

# Trade Agreements, Bargaining and Economic Growth

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## **Abstract**

Rebelo's two-sector endogenous growth model is embedded within a two-country international trade framework. The two countries bargain over a trade agreement that specifies: (i) the size of the foreign aid that the richer country gives to the poorer one; (ii) the terms of the international trade that takes place after the aid is given. Foreign aid is given not because of generosity, but because it improves the capital allocation across the world and thus raises total world production. This world production surplus enables the rich country to raise its equilibrium consumption and welfare beyond their no-aid levels. To ensure it, the rich country uses a trade agreement to condition the aid on favorable terms of trade.

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## 1. Introduction

This paper explores the underlying economic rationale for linking foreign aid to trade agreements among developed and developing countries. We analyze a theoretical model in which two countries bargain over a trade agreement. The agreement specifies the size of the foreign aid to be given by a rich country to a poorer one, and the terms of the trade that take place between the two countries after the aid is given. The aid in our analysis is given not because of any assumed generosity on the part of the rich country, but because it improves the capital allocation across the world and raises total world production. This world production surplus enables the rich country, through international trade, to raise its equilibrium consumption and welfare beyond their no-aid levels. To ensure it, and to push consumption and welfare as high as possible, the rich country uses a trade agreement to condition the aid on favorable terms of trade.

An important assumption in our model is that international loan markets are imperfect.<sup>1</sup> It is due to this assumption that aid can improve the capital allocation across the world and raise total world production. We also show how due to this increased world production it is possible that the rich country may benefit from giving the aid even if it is merely a gift in the sense that after the aid is given the trade between the two countries is perfectly free, rather than subject to the stipulations of an agreement. In contrast, when international markets for loans work perfectly, an efficient allocation of production factors can be supported by lending and borrowing, eliminating the potential economic

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<sup>1</sup> This assumption reflects both theoretical and empirical findings. Bulow and Rogoff (2005) justify theoretically why development banks give grants rather than loans to developing countries. Cohen, Jacquet and Reisen (2006) show that bilateral donors have favored grants over loans during the past three decades, and that in recent years, this preference has been emulated by multilateral aid agencies as well.

benefits from giving aid. For the same reason we also assume that international labor mobility is imperfect.

The model is based on the two-sector growth model of Rebelo (1991) and on its two-country international trade extension developed by Felbermayr (2007). This model has several realistic virtues. First, it generates the empirically observed decline over time in the relative price of capital goods in terms of consumption.<sup>2</sup> Second, in the equilibrium of this model the developed country exports capital goods and the developing one exports consumption goods, as is typically the case in rich-poor countries trade relationships.<sup>3</sup> The Rebelo model provides us not only with endogenous growth but also supplies a deviation from the Inada conditions, which is mandatory for specialization to emerge in equilibrium. Thus, this deviation is also important for the possibility that aid can raise total world production.

We model these negotiations according to the Nash Bargaining mechanism presented in Nash (1950). This axiomatic mechanism alleviates the need to specify the procedure and structure of the negotiations. Consequently, it predicts an outcome which depends only on feasible allocations of the surplus to be created by the agreement and on the consequences of non-agreement. In that sense this Nash bargaining mechanism is better for our purposes than other bargaining mechanisms, for example – the non-cooperative ones of the type studied by Rubinstein (1982).

The results of this paper shed some light, then, on how developed countries manage to gain more than developing countries from establishing bilateral trade relationships, as seem to be indicated by World Trade Organization (WTO) empirical

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<sup>2</sup> See Cummins and Violante (2002) who calculate a decline of the relative price of capital goods in the United States at a rate of 3%-4% since 1974.

<sup>3</sup> See the evidence in Felbermayr (2007).

evidence. Computational general equilibrium analysis of the outcomes of the Uruguay Round agreements show, for example, a disproportional GDP benefit to developed countries, compared to that enjoyed by developing ones (Ackerman, 2005). Furthermore, Stiglitz (2002) argues that through the Uruguay Round developed countries have set a lopsided division of profits generated by globalization in their own favor, either through maintaining agricultural subsidies given to farmers in the developed countries, or by legislating property rights that reflect solely the interests of firms in the developed world. Thus, understanding the economic forces behind such agreements can help interpreting their outcomes.

An important feature of the model is that total world production level of consumption goods is higher under a trade agreement than it is with free trade. On the other hand, the growth rates of this output are lower in the trade agreement equilibrium compared to free trade. It is important to note, however, that the relatively slower growth with a trade agreement is due to improved allocation of factors among the trading parties under diminishing marginal productivity. This is fundamentally different from results about growth rates in models where two countries interact strategically in a non-cooperative manner. For example, in Devereux (1997) a tariff war mechanism reduces the world-wide growth rates compared to free trade, as in our model, but this is due to distortions inflicted by the tariffs that have adverse effects on production.

