127	1	0.205658755	0
32	0.235699094	0.727430659	1	0.121723526	0
64	0.18855043	0.677659754	1	0.109163147	0
128	0.163515509	0.722121472	1	0.073980178	0

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.44658739	0.6843255	1	0.448155221	0
8	0.378197466	0.729225791	1	0.343100554	0
16	0.372793727	0.682541127	1	0.182328623	0
32	0.264870377	0.727430659	1	0.127422061	0
64	0.255804321	0.677659754	1	0.145643414	0
128	0.220284078	0.722121472	1	0.095278906	0

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.327028367	0.6843255	1	0.048450576	0
8	0.159222265	0.729225791	1	0.018300437	0
16	0.081565117	0.682541127	1	0.005499285	0
32	0.041151342	0.727430659	1	0.003211345	0
64	0.021039523	0.677659754	1	0.00112444	0
128	0.010635594	0.722121472	1	0.000472157	0

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.158842317	0.6843255	1	0.017637854	0
8	0.079197917	0.729225791	1	0.005753561	0
16	0.039770405	0.682541127	1	0.00242448	0
32	0.020161318	0.727430659	1	0.001085141	0
64	0.010157021	0.677659754	1	0.000432423	0
128	0.005085081	0.722121472	1	0.000187426	0

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.603995747	0.6843255	1	0.073571321	0
8	0.605193372	0.729225791	1	0.073254928	0
16	0.541178311	0.682541127	1	0.059143809	0
32	0.56976334	0.727430659	1	0.057255324	0
64	0.533884226	0.677659754	1	0.058332416	0
128	0.581364957	0.722121472	1	0.056523064	0

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.340532204	0.6843255	1	0.517172119	0
8	0.241614846	0.729225791	1	0.222813939	0
16	0.195775458	0.682541127	1	0.16003851	0
32	0.168414597	0.727430659	1	0.105866815	0
64	0.137654443	0.677659754	1	0.081672155	0
128	0.108799352	0.722121472	1	0.081097603	0

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.603995747	0.6843255	1	0.073571321	0
8	0.605193372	0.729225791	1	0.073254928	0
16	0.541178311	0.682541127	1	0.059143809	0
32	0.56976334	0.727430659	1	0.057255324	0
64	0.533851538	0.677659754	1	0.058250464	0
128	0.581364957	0.722121472	1	0.056523064	0

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.467443999	0.6843255	1	0.498547259	0
8	0.407292403	0.729225791	1	0.370817719	0
16	0.365709292	0.682541127	1	0.247023771	0
32	0.275695581	0.727430659	1	0.141064986	0
64	0.229705474	0.677659754	1	0.152837835	0
128	0.193312337	0.722121472	1	0.098750881	0

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.164534575	0.6843255	1	0.028689831	0
8	0.080675469	0.729225791	1	0.007989287	0
16	0.040455465	0.682541127	1	0.003211847	0
32	0.020546445	0.727430659	1	0.001473009	0
64	0.010286967	0.677659754	1	0.000613347	0
128	0.005112415	0.722121472	1	0.00024559	0

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.598872195	0.6843255	1	0.103682064	0
8	0.290758433	0.729225791	1	0.025309854	0
16	0.527536195	0.682541127	1	0.068189546	0
32	0.269140881	0.727430659	1	0.027676736	0
64	0.513153263	0.677659754	1	0.054661083	0
128	0.268949355	0.722121472	1	0.028167516	0

FOCS results

	N=128	N=64	N=32	N=16	N=8	N=4
EV	1.348113703	1.290167656	1.242699815	1.184790336	1.140537772	1.099457313
UV	1.574484088	1.520677753	1.458920089	1.369973869	1.261771655	1.151974335
AFR	0.745001553	0.703353616	0.744813989	0.700452105	0.738621149	0.690716427
SFR	0.617519446	0.56483539	0.596549887	0.568386857	0.619992611	0.614511083

F Results – Tel-Aviv Map with hotspot noise parameter 0.6

Egalitarian Results

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.093157839	0.836468329	1.093157839	1	0.17583266	0.263973097	0.17583266
8	1.094830146	0.776935341	1.095671491	1	0.107274021	0.075884965	0.104159374
16	1.115552491	0.751637555	1.115598949	1	0.119520361	0.033985768	0.119429259
32	1.109312968	0.740925864	1.110210295	1	0.105702913	0.018034729	0.105260623
64	1.122211258	0.736077699	1.122788492	1	0.09207819	0.008839827	0.092405167
128	1.139448896	0.733866889	1.139491072	1	0.092582454	0.003146253	0.092624545

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.072852686	0.836468329	1.072852686	1	0.112345828	0.263973097	0.112345828
8	1.067281286	0.776935341	1.068851357	1	0.079123673	0.075884965	0.076987869
16	1.063300736	0.751637555	1.065792445	1	0.065088406	0.033985768	0.063907319
32	1.06295697	0.740925864	1.065323751	1	0.054483912	0.018034729	0.058717904
64	1.050217109	0.736077699	1.059185699	1	0.033163875	0.008839827	0.037682865
128	1.041692064	0.733866889	1.051969812	1	0.024669958	0.003146253	0.035213161

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.112233335	0.836468329	1.112872448	1	0.204404697	0.263973097	0.202950647
8	1.128616881	0.776935341	1.129162826	1	0.139152493	0.075884965	0.141234392
16	1.143424222	0.751637555	1.145487261	1	0.121070735	0.033985768	0.116896877
32	1.140097352	0.740925864	1.142202677	1	0.117615639	0.018034729	0.11773121
64	1.165477672	0.736077699	1.166312734	1	0.104398343	0.008839827	0.104127391
128	1.187069879	0.733866889	1.190232	1	0.101716167	0.003146253	0.099575619

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.114947431	0.836468329	1.115586544	1	0.197126284	0.263973097	0.195526478
8	1.13047964	0.776935341	1.13093126	1	0.13491336	0.075884965	0.13709562
16	1.148823329	0.751637555	1.1490147	1	0.123315843	0.033985768	0.122646323
32	1.148679539	0.740925864	1.149528278	1	0.113745147	0.018034729	0.111772599
64	1.176173748	0.736077699	1.177311385	1	0.102965896	0.008839827	0.102337354
128	1.189512555	0.733866889	1.190557518	1	0.106197142	0.003146253	0.104374003

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.117826377	0.836468329	1.118279936	1	0.193724152	0.263973097	0.192416696
8	1.131506847	0.776935341	1.131506847	1	0.132201996	0.075884965	0.132201996
16	1.15279781	0.751637555	1.153717582	1	0.118796991	0.033985768	0.119976999
32	1.139628435	0.740925864	1.140482366	1	0.126435912	0.018034729	0.127583422
64	1.157662262	0.736077699	1.162421074	1	0.105479791	0.008839827	0.101150473
128	1.192309677	0.733866889	1.195067534	1	0.097274369	0.003146253	0.096139435

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.074975169	0.836468329	1.080858409	1	0.10682423	0.263973097	0.109332697
8	1.079937517	0.776935341	1.087163856	1	0.076821552	0.075884965	0.085479405
16	1.094985001	0.751637555	1.099865153	1	0.068005816	0.033985768	0.075627872
32	1.094093583	0.740925864	1.097525233	1	0.057615865	0.018034729	0.060776558
64	1.100628515	0.736077699	1.102081042	1	0.050693213	0.008839827	0.052305954
128	1.101826851	0.733866889	1.104188285	1	0.035299284	0.003146253	0.036190194

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.098241377	0.836468329	1.100277799	1	0.208478726	0.263973097	0.20391009
8	1.114343621	0.776935341	1.116005666	1	0.138800098	0.075884965	0.135876639
16	1.135376715	0.751637555	1.137357106	1	0.125028232	0.033985768	0.121105127
32	1.138046154	0.740925864	1.140149112	1	0.120039041	0.018034729	0.117621039
64	1.167568105	0.736077699	1.169770154	1	0.10403724	0.008839827	0.10466856
128	1.18677323	0.733866889	1.189302851	1	0.10074639	0.003146253	0.095770212

