

The Open University of Israel

Department of Mathematics and Computer Science

MMM - Multi-Channel TDMA with MPR Capabilities for MANETs

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Shimon Avadis

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Abstract

Many Ad-hoc networks for Military and Public Safety environments are characterized by a large number of nodes in the same area (which means that frequency spatial reuse is less applicable), crucial situation awareness (SA) (which implies periodic frequent location updates, mission status, etc.), or high propagation delay (for example, acoustic or airborne networks).

In order to support such networks, an efficient Medium Access Control (MAC) broadcast algorithm is essential. Using one shared channel with only one packet reception at a time capability may not be scalable and therefore Multi-Packet Reception (MPR) techniques are more suitable. Recent technological developments (patent pending [18]) enable all nodes to receive messages simultaneously in many, even hundreds of channels in each node.

In SA applications, the most important indices are the packet reception rate from each node and the maximum delay between two consecutive updates from each node in the surrounding zone. SA messages are very important, for instance, in avoiding airplane collisions and friendly fire.

In this work, we define two indices to evaluate a scheduling of nodes in an MPR setup. The first – rate index: This index considers the worst-case packet reception rate between any pair of nodes. The second – delay index: considers the worst-case delay between two consecutive receptions of any pair of nodes.

Then we propose three multi-channels TDMA MAC based scheduling algorithms and we analyse the performance and the output of the algorithms. We show that each one of the algorithms performs better with respect to the different indices and traffic assumptions.

In this work, we consider the best scenario that is, the simple case of full mesh. Some of the results in this work were published in [1]. The full paper is given in Appendix A.

1. Introduction

1.1. Motivation

One of the major applications in military and public wireless safety networks is Situation Awareness (SA). SA allows each node to be aware of the location of other nodes in the same zone. The theory of situation awareness in dynamic systems can be found in a paper by Endsley [2]. SA applications generate periodic messages which are broadcast to all nodes in the network. The periodic messages are generated at a very high rate, so it can be assumed that each node always has a packet to transmit (heavy load traffic). In crowded networks the SA information is too large to be transmitted in a single time slot so it is crucial to get very high update rate time slots.

We refer to networks, such as airplane control [3] or emergency detection, in which SA is crucial and the propagation delay is not negligible. To support updated SA information, frequent messages are sent, so naturally, TDMA schemes are usually chosen. In SA applications, the most important indices are the packet reception rate from each node and the maximum delay between two consecutive updates from each node in the surrounding zone. For instance, SA messages are very important, for avoiding airplane collision and friendly fire. It is crucial to receive all the nodes in the radio 1-hop range. The packet reception rate is defined by the ratio between the number of packets received in a period of time over the same period of time.

MPR (Multi-packet Reception) technology enables nodes to receive messages simultaneously in many channels. Based on this technology, opportunities to develop new scheduling algorithms, which might improve performance the aforementioned networks, compared to single channel reception algorithms.

In this work, we design multi-channel TDMA scheduling algorithms that take advantage of MPR capability. We assume half duplex radio channels with multi-packet reception capability from each channel.

1.2. Literature review/Background

1.2.1. MAC algorithms

MAC algorithms can be roughly divided into two main categories: contention based algorithms and contention free algorithms. Contention based algorithms are either based on random access (ALOHA, slotted-ALOHA), on carrier sense access (CSMA), or on collision avoidance with handshaking access (MACA, MACAW). A detailed survey of contention based algorithms can be found in an article by den Hevel-Romaszko and Blondia [4] and in the work of Kumar, Raghavan and Deng [5]. Although contention based algorithms might be suitable for bursty traffic loads, they suffer from low throughput in high load networks, due to the increased number of collisions in high load traffic (see for example [4] and Zhvand and Zhou [19]).

The most popular contention free MAC algorithms combine TDMA, FDMA and CDMA. Standard TDMA algorithms for broadcast purposes usually assume reception of one packet at a time, and usually assume that the influence of the propagation delay is negligible (for examples see work by Bao and Aceves [6] and [7]).

1.2.2. Multi-channel MAC algorithms

Multi-channel MAC algorithms, without MPR capabilities, are usually used for simultaneous multiple number unicast sessions, or for simultaneous multiple number multicast sessions in a multi-hop spatial reuse network.

Such algorithms usually assume that only one packet can be received at any given time. To support the multi-channel property, in most of the existing networks (example, IEEE 802.11), the nodes swap between the channels using the same antenna.

Most of the Multi-channel MAC algorithms are designed for unicast messages and are based on IEEE 802.11. Most of them require a control channel for access negotiation, that is, for choosing the channel for the data communication. A survey can be found in [4].

Clustering is yet another technique to obtain simultaneous multi-channelling, i.e., one

channel per cluster. This structure is hierarchical, and cluster heads and backbone communication algorithms need to be defined. A survey of clustering schemes is described by Yu and Chong [8].

1.2.3. Multi-channel MAC algorithms with MPR

With the advent of sophisticated signal processing techniques, it is possible to achieve Multi-Packet Reception (MPR) simultaneously in different channels. The potential improvement of a network's performance by using MPR is shown in the work of Aceves, Sadjadpour and Wang [10] and [11]. Crichigno, Wu and Shu [9] suggest the use of several radio antennas for unicast applications, in order to achieve MPR capabilities. The problem defined there was dynamic channel assignment when the number of antennas is smaller than the number of channels defined by IEEE-802.11.

In this work we assume MPR capabilities: several messages can be received simultaneously. We also assume that there are half duplex nodes. A half-duplex node is a node which cannot receive and transmit at the same time. The same assumptions were made for example, by Chlamtac and Faago [12], and in [10] and [11]. The GAFT algorithm in [12] and the algorithm suggested by Shrader and Giles [13] assume similar conditions. These algorithms were designed for unicast messages. Cai and Lu [14] further assume the ability to receive messages during transmission in different channels, which is technically harder to achieve.

The COMB algorithm [15] divides space into hexagonal cells. Each cell is allocated a code, which is a combination of twelve orthogonal CDMA codes. The codes are spatially reused between cells that are far enough from each other (separated by at least three cells). The intra-cell algorithm, SOTDMA [16], uses the cell's code. The drawback to this algorithm is the different density of the various cells.

In CDMA schemes for cellular networks, only the base station has MPR capabilities, and these capabilities rely on adaptive power control. Furthermore, the number of simultaneous receptions is limited. This makes it less suitable for ad-hoc networks, as ad-hoc networks are many-to-many communication networks.

1.3. Single wide band channel versus multiple channels

A Medium Access Control (MAC) algorithm is an algorithm which coordinates the access to the transmission medium among different nodes. For a wireless network, the medium is usually defined by a range of frequency bands (channels) that a node can use to transmit information. A natural question is what are the benefits of using multiple narrow channels versus using one wideband channel.

[17], the authors proved that the capacity of a network with one wideband channel is not less than the capacity of a network in which the spectrum is divided into m channels. However, their model differs from our model in several aspects: they assume unicast messages, single channel reception and negligible propagation delay time.

In our model the propagation time is crucial and not negligible and therefore we utilize MPR capability.

There are three main reasons to use multiple channels:

1. Usually, the available spectrum is not continuous so it is divided by its nature into sub-channels like 802.11 channels in ISM bands. The authors in [17] mention two administrative reasons for that:
 - a. "Using different channels for separate networks allows the networks to coexist without interference. If only one channel is available, nodes from different networks must coordinate with each other so that they do not transmit simultaneously or else the network performance can degrade substantially. Coordination among nodes from separate networks is very difficult when the interfering links across the networks are weak".
 - b. "Due to historical and political reasons, it is often impossible to assign the same frequency band in every country. Consequently, a wireless network technology is designed to operate at one of several possible frequency bands depending on location. Furthermore, the bandwidth available varies from one country to another. Instead of defining a different standard for every region, multiple channels are defined in the same standard. A subset of these channels can be used in each region".
2. The cost of the power amplifier at each node is proportional to the bandwidth and therefore a wider channel means more expensive equipment at each node.
3. As the propagation delay does not depend on the width of the channel, multi-channel architectures suffer less from high propagation delay. Therefore, in networks that operate in a high propagation

delay environment, like airborne or acoustic networks, using multi-channel might improve the performances.

In multi-channel TDMA networks, the transmission time on each channel is divided into time slots, which are in turn grouped into frames. The frames on all the channels are synchronized with each other. The time-aligned frames on all of the channels are grouped into a multi-channel TDMA frame.

We use the following notations:

T - The transmission time in a single (wideband) channel;

P - The maximum propagation delay between any pair of nodes in the network;

m - The number of channels; and T_p - The transmission time in one narrowband channel ($T_p = m * T$). The size of the time-slot is therefore $mt + P$.

Most of the TDMA MAC algorithms assume that P is negligible. Nevertheless, in airborne/acoustic networks P is significant. It is easy to show that the transmission rate is $\frac{m}{mt+P}$. Therefore, as long as P is not negligible, the transmission rate is an increasing function of m . The transmission time (T_p) of a packet is also an increasing function of m while the propagation delay time is a constant. Therefore, the percentage of time wasted on propagation delay is decrease as m increase, as described in Figure 1.