The rest of the paper is structured as follows. Section 2 offers a survey of the relevant literature on trade agreements and their outcomes. Section 3 sets up the basic growth and trade model. Section 4 describes the free trade scenario. Section 5 analyzes the bargaining-based trade agreement equilibrium, and section 6 concludes.

## 2. A survey of the literature

The economic relations between developing and developed countries are complex by nature. These relations are based mostly on two channels. The first is the transfer of resources, in the form of a loan or foreign aid, from the developed country to the developing one. The second is the trade between the two countries. These two channels are implicitly linked, as developed countries may tie the aid (or loan) to changing the terms of trade in their favor. Such links may improve the donor country's welfare at the expense of the developing country.

Foreign aid can affect welfare through impact on international trade, economic growth, or merely by increasing income in the recipient country.<sup>4</sup> However, the linkage of foreign aid to trade and growth outcomes has rarely been studied in the literature.

Several studies explore the connection between aid and trade.<sup>5</sup> Among them, the theoretical ones typically assume that the trade policies of both countries, as well as the size of the transfer, are exogenous. They also assume that when foreign aid is tied to some policy changes in the recipient country, the tying rule is imposed exogenously, and usually link the aid to some measure of the poorer country's expenditures rather than to its trade policies. The few articles who abstract from such assumptions use static models, which necessarily abstract from considering the growth implications of foreign aid and trade. Often, these articles study tariff wars rather than trade agreements as a means of

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<sup>4</sup> Sometimes foreign aid might cause a *decline* in welfare in the recipient country. This phenomenon is the well-known 'transfer paradox'. This paradox is not analyzed in the paper.

<sup>5</sup> For a full survey of the linkage between aid and trade see Suwa-Eisenmann and Verdier (2007).

allocating surplus.<sup>6</sup> In contrast, in this article we study a two-country growth model where foreign aid is tied to trade policies by an agreement between the two countries.

We focus on bilateral trade agreements signed between a developing country and a developed one, akin to the kind of regional bilateral trade agreements that were common during the 1990's.<sup>7</sup> Both parties to such agreements typically have to make concessions on different issues, including the complete abolition or weakening of protectionist policies that were in force prior the agreement. While such agreements have become more important and more widespread in recent years, there are still only few theoretical studies that attempted to study their general properties. Most of these studies concentrated on how bigger countries tend to win tariff wars, and typically employ static models, (e.g, Kennan and Riezman, 1988). Studies using dynamic models, like Devereux (1997), show that tariff wars reduce the world-wide growth rates compared to free trade, due to distortions inflicted by the tariffs.

Trade agreements typically include restrictions on industrial and development policies that each country can use. Although such policies do not have the same direct impact on trade that tariffs and export subsidies have, they nevertheless can affect trade indirectly through their impacts on production activities. Wade (2003) argues that the agreements that arose from the Uruguay Round – TRIMS, GATS and TRIPS on investment, trade in services, and property rights respectively - benefit the block of the developed countries at the expense of the block of the developing countries. This

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<sup>6</sup> For a more detailed survey of this strand of the literature see the introduction in Lahiri, Raimondos-Moller, Wong and Woodland (2002).

<sup>7</sup> For instance, since the early 1990s the European Free Trade Association (EFTA) has established an extensive network of contractual free trade relations all over the world, including Singapore, Egypt, Israel, Chile, Mexico, Croatia, Colombia and Lebanon. For more details see <http://www.efta.int/content/free-trade/fta-countries>.

outcome occurs not through direct control of relative prices of commodities and trading volumes, but by restricting the development measures that developing countries can employ.

Multilateral trade agreements can often take resemblance to a bilateral agreement between developed and developing countries with conflicting interests (as suggested in the last paragraph). Most disputes preventing a new multilateral trade agreement among WTO members are between the block of developed countries led by European Union, USA and Japan, and the block of developing countries led by India, Brazil, China and South Africa. Clearly, the leading developed countries involved are those that also contribute most of the foreign aid. Hence, as we demonstrate in this study, important insights about the links between foreign aid and trade agreements can be gained by considering them jointly in the context of a dynamic equilibrium model.<sup>8</sup>

### 3. The Basic Model

Consider a world consisting of two economies, North and South, denoted  $N$  and  $S$ .<sup>9</sup> North is richer than South in the sense that it has a higher initial endowment of capital. Both economies have the same constant population size. A representative agent in each economy seeks to maximize the following utility function:

$$(1) \quad U^i = \int_0^{\infty} e^{-\rho t} \cdot u(c(t)) dt = \int_0^{\infty} e^{-\rho t} \cdot \frac{c^i(t)^{1-\theta}}{1-\theta} dt,$$

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<sup>8</sup>While Alesina and Dollar (2000) argue that political rather than economic considerations underlie the aid given by developed countries in some cases, other studies, such as Asante (1985) claim that economic considerations typically motivate foreign aid.