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.094595374	0.836468329	1.095048933	1	0.203475387	0.263973097	0.202768725
8	1.102693385	0.776935341	1.106894857	1	0.13659551	0.075884965	0.134438531
16	1.124318605	0.751637555	1.127883936	1	0.112799239	0.033985768	0.113385494
32	1.120791516	0.740925864	1.121533427	1	0.122474098	0.018034729	0.122617927
64	1.140022915	0.736077699	1.142050951	1	0.093464921	0.008839827	0.095305724
128	1.155728317	0.733866889	1.156888649	1	0.099792207	0.003146253	0.099742084

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.098241377	0.836468329	1.100277799	1	0.208478726	0.263973097	0.20391009
8	1.110805513	0.776935341	1.111790287	1	0.141324619	0.075884965	0.138896315
16	1.135140724	0.751637555	1.137275883	1	0.124431362	0.033985768	0.120059017
32	1.135928733	0.740925864	1.136648233	1	0.113593134	0.018034729	0.112164131
64	1.164398725	0.736077699	1.168286179	1	0.102712253	0.008839827	0.100599294
128	1.17687522	0.733866889	1.181145724	1	0.095567411	0.003146253	0.089663743

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.069953551	0.836468329	1.073842769	1	0.095951988	0.263973097	0.102238748
8	1.066444458	0.776935341	1.070738683	1	0.071732774	0.075884965	0.076535991
16	1.06741981	0.751637555	1.072858017	1	0.071915772	0.033985768	0.081900004
32	1.066110645	0.740925864	1.071238797	1	0.057363966	0.018034729	0.061943522
64	1.051062298	0.736077699	1.056216482	1	0.03589726	0.008839827	0.045160257
128	1.041692064	0.733866889	1.0492968	1	0.024669958	0.003146253	0.040023449

Utilitarian Results

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.16880958	1.006792873	1.16880958	1	0.155011521	0.255473132	0.155011521
8	1.21872364	1.006112414	1.219949398	1	0.098048094	0.183359995	0.096814474
16	1.258487101	0.999944578	1.2588486	1	0.057563533	0.106959665	0.055481942
32	1.274498965	1.005079623	1.27507176	1	0.032725154	0.069208612	0.032664908
64	1.287773538	0.994192431	1.288155266	1	0.022982176	0.055416576	0.022920574
128	1.295805134	0.999776112	1.296351103	1	0.010900767	0.042957721	0.01099913

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.126633752	1.006792873	1.126633752	1	0.13845518	0.255473132	0.13845518
8	1.166080912	1.006112414	1.166728621	1	0.081496315	0.183359995	0.082448249
16	1.192259679	0.999944578	1.193405207	1	0.048444826	0.106959665	0.049692263
32	1.207570688	1.005079623	1.208584438	1	0.028044341	0.069208612	0.027662744
64	1.213800581	0.994192431	1.215722325	1	0.019077578	0.055416576	0.019050407
128	1.221267589	0.999776112	1.223192053	1	0.011690603	0.042957721	0.011607844

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.188132323	1.006792873	1.189010682	1	0.185260366	0.255473132	0.184432346
8	1.26145022	1.006112414	1.262419424	1	0.123814581	0.183359995	0.123262117
16	1.321949877	0.999944578	1.324314344	1	0.07284411	0.106959665	0.068873431
32	1.361678028	1.005079623	1.364512864	1	0.045247016	0.069208612	0.041459596
64	1.403435639	0.994192431	1.405986662	1	0.037967864	0.055416576	0.034551762
128	1.437397099	0.999776112	1.440349821	1	0.025600397	0.042957721	0.021889723

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.189732673	1.006792873	1.190611031	1	0.183614655	0.255473132	0.182699683
8	1.263208448	1.006112414	1.26381489	1	0.121629278	0.183359995	0.122603803
16	1.327297995	0.999944578	1.327771745	1	0.06813294	0.106959665	0.066619965
32	1.366223182	1.005079623	1.367596881	1	0.03702087	0.069208612	0.035822769
64	1.406532319	0.994192431	1.408025229	1	0.031868792	0.055416576	0.030748546
128	1.439316967	0.999776112	1.441079859	1	0.024254671	0.042957721	0.022143267

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.19097733	1.006792873	1.191758298	1	0.183258704	0.255473132	0.182378081
8	1.258717135	1.006112414	1.259916495	1	0.122810525	0.183359995	0.121658323
16	1.322468822	0.999944578	1.32360603	1	0.072602779	0.106959665	0.069651282
32	1.352723092	1.005079623	1.356951114	1	0.051449425	0.069208612	0.046114979
64	1.390792409	0.994192431	1.396628774	1	0.054334478	0.055416576	0.043041026
128	1.424008859	0.999776112	1.430758455	1	0.042106869	0.042957721	0.031853953

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.143648258	1.006792873	1.149481792	1	0.138205387	0.255473132	0.137724398
8	1.197486583	1.006112414	1.20689543	1	0.08173686	0.183359995	0.087555974
16	1.261152711	0.999944578	1.280519567	1	0.064209753	0.106959665	0.061688791
32	1.296232813	1.005079623	1.317241822	1	0.036172957	0.069208612	0.029741458
64	1.324437983	0.994192431	1.354366215	1	0.029299045	0.055416576	0.029253214
128	1.343770855	0.999776112	1.380555243	1	0.024930689	0.042957721	0.018847709

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.169569932	1.006792873	1.171032807	1	0.190591717	0.255473132	0.188787305
8	1.243431938	1.006112414	1.248544407	1	0.123817685	0.183359995	0.120234044
16	1.308381478	0.999944578	1.312190374	1	0.067508274	0.106959665	0.062699922
32	1.350113047	1.005079623	1.355234638	1	0.042487955	0.069208612	0.039565616
64	1.390942929	0.994192431	1.397076457	1	0.034520568	0.055416576	0.031260538
128	1.425169896	0.999776112	1.431756529	1	0.021766264	0.042957721	0.019923427

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.161366975	1.006792873	1.162647735	1	0.199831859	0.255473132	0.200161026
8	1.231124886	1.006112414	1.237511895	1	0.142367665	0.183359995	0.136562823
16	1.299264619	0.999944578	1.303084208	1	0.081438012	0.106959665	0.075229396
32	1.334898459	1.005079623	1.337944247	1	0.047271075	0.069208612	0.045751357
64	1.364594299	0.994192431	1.367372173	1	0.038747445	0.055416576	0.038399586
128	1.386736171	0.999776112	1.389775043	1	0.022195048	0.042957721	0.020938813

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.169569932	1.006792873	1.171032807	1	0.190591717	0.255473132	0.188787305
8	1.239789268	1.006112414	1.243943897	1	0.121446401	0.183359995	0.118582218
16	1.308758935	0.999944578	1.313176936	1	0.068460363	0.106959665	0.061532206
32	1.34768679	1.005079623	1.352670763	1	0.040090925	0.069208612	0.037674211
64	1.388918848	0.994192431	1.395351862	1	0.032814567	0.055416576	0.030552929
128	1.422846896	0.999776112	1.42988794	1	0.020172206	0.042957721	0.018793113

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.129708531	1.006792873	1.132041317	1	0.129344252	0.255473132	0.133548089
8	1.171925101	1.006112414	1.179605089	1	0.070473796	0.183359995	0.072269812
16	1.214121432	0.999944578	1.22319474	1	0.053691703	0.106959665	0.05459717
32	1.233001487	1.005079623	1.242655706	1	0.029376521	0.069208612	0.027842807
64	1.234894764	0.994192431	1.249036308	1	0.025548555	0.055416576	0.024143071
128	1.242074103	0.999776112	1.257313571	1	0.020900274	0.042957721	0.016105205