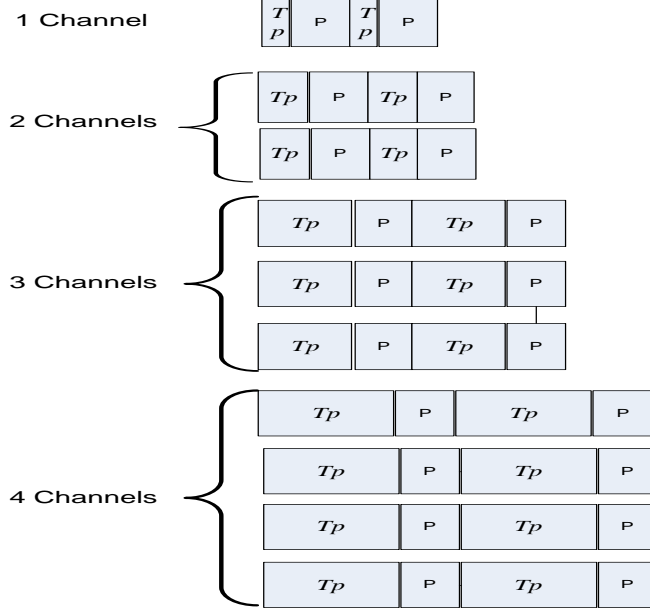


Figure 1: The size of a time slot as a function of m

In this work, we state the necessary and sufficient conditions where multi-channel models achieve a better performance than a single (wideband) channel model, as a function of the number of nodes, the packet transmission time, and the propagation delay. We compare the performance using the following two indices: R -index and D -index described in 2.2.

1.4. Our results

In this work, we examine and compare two kinds of models: cooperative and non-comparative. Let n be the number of nodes, and m be the number of channels.

1. Non-comparative model

In this model, only one packet can transmit in each access. In this model, there are approaches to optimization: the first approach is designed to maximize the reception rate between any pair of nodes, the second approach designed to minimize the delay between two consecutive updates over all the pair of nodes. We prove the following:

- a. Upper bound of the minimum packet reception rate over all pairs of nodes. The minimum packet reception rate over all pairs of nodes is equal or less than to the following bound:

$$\frac{(n-m) \times m}{(n-1) \times n \times (mT+P)}. \text{ We present an algorithm which equals this bound.}$$

b. Upper bounds for the maximum delay between two consecutive updates over all pairs of nodes.

- i. The maximum delay is less or equal $2 \left\lceil \frac{n}{m} \right\rceil (\lceil \log m \rceil + 1)(mT + P)$.
- ii. The maximum delay is less or equal $2 \lceil \sqrt{n} \rceil (mT + P)$ if $n \leq m^2$
- iii. The maximum delay is less or equal $2 \lceil n/m \rceil (mT + P)$ if $n \geq m^2$

In each of the above cases, we show a linear time algorithm that produces a schedule of a maximum size equal to the upper bound.

c. Lower bounds for the maximum delay between two consecutive updates over all pairs of nodes.

- i. The maximum delay is higher or equal $\lceil 2n/m \rceil (mT + P)$
- ii. The maximum delay is higher or equal $\lceil \log n \rceil (mT + P)$

We show a linear time algorithm which reaches the lower bound when $n \geq m^2$

2. comparative model

In this model, we assume full cooperation, i.e., the algorithm assumes that nodes will use the first half of their transmission time for their own messages, and the second half for forwarding messages originating in another node. The time slot duration is bigger than non-comparative model algorithms. We show an algorithm with the following bounds assuming $m \leq \frac{n}{2}$.

a. The lower bound of the minimum packet reception rate over all pairs of nodes is:

$$\frac{m}{n \times (2 * m * T + P)}$$

b. The upper bound for the maximum delay between two consecutive updates over all pairs of nodes is $\left(\left\lceil \frac{n}{m} \right\rceil + 1 \right) \times (2mT + P)$.

In this work, we state the conditions where multi-channel algorithms achieve a better performance than a single (wideband) channel model.

This work is organized as follows. In Section 2 we describe the model and define the performance Indices. The Indices bounds proof and scheduling algorithms are presented and analysed in Section 3, and the optimal number of channels is calculated. In section 4 we compare the performance of the proposed algorithms. Finally, we conclude and present several suggestions for future developments in Section 5.

2. Models and Performance Indices

2.1. Model description

We assume a full mesh MANET with unlimited frequent status updates. The network consists of N homogeneous nodes and can support multiple packet reception (in different channels). All channels are accessible to all nodes. The nodes communicate by sending periodic equal length packets. The system restrictions are as follows:

1. A node cannot transmit and receive at the same time (half duplex transmitters).
2. A node cannot transmit on more than one channel at the same time.
3. If more than one node transmits in the same channel at the same time, the packets collide.

In this work we do not consider the effect of multiple access interference caused by imperfect orthogonality of channels. We also do not consider the guard band between any two channels. The nodes are synchronized timewise. The number of channels used might change dynamically according to the topology condition.

We use the following notations:

- $N = \{1, \dots, n\}$ - The set of nodes.
- $M = \{1, \dots, m\}$ - The set of available channels.
- P - The maximum propagation delay between two neighboring nodes in the network.
- T - The transmission time in a single (wideband) channel.

T_{Slot} - The size of a time slot, that is, the transmission time plus and the propagation delay time. Therefore, T_{Slot} is $mT + P$.

2.2. Performance Indices

We define two performance indices:

1. ***R-index*** - The global minimum packet reception rate. Denote by $r(i, j)$ - The average packet reception rate per node i , from node j , i.e. the ratio between the number of packets nodes i received from node j , in a period of time over the same period of time. Note that $r(i, j)$ does not necessary equal $r(j, i)$. *R-index* is then defined as:

$R = \min_{i,j}\{r(i,j)\}$. This index is a worst case packet reception rate between **any** two nodes.

2. **D-index** - The global maximum delay between two consecutive receptions of any pair of nodes. Denote by $d(i,j)$ – the maximum delay between two consecutive receptions from node j by node i . Note that $d(i,j)$ does not necessarily equal $d(j,i)$. *D-index* is then defined as: $D = \max_{i,j}\{d(i,j)\}$. That is, the worst delay case between two consequent receptions between any two nodes.

3. Scheduling Algorithms

3.1. The algorithm problem

Given N and M , we are interested in designing a periodic transmission strategy (namely, a periodic schedule). This means that every $i \in N$ is provided with a finite increasing time-slot sequence $t_1^i, \dots, t_{k_i}^i$. Let $d = \max_i t_{k_i}^i$ be the schedule length (frame) of the sequence. The time slots when i can transmit are $t_1^i, \dots, t_{k_i}^i, t_1^i + d, \dots, t_{k_i}^i + d$. Note that the parameter d depends on the transmission strategy, but it is the same for all nodes. A reception is considered successful if the receiving node is idle during the time-slot of the transmission. Hence we have the following requirement.

1. For every $(i, j) \in N \times N$ there will be an “ (i, j) - transmission” time slot, when i transmits and j does not transmit.
2. At most, m nodes can transmit simultaneously at the same time slot.

Definition: A schedule S of N is a sequence of subsets of N such that the following holds.

1. The union of the sets in S is N .
2. S is an m -schedule if all the sets in S have a maximum size of m .

In our scheduling algorithms, we illustrate a schedule S (frame) in a table format. Each row corresponds to a different channel, and each column corresponds to a different time-slot.

We thus consider the following algorithmic problems:

Given a set $N = \{1, \dots, n\}$ of nodes and an integer m (the number of channels). Find an m -schedule S of N that minimizes D -index.

Given a set $N = \{1, \dots, n\}$ of nodes and an integer m (the number of channels). Find an m -schedule S of N that maximizes R -index

We consider 3 different multi-channel TDMA scheduling algorithms. We describe each algorithm, its performance, the optimal number of channels to use and algorithm examples. As a reference point, we first analyze the basic TDMA algorithm with a single (wideband) channel.

3.2. Single Channel TDMA Algorithm – Algorithm SC

The basic algorithm uses a single channel ($m=1$), with a TDMA policy, as demonstrated in Table 1. Formally, $\text{Frame}(t) = (t \bmod n) + 1, t = \{0,1, \dots\}$. In algorithm SC, $T_{slot} = T + P$.

Table 1: Basic TDMA scheduling Frame

Ch ₁	1	2	3	$n-1$	n	1	2	...
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3.2.1. Performance analyzes:

1. $R_{SC} = \frac{1}{n \times (T + P)}$, each node transmits every n time slot and is received by all the nodes in the network (only one node transmits at a time).
2. $D_{SC} = n \times (T + P)$, the packet might wait in the queue of n time slots.

3.3. Maximum-Rate m -channel Algorithm - Algorithm A

This algorithm is designed to optimize R -index. The channel is divided into m equal-bandwidth channels. All the m -combinations of n nodes are calculated. Each combination of m nodes is assigned one time slot, and each node in the combination is then allocated one channel from the m channels to transmit during the assigned time slot. In this way, the set of all combinations defines a TDMA cyclic frame. The frame length is $\binom{n}{m}$ time slots. An example frame is shown in Table 2. In this example we construct the combination with lexicographic order. In algorithm A, $T_{slot} = mT + P$.

We suggest a recursive algorithm, that in each step, the next combination is calculated based on the previous combination. Assume that the first permutation $p(1)$ is the

natural permutation (1,2,...,m). Given p(k), p(k+1) is calculated as follows:

1. Find the largest i such that $p(k)[i] \neq n - m + i$.
2. For $j= 1$ to $i-1$ $p(k + 1)[j] = p(k)[j]$.
3. $p(k + 1)[i] = p(k)[i] + 1$.
4. For $j = i+1$ to m $p(k + 1)[j] = p(k + 1)[j - 1] + 1$.

The resultant schedule is shown in Table 2.

Table 2: An example of a TDMA frame of Algorithm A (lexicographic order) with $n=6$ and $m=3$

Ch ₁	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	4	...
Ch ₂	2	2	2	2	3	3	3	4	4	5	3	3	3	4	4	5	4	4	5	5	...
Ch ₃	3	4	5	6	4	5	6	5	6	6	4	5	6	5	6	6	5	6	6	6	...