<sup>9</sup> These economies may be either two countries or two blocks of countries, as in the case of WTO negotiations. Without any loss of generality, we do not distinguish here between the two options.

where  $c^i(t)$  is per-capita consumption at economy  $i$  at time  $t$ ,  $i \in \{N, S\}$ ,  $\rho$  and  $\theta$  are constants satisfying  $0 < \rho < 1$  and  $0 < \theta < 1$ . The agent has one unit of labor which is supplied inelastically, owns the capital in the economy and continuously rents it to firms. The lifetime budget constraint of the representative agent in each economy  $i$  is given by:

$$(2) \quad \int_0^{\infty} c^i(t) e^{-r^i(t)t} dt = P_q^i(0) k^i(0) + \int_0^{\infty} w^i(t) e^{-r^i(t)t} dt$$

where  $P_q^i(t)$  is the relative price of capital in terms of consumption goods in country  $i$  at time  $t$ , and  $r^i(t)$ ,  $k^i(t)$  and  $w^i(t)$  are, respectively, the interest rate, capital and real wage rate.

Each country has two competitive production sectors, one for consumption goods and the other for capital goods. Consumption goods (per capita) produced in country  $i$  at time  $t$ , denoted by  $c_p^i(t)$ , are given by:

$$(3) \quad c_p^i(t) = B [k_C^i(t)]^\alpha,$$

where  $0 < \alpha < 1$ ,  $k_C^i(t)$  is the amount of capital employed in producing consumption goods in country  $i$  at time  $t$  and  $B$  is a technology productivity factor. The subscript  $P$  denotes production.



Capital goods are producible factors of production. The total amount of new capital goods in country  $i$  at time  $t$ , is denoted by  $q^i(t)$ . The country  $i$  at time  $t$  local production of these capital goods is denoted by  $q_P^i(t)$  satisfying:

$$(4) \quad q_P^i(t) = A[k^i(t) - k_C^i(t)],$$

where  $A$  is a technology productivity factor and  $k^i(t)$  is the per-capita amount of capital in country  $i$  at time  $t$ . With capital depreciation rate  $\delta$ , the capital stock in each country evolves through time according to:

$$(5) \quad \dot{k}^i(t) = q^i(t) - \delta k^i(t).$$

In a competitive equilibrium all markets clear at each point in time; firms maximize current profits, while the representative household rents labor and capital to firms, and chooses consumption so as to maximize the lifetime utility in (1).

The analysis is carried out under the following parametric assumption:

*Assumption 1:*  $\alpha(1 - \theta)(A - \delta) < \rho < A - \delta$ .

The first inequality in *Assumption 1* suffices to satisfy the transversality condition ensuring that utility is bounded. The second inequality is necessary for positive growth of consumption and capital.

### 3.1 Autarky Equilibrium

We start with the case of autarky, to be used as a benchmark for evaluating free trade and trade agreements outcomes later on. Under autarky consumption and investment are based on local production alone, implying  $c^i(t) = c_p^i(t)$  and  $q^i(t) = q_p^i(t)$ . Since this case was already analyzed by Rebelo (1991), results are presented here without proof.

In Equilibrium, profits maximizing firms are indifferent at the margin between employing capital for producing consumption and capital goods. That is:

$$(6) \quad P_q^i(t)A = \alpha B [k_C^i(t)]^{\alpha-1}.$$

Each economy experiences no transitional dynamics, and grows along a Balanced Growth Path (BGP) with a constant interest rate, and with capital, consumption and the relative price of capital growing at constant rates, denoted  $g_k$ ,  $g_c$  and  $g_p$  respectively. Along the BGP,  $g_k$  and  $g_c$  are positive while  $g_p$  is negative, implying that consumption and capital grow over time while the relative price of capital falls over time. These constant growth rates do not depend on initial capital stocks, and hence will be the same in both countries.