Largest Envy Results

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.014902007	1.506127389	1.014902007	1	0.088118144	0.465866234	0.088118144
8	1.073210223	1.696383734	1.072724314	1	0.153747218	0.084139732	0.153398335
16	1.109200239	1.777345688	1.109134031	1	0.12305526	0.033293547	0.123056369
32	1.133325779	1.846942278	1.133151833	1	0.11859083	0.023036491	0.119197402
64	1.116471514	1.890549135	1.116471514	1	0.098864035	0.018985498	0.098864035
128	1.123586849	1.924786229	1.123123415	1	0.096067203	0.010352561	0.096775817

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.02228067	1.506127389	1.02228067	1	0.114447608	0.465866234	0.114447608
8	1.073751587	1.696383734	1.073461574	1	0.101023448	0.084139732	0.101867824
16	1.105971495	1.777345688	1.104191829	1	0.091298487	0.033293547	0.089845478
32	1.132832459	1.846942278	1.131981462	1	0.096733858	0.023036491	0.098666117
64	1.139325781	1.890549135	1.132945588	1	0.063715301	0.018985498	0.059552985
128	1.153679559	1.924786229	1.140310625	1	0.0643559	0.010352561	0.068836079

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.021721422	1.506127389	1.020408473	1	0.13334803	0.465866234	0.131390748
8	1.081331464	1.696383734	1.080650388	1	0.210101948	0.084139732	0.208793405
16	1.153510002	1.777345688	1.149949298	1	0.180633008	0.033293547	0.167363859
32	1.225789624	1.846942278	1.220229532	1	0.262226812	0.023036491	0.262068222
64	1.258761476	1.890549135	1.255920429	1	0.224865008	0.018985498	0.220656456
128	1.292356705	1.924786229	1.286204	1	0.20742036	0.010352561	0.204884338

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.012951639	1.506127389	1.01163869	1	0.096094445	0.465866234	0.092075956
8	1.069137547	1.696383734	1.068582403	1	0.184735766	0.084139732	0.183127396
16	1.117985594	1.777345688	1.117985594	1	0.149286226	0.033293547	0.149286226
32	1.175272024	1.846942278	1.175272024	1	0.151889126	0.023036491	0.151889126
64	1.23329527	1.890549135	1.229907616	1	0.198607036	0.018985498	0.197090166
128	1.279801908	1.924786229	1.278040676	1	0.209358395	0.010352561	0.207800545

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.013953746	1.506127389	1.012873194	1	0.104268442	0.465866234	0.100546368
8	1.077436565	1.696383734	1.07647142	1	0.172945759	0.084139732	0.169903138
16	1.153283091	1.777345688	1.153066071	1	0.280370858	0.033293547	0.280667221
32	1.283948667	1.846942278	1.28162201	1	0.364309462	0.023036491	0.366609001
64	1.362400905	1.890549135	1.35588357	1	0.473517872	0.018985498	0.473046579
128	1.406737147	1.924786229	1.405222457	1	0.408223436	0.010352561	0.406797159

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.046091568	1.506127389	1.03100683	1	0.185369639	0.465866234	0.163429527
8	1.155385178	1.696383734	1.13560586	1	0.194114212	0.084139732	0.198816473
16	1.408557616	1.777345688	1.393033233	1	0.260290437	0.033293547	0.261653395
32	1.459486965	1.846942278	1.450059144	1	0.213608473	0.023036491	0.217347675
64	1.778149021	1.890549135	1.768003543	1	0.3193242	0.018985498	0.325102265
128	1.830374491	1.924786229	1.825395414	1	0.23959416	0.010352561	0.23977913

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.030693847	1.506127389	1.027031293	1	0.174315226	0.465866234	0.170557049
8	1.11680121	1.696383734	1.106940085	1	0.174703034	0.084139732	0.16081242
16	1.154744473	1.777345688	1.149214188	1	0.167156101	0.033293547	0.15464367
32	1.242634757	1.846942278	1.231874173	1	0.226086497	0.023036491	0.222751665
64	1.230859325	1.890549135	1.225549605	1	0.152066301	0.018985498	0.152784183
128	1.273069434	1.924786229	1.270104406	1	0.160866174	0.010352561	0.159157524

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.040502954	1.506127389	1.037576209	1	0.208659031	0.465866234	0.208864469
8	1.132199505	1.696383734	1.126994941	1	0.255879718	0.084139732	0.252614659
16	1.213478945	1.777345688	1.208115044	1	0.289472828	0.033293547	0.284759995
32	1.287435877	1.846942278	1.283963209	1	0.319482722	0.023036491	0.311518979
64	1.328904907	1.890549135	1.323303579	1	0.314877123	0.018985498	0.319449335
128	1.390228238	1.924786229	1.372701937	1	0.321555205	0.010352561	0.310620502

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.030693847	1.506127389	1.027031293	1	0.174315226	0.465866234	0.170557049
8	1.114633359	1.696383734	1.106673202	1	0.173658865	0.084139732	0.15991938
16	1.165045888	1.777345688	1.158795069	1	0.200450269	0.033293547	0.181866293
32	1.24070039	1.846942278	1.234324842	1	0.206737734	0.023036491	0.20825543
64	1.270199183	1.890549135	1.265174283	1	0.204605045	0.018985498	0.20417332
128	1.325127669	1.924786229	1.321152764	1	0.154817152	0.010352561	0.160642858

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.026154867	1.506127389	1.019973282	1	0.121963945	0.465866234	0.107779662
8	1.151468851	1.696383734	1.141089984	1	0.268107481	0.084139732	0.264086661
16	1.358749929	1.777345688	1.334790146	1	0.321977843	0.033293547	0.345254021
32	1.479460195	1.846942278	1.448324054	1	0.305428275	0.023036491	0.294827855
64	1.71759258	1.890549135	1.680374788	1	0.46682215	0.018985498	0.469404476
128	1.928222102	1.924786229	1.884885943	1	0.375289366	0.010352561	0.38601752

Average Face Ratio Results

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.30952381	0.79356688	1	6.01E-17	0
8	0.154761905	0.627873056	1	0	0
16	0.077380952	0.52643403	1	5.86E-18	9.90E-17
32	0.038690476	0.513115304	1	2.80E-18	6.86E-17
64	0.019345238	0.450821173	1	0	2.74E-17
128	0.009672619	0.54456221	1	6.79E-19	3.11E-17

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.201923077	0.79356688	1	2.12E-17	0
8	0.100961538	0.627873056	1	1.12E-17	0
16	0.050480769	0.52643403	1	4.64E-18	9.90E-17
32	0.025240385	0.513115304	1	1.00E-18	6.86E-17
64	0.012620192	0.450821173	1	9.50E-19	2.74E-17
128	0.006310096	0.54456221	1	7.15E-19	3.11E-17

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.562438948	0.79356688	1	0.479317124	0
8	0.515199214	0.627873056	1	0.218337455	0
16	0.450785373	0.52643403	1	0.115380646	9.90E-17
32	0.447891854	0.513115304	1	0.092762708	6.86E-17
64	0.455255821	0.450821173	1	0.057040943	2.74E-17
128	0.47595762	0.54456221	1	0.036943529	3.11E-17

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.566727108	0.79356688	1	0.512880167	0
8	0.512014739	0.627873056	1	0.245831688	0
16	0.431855117	0.52643403	1	0.125613133	9.90E-17
32	0.418132084	0.513115304	1	0.097924517	6.86E-17
64	0.428765063	0.450821173	1	0.071679307	2.74E-17
128	0.440241038	0.54456221	1	0.054927917	3.11E-17