Note: In the special case of $m = 1$ we get a basic TDMA as shown in section 3.1

3.3.1. R-index performance

Claim 1:

$$R_A = \frac{(n-m) \times m}{(n-1) \times n \times (mT+P)}$$

Proof of claim 1:

Each node transmits exactly $\binom{n}{m} \times \frac{m}{n}$ times in each frame (symmetry);

The number of times per frame that each pair of nodes transmits in the same time slot is $\binom{n-2}{m-2}$.

Therefore, for any two nodes i and j , the number of times per frame node i receives node j packet is:

$$\binom{n}{m} \times \frac{m}{n} - \binom{n-2}{m-2} = \binom{n}{m} \times \frac{m}{n} \times \left(\frac{n-m}{n-1}\right)$$

Therefore,

$$R_A = \frac{\binom{n}{m} \times \frac{m}{n} \times \left(\frac{n-m}{n-1}\right)}{\binom{n}{m} \times (mT + P)} = \frac{(n-m) \times m}{(n-1) \times n \times (mT + P)}$$

The number of channels (m) that maximizes R_A is then:

$$\frac{\partial R_A}{\partial m} = \frac{\partial}{\partial m} \frac{(n-m) \times m}{(n-1) \times n \times (mT + P)} = 0 \Rightarrow m = \frac{-P + \sqrt{P^2 + PTn}}{T}.$$

Corollary: As one could expect, m is an increasing function of n and of P . Therefore, for dense networks and for high propagation delay (P) networks, bigger m 's are advised. m is also a decreasing function of T , so for long transmission times (long packets for example) fewer channels will have a smaller R -index.

Claim2:

Given that one packet is transmitted in each time slot, algorithm A achieves the optimal (upper bound) R -index.

Proof of claim 2:

In algorithm A, the number of packets transmitted by each node per frame is the same, and $\forall i, j \in N, r(i, j) = R_A$. Let us assume, by contradiction, there is another algorithm, say algorithm B which $R_B > R_A$. Let T be the frame size of algorithm B. The general number of all packet reception in a frame is $m(n-m)T$, so the average packet reception rate of any node by any node is $\frac{m(n-m)}{n(n-1)(mT+P)}$ which it equal to R_A . Therefore, if $\exists i, j$ such that $r_B(i, j) > R_A$ then there must be at least two nodes, say nodes k and l such that $r_B(k, l) < R_A$. Since $R_B = \min_{i,j} \{r_b(i, j)\} < R_A$, which contradict the assumption of $R_B > R_A$ therefore R_A is optimal.

3.3.2. D-index performance

When $m=1$, $D_A = n \times (T + P)$, but when $m>1$, D_A is tightly dependant on the combination generation order, the following claims assume a general combination method.

Claim 3:

$$D_A \leq \left(\binom{n}{m} \left(1 - \frac{m}{n}\right) + \binom{n-2}{m-2} + 1 \right) \times (mT + P)$$

Proof of claim 3:

The number of times per frame node i receives node j for any two nodes is: $\binom{n}{m} \times \frac{m}{n} - \binom{n-2}{m-2}$. Let us assume, by contradiction, that $\exists i, j$ such that $d(i, j) > \left(\binom{n}{m} \left(1 - \frac{m}{n}\right) + \binom{n-2}{m-2} + 1 \right)$ time slots. Its means that the remaining of the receptions of node j by node i ($\binom{n}{m} \times \frac{m}{n} - \binom{n-2}{m-2} - 2$) should be in the remaining of $\binom{n}{m} - d(i, j)$ time slots. Since $\binom{n}{m} - d(i, j) < \left(\binom{n}{m} - \left(\binom{n}{m} \left(1 - \frac{m}{n}\right) + \binom{n-2}{m-2} + 2 \right) \right) = \left(\binom{n}{m} \times \frac{m}{n} - \binom{n-2}{m-2} - 2 \right)$ the assumption is not fulfilled and therefore the maximum delay is:

$$D_A \leq \left(\binom{n}{m} \left(1 - \frac{m}{n}\right) + \binom{n-2}{m-2} + 1 \right) \text{ time slots.}$$

Explanation: The worst case delay between any given two given nodes i, j occurs when node i transmits continuously for $\binom{n}{m} \times \frac{m}{n}$ time slots and node j transmits continuously simultaneously with node i in the first $\binom{n-2}{m-2}$ time slot (The number of times per frame that each pair of nodes transmits in the same time slot).

The lexicographic order combination as shown above is the worst case delay . Table 2 illustrates the schedule of $m=3$ and $n=6$ with a cycle of 20 time slots. Consider time slot 16 (allocated to nodes 2,5 and 6). During this time slot, node 1 successfully recieves

transmission from node 2. The next time that node 1 successfully receives transmission from node 2 is on time slot 11, that is, a delay of 15 time slots.

We showed above the worst upper bound for any given permutation order. When $m^2 \leq n$ (which is the common state) and n divided by m , this bound might be improved. We show that if we could impose a certain order of the permutations, such that the following two conditions hold, then the upper bound would be significantly improved. We divide the set of all the permutations into several cycles, of size $\frac{n}{m}$ such that:

1. Each node is allocated exactly one time slot for transmission in each cycle
2. If two nodes transmit simultaneously in a given cycle, they will not transmit simultaneously in the consecutive cycle.

In this case, the upper bound is reduced to:

$$D_A \leq (3 * \frac{n}{m} - 1) \times (mT + P)$$

$3 * \frac{n}{m} - 1$ time slots are the maximum gap between two successive reception of 2 nodes. Consider any two nodes i and j . The worst case is when node j receives the transmission of node i in the first time slot of cycle A. Then, in cycle A+1 they both transmit together; and in cycle A+2 node j receives node's i transmission in the last time slot of this cycle. Therefore the delay between these 2 nodes is $3 * \frac{n}{m} - 1$ time slots, which is $(3 * \frac{n}{m} - 1) \times (mT + P)$. Table 3 illustrates the schedule of $m=2$ and $n=6$ which fulfil the two conditions.

Table 3: An example of a TDMA frame $m=2$ and $n=6$.

Ch ₁	1	2	3	1	4	2	1	5	4	1	6	5	1	3	6	...
Ch ₂	4	5	6	5	6	3	6	3	2	3	2	4	2	4	5	...

We now prove the upper bound for some special cases.

1. Case I: $m=2$.

1. We form the frame, subdivided into several cycles, each cycle with $n/2$ time slots, as follows: In the first cycle we schedule all nodes by their sequential numbers, that is, nodes $1, \dots, \frac{n}{2}$ are assigned to the first channel; and nodes $\frac{n}{2} + 1, \dots, n$ are assigned to the second channel. Let A be the first cycle of the frame, $A =$

$$\begin{cases} A_{0,t} = t + 1 \\ A_{1,t} = \frac{n}{2}t + 1 \end{cases}, t = \{0, 1, \dots, \frac{n}{2} - 1\}$$

2. Each cycle is derived from the previous cycle by cyclic shifting of all nodes clockwise except for the first node in the first channel, which stays static. Let \bar{A}

be the next cycle so $\bar{A} = \begin{cases} \bar{A}_{0,t} = A_{0,t-1}, t = \{2 \leq t \leq \frac{n}{2} - 1\} \\ \bar{A}_{1,t} = A_{0,t+1}, t = \{0 \leq t \leq \frac{n}{2} - 2\} \\ \bar{A}_{0,0} = A_{0,0} \\ \bar{A}_{0,1} = A_{1,0} \\ \bar{A}_{1,\frac{n}{2}-1} = A_{0,\frac{n}{2}-1} \end{cases}$

3. The frame consists of $n-1$ cycles. Each node is scheduled in a different position in each cycle except the position of the first time slot in the first channel which is assigned to node 1 only.

Claim 4: In case I, $D_A \leq (2 * \lceil \frac{n}{2} \rceil + 2) \times (mT + P)$.

Proof of claim 4:

1. Each node transmits in each cycle exactly once.
2. Each node transmits with any other node exactly once in each frame.
3. Therefore, in every 3 cycles it is certain that each node receives any other node at least twice.
4. Since each node shifts only one time slot each cycle, the worst case for the delay is two cycles plus 2 time slots between 2 consecutive receptions. Therefore, $D_A \leq 2 * \lceil \frac{n}{m} \rceil + 2$ time slots

In Table 4 we show such a cyclic construction for $m=2$ and $n=8$.

Table 4: An example of a TDMA frame special case ($m=2$) and $n=8$.

Ch ₁	1	2	3	4	1	5	2	3	1	6	5	2	1	7	6	5	1	8	7	6	1	4	8	7	1	3	4	8	...
Ch ₂	5	6	7	8	6	7	8	4	7	8	4	3	8	4	3	2	4	3	2	5	3	2	5	6	2	5	6	7	...

2. Case II: $m^2 = n, m \in \{2, 3, 5, 7\}$

The schedule does not necessarily use all the combinations but is still bounded by R_A and $D_A \leq (3 * \frac{n}{m} - 1)$ time slots.

The schedule can be described as follow:

1. The frame is divided into cycles of m time slots and m channels.
2. In each cycle, each node is allocated exactly one access.
3. In each frame, each node transmits simultaneously with each other node exactly once.
4. Each cycle is represented by a square $m \times m$ matrix
5. The assignment in each cycle depends on the previous cycle.
6. A is the current cycle matrix and \bar{A} is the next cycle matrix and the transformation function is : $\bar{A}_{j,i} = A_{(i,(i+j) \bmod(m))}$, $0 \leq i \leq m - 1, 0 \leq j \leq m - 1$
7. The number of the cycles is $\frac{n-1}{m-1} = m + 1$.
8. The transformation function is operated m times to form the m cycles. In the $m+1$ cycle the m nodes that have not been transmitting simultaneously in the previous cycles are assigned for transmission in the same time slot.