In an autarkic equilibrium, a constant fraction of capital,  $\gamma$ , is allocated to producing consumption goods, so that the consumption in country  $i$  is given by:

$$(7) \quad c_A^i(t) = B [\gamma k^i(t)]^\alpha,$$

where the subscript  $A$  refers to autarky, and  $\gamma$  is given by:

$$\gamma \equiv \frac{\rho - \alpha(1 - \theta)(A - \delta)}{A[1 - \alpha(1 - \theta)]}.$$

The difference in initial amount of capital, therefore, manifests itself through the levels of consumption and capital and not via their growth rates.

#### 4. The Model with Free Trade

In this section we study the case where the two countries freely trade with one another. Specifically we assume that at  $t=0$  the two economies unexpectedly start trading with each other and that from then on both countries face a common relative price between the two goods. Indeed, capital in the Rebelo (1991) model should not be taken literally as physical capital. Instead the model should be viewed as a reduced form of more elaborate mechanisms of endogenous growth, such as learning-by-doing or endogenous technological change. Treating knowledge outcomes as physical capital, we implicitly assume that knowledge can be protected by its owner on the one hand, but can also be surrendered to others.

The following clearing market condition must hold at all times, reflecting the result that with free trade South specializes in producing consumption goods:

$$(8) \quad c_{FT}^N(t) + c_{FT}^S(t) = B[k_C^N(t)]^\alpha + B[k^S(t)]^\alpha,$$

where the  $FT$ -subscript represents free trade. We adopt an extreme form of imperfection in international capital markets that precludes all lending and borrowing in capital. We also assume that labor is immobile across countries. This international trade extension of the Rebelo (1991) model was already studied by Felbermayr (2007), who established the results reported in the following sub-section 4.1.

#### 4.1 The free trade equilibrium

Without trade, the price of capital goods in the North is lower than in the South. Therefore, with trade the South imports capital goods, and exports consumption goods.

At all times the North produces both capital and consumption goods and producers in the North are indifferent at the margin between producing capital and consumption goods. Thus (6) holds for the North at all times. In contrast, in the South there are two possibilities depending on initial conditions. If  $k^S(0)$  is sufficiently large, given  $k^N(0)$ , then (6) holds and both goods are produced in the South too. In that case the world is on a BGP. Otherwise, if  $k^S(0)$  is sufficiently smaller than  $k^N(0)$ , then South specializes in producing consumption goods and refrains from producing investment goods. We focus on the latter case from here onwards, and present below an explicit expression for the threshold level of  $k^S(0)$  that distinguishes the two cases.

The specialization starts at  $t=0$  and from then on this two-country world experiences transitional dynamics towards a balanced growth path in which capital and consumption in each country grow at a constant rate. The specialization of the South in consumption goods persists throughout these dynamics. The specialization in the South implies that the world equilibrium relative price of capital goods satisfies:

$$(9) \quad P_q(t) \leq \frac{\alpha B}{A} [k^S(t)]^{\alpha-1}.$$

The inequality in (9) is strict everywhere along the convergence to BGP, approaches equality asymptotically, and holds with equality on a BGP.

#### 4.2. The balanced growth path

As Felbermayr (2007) shows, along the BGP capital and consumption in both countries grow at constant rates which are the same as in autarky. The interest rates are equal in both countries which in turn implies equal marginal products of capital in producing consumption, so that  $k_C^N(t) = k^S(t)$ .

Going further beyond the results in Felbermayr (2007), in appendix A we show that if  $k^S(0)$  and  $k^N(0)$  are such that the world is on its BGP already at  $t=0$  then consumption in each country satisfies:

$$(10) \quad c_{FT}^N(0) = \gamma A P_q(0) k^N(0) + (1 - \alpha) B [k_C^N(0)]^\alpha$$

and

$$(11) \quad c_{FT}^S(0) = \gamma A P_q(0) k^S(0) + (1 - \alpha) B [k^S(0)]^\alpha$$

Using (6), (10), (11) and  $k_C^N(t) = k^S(t)$  in (8) yields in this case:

$$(12) \quad k^S(0) = \frac{\gamma}{2-\gamma} k^N(0).$$

The RHS of (12) provides us therefore a specific formula for the threshold for specialization that its existence was identified by Felbermayr (2007). Note that this threshold is smaller than  $k^N(0)$  since  $0 < \gamma < 1$ .

The following Lemma establishes the *productive efficiency* of the BGP, a property we use in analyzing foreign aid tied to trade in a trade agreement. The lemma looks at the different allocations of a given amount of an initial total world capital to  $k^S(0)$  and  $k^N(0)$ . As the lemma shows, the allocation that puts the world on a BGP also maximizes the total world production of consumption goods at each point in time.