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.596576543	0.79356688	1	0.485028349	0
8	0.532731008	0.627873056	1	0.246237716	0
16	0.441366399	0.52643403	1	0.110894928	9.90E-17
32	0.447265116	0.513115304	1	0.083236714	6.86E-17
64	0.463432089	0.450821173	1	0.066779203	2.74E-17
128	0.492487094	0.54456221	1	0.054704905	3.11E-17

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.73722372	0.79356688	1	0.055576874	0
8	0.4498558	0.627873056	1	0.013980018	0
16	0.518447436	0.52643403	1	0.049104697	9.90E-17
32	0.394591296	0.513115304	1	0.020507449	6.86E-17
64	0.413803859	0.450821173	1	0.02695205	2.74E-17
128	0.380385766	0.54456221	1	0.014873766	3.11E-17

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.768931007	0.79356688	1	0.104173576	0
8	0.649400023	0.627873056	1	0.020912783	0
16	0.559277459	0.52643403	1	0.024237805	9.90E-17
32	0.522988754	0.513115304	1	0.013213645	6.86E-17
64	0.449601487	0.450821173	1	0.020937832	2.74E-17
128	0.50022815	0.54456221	1	0.007090739	3.11E-17

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.665152962	0.79356688	1	0.454524903	0
8	0.52961379	0.627873056	1	0.261286652	0
16	0.345793714	0.52643403	1	0.033618062	9.90E-17
32	0.232175086	0.513115304	1	0.018715575	6.86E-17
64	0.153773897	0.450821173	1	0.018770966	2.74E-17
128	0.132883178	0.54456221	1	0.023922113	3.11E-17

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.768931007	0.79356688	1	0.104173576	0
8	0.673433073	0.627873056	1	0.046833732	0
16	0.565245516	0.52643403	1	0.033442956	9.90E-17
32	0.567837577	0.513115304	1	0.02827429	6.86E-17
64	0.58919999	0.450821173	1	0.023431873	2.74E-17
128	0.596050948	0.54456221	1	0.010214869	3.11E-17

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.44944911	0.79356688	1	0.043937337	0
8	0.237307703	0.627873056	1	0.049840959	0
16	0.133214685	0.52643403	1	0.032637628	9.90E-17
32	0.071485884	0.513115304	1	0.02423269	6.86E-17
64	0.035627541	0.450821173	1	0.010266725	2.74E-17
128	0.018454127	0.54456221	1	0.004998466	3.11E-17

Smallest Face Ratio Results

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.213244289	0.540928362	1	0.081360914	0
8	0.082006896	0.421454629	1	0.023869412	0
16	0.039349548	0.253117496	1	0.008911868	0
32	0.019857489	0.16029637	1	0.003346334	0
64	0.009634103	0.082591238	1	0.001307389	0
128	0.004734959	0.067670944	1	0.000463862	0

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.094029202	0.540928362	1	0.015944965	0
8	0.042454379	0.421454629	1	0.004655923	0
16	0.020811203	0.253117496	1	0.001913901	0
32	0.010184013	0.16029637	1	0.000745893	0
64	0.005024827	0.082591238	1	0.000245466	0
128	0.002508463	0.067670944	1	8.36E-05	0

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.327362531	0.540928362	1	0.448684785	0
8	0.201008833	0.421454629	1	0.173728928	0
16	0.158292076	0.253117496	1	0.103305652	0
32	0.115464296	0.16029637	1	0.08223655	0
64	0.086399787	0.082591238	1	0.041058025	0
128	0.06293819	0.067670944	1	0.030930523	0

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.335208397	0.540928362	1	0.501159627	0
8	0.181026677	0.421454629	1	0.183737905	0
16	0.140933115	0.253117496	1	0.104783175	0
32	0.093161449	0.16029637	1	0.062508679	0
64	0.066998314	0.082591238	1	0.042893305	0
128	0.053948114	0.067670944	1	0.019405151	0

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.358225667	0.540928362	1	0.489913972	0
8	0.22257701	0.421454629	1	0.225336232	0
16	0.155214512	0.253117496	1	0.101361888	0
32	0.102552178	0.16029637	1	0.068023673	0
64	0.087009387	0.082591238	1	0.048308237	0
128	0.056379028	0.067670944	1	0.027786528	0

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.560836754	0.540928362	1	0.116634253	0
8	0.099149548	0.421454629	1	0.005844283	0
16	0.140863516	0.253117496	1	0.009287209	0
32	0.056911686	0.16029637	1	0.002944428	0
64	0.066614045	0.082591238	1	0.005961108	0
128	0.029202359	0.067670944	1	0.004121086	0

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.551317848	0.540928362	1	0.111472606	0
8	0.318169566	0.421454629	1	0.046029267	0
16	0.165099504	0.253117496	1	0.056391764	0
32	0.113406415	0.16029637	1	0.004336943	0
64	0.083306035	0.082591238	1	0.012818689	0
128	0.050092511	0.067670944	1	0.002393023	0

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.365713501	0.540928362	1	0.701893606	0
8	0.157666818	0.421454629	1	0.122547342	0
16	0.094514491	0.253117496	1	0.016902544	0
32	0.047034585	0.16029637	1	0.011578971	0
64	0.022739755	0.082591238	1	0.005394288	0
128	0.01127147	0.067670944	1	0.002223415	0

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.551317848	0.540928362	1	0.111472606	0
8	0.214349713	0.421454629	1	0.096537487	0
16	0.190650119	0.253117496	1	0.09875158	0
32	0.13225861	0.16029637	1	0.068356781	0
64	0.078545891	0.082591238	1	0.02074366	0
128	0.057448624	0.067670944	1	0.011261579	0

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.097970591	0.540928362	1	0.017825447	0
8	0.047572734	0.421454629	1	0.006511023	0
16	0.023145513	0.253117496	1	0.002400423	0
32	0.011333875	0.16029637	1	0.000800163	0
64	0.005505198	0.082591238	1	0.000304223	0
128	0.002725313	0.067670944	1	7.85E-05	0

FOCS results

	N=128	N=64	N=32	N=16	N=8	N=4
EV	1.239305579	1.210260851	1.177647112	1.171794015	1.144674057	1.121786703
UV	1.44371843	1.411511139	1.371048875	1.330964161	1.266580447	1.192755399
AFR	0.687681844	0.670721111	0.629063099	0.634724715	0.677872227	0.774795024
SFR	0.164869885	0.217717514	0.249072903	0.299968338	0.37893085	0.580947397

G Results – New-Zealand Map with uniform noise parameter 0.6

Egalitarian Results

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000993177	0.998010555	1.001053429	1	0.001619372	0.004657505	0.001683432
8	1.001567813	0.995977207	1.001659729	1	0.001656622	0.004027357	0.001592385
16	1.002134614	0.992310611	1.002554363	1	0.001546218	0.005559627	0.001879909
32	1.002912876	0.986161336	1.003768371	1	0.001721962	0.006327717	0.002023624
64	1.004311497	0.977521383	1.005546509	1	0.001819478	0.007623994	0.002145606
128	1.005730645	0.964189987	1.007360051	1	0.002089757	0.010836674	0.003256497

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000914753	0.998010555	1.000939253	1	0.001665347	0.004657505	0.001621661
8	1.001400174	0.995977207	1.00157508	1	0.001433761	0.004027357	0.001519451
16	1.002118833	0.992310611	1.002537602	1	0.001495938	0.005559627	0.001968527
32	1.00303797	0.986161336	1.003858306	1	0.002036303	0.006327717	0.002331003
64	1.004133944	0.977521383	1.005384612	1	0.002278468	0.007623994	0.002910035
128	1.005768781	0.964189987	1.007781169	1	0.002598851	0.010836674	0.002971471