In Table 5-9, we show examples for $m=2, 3$ and 5 respectively.

Table 5: An example of a TDMA frame of algorithm A special case $m^2 = n$ ($m=2$).

Ch ₁	1	3	1	2	1	4	...
Ch ₂	2	4	4	3	3	2	...

Table 6: An example of a TDMA frame of algorithm A special case $m^2 = n$ ($m=3$)

Ch ₁	1	2	3	1	4	7	1	8	6	1	9	5	...
Ch ₂	4	5	6	8	2	5	9	4	2	3	8	4	...
Ch ₃	7	8	9	6	9	3	5	3	7	2	7	6	...

Table 7: An example of a TDMA frame of algorithm A special case $m^2 = n$ ($m=5$)

Ch ₁	1	6	11	16	21	1	2	3	4	5	1	10	14	18	22
Ch ₂	2	7	12	17	22	10	6	7	8	9	23	2	6	15	19
Ch ₃	3	8	13	18	23	14	15	11	12	13	20	24	3	7	11
Ch ₄	4	9	14	19	24	18	19	20	16	17	12	16	25	4	8
Ch ₅	5	10	15	20	25	22	23	24	25	21	9	13	17	21	5

Ch ₁	1	23	20	12	9	1	13	25	7	19	1	2	3	4	5	...
Ch ₂	13	10	2	24	16	11	23	10	17	4	8	9	10	6	7	...
Ch ₃	25	17	14	6	3	21	8	20	2	14	15	11	12	13	14	...
Ch ₄	7	4	21	18	15	6	18	5	12	24	17	18	19	20	16	...
Ch ₅	19	11	8	5	22	16	3	15	22	9	24	25	21	22	23	...

3.3.3. The optimal number of channels

The optimal number of channels for algorithm A in order to maximize the R -index is

$$m = \text{round} \left(\frac{-P + \sqrt{P^2 + PTn}}{T} \right). \text{ Therefore, } R_A > R_{SC}, \text{ whenever}$$

$$\text{round} \left(\frac{-P + \sqrt{P^2 + PTn}}{T} \right) > 1 \text{ is satisfied.}$$

In the special case II above, where the upper bound of the delay is less or equal to $D_A \leq (3 * \frac{n}{m} - 1) \times (mT + P)$. D_{SC} of single (wideband) channel is $n(T + P)$. Therefore, $D_{SC} > D_A$, whenever $\frac{P(nm-3n+m)}{2nm-m^2} > T$.

Remarks:

In situations where the optimization of the *R-index* is crucial, it is beneficial to use algorithm A. The algorithm keeps fairness of packet reception rate between any two nodes, similar to the one channel TDMA. The fairness is important for SA applications QoS. The main disadvantage of this algorithm is its high delay and the complexity of constructing a general scheduling to reduce the delay.

3.4. Minimum-Delay m-channel Algorithm - Algorithm B

In this section, we focus on algorithms that minimize the *D-index*. In our published paper (Appendix A), we proved close upper and lower bounds of the delay and had shown linear complexity of the running time of the algorithms. The results are summarized in the following theorems (time is measured in terms of time slots units when $T_{Slot} = mT + P$).

3.4.1. D-index performance

In Appendix A, we proved the following two theorems:

Theorem 1

- $D_B \leq 2 \left\lceil \frac{n}{m} \right\rceil (\lceil \log m \rceil + 1)$
- $D_B \leq 2 \lceil \sqrt{n} \rceil$ if $n \leq m^2$
- $D_B \leq 2m \left\lceil \frac{n}{m^2} \right\rceil$ if $n \geq m^2$

Theorem 2 For any positive integers $n \geq m \geq 2$ the following holds.

- $D_B \geq \left\lceil \frac{2n}{m} \right\rceil$.
- $D_B \geq \lceil \log n \rceil$.

Since the paper in Appendix A was published, we found an algorithm that reaches the lower bound for every $n \geq m^2$. So in this section we concentrate on the additional algorithm that was not included in Appendix A.

We now show that Algorithm which reaches the lower bound delay when $n \geq m^2$, i.e., $D_B = \left\lceil \frac{2n}{m} \right\rceil$ (time is measured in terms of time slots units).

Algorithm B:

- Let A be a matrix which represents the scheduling frame. The columns represent channels and the rows represent time slots.
- The frame scheduling takes two phases. In each phase we assign each of the n nodes only once.

Phase I:

1. $A_{(i,j)} = (1 + i + j * m) \bmod n$, if $1 + i + j * m \leq n$, $0 \leq i \leq m - 1$, $0 \leq j \leq \left\lceil \frac{n}{m} \right\rceil - 1$.

Phase II:

1. Find from phase I, $index_i$ and $index_j$ such that $A_{(index_i, index_j)} = n$
2. If $(index_i + 1) \bmod m = 0$ $index_i = 0$; $index_j = index_j + 1$
3. Else $index_i = index_i + 1$
4. for $i=0$ to $m-1$
 - a. for $j=0$ to $\text{ceil}(n/m)-1$
 - i. if $(i + j * m + 1 \leq n)$
 1. $A_{(index_i, index_j)} = 1 + i + j * m$
 2. If $(index_i + 1) \bmod m = 0$ $index_i = 0$; $index_j = index_j + 1$
 3. Else $index_i = index_i + 1$

Note that in phase II we follow the assignment that was formed in the previous phase row-wise, and assign in the A matrix all n nodes column-wise, starting after the last assignment of the first phase

In Tables 8-9 we illustrate the scheduling formed by Algorithm B for different values of n and m .

Table 8: An example of a TDMA frame of algorithm B: $n=7$ and $m=3$.

Ch ₁	1	4	7	7	3
-----------------	---	---	---	---	---

Ch ₂	2	5	1	2	6
Ch ₃	3	6	4	5	

Table 9 relates to the same values we used to illustrate Algorithm A (Table 6). In Algorithm B, the *R-index* is 1/6 time slots (versus 1/4 in Algorithm A) the *D-index* is 6 time slots (versus 8 time slots in Algorithm A).

Table 9: An example of a TDMA frame of algorithm B: $n=9$ and $m=3$

Ch ₁	1	4	7	1	2	3
Ch ₂	2	5	8	4	5	6
Ch ₃	3	6	9	7	8	9

Claim 5 : $D_B = \left\lceil \frac{2n}{m} \right\rceil \times (mT + P)$, that is, Algorithm B reaches the best lower bound.

Proof of claim 5:

1. The frame size is $\left\lceil \frac{2n}{m} \right\rceil$ time slots, since each node is assigned exactly twice and the number of channels is m . Therefore, the number of time slots per frame is $\left\lceil \frac{2n}{m} \right\rceil$ time slots.
2. Each node transmits simultaneously with any other node at most once per frame since the row size of the first phase is less than the column size ($n \geq m^2$), this guarantees that if two nodes transmit simultaneously in the first stage they will not transmit simultaneously in the second stage.
3. Each node receives a message from every other node at least once per frame. Therefore, $D_b \leq \left\lceil \frac{2n}{m} \right\rceil$ time slots, but from the lower bound proof (in Appendix A) it follows that $D_b \geq \left\lceil \frac{2n}{m} \right\rceil$ and therefore the following equation must hold:

$$D_b = \left\lceil \frac{2n}{m} \right\rceil.$$

3.4.2. *R-index* performance

Claim 6 : $R_B \geq \frac{1}{\lceil \frac{2n}{m} \rceil \times (mT+P)}$, when $n \geq m^2$

Proof of Claim 6:

As *R-index* is the worst case scenario and as Algorithm B is designed such that in each frame each node receives a message from any other node at least once, therefore *R-index* equals 1 over the frame's size:

$$R_B \geq \frac{1}{\lceil 2n/m \rceil \times (m * T + P)}$$

3.4.3. The optimal number of channels

We proved that if $n \leq m^2$ then $D_B \leq 2\lceil \sqrt{n} \rceil (mT + P)$. Therefore, D_B is an increasing function of m . The minimum delay is thus achieved for minimum value of m that is, $m = \lceil \sqrt{n} \rceil$. $D_{SC} = n(T + P)$, therefore multi-channel with $\lceil \sqrt{n} \rceil$ channels achieves lower delay than on single channel ($D_B \leq D_{SC}$) whenever $P(\frac{n-2\lceil \sqrt{n} \rceil}{n}) > T$.

We proved that if $n \geq m^2$ then $D_B = \lceil \frac{2n}{m} \rceil (mT + P)$. Therefore, D_B is a decreasing function of m . The minimum delay is thus achieved for maximum value of m that is, $m = \lfloor \sqrt{n} \rfloor$. Consequently, multi-channel with $\lfloor \sqrt{n} \rfloor$ channels achieves lower delay than on single channel ($D_B \leq D_{SC}$) whenever $P(\frac{n-2\lfloor \sqrt{n} \rfloor}{n}) > T$.

Remarks:

Algorithm B focuses on *D-index* minimization. The algorithm guarantees a minimum delay between consecutive packets for any two nodes. The packet reception rate between any two nodes is not homogenous, i.e., some nodes might receive more packets from some nodes and fewer packets from other nodes. *R-index* in algorithm B is not optimal (proof was given in algorithm A).

