ON GROUP GRADINGS ON DIVISION ALGEBRAS

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Finite group gradings on finite dimensional k-central division algebras and more generally on k-central simple algebras play a key role in their study and also in the study of the Brauer group of k. For instance the crossed product structure, where each homogeneous component is of dimension n over k (= order of the grading group G), gives rise to the well known isomorphism of $Br(k)$ with the second cohomology group $H^2(G_k, k_s^*)$ where G_k is the absolute Galois group of the field k and k_s is the separable closure of k. A different grading on central simple algebras, namely with the group $H \cong \mathbb{Z}_n \times \mathbb{Z}_n$, gives rise to the symbol algebra construction where each homogeneous component is of dimension 1 over k, and is key in the relation between $Br(k)$ and K_2. It turns out that every grading on a division algebra is a combination of these two gradings, called *elementary* and *fine*, respectively. More precisely every faithful grading on a division algebra, say by a group Γ, gives rise to a group extension

$$\beta : 1 \to H \to \Gamma \to G \to 1$$

where for instance, for the crossed product grading we have $H = \{1\}$, $\Gamma \cong G$ and for the symbol algebra grading $H = \mathbb{Z}_n \times \mathbb{Z}_n$, $G = \{1\}$. The main question we are interested in is which group extensions β are *realizable* in this way and in particular which groups H may occur (it is well known every finite group G occurs in a G-crossed product division algebra). We show every extension with abelian kernel is realizable. As for extensions β with nonabelian kernel (may occur in general), we show the group H must be solvable of very special type. In collaboration with (1) D. Haile and Y. Karasik, (2) D. Haile.