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001462058	0.998010555	1.001462058	1	0.001867705	0.004657505	0.001867705
8	1.001874668	0.995977207	1.001954864	1	0.001446967	0.004027357	0.001410888
16	1.002847247	0.992310611	1.003134697	1	0.001579698	0.005559627	0.001621056
32	1.004161721	0.986161336	1.004708816	1	0.001839484	0.006327717	0.002033908
64	1.005744597	0.977521383	1.006888989	1	0.00204821	0.007623994	0.002891362
128	1.007777449	0.964189987	1.009090657	1	0.002688711	0.010836674	0.003472128

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000989414	0.998010555	1.000997002	1	0.001852243	0.004657505	0.001888993
8	1.00125221	0.995977207	1.001375299	1	0.001440139	0.004027357	0.00153202
16	1.00201465	0.992310611	1.002379084	1	0.001513851	0.005559627	0.0019712
32	1.002952094	0.986161336	1.003856738	1	0.001453168	0.006327717	0.002001517
64	1.004128527	0.977521383	1.005576558	1	0.002221315	0.007623994	0.00315173
128	1.005720802	0.964189987	1.007553282	1	0.002442189	0.010836674	0.003766442

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.00096214	0.998010555	1.000995504	1	0.001643866	0.004657505	0.001741007
8	1.001366122	0.995977207	1.001609461	1	0.001634406	0.004027357	0.001885708
16	1.002129613	0.992310611	1.002486353	1	0.001476108	0.005559627	0.001703248
32	1.003171109	0.986161336	1.003692483	1	0.002043304	0.006327717	0.002110664
64	1.004262208	0.977521383	1.005548	1	0.001945843	0.007623994	0.002926182
128	1.006138486	0.964189987	1.007707183	1	0.002396379	0.010836674	0.003153217

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000914753	0.998010555	1.000939253	1	0.001665347	0.004657505	0.001621661
8	1.001400174	0.995977207	1.00157508	1	0.001433761	0.004027357	0.001519451
16	1.002150991	0.992310611	1.002615915	1	0.001637326	0.005559627	0.001562865
32	1.003080889	0.986161336	1.003721264	1	0.00177975	0.006327717	0.002237924
64	1.004134738	0.977521383	1.005456827	1	0.002348735	0.007623994	0.003149479
128	1.005605589	0.964189987	1.007852634	1	0.002282464	0.010836674	0.00325033

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001141149	0.998010555	1.00116254	1	0.001952129	0.004657505	0.001953692
8	1.001437046	0.995977207	1.001569934	1	0.001592631	0.004027357	0.001637038
16	1.002108613	0.992310611	1.002363131	1	0.001460305	0.005559627	0.00161352
32	1.002819046	0.986161336	1.003736433	1	0.001463041	0.006327717	0.002371844
64	1.004384448	0.977521383	1.005423477	1	0.002344558	0.007623994	0.002567708
128	1.005781222	0.964189987	1.00735379	1	0.002230743	0.010836674	0.003593125

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001379169	0.998010555	1.001386717	1	0.00189415	0.004657505	0.001914362
8	1.001971599	0.995977207	1.002042734	1	0.0015957	0.004027357	0.001625242
16	1.002719579	0.992310611	1.003053125	1	0.0014683	0.005559627	0.001925851
32	1.003825119	0.986161336	1.00438019	1	0.001935504	0.006327717	0.002466319
64	1.005266147	0.977521383	1.006252943	1	0.002640584	0.007623994	0.003032676
128	1.007159683	0.964189987	1.009262738	1	0.002854263	0.010836674	0.003916507

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001454673	0.998010555	1.001454673	1	0.001934655	0.004657505	0.001934655
8	1.00206257	0.995977207	1.002116279	1	0.001693938	0.004027357	0.001741655
16	1.00296508	0.992310611	1.003215901	1	0.001620523	0.005559627	0.0017341
32	1.004212599	0.986161336	1.004865246	1	0.002059972	0.006327717	0.00219367
64	1.006146952	0.977521383	1.007220417	1	0.002383625	0.007623994	0.003013877
128	1.007742932	0.964189987	1.009556655	1	0.00346837	0.010836674	0.003888875

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001141149	0.998010555	1.00116254	1	0.001952129	0.004657505	0.001953692
8	1.001460341	0.995977207	1.001566223	1	0.001526437	0.004027357	0.001419351
16	1.00204039	0.992310611	1.002526093	1	0.001573944	0.005559627	0.002058624
32	1.003073539	0.986161336	1.003793582	1	0.001700265	0.006327717	0.002191504
64	1.004435336	0.977521383	1.005368817	1	0.002370768	0.007623994	0.002888526
128	1.005778627	0.964189987	1.007615941	1	0.002516854	0.010836674	0.002884322

Utilitarian Results

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001783939	1.00012887	1.001862635	1	0.001756438	0.003470284	0.001717448
8	1.003421304	1.000262356	1.003717687	1	0.00167006	0.003044043	0.001493837
16	1.005562291	0.999944547	1.00651825	1	0.001493673	0.002908515	0.001187126
32	1.008641486	0.999913458	1.010668633	1	0.001257407	0.002389188	0.001321081
64	1.012993307	0.999931698	1.017235273	1	0.001051123	0.002195801	0.001210951
128	1.018915356	1.000153969	1.026181194	1	0.001164088	0.002222639	0.001157837

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.00179744	1.00012887	1.001860407	1	0.002006424	0.003470284	0.001985154
8	1.003415481	1.000262356	1.003712764	1	0.001835566	0.003044043	0.001706928
16	1.005646023	0.999944547	1.006559011	1	0.001880223	0.002908515	0.001567603
32	1.00878209	0.999913458	1.010719526	1	0.001456436	0.002389188	0.001459008
64	1.013116396	0.999931698	1.017306132	1	0.001239853	0.002195801	0.001220239
128	1.019338773	1.000153969	1.026835832	1	0.001385239	0.002222639	0.001007411

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.002195451	1.00012887	1.002195451	1	0.001972347	0.003470284	0.001972347
8	1.003876639	1.000262356	1.004089402	1	0.001604645	0.003044043	0.001655448
16	1.00635126	0.999944547	1.006871731	1	0.001501494	0.002908515	0.001329075
32	1.009792681	0.999913458	1.011358308	1	0.001385725	0.002389188	0.001103202
64	1.015066133	0.999931698	1.018173993	1	0.001474396	0.002195801	0.001385617
128	1.022064811	1.000153969	1.02833008	1	0.001328549	0.002222639	0.001052861

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001844371	1.00012887	1.00186995	1	0.001854059	0.003470284	0.001815573
8	1.00333463	1.000262356	1.00368803	1	0.001815041	0.003044043	0.001694842
16	1.005571827	0.999944547	1.006571416	1	0.001617558	0.002908515	0.001495364
32	1.008695913	0.999913458	1.010719614	1	0.001675645	0.002389188	0.001415833
64	1.013147724	0.999931698	1.017301554	1	0.001341991	0.002195801	0.001169218
128	1.019370928	1.000153969	1.027275242	1	0.001300367	0.002222639	0.001003414

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001749529	1.00012887	1.001816529	1	0.001783911	0.003470284	0.001918517
8	1.003291564	1.000262356	1.003748777	1	0.001862359	0.003044043	0.001860032
16	1.005652047	0.999944547	1.006484239	1	0.001281749	0.002908515	0.001336984
32	1.008797042	0.999913458	1.010807065	1	0.00161371	0.002389188	0.001331442
64	1.013384121	0.999931698	1.017716086	1	0.001378517	0.002195801	0.000963893
128	1.019515464	1.000153969	1.027678116	1	0.001194191	0.002222639	0.001114064

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.00179744	1.00012887	1.001860407	1	0.002006424	0.003470284	0.001985154
8	1.003415481	1.000262356	1.003712764	1	0.001835566	0.003044043	0.001706928
16	1.005649282	0.999944547	1.006574333	1	0.001857546	0.002908515	0.001566645
32	1.008802529	0.999913458	1.010786553	1	0.001395249	0.002389188	0.001486285
64	1.013103504	0.999931698	1.017290315	1	0.001271022	0.002195801	0.001230136
128	1.01932298	1.000153969	1.02680271	1	0.001328245	0.002222639	0.001030943