3.5. Relay m -channel Algorithm - Algorithm C

In Algorithm C, we assume full cooperation, i.e., the algorithm assumes that nodes will use the first half of their transmission time for their own messages, and the second half for forwarding messages originated in another node. In each time slot, m nodes are transmitting. Each of them transmits two messages: in the first half – their own message; and in the second half – the message received in the first half of the previous time slot in the same channel. The time slot duration is bigger than in all other algorithms. In this algorithm we assume $m \leq \frac{n}{2}$.

Algorithm C is actually an extension of the unicast algorithm GAFT, described in [12], that applies to broadcast messages. The main modification is due to the fact that in Algorithm C we expect all nodes to finally receive all messages while GAFT messages are directed to a single destination.

Therefore, in Algorithm C we retransmit all the messages while in GAFT we need to retransmit $\frac{m-1}{n-1}$ from the messages.

In algorithm C, $T_{Slot} = 2mT + P$. Time slots are grouped into frames. Each frame contains n time slots.

The scheduling procedure:

The scheduled algorithm frame is constructed as a matrix of size $m \times n$ as follows: Assign node's id numbers sequentially column-wise until all the matrix's cells are assigned. That is, the first m nodes are assigned to the first column (the first time slot), etc.

Formally, Each node i acts according to the following rule in time slot $t = \{0, 1, \dots\}$:

1. if there is a $j \in \{1, \dots, m\}$ such that $(tm + j) \bmod n = i$, node i can transmit on channel j .
2. if node i can transmit in a time slot t via channel j , node i also relays the packet which it received at time slot $t-1$ via channel j (packing to its own packet).

A scheduling example can be found in Table 10.

Table 10: An example of a TDMA frame of algorithm C: $n=9$ and $m=4$

Ch ₁	1 (+6)	5 (+1)	9 (+5)	4 (+9)	8 (+4)	3 (+8)	7 (+3)	2 (+7)	6 (+2)	...
Ch ₂	2 (+7)	6 (+2)	1 (+6)	5 (+1)	9 (+5)	4 (+9)	8 (+4)	3 (+8)	7 (+3)	...
Ch ₃	3 (+8)	7 (+3)	2 (+7)	6 (+2)	1 (+6)	5 (+1)	9 (+5)	4 (+9)	8 (+4)	...
Ch ₄	4 (+9)	8 (+4)	3 (+8)	7 (+3)	2 (+7)	6 (+2)	1 (+6)	5 (+1)	9 (+5)	...

3.5.1. R-index performance

Since in each frame, each node receives m packets from every node (directly or relayed) the R -index is:

$$R_C \geq \frac{m}{n \times (2 * m * T + P)}$$

3.5.2. D-index performance

Each node is assigned to transmit in the worse case, every $\left\lceil \frac{n}{m} \right\rceil$ time slot. As we assumed that $2m \leq n$, a node will not transmit in two consequent time slots. Therefore, nodes that transmit simultaneously have to wait one time slot to receive the packet by relay, so the worst case delay is $\left\lceil \frac{n}{m} \right\rceil + 1$ time slots. Thus:

$$D_C \leq \left(\left\lceil \frac{n}{m} \right\rceil + 1 \right) \times (2mT + P).$$

3.5.3. The optimal number of channels

As $m \leq \frac{n}{2}$ and as R_C is a monotonically increasing function of m , the maximization of R_C occurs when $m = \frac{n}{2}$. $R_{SC} = \frac{1}{n(T+P)}$. Therefore, when $m = \frac{n}{2}$, $R_{SC} < R_C$, whenever

$$\frac{P(n-2)}{2} > T.$$

$$D_{SC} = n(T + P). \text{ Therefore, } D_{SC} > D_C, \text{ whenever } P \times \left(\frac{(n-1)m-n}{2m^2+mn} \right) > T.$$

Remarks

The main disadvantage of Algorithm C is the overhead that results from the fact that each message is sent twice: once by the originator and once as a relayed message. Note that the cost of the two transmissions considers only the transmission time and not the propagation delay that is "paid" only once for the two packets.

This disadvantage can be regarded as an advantage as it is more resilient with regard to packet loss.

4. Algorithms Performance Comparison

4.1. *D-index*

We use the following *D-index* measures for comparison between the performance of algorithms A, B and C:

1. $D_A = (3 \frac{n}{m} - 1)(mT + P)$
2. $D_B = \lceil \frac{2n}{m} \rceil \times (mT + P)$
3. $D_C = (\lceil \frac{n}{m} \rceil + 1) \times (2mT + P)$

Note that measures 2 and 3 were proved above, while measure 1 was proved subject to several conditions

These are the comparison results:

1. $D_B < D_A$ always.
2. $D_B < D_C$ whenever $P \times \left(\frac{n-m}{2m^2} \right) < T$ (for simplicity we assumed that n is divided by m).
3. $D_C < D_A$ always.

4.2. *R-index*

We use the following *R-index* measures for comparison between performance of algorithms A, B and C:

1. $R_A = \frac{(n-m) \times m}{(n-1) \times n \times (mT+P)}$
2. $R_B = \frac{1}{\lceil \frac{2n}{m} \rceil \times (mT+P)}$
3. $R_C = \frac{m}{n \times (2mT+P)}$

These are the comparisons results:

1. $R_B < R_A$ always.
2. $R_C < R_A$ whenever $P \times \left(\frac{m-1}{m+nm-2m^2} \right) < T$
3. $R_B < R_C$ always.

5. Conclusions and open problems

Based on the Multi-Packet Reception (MPR) technique, that enables nodes in ad hoc networks to receive messages simultaneously in many channels, we have formalized a new algorithmic scheduling problem. Given the number of nodes in the network, packet transmission time, propagation delay time and the number of channels, we derived upper and lower bounds for two indices: maximum delay between updates over all pairs of nodes and the minimum update rate over all pairs of nodes. We proposed three multi-channel TDMA MAC algorithms for broadcasting of periodic messages and analysed their performance. We also showed the conditions whenever multi-channel achieves better performances than a single (wideband) channel. In general as the propagation delay time increases the multi-channel algorithms achieve better performance than the single channel. We showed that each one of the algorithms performs better with respect to the different indices and traffic assumptions.

In this work we focused on full mesh topology. Other topologies such as multi-hop networks might also be considered. We also assumed homogeneous networks which means that each node has the same weight for transmitting and receiving. Heterogeneous networks with weights (edges or nodes) might be considered. It might also be interesting to extend this work by evaluating the influence of a packet loss on the various algorithms.

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Appendix A

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MMM: multi-channel TDMA with MPR capabilities for MANETs

Shimon Avadis · Anat Lerner · Zeev Nutov

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Abstract Many Ad-hoc networks for military and public safety environments are characterized by: large number of nodes in the same area (that means that frequency spatial reuse is less applicable), crucial situation awareness (which implies periodically frequent location updates, mission status, etc.), or high propagation delay (for example, acoustic or airborne networks). In order to support such networks, an efficient medium access control broadcast protocol is essential. Obviously, using one shared channel with only one packet reception at a time is not scalable and therefore multi-packet reception techniques are more suitable. Recent technological developments (patent pending) enable nodes to receive messages simultaneously in many, even hundreds of channels. In this paper we study the impact of the new multi packet reception capabilities. In order to compute close upper and lower bounds on the maximum delay, we consider the best scenario that is, the simple case of full mesh. We then propose algorithms that achieve a close to the best possible maximum delay between updates over all pairs of nodes. This is done by providing close upper and lower bounds on the maximum delay and giving simple algorithms that meet the upper bound. For theoretical completeness we study bounds for all possible relations between the number of nodes and the number of channels.

S. Avadis
Rafael Ltd., P.O.B 2250, 31021 Haifa, Israel
e-mail: ashimon@rafael.co.il

A. Lerner (✉) · Z. Nutov
Mathematics and Computer Science Department,
The Open University of Israel, 1 university Rd.,
Raanana, Israel
e-mail: anat@cs.openu.ac.il

Z. Nutov
e-mail: nutov@openu.ac.il

Keyword Multi-channel · Ad-hoc networks (MANET) · Multi-packet reception (MPR) · Medium access control (MAC) algorithms

1 Introduction

One of the major applications in military and public wireless safety networks is situation awareness (SA). SA allows each node to be aware of the location of other nodes in the same zone. Theory of situation awareness in dynamic systems can be found in [8]. SA applications generate periodical messages which are broadcasted to all nodes in the network. The periodical messages are generated in a very high rate, so it can be assumed that each node always has a packet to transmit.

We refer to networks, such as airplane control [15] or emergency detection, in which SA is crucial. To support updated SA information, frequent messages are sent, so naturally, TDMA schemes are usually chosen. In SA applications, one of the most important performance parameters is the delay between two consecutive updates. Since the messages are generated and transmitted periodically, reliability is less relevant.

In this paper we study algorithms for the MAC protocol, designed for the SA application. The algorithms aim to minimize the maximum delay between two consequent updates of any pair of nodes.

1.1 MAC protocols

MAC protocols can be roughly divided into two main categories: contention based protocols and contention free protocols. A detailed survey of contention based protocols

can be found in [7, 11]. Contention based protocols are not suitable for high load networks.

The most popular contention free MAC protocols combine TDMA, FDMA and CDMA. Standard TDMA protocols for broadcast purposes usually assume reception of one packet at a time, and usually assume that the influence of the propagation delay is negligible. (See for example: [1], and [6]).