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.001937268	1.00012887	1.001990846	1	0.002072102	0.003470284	0.002111316
8	1.003466012	1.000262356	1.00375941	1	0.00168958	0.003044043	0.001507965
16	1.00559812	0.999944547	1.006445557	1	0.001445703	0.002908515	0.001290454
32	1.008566362	0.999913458	1.010831402	1	0.001344901	0.002389188	0.001179232
64	1.013245529	0.999931698	1.017565846	1	0.00142062	0.002195801	0.001141049
128	1.019425162	1.000153969	1.027680588	1	0.001338562	0.002222639	0.001293301

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.002230202	1.00012887	1.002234886	1	0.002038803	0.003470284	0.002043497
8	1.004081504	1.000262356	1.004261472	1	0.001517104	0.003044043	0.001533611
16	1.006493016	0.999944547	1.007109269	1	0.001438721	0.002908515	0.001498215
32	1.009880456	0.999913458	1.011340866	1	0.001187078	0.002389188	0.001136793
64	1.015049854	0.999931698	1.018230923	1	0.001500481	0.002195801	0.00138756
128	1.022077403	1.000153969	1.028569387	1	0.001371658	0.002222639	0.001037768

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.0022655	1.00012887	1.002269008	1	0.001981035	0.003470284	0.001969959
8	1.004178569	1.000262356	1.004260216	1	0.001496289	0.003044043	0.001536553
16	1.006729221	0.999944547	1.007214616	1	0.001302048	0.002908515	0.001384633
32	1.010541529	0.999913458	1.011719295	1	0.001428936	0.002389188	0.001317056
64	1.015831797	0.999931698	1.018572315	1	0.001059462	0.002195801	0.001295264
128	1.023142138	1.000153969	1.028617815	1	0.001289511	0.002222639	0.001087999

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001937268	1.00012887	1.001990846	1	0.002072102	0.003470284	0.002111316
8	1.003447412	1.000262356	1.003735921	1	0.001604706	0.003044043	0.001422982
16	1.005526029	0.999944547	1.006435863	1	0.001481759	0.002908515	0.001223642
32	1.008754997	0.999913458	1.010938165	1	0.001417034	0.002389188	0.001365072
64	1.013254706	0.999931698	1.017725823	1	0.001366115	0.002195801	0.00144362
128	1.019627299	1.000153969	1.027906524	1	0.001422716	0.002222639	0.00110721

Nash Social Results

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001783608	1.000127147	1.001862299	1	0.001756086	0.003470662	0.00171714
8	1.003420231	1.000258016	1.003716392	1	0.001669118	0.003043418	0.001492877
16	1.005559653	0.999935469	1.006514761	1	0.001492853	0.002909342	0.001186596
32	1.00863573	0.999891597	1.010661062	1	0.001255935	0.002389966	0.001320302
64	1.012981988	0.999888637	1.0172176	1	0.001049697	0.002194182	0.001209242
128	1.018891871	1.000065144	1.026145449	1	0.001161517	0.002226888	0.001155906

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001797083	1.000127147	1.001860039	1	0.00200603	0.003470662	0.00198465
8	1.003414389	1.000258016	1.003711526	1	0.001834719	0.003043418	0.001706318
16	1.005643202	0.999935469	1.006555727	1	0.001878383	0.002909342	0.001566092
32	1.008776485	0.999891597	1.010711557	1	0.001455616	0.002389966	0.001457789
64	1.013104828	0.999888637	1.017287575	1	0.001238176	0.002194182	0.001218258
128	1.019315292	1.000065144	1.026799244	1	0.001382111	0.002226888	0.001010036

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.002195208	1.000127147	1.002195208	1	0.001972187	0.003470662	0.001972187
8	1.003875543	1.000258016	1.004088185	1	0.001603864	0.003043418	0.001654537
16	1.006348555	0.999935469	1.006868668	1	0.001500382	0.002909342	0.001328398
32	1.009786793	0.999891597	1.011350792	1	0.001384586	0.002389966	0.001102267
64	1.015053802	0.999888637	1.018157201	1	0.001472017	0.002194182	0.001381381
128	1.02203878	1.000065144	1.028290325	1	0.001323319	0.002226888	0.001049114

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001844007	1.000127147	1.001869583	1	0.001853699	0.003470662	0.001815234
8	1.00333341	1.000258016	1.003686616	1	0.0018141	0.003043418	0.001694165
16	1.005569222	0.999935469	1.006568128	1	0.001616456	0.002909342	0.001494507
32	1.008690109	0.999891597	1.010711502	1	0.001673325	0.002389966	0.001413342
64	1.013135338	0.999888637	1.017283933	1	0.001340079	0.002194182	0.001167759
128	1.019346271	1.000065144	1.027233934	1	0.001296167	0.002226888	0.001002913

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001749207	1.000127147	1.00181617	1	0.001783673	0.003470662	0.001918121
8	1.003290396	1.000258016	1.003747317	1	0.001861269	0.003043418	0.001858971
16	1.005649279	0.999935469	1.006480946	1	0.001280248	0.002909342	0.001335428
32	1.008791331	0.999891597	1.010799009	1	0.001612809	0.002389966	0.001330819
64	1.013371436	0.999888637	1.017697873	1	0.001376797	0.002194182	0.000962488
128	1.019489782	1.000065144	1.027635497	1	0.001192178	0.002226888	0.00110873

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001797083	1.000127147	1.001860039	1	0.00200603	0.003470662	0.00198465
8	1.003414389	1.000258016	1.003711526	1	0.001834719	0.003043418	0.001706318
16	1.0056465	0.999935469	1.006571071	1	0.001855642	0.002909342	0.001565131
32	1.008796991	0.999891597	1.010778458	1	0.001394538	0.002389966	0.001485097
64	1.013091796	0.999888637	1.017271751	1	0.001269604	0.002194182	0.001227851
128	1.019299299	1.000065144	1.026766097	1	0.001324841	0.002226888	0.00103161

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.001936978	1.000127147	1.001990541	1	0.002071776	0.003470662	0.002110964
8	1.0034648	1.000258016	1.003758089	1	0.001688906	0.003043418	0.001507519
16	1.005595389	0.999935469	1.006442192	1	0.00144475	0.002909342	0.001289378
32	1.008561075	0.999891597	1.010823223	1	0.001343193	0.002389966	0.001176651
64	1.013233252	0.999888637	1.017547017	1	0.001418621	0.002194182	0.001138994
128	1.019399715	1.000065144	1.027637484	1	0.001333325	0.002226888	0.001289748

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.002229877	1.000127147	1.00223456	1	0.002038542	0.003470662	0.002043241
8	1.004080213	1.000258016	1.004260101	1	0.001516389	0.003043418	0.001532835
16	1.006489907	0.999935469	1.007105849	1	0.001437527	0.002909342	0.00149691
32	1.009873846	0.999891597	1.01133346	1	0.00118617	0.002389966	0.001136127
64	1.015035641	0.999888637	1.018213107	1	0.001498576	0.002194182	0.00138429
128	1.022047369	1.000065144	1.02852819	1	0.00136761	0.002226888	0.001036734

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.002265185	1.000127147	1.002268694	1	0.001980925	0.003470662	0.001969847
8	1.00417738	1.000258016	1.004259054	1	0.001495733	0.003043418	0.001536076
16	1.006726322	0.999935469	1.007211437	1	0.001301473	0.002909342	0.001383887
32	1.010534845	0.999891597	1.01171214	1	0.001427538	0.002389966	0.001316254
64	1.015818153	0.999888637	1.018556029	1	0.001059428	0.002194182	0.001294119
128	1.023114672	1.000065144	1.028581167	1	0.001286187	0.002226888	0.001084326

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001936978	1.000127147	1.001990541	1	0.002071776	0.003470662	0.002110964
8	1.003446194	1.000258016	1.003734586	1	0.001603889	0.003043418	0.001422175
16	1.005523395	0.999935469	1.006432665	1	0.001480942	0.002909342	0.001223105
32	1.00874947	0.999891597	1.010930214	1	0.001415511	0.002389966	0.001362895
64	1.013242708	0.999888637	1.017706208	1	0.001363414	0.002194182	0.00144119
128	1.019602044	1.000065144	1.027862436	1	0.001419224	0.002226888	0.001105749

Largest Envy Results

Ver

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001276331	1.00486987	1.001130634	1	0.003420141	0.006496919	0.003492882
8	1.00435479	1.01024382	1.003967051	1	0.00470685	0.005721355	0.004607624
16	1.008424238	1.017865787	1.007862653	1	0.004631694	0.006242783	0.004888797
32	1.01603295	1.031543048	1.014725728	1	0.005775226	0.009580758	0.006581087
64	1.027753899	1.050830562	1.025700229	1	0.006913865	0.012834345	0.007097628
128	1.042282247	1.079131401	1.038261021	1	0.007326986	0.013460231	0.009930255

SmallestHalfCut

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001125325	1.00486987	1.000931681	1	0.004084813	0.006496919	0.003829234
8	1.004022365	1.01024382	1.003714461	1	0.004306716	0.005721355	0.004272998
16	1.00900414	1.017865787	1.008482612	1	0.0036006	0.006242783	0.003838588
32	1.017064461	1.031543048	1.016254946	1	0.006320776	0.009580758	0.007026733
64	1.02800995	1.050830562	1.025801468	1	0.006637042	0.012834345	0.005162792
128	1.043675712	1.079131401	1.040561872	1	0.00973554	0.013460231	0.010509746

LargestMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000871984	1.00486987	1.000871984	1	0.003283752	0.006496919	0.003283752
8	1.003839758	1.01024382	1.003500684	1	0.004394082	0.005721355	0.004197119
16	1.008561428	1.017865787	1.007999492	1	0.005116234	0.006242783	0.005154756
32	1.016026999	1.031543048	1.015110731	1	0.006325059	0.009580758	0.006384434
64	1.028772517	1.050830562	1.026147081	1	0.009920064	0.012834345	0.010798526
128	1.045961666	1.079131401	1.041953345	1	0.012334139	0.013460231	0.014189736

SmallestPiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001020232	1.00486987	1.000957163	1	0.003744459	0.006496919	0.003803984
8	1.004463874	1.01024382	1.004158805	1	0.00400139	0.005721355	0.004283537
16	1.009019649	1.017865787	1.008541427	1	0.004365222	0.006242783	0.00468594
32	1.017307402	1.031543048	1.016026541	1	0.005586847	0.009580758	0.006541914
64	1.029338309	1.050830562	1.026558667	1	0.009232908	0.012834345	0.00917466
128	1.045676006	1.079131401	1.041319305	1	0.008149445	0.013460231	0.009471954

VerHor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001387986	1.00486987	1.001268067	1	0.004206586	0.006496919	0.004110932
8	1.004946558	1.01024382	1.004229154	1	0.004863875	0.005721355	0.004559029
16	1.009462333	1.017865787	1.008898558	1	0.005290885	0.006242783	0.005738128
32	1.018393323	1.031543048	1.017175571	1	0.007522583	0.009580758	0.008262154
64	1.028655646	1.050830562	1.026497167	1	0.007812539	0.012834345	0.007684614
128	1.047810966	1.079131401	1.042979932	1	0.011467619	0.013460231	0.011093441

Hor

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.001125325	1.00486987	1.000931681	1	0.004084813	0.006496919	0.003829234
8	1.004022365	1.01024382	1.003714461	1	0.004306716	0.005721355	0.004272998
16	1.008917714	1.017865787	1.008325561	1	0.003837842	0.006242783	0.003993382
32	1.017125284	1.031543048	1.016160349	1	0.006256891	0.009580758	0.00699615
64	1.02810825	1.050830562	1.02557423	1	0.007183209	0.012834345	0.006304079
128	1.044003714	1.079131401	1.039923547	1	0.009152857	0.013460231	0.010642036

HorVer

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI

4	1.000898026	1.00486987	1.000774382	1	0.003454418	0.006496919	0.003375531
8	1.004178839	1.01024382	1.003877587	1	0.004154849	0.005721355	0.004075373
16	1.008851983	1.017865787	1.008196646	1	0.004271572	0.006242783	0.003966703
32	1.016283774	1.031543048	1.014627498	1	0.006603497	0.009580758	0.00619885
64	1.029160016	1.050830562	1.026419683	1	0.006979409	0.012834345	0.007987425
128	1.049861017	1.079131401	1.04411713	1	0.012775935	0.013460231	0.01344438

HighestScatter

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000887541	1.00486987	1.000876667	1	0.003663494	0.006496919	0.003682166
8	1.00346755	1.01024382	1.003111654	1	0.004405446	0.005721355	0.00401554
16	1.008296973	1.017865787	1.007356673	1	0.004856514	0.006242783	0.004250275
32	1.016500768	1.031543048	1.015442863	1	0.006688994	0.009580758	0.006261545
64	1.027774247	1.050830562	1.025027626	1	0.006478087	0.012834345	0.007310385
128	1.046458948	1.079131401	1.041887349	1	0.011341018	0.013460231	0.012369542

MostValuableMargin

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000682863	1.00486987	1.000682863	1	0.002979511	0.006496919	0.002979511
8	1.003567942	1.01024382	1.003511803	1	0.004795964	0.005721355	0.00475802
16	1.008191355	1.017865787	1.007546827	1	0.004693526	0.006242783	0.004717468
32	1.015342907	1.031543048	1.014582887	1	0.006740622	0.009580758	0.006794334
64	1.026028288	1.050830562	1.024243187	1	0.007863731	0.012834345	0.008634139
128	1.045399842	1.079131401	1.040993027	1	0.01118345	0.013460231	0.011828772

SquarePiece

#Agents	EvenPaz	Assessor	EvenPaz+TTC	Selling	EvenPaz CI	Assessor CI	EvenPaz+TTC CI
4	1.000898026	1.00486987	1.000774382	1	0.003454418	0.006496919	0.003375531
8	1.004161156	1.01024382	1.003939114	1	0.003981649	0.005721355	0.003818291
16	1.008995731	1.017865787	1.008314522	1	0.004723055	0.006242783	0.004641325
32	1.01678742	1.031543048	1.015240381	1	0.006926171	0.009580758	0.006834156
64	1.031020915	1.050830562	1.027818929	1	0.009235883	0.012834345	0.008533362
128	1.050059158	1.079131401	1.045463576	1	0.011965199	0.013460231	0.01397528

Average Face Ratio Results

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.17114094	0.558331937	1	4.60E-17	1.24E-16
8	0.08557047	0.431197869	1	2.17E-17	3.60E-17
16	0.042785235	0.452647497	1	6.95E-18	2.92E-17
32	0.021392617	0.383458149	1	5.12E-18	5.12E-17
64	0.010696309	0.417685542	1	2.56E-18	6.03E-17
128	0.005348154	0.365192052	1	1.25E-18	6.37E-17

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.365196078	0.558331937	1	8.89E-17	1.24E-16
8	0.182598039	0.431197869	1	4.03E-17	3.60E-17
16	0.111082696	0.452647497	1	5.84E-05	2.92E-17
32	0.061905438	0.383458149	1	9.35E-05	5.12E-17
64	0.035630984	0.417685542	1	2.42E-05	6.03E-17
128	0.017815646	0.365192052	1	9.17E-06	6.37E-17