1.2 Multi-channel MAC protocols

Multi-channel MAC protocols usually assume that only one packet can be received at a given time. It is assumed that a node has a tunable radio allowing it to listen or transmit on one channel at a time. To ensure high connectivity, all nodes tune their radio to the same channel. But as a result the node density increases, and therefore the available bandwidth per node decreases. To overcome this problem the nodes are usually divided into clusters. Each cluster is then allocated a dedicated channel that can be reused by further clusters. The bandwidth allocated to each cluster can be relatively high, but the transmission rate of messages between clusters is very low. A survey of clustering schemes is described in [16]. Most of the Multi-channel MAC protocols are dedicated to unicast messages and are based on IEEE 802.11. A survey can be found in [5]. The solution proposed in [5] suggests equipping each node with an IEEE 802.11 radio interface (with small bandwidth). Therefore, there is a need for dynamic channel assignment between the transmitting node and the receiving node.

1.3 Multi-channel MAC protocols with MPR

With the advent of sophisticated signal processing techniques, it is possible to achieve multi-packet reception (MPR) simultaneously in different channels. The potential improvement of network's performance by using MPR is shown in [10, 13].

In this paper we assume MPR capabilities, that is, more than one message can be received simultaneously. We also assume half duplex nodes, that is a node can not receive and transmit at the same time. Same assumptions were made for example, in [3, 10] and [13]. The GAFT protocol for example [3] and the protocol suggested in [14] assume similar assumptions. These protocols were designed for unicast messages. In [2] they further assume the ability to receive messages during transmission in different channels, which is technically harder to achieve.

COMB protocol [9] divide the space into hexagonal cells. Each cell is allocated a code, that is combined of twelve orthogonal CDMA codes. The codes are spatially reused between cells that are far enough from each other

(separated by at least three cells). The intra-cell protocol, SOTDMA [12], uses the cell's code. The drawback of this protocol is the different density of the various cells.

In CDMA schemes for cellular networks, only the base station has MPR capabilities, and these capabilities rely on adaptive power control. Furthermore, the number of simultaneous receptions is limited. This makes it less suitable for ad-hoc networks, as ad-hoc networks are many-to-many communication networks.

New technology (patent pending) enables nodes to receive messages simultaneously in many, even hundreds of channels. Based on this technology, we propose protocols that use this new capability to achieve a close to the best possible maximum delay between updates over all pairs of nodes. This is done by providing close upper and lower bounds on the maximum delay and giving simple algorithms that meet the upper bound.

1.4 Single wide band channel versus multiple channels

Given the range of the available frequencies, the entire range can be used as a wide band channel or as a single super-channel. There are three main reasons to use multiple channels.

- Usually, the available spectrum is not continuous so it is divided to sub-channels by its nature.
- The expense of the power amplifier at each node is proportional to the bandwidth of the channel and therefore a wider channel means more expensive equipment at each node.
- As the propagation delay does not depend on the width of the channel, multi-channel architectures suffer less from high propagation delay. Therefore, in networks that operate in a high propagation delay environment, like airborne or acoustic networks, multi-channel can increase the performance.

1.5 Our results

In this paper we prove an upper bound for the maximum delay between two consecutive updates over all pairs of nodes. Let n be the number of nodes, and m be the number of channels, we show that: The maximum delay is less or equal $2\lceil n/m \rceil (\lceil \lg m \rceil + 1)$. We further show that if $n \leq m^2$ then the delay is less or equal $2\lceil \sqrt{n} \rceil$, and if $n \geq m^2$ then the delay is less or equal $2\lceil n/m \rceil$. Furthermore, in each one of the cases, there exist a linear time algorithm that produces a schedule of size at most the upper bound stated.

We also prove two lower bounds of the maximum delay, that is, the maximum delay is higher or equal both $\lceil 2n/m \rceil$ and $\lceil \lg n \rceil$.

We then present 3 algorithms that create scheduling cycle with a maximum delay lower or equal the upper bound. Where in the case of $n \geq m^2$ the upper and lower bounds coincide.

We proceed as follows. In Sect. 2 we describe the model and basic assumptions. The algorithmic problem is formalized in Sect. 3. The scheduling algorithms are presented, analyzed and compared in Sect. 4. In Sect. 5 we illustrate the algorithms with some examples. In Sect. 6 we compute the optimal number of channels and characterize when the multi-channel model achieves a better performance than the one super channel model. We then conclude in Sect. 7.

2 Model description

We assume a full mesh MANET with frequent status updates. The network consists of n homogeneous nodes and can support multiple packets reception (in different channels). All channels are accessible to all nodes. The nodes communicate by sending periodical equal length packets. The system restrictions are as follows.

- (a) A node cannot transmit and receive at the same time (half duplex transmitters).
- (b) A node cannot transmit on more than one channel at the same time.
- (c) If more than one node transmits in the same channel at the same time, the packets collide.

In this paper we do not consider the effect of multiple access interference caused by imperfect orthogonality of channels. We also do not consider guard band between any two channels. The nodes are synchronized time wise. The number of channels used might change dynamically according to the topology condition.

We use the following notation.

- $N = \{1, \dots, n\}$ denotes the set of nodes.
- $M = \{1, \dots, m\}$ denotes the set of available channels.
- p is the maximum propagation delay between two neighboring nodes in the network.
- t is the transmission time in a single super-channel.

The size of a time-slot is therefore $mt + p$. It is easy to show that the transmission rate is $m/(mt + p)$. Therefore, as long as p is not negligible, the transmission rate is an increasing function of m .

3 The algorithmic problem

Given N and M , we are interested to design a *periodical* transmission strategy (namely, a periodical schedule). This

means that every $i \in N$ is provided with a finite increasing time-slot sequence t_1^i, \dots, t_k^i . Let $d = \max_i t_k^i$ be the *schedule length* of the sequence. The time slots when i can transmit are

$$t_1^i, \dots, t_k^i, t_1^i + d, \dots, t_k^i + d, \dots$$

Note that the parameter d depends on the transmission strategy, but it is the same for all nodes. A reception is considered successful if the receiving node is idle during the time-slot of the transmission. Hence we have the following requirement.

- (R1) For every $(i, j) \in N \times N$ there will be an “ (i, j) -transmission” time slot, when i transmits and j does not transmit.

Then, since the transmission schedule is periodical, for any $(i, j) \in N \times N$ we have that d is an upper bound on the “ (i, j) -delay”—the number of time slots between two consecutive (i, j) -transmissions. Moreover, if for some (i, j) there is only one (i, j) -transmission time slot, then d equals the (i, j) -delay; again, since the transmission is periodical, then w.l.o.g. we may assume that this is the case.

In addition, since the number of channels is bounded by m , we require that

- (R2) At most m nodes can transmit simultaneously at the same time slot.

The discussion above motivates the following definition.

Definition 1 A *schedule* \mathcal{S} of N is a sequence of subsets of N such that the following holds.

- (R1) The union of the sets in \mathcal{S} is N .

\mathcal{S} is an *m -schedule* if the following holds.

- (R2) All the sets in \mathcal{S} have size at most m .

The *length* d of a schedule \mathcal{S} is the length of \mathcal{S} , namely, $d = |\mathcal{S}|$.

In Table 1 we illustrate a schedule \mathcal{S} in a table format. Each row corresponds to a different channel, and each column corresponds to a different time-slot.

We thus consider the following algorithmic problem:

Given a set $N = \{1, \dots, n\}$ of nodes and an integer m (the number of channels), find an m -schedule \mathcal{S} of N of minimum size (delay).

One can further observe, that the order of the sets in a schedule \mathcal{S} is irrelevant for requirements (R1) and (R2); namely, any permutation of an m -schedule \mathcal{S} is also an

Table 1 An example of an m -schedule S for $N = \{1, 2, 3, 4, 5, 6\}$ and $m = 3$

ch ₁	1	4	1	3	1	2
ch ₂	2	5	2	5	3	4
ch ₃	3	6	4	6	5	6

m -schedule. This motivates the following definition, which without the restriction (R2), is the case $k = 1$ of k -resilient families considered in [4].

Definition 2 We say that a set-family \mathcal{F} on a groundset N is *resilient* if for any $(u, v) \in N \times N$ with $u \neq v$, there is $S \in \mathcal{F}$ with $u \in S$ and $v \notin S$. We say that \mathcal{F} is an m -family if $|S| \leq m$ for all $S \in \mathcal{F}$. For positive integers m, n , let $\rho(n, m)$ denote the minimum size of a resilient m -family on a groundset of n elements.

Now we see that our problem is reduced to the following optimization problem.

Minimum Resilient m -Family
 Given positive integers n, m find a minimum size resilient m -family on $N = \{1, \dots, n\}$.

We note that the problem of determining the parameter $\rho(n, m)$ and efficiently computing an optimal (minimum size) resilient m -family is of independent interest in Combinatorics, and it may have many other applications (e.g., in other network scheduling problems). In the next section we will consider this important problem.

4 Bounds and algorithms

In this section we provide close upper and lower bounds on $\rho(n, m)$.

The upper bounds are proved by providing algorithms that compute a resilient m -family within the stated upper bound. Our algorithms are efficient, and run in linear time. These results are summarized in the following theorem.

Theorem 1

- (i) $\rho(n, m) \leq 2\lceil n/m \rceil (\lceil \lg m \rceil + 1)$.
- (ii) $\rho(n, m) \leq 2\lceil \sqrt{n} \rceil$ if $n \leq m^2$.
- (iii) $\rho(n, m) \leq 2\lceil n/m \rceil$ if $n \geq m^2$.

Furthermore, in each one of the cases, there exist a linear time algorithm that produces a resilient family of size at most the upper bound stated.

Note that bound (iii) is always better than bounds (i) and (ii), but it applies only in the range $n \geq m^2$; this is usually the case in practical problems. In the case $n < m^2$, bound (i) is usually better than bound (ii) for very large values of m , say $m \geq 2^9$. The reason for that is that although asymptotically $\lg m \ll \sqrt{m}$, for small values of m we have $\sqrt{m} \approx \lceil \lg m \rceil + 1$. For example, for $m = 2^6 = 64$, we have $\sqrt{m} = 8$ and $\lceil \lg m \rceil + 1 = 8$.