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.415952942	0.558331937	1	0.307620577	1.24E-16
8	0.287801124	0.431197869	1	0.13325211	3.60E-17
16	0.307803425	0.452647497	1	0.163314187	2.92E-17
32	0.330544299	0.383458149	1	0.085756549	5.12E-17
64	0.357717618	0.417685542	1	0.087233126	6.03E-17
128	0.387645884	0.365192052	1	0.069131817	6.37E-17

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.552922788	0.558331937	1	0.000268509	1.24E-16
8	0.276447603	0.431197869	1	9.17E-05	3.60E-17
16	0.138225352	0.452647497	1	3.15E-05	2.92E-17
32	0.069114191	0.383458149	1	1.21E-05	5.12E-17
64	0.046540871	0.417685542	1	2.59E-05	6.03E-17
128	0.025314618	0.365192052	1	2.58E-05	6.37E-17

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.526254222	0.558331937	1	0.000433058	1.24E-16
8	0.297471035	0.431197869	1	0.000187112	3.60E-17
16	0.390607188	0.452647497	1	0.00068592	2.92E-17
32	0.28282469	0.383458149	1	0.000349363	5.12E-17
64	0.374622989	0.417685542	1	0.000682465	6.03E-17
128	0.323665345	0.365192052	1	0.000416012	6.37E-17

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.365196078	0.558331937	1	8.89E-17	1.24E-16
8	0.182598039	0.431197869	1	4.03E-17	3.60E-17
16	0.09129902	0.452647497	1	1.60E-17	2.92E-17
32	0.04564951	0.383458149	1	6.67E-18	5.12E-17
64	0.022824755	0.417685542	1	3.77E-18	6.03E-17
128	0.011412377	0.365192052	1	1.87E-18	6.37E-17

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.558316927	0.558331937	1	0.000233486	1.24E-16
8	0.360846484	0.431197869	1	0.000242624	3.60E-17
16	0.331624716	0.452647497	1	0.000346437	2.92E-17
32	0.336041573	0.383458149	1	0.00052267	5.12E-17
64	0.317212082	0.417685542	1	0.000418492	6.03E-17
128	0.355341006	0.365192052	1	0.000837287	6.37E-17

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.419200723	0.558331937	1	0.315172427	1.24E-16
8	0.293532814	0.431197869	1	0.134173791	3.60E-17
16	0.305968012	0.452647497	1	0.145545511	2.92E-17
32	0.339750576	0.383458149	1	0.093488114	5.12E-17
64	0.368146554	0.417685542	1	0.058900549	6.03E-17
128	0.415459712	0.365192052	1	0.04366358	6.37E-17

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.392101405	0.558331937	1	0.427519382	1.24E-16
8	0.271320704	0.431197869	1	0.194128449	3.60E-17
16	0.246226463	0.452647497	1	0.151774397	2.92E-17
32	0.228673374	0.383458149	1	0.104461185	5.12E-17
64	0.237933166	0.417685542	1	0.108201512	6.03E-17
128	0.255087482	0.365192052	1	0.077611551	6.37E-17

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.558316927	0.558331937	1	0.000233486	1.24E-16
8	0.388895745	0.431197869	1	0.000304371	3.60E-17
16	0.409476414	0.452647497	1	0.000422167	2.92E-17
32	0.4879351	0.383458149	1	0.000581562	5.12E-17
64	0.530510308	0.417685542	1	0.000783934	6.03E-17
128	0.584430182	0.365192052	1	0.000708286	6.37E-17

Smallest Face Ratio Results

Ver

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.03598322	0.246301227	1	0.00016309	0
8	0.016465608	0.09338135	1	7.86E-05	0
16	0.007600817	0.040181471	1	4.88E-05	0
32	0.003760944	0.042319339	1	3.10E-05	0
64	0.00186216	0.02228519	1	1.75E-05	0
128	0.000928096	0.019324674	1	9.48E-06	0

SmallestHalfCut

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.073513044	0.246301227	1	0.00052702	0
8	0.031251818	0.09338135	1	0.000164939	0
16	0.015178208	0.040181471	1	8.77E-05	0
32	0.007377741	0.042319339	1	6.62E-05	0
64	0.003638736	0.02228519	1	2.82E-05	0
128	0.001806058	0.019324674	1	1.98E-05	0

LargestMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.111233564	0.246301227	1	0.265610879	0
8	0.06439546	0.09338135	1	0.095641816	0
16	0.041160625	0.040181471	1	0.03818716	0
32	0.032097592	0.042319339	1	0.025260917	0
64	0.018614427	0.02228519	1	0.010967184	0
128	0.013473722	0.019324674	1	0.008049341	0

SmallestPiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.07557239	0.246301227	1	0.000331509	0
8	0.033336965	0.09338135	1	0.000159942	0
16	0.015984109	0.040181471	1	9.71E-05	0
32	0.007825251	0.042319339	1	6.48E-05	0
64	0.003902878	0.02228519	1	3.59E-05	0
128	0.001908947	0.019324674	1	2.23E-05	0

VerHor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.263573133	0.246301227	1	0.000213019	0
8	0.036886006	0.09338135	1	0.000163319	0
16	0.041666962	0.040181471	1	0.000125958	0
32	0.020721602	0.042319339	1	0.000149471	0
64	0.021947909	0.02228519	1	0.000122372	0
128	0.008073279	0.019324674	1	0.000150764	0

Hor

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.073513044	0.246301227	1	0.00052702	0
8	0.031251818	0.09338135	1	0.000164939	0
16	0.015178208	0.040181471	1	8.77E-05	0
32	0.007377741	0.042319339	1	6.62E-05	0
64	0.003638736	0.02228519	1	2.82E-05	0
128	0.001806058	0.019324674	1	1.98E-05	0

HorVer

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.246285698	0.246301227	1	0.000496857	0
8	0.129188877	0.09338135	1	0.000497161	0
16	0.052388211	0.040181471	1	0.000235057	0
32	0.061720627	0.042319339	1	0.000232424	0
64	0.016745143	0.02228519	1	0.000142854	0
128	0.017635099	0.019324674	1	0.000109429	0

HighestScatter

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.103779862	0.246301227	1	0.240704371	0
8	0.060080258	0.09338135	1	0.067749258	0
16	0.039751654	0.040181471	1	0.023398743	0
32	0.031163914	0.042319339	1	0.022538732	0
64	0.020729088	0.02228519	1	0.01161559	0
128	0.01417952	0.019324674	1	0.007826323	0

MostValuableMargin

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.100643819	0.246301227	1	0.264533602	0
8	0.046740017	0.09338135	1	0.080005072	0
16	0.026930932	0.040181471	1	0.031632327	0
32	0.013798887	0.042319339	1	0.014896731	0
64	0.007724487	0.02228519	1	0.008428824	0
128	0.004333221	0.019324674	1	0.004709386	0

SquarePiece

#Agents	EvenPaz	Assessor	Selling	EvenPaz CI	Assessor CI
4	0.246285698	0.246301227	1	0.000496857	0
8	0.129188877	0.09338135	1	0.000497161	0
16	0.105903443	0.040181471	1	0.000756257	0
32	0.078997703	0.042319339	1	0.000492221	0
64	0.052082187	0.02228519	1	0.000514127	0
128	0.028268832	0.019324674	1	0.005039949	0

FOCS results

	N=128	N=64	N=32	N=16	N=8	N=4
EV	1.014921061	1.01038763	1.006866426	1.004421482	1.002770939	1.001654864
UV	1.029586864	1.019418343	1.012326886	1.007787923	1.004685801	1.002472173
AFR	0.651848524	0.591791581	0.542051849	0.506856634	0.431102755	0.558316927
SFR	0.087745087	0.106219067	0.118833565	0.149122412	0.163680427	0.263573133