However, for large values of m , bound (i) is better than bound (ii), unless $(m/\lg m)^2 \leq n \leq m^2$. We will illustrate this in more details in Sect. 5.

In addition, to indicate that our algorithms are either optimal or close to being optimal, we provide lower bounds on $\rho(n, m)$ that almost match our upper bounds. These results are summarized in the following theorem.

Theorem 2 For any positive integers $n \geq m \geq 2$ the following holds.

- (i) $\rho(n, m) \geq \lceil 2n/m \rceil$.
- (ii) $\rho(n, m) \geq \lceil \lg n \rceil$.

Note that for the most important “practical” case $n \geq m^2$ our upper bound $2\lceil n/m \rceil$ and our lower bound $\lceil 2n/m \rceil$ almost coincide, hence our algorithm is almost optimal in this case. In the other cases, our upper and lower bounds on $\rho(n, m)$ coincide up to an $O(\lg m)$ factor. We will elaborate on this in more details in Sect. 5.

Theorems 1 and 2 are proved in the following two sections, 4.1 and 4.2, respectively.

4.1 Upper bounds and algorithms (Theorem 1)

The following statement is easily verified.

Lemma 3 For $i = 1, \dots, q$, let \mathcal{F}_i be a resilient family on N_i . Then $\cup_{i=1}^q \mathcal{F}_i$ is a resilient family on $\cup_{i=1}^q N_i$. Thus $\rho(n, m) \leq \sum_{i=1}^q \rho(n_i, m)$ for any positive integers n_1, \dots, n_q with $\sum_{i=1}^q n_i = n$.

The three parts of Theorem 1 are proved in the following three lemmas, respectively.

Lemma 4 Let d be an integer such that $2^{d-1} \leq m$. Then $\rho(n, m) \leq 2d \cdot \lceil n/2^d \rceil$. In particular, for $d = \lceil \lg m \rceil + 1$ we obtain that

$$\rho(n, m) \leq 2(\lceil \lg m \rceil + 1) \cdot \lceil n/2^{\lceil \lg m \rceil + 1} \rceil \leq 2(\lceil \lg m \rceil + 1) \cdot \lceil n/m \rceil.$$

Proof 1 Let N be a groundset of n elements. We first prove the statement for the case $n \leq 2^d$, by constructing in this case a resilient m -family \mathcal{F} of size $|\mathcal{F}| \leq 2d$ as follows.

Procedure 1

1. Let Q be a cube of dimension d with vertices in $\{-1, 1\}^d$. Distribute the members of V in the 2^d unit cells of Q (unit subcubes of Q with vertices in $\{-1, 0, 1\}^d$), at most one member in each cell.
2. Every hyperplane $x_i = 0$ cuts V into two sets S'_i, S''_i of size $2^{d-1} \leq m$ each. Let $\mathcal{F} = \bigcup_{i=1}^d \{S'_i, S''_i\}$.

Clearly, $|\mathcal{F}| = 2d$. Since any two cells are separated by some hyperplane $x_i = 0$, \mathcal{F} is a resilient family. For $n \geq m$ arbitrary \mathcal{F} is defined as follows.

Algorithm 1

1. Partition N into $\lceil n/2^d \rceil$ sets of size at most 2^d each.
2. For each part N' as a groundset, use **Procedure 1** to compute a resilient 2^{d-1} -family \mathcal{F}' of size $|\mathcal{F}'| \leq 2d$. Let \mathcal{F} be the union of these families.

By the definition, $|\mathcal{F}| \leq 2d \cdot \lceil n/2^d \rceil$. By Lemma 3, \mathcal{F} is a resilient family. □

Lemma 5 *If $n \leq m^2$ then $\rho(n, m) \leq 2\lceil\sqrt{n}\rceil$.*

Proof 2 We prove the lemma by constructing a resilient family \mathcal{F} of size $|\mathcal{F}| \leq 2\lceil\sqrt{n}\rceil$ as follows.

Algorithm 2

1. Let Q be an $\lceil\sqrt{n}\rceil \times \lceil\sqrt{n}\rceil$ matrix. Distribute the members of N in the cells of Q , at most one member in each cell.
2. Let \mathcal{F} be the family of sets formed by the (non-empty) rows and columns of Q .

Clearly, $|\mathcal{F}| \leq 2\lceil\sqrt{n}\rceil$. Also, $|S| \leq \sqrt{n} \leq m$ for all $S \in \mathcal{F}$, hence \mathcal{F} is an m -family. Since every entry of Q is uniquely determined by its coordinates, \mathcal{F} is a resilient family on N . □

Lemma 6 *If $n \geq m^2$ then $\rho(n, m) \leq 2\lceil n/m \rceil$.*

Proof 3 We prove the lemma by constructing a resilient family \mathcal{F} of size $|\mathcal{F}| \leq 2\lceil n/m \rceil$ as follows.

Algorithm 3

1. Partition N into $\lceil n/m^2 \rceil$ sets of size at most m^2 each.
2. For each part N' as a groundset, use **Algorithm 2** to compute a resilient m -family \mathcal{F}' of size $|\mathcal{F}'| \leq 2\lceil\sqrt{|N'|}\rceil \leq 2m$. Let \mathcal{F} be the union of these families.

By the definition, $|\mathcal{F}| \leq \lceil n/m^2 \rceil \cdot 2m \leq 2\lceil n/m \rceil$. By Lemma 3, \mathcal{F} is a resilient family. □

It is not hard to verify that our algorithms can be implemented to run in linear time.

The proof of Theorem 1 is complete.

4.2 Lower bounds (Theorem 2)

It is easy to see that $\rho(n, m) \geq \lceil n/m \rceil$. This follows from the observation that if \mathcal{F} is a resilient family on N , then the union of the sets in \mathcal{F} is N . Hence if \mathcal{F} is an m -family, then $|\mathcal{F}| \geq \lceil n/m \rceil$.

Proving the bound $\rho(n, m) \geq \lceil 2n/m \rceil$ is more tricky, and requires the following observation.

Claim *Let \mathcal{F} be a resilient family on N and let $v \in N$. Then either $\{v\} \in \mathcal{F}$ or v belongs to at least two sets in \mathcal{F} .*

Proof 4 Since \mathcal{F} is a resilient family on N , there is $S \in \mathcal{F}$ such that $v \in S$. If $S = \{v\}$ then we are done. Otherwise, S contains some $u \in N$ distinct from v . Since \mathcal{F} is a resilient family on N , there must be $S' \in \mathcal{F}$ such that $v \in S'$ and $u \notin S'$. Note that $S' \neq S$ (since $u \in S$ and $u \notin S'$), and that each of S, S' contains v . Hence v belongs to at least two sets in \mathcal{F} , as claimed. □

Now we show that $\rho(n, m) \geq \lceil 2n/m \rceil$. Let \mathcal{F} be a resilient m -family on N . Let q be the number of singleton sets (sets of size 1) in \mathcal{F} . Then, since \mathcal{F} is an m -family and since $m \geq 2$, from Claim 4.2 it follows that

$$|\mathcal{F}| \geq q + \lceil 2(n - q)/m \rceil \geq \lceil 2n/m \rceil.$$

This concludes the proof of the lower bound $\rho(n, m) \geq \lceil 2n/m \rceil$.

Finally, we prove that $\rho(n, m) \geq \lceil \lg n \rceil$. Let us say that a set-family on N is *weakly resilient* if for any pair $\{u, v\} \subseteq N$ with $u \neq v$, there is $S \in \mathcal{F}$ such that $|\{u, v\} \cap S| = 1$. Let $\rho^*(n, m)$ denote the minimum size of a weakly resilient m -family on a groundset of n elements. From the definitions it follows that any resilient family is weakly resilient (but the inverse is not true in general), and that $\rho^*(n, m)$ is a decreasing function of m . Hence

$\rho(n, m) \geq \rho^*(n, m) \geq \rho^*(n, \infty)$. We will prove that $\rho^*(n, \infty) \geq \lceil \lg n \rceil$.

Let \mathcal{F} be a weakly resilient family on N . Let S_1, \dots, S_h be an ordering of \mathcal{F} . Define an auxiliary binary rooted tree T with $h + 1$ levels as follows, where each node $a \in T$ represents a subset $S(a)$ of N . The root of T represents N and has level 0. Any other level i is defined inductively by the set S_i as follows. If a is a node at level $i - 1$, then the right child of a represents the set $S(a) \cap S_i$, and the left child of A represents the set $S(a) \setminus S_i$. Since \mathcal{F} is weakly resilient, for every $v \in N$ there is a leaf a of T such that $S(a) = \{v\}$. Thus T has at least $\ell \geq n$ leaves. As any binary tree with ℓ leaves has height at least $\lceil \lg \ell \rceil + 1$, this implies $h + 1 \geq \lceil \lg n \rceil + 1$, as claimed.

5 Examples and discussion

Let us now consider some examples of practical values of m, n . Suppose that $n = \alpha^2 m$ for some $\alpha > 1$ (α may not be integer). Then $n/m = \alpha^2$, and the condition $n \geq m^2$ becomes $\alpha^2 \geq m$. Consequently, we get the following bounds from Theorem 1.

- (i) $\rho(n, m) \leq 2\lceil n/m \rceil (\lceil \lg m \rceil + 1) = 2\lceil \alpha^2 \rceil (\lceil \lg m \rceil + 1)$
- (ii) $\rho(n, m) \leq 2\lceil \sqrt{n} \rceil = 2\lceil \alpha\sqrt{m} \rceil$ if $\alpha^2 \leq m$.
- (iii) $\rho(n, m) \leq 2\lceil n/m \rceil = 2\lceil \alpha^2 \rceil$ if $\alpha^2 \geq m$.

Example 1 The following example illustrates that in most practical cases, the third and the second bounds in Theorem 1 are the relevant ones. Let $m = 64$. Note that then $\lg m = 7$ and $\sqrt{m} = 8$. We get the following bounds from Theorem 1.

- (i) $\rho(n, m) \leq 2\lceil \alpha^2 \rceil (\lceil \lg m \rceil + 1) = 16\lceil \alpha^2 \rceil$.
- (ii) $\rho(n, m) \leq 2\lceil \alpha\sqrt{m} \rceil \leq 16\lceil \alpha \rceil$ if $\alpha \leq 8$.
- (iii) $\rho(n, m) \leq 2\lceil \alpha^2 \rceil$ if $\alpha \geq 8$.

One can now see that for $m = 64$, bound (ii) is strictly better than bound (i). Whether bound (ii) or (iii) applies depends on the value of α . Note also that for $\alpha = 8$ both bounds coincide.

Example 2 The following example illustrates that theoretically, for large values of m , the first bound in Theorem 1 can be better than the second bound. Let $m = 2^{10}$. Note that then $\lg m = 10$ and $\sqrt{m} = 2^5 = 32$. We get the following bounds from Theorem 1.

- (i) $\rho(n, m) \leq 2\lceil \alpha^2 \rceil (\lceil \lg m \rceil + 1) = 22\lceil \alpha^2 \rceil$.
- (ii) $\rho(n, m) \leq 2\lceil \alpha\sqrt{m} \rceil \leq 64\lceil \alpha \rceil$ if $\alpha \leq 32$.
- (iii) $\rho(n, m) \leq 2\lceil \alpha^2 \rceil$ if $\alpha \geq 32$.

One can see that here bound (i) is strictly better than bound (ii) when $\alpha < 64/22$. This case however can rarely appear in practical problems.

Summary The above two examples illustrate that in most practical problems, in Theorem 1 the relevant bounds are (ii) and (iii).

Example illustrating Algorithms 2 The following simple example illustrates Algorithm 2. Note that a simple way to obtain an m -schedule in Algorithm 2 is after computing a cubic matrix Q as in the algorithm, to compute the transpose matrix Q^T of Q , and “gluing” Q and Q^T ; see Tables 2, 3, and 4 for illustration, where $N = \{1, \dots, 8\}$ and $m = 3$.

Example illustrating Algorithms 3 The following simple example illustrates Algorithm 3. Let $N = \{1, \dots, 18\}$ and $m = 3$. Then, at the first step of the algorithm we partition the set of nodes into two sets: $N_1 = \{1, \dots, 9\}$ and $N_2 = \{10, \dots, 18\}$. We apply Algorithm 2 on each set, and then unify the two outputs. The resulting schedule is shown in Table 5.

6 The optimal number of channels

In this section we address the following question.

Minimum Delay
Given: The number n of nodes, the packet transmission time t , and the propagation delay p .
Find: The number of channels m such that the delay is minimized.

We separate the discussion to two different ranges: $(m/\lg m)^2 \leq n \leq m^2$ and $n \geq m^2$. Note that the range where

Table 2 The matrix Q formed in the first step of Algorithm 2; here $n = 8$ and $m = 3$

1	4	7
2	5	8
3	6	

Table 3 The matrix Q^T

1	2	3
4	5	6
7	8	

Table 4 The 3-schedule S computed by Algorithm 2 is obtained by “gluing” the matrices Q and Q^T

ch ₁	1	4	7	1	2	3
ch ₂	2	5	8	4	5	6
ch ₃	3	6		7	8	

Table 5 An example of a 3-schedule \mathcal{S} computed by Algorithm 3; $n = 18, m = 3$

ch ₁	1	4	7	1	2	3	10	13	16	10	11	12
ch ₂	2	5	8	4	5	6	11	14	17	13	14	15
ch ₃	3	6	9	7	8	9	12	15	18	16	17	18

$(m/\lg m)^2 > n$ is not practical, as explained in Example 2 in Sect. 5.

The range $(m/\lg m)^2 \leq n \leq m^2$ In Theorem 1(ii) we proved that if $n \leq m^2$ then $\rho(n, m) \leq 2\lceil\sqrt{n}\rceil$. We showed that Algorithm 2 satisfies this bound. The delay (time wise) of Algorithm 2 is then an order of $(mt + p) \times 2\lceil\sqrt{n}\rceil$. That is, the delay is an increasing function of m . The minimum delay is thus achieved for the minimum value of m that is, $m = \lceil\sqrt{n}\rceil$.

The delay in one super channel is $n(t + p)$. Therefore, Multi-channel with $\lceil\sqrt{n}\rceil$ channels achieves lower delay than one super channel whenever $p \geq \frac{nt}{n-2\lceil\sqrt{n}\rceil}$.

The range $n \geq m^2$ In Theorem 1(iii) we proved that if $n \geq m^2$ then $\rho(n, m) \leq 2\lceil n/m \rceil$. In this case the solution constructed by Algorithm 3 has delay $2\lceil n/m \rceil(mt + p)$. That is, the delay is a decreasing function of m . Therefore, the minimum delay is achieved when m is maximized that is, $m = \lfloor\sqrt{n}\rfloor$. Consequently, Multi-channel with $\lfloor\sqrt{n}\rfloor$ channels achieves lower delay than one super channel whenever $p \geq \frac{nt}{n-2\lfloor\sqrt{n}\rfloor}$.

Summary We can conclude that in the practical range, that is, $(m/\lg m)^2 < n$, if $p \geq \frac{nt}{n-2\sqrt{n}}$, then the minimum delay is achieved by using \sqrt{n} channels. If the propagation delay is lower than that, the one super channel achieves lower delay than any multi channel algorithm.

7 Conclusions

Based on a new technology (patent pending), that enables nodes to receive messages simultaneously in many channels, we have formalized a new algorithmic scheduling problem. Given the number of nodes in the network and the number of channels, we derived close to tight upper and lower bounds. We proposed three multi-channel TDMA MAC algorithms for broadcasting of periodical messages and analyzed their performance. The three algorithms achieve a close to the best possible maximum delay between updates over all pairs of nodes. We then compared the delay between multi-channel and a single super channel. Our conclusion is that in all practical implementations, if $p \geq \frac{nt}{n-2\sqrt{n}}$ then multi channel achieves lower delay than a single channel, and the lowest delay is achieved when the

number of channels is the square root of the number of nodes.

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תקציר

רשתות אד הוק עבור תקשורת צבאית או עבור ארגוני החירום מאופיינות בכמה מאפיינים, ביניהם:

1. מספר רב של צמתים באותו אזור, כלומר שימוש חוזר בתדרים אינו אפשרי.
 2. הצמתים נדרשים לתמוך באפליקציות של מודעות מיקום – SA - Situation Awareness (כדוגמת שליחת הודעות הפצה מחזוריות וקריטיות של מיקומים וסטאטוסים)
 3. זמן התפשטות גבוה כדוגמת רשתות מטוסים או רשתות אקוסטיות.
- אפליקציות הפצה מחזוריות דורשות העברת מידע באופן תכוף לכלל הצמתים בהשגיה נמוכה כגון התרעת התנגשות במטוסים/מסוקים. מאחר וקצב ההודעות גבוה ומיועדים לכלל הצמתים, חשוב להשתמש בגישה לערוץ המונעת התנגשויות (contention free) כגון TDMA ולא גישה שאינה מונעת התנגשות שהנצילות שלה נמוכה ומתאימה יותר להודעות מסוג מזדמן וממוען.
- טכנולוגיה חדשה וישימה שהתפתחה בשנים האחרונות המשתמשת בדגימה ישירה של כלל הספקטרום מאפשרת לכל צומת לקלוט בו זמנית את כלל הערוצים בספקטרום. טכנולוגיה זו מיישמת יכולת MPR (Multi Packet Reception). טכנולוגיה זו מאפשרת לעצב אלגוריתמים חדשים המתאימים לסוג אפליקציות אלו.
- עבודה זו בדקנו את שילוב יכולת MPR לשפור ביצועים עבור סוג אפליקציות של מודעות מצבית בהיבט של קצב קליטה מקסימאלי בין כל זוג צמתים ברשת והשגיה מינימאלית בין הודעות עוקבות בין כל זוג צמתים ברשת.
- בעבודה זו אנו מציעים שלושה אלגוריתמים המבוססים על Multi-channel TDMA שעושים שימוש ביכולת הקליטה הרב ערוצית. אנו מנתחים את ביצועי האלגוריתמים ביחס ל- 2 המדדים, ומראים באילו מצבים כל אחד מהם עדיף על האחרים.
- בנוסף אנו בוחנים בעבודה באילו תנאים כדאי להשתמש ביכולת הקליטה הבו-זמנית (שימוש בערוץ רחב סרט יחיד לעומת מספר ערוצים צרי סרט) ובכמה ערוצים כדאי להשתמש עבור האופטימיזציה של כל אחד מהמדדים. עבודה זו מתרכזת בטופולוגיות full mesh.

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תודות

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האוניברסיטה הפתוחה
המחלקה למתמטיקה ולמדעי המחשב

עבודת תזה לתואר שני במדעי המחשב

תזמון הודעות הפצה מחזוריות עם יכולת קליטה מרובת ערוצים

שמעון עבדיס

ת"ז 029576246

מנחה: ד"ר ענת לרנר

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