*Lemma 1: For any given initial aggregate stock of capital  $K > 0$ , world-wide consumption for all  $t \geq 0$ ,  $c^N(t) + c^S(t)$ , is maximized relative to all other free trade equilibrium allocations when  $k^S(0)$  and  $k^N(0)$  satisfy:*

$$(i) \quad k^S(0) + k^N(0) = K$$

$$(ii) \quad k^S(0) = \frac{\gamma}{2-\gamma} k^N(0)$$

*Proof* Condition (ii) places the world on its BGP at  $t=0$ , implying that (9) holds as an equality and that  $k^S(t) = k_C^N(t)$ , ensuring equal marginal products of capital in

producing consumptions across North and South at each point in time. This proves the claim given identical and concave production technologies in both countries. ■

### 4.3 Free trade with aid

As mentioned earlier, we focus on the case where initially South is sufficiently poorer than North. In that case, due to diminishing marginal product of capital in producing consumption goods, moving capital from North to South would increase world-wide consumption without reducing future world-wide capital stocks. In the absence of international capital markets, this capital reallocation can only be done through foreign aid. In particular, the aid can be such that it puts the world on a BGP. Let  $T_k$  denote the size of the aid that indeed places the world on the BGP. Also let  $k_0^N$  and  $k_0^S$  denote the initial *pre-aid* values of capital in North and South and let  $k^N(0)$  and  $k^S(0)$  denote the *post-aid* initial capital stocks in the two countries. Thus,  $k^S(0) = k_0^S + T_k$  and  $k^N(0) = k_0^N - T_k$ . Then, from (12) we get:

$$(13) \quad T_k = \gamma \frac{k_0^N + k_0^S}{2} - k_0^S.$$

## 5. The Bargaining-Trade Equilibrium

In the previous section we showed that if initially South is sufficiently poorer than North then the world is on a dynamic path in which South specializes in producing consumption

goods. In this section we show the equilibrium in case that trade but subject to limitations imposed by a trade agreement.

These compensating trade policies can take many different forms, including tariffs, trade quotas, subsidies, or other policy tools affecting trade indirectly through their impact on production decisions. Since under perfect knowledge and rationality the outcome of a mechanism based on such tools is, at its bottom line, a division of surplus generated by cooperation, we do not specifically describe any of these policy measures. Instead, we assume that these compensating trade arrangements can be represented by a welfare transfer from South to North. This allows us to use the bargaining mechanism as a solution concept for analyzing how the North can be compensated for giving foreign aid to the South, without invoking non-economic (e.g. altruistic or political) justifications. While the role of non-economic considerations is obvious and can be considerable, we want in this paper to examine how far purely economic considerations can go towards explaining observed ties between aid and trade policies.

Since the agreement is about the division of worldwide welfare, it is optimal for the two countries to first maximize the world production of consumption. Based on *Lemma 1*, the two countries therefore would agree on the aid magnitude given by (13), which also places the world on the BGP. The actual bargaining is then over trade policies that divide the surplus created by the aid transfer, as we analyze in the next sub-section

### **5.1 The Bargaining Setup**

We employ the Nash (1950) axiomatic bargaining approach. This approach stipulates a compact and convenient way to find a solution to a bargaining problem that satisfies four



properties: invariance to affine transformations, Pareto efficiency, symmetry and independence of irrelevant alternatives. Following Chan (1988), we make the following two assumptions that justify our search for a Pareto efficient and symmetric solution.

Assumption 2 The two countries have full information about preferences and technologies of both themselves and their trading partners.

This assumption implies that the bargaining solution is Pareto efficient.

Assumption 3 Negotiators from each country have the same bargaining skill.

Assumption 3, together with the fact that interest rates in both countries are equal along the BGP, imply that the bargaining solution should be symmetric in the sense that if the two countries are identical in *all* respects, their equilibrium payoffs should be equal.

The Nash-bargaining solution solves the following problem:

$$(14) \quad \underset{c^N, c^S}{Max} \left\{ [U^N(c^N) - \bar{U}^N] \cdot [U^S(c^S) - \bar{U}^S] \right\}$$

s.t.

$$(15) \quad c^N + c^S = 2B(k^S)^\alpha,$$

where  $c^i$  represents the consumption level in country  $i$  at time 0 resulting from the Nash bargaining mechanism,  $k^S$  is the post-transfer capital in the South at  $t=0$ , while  $U^i(\cdot)$  and  $\bar{U}^i$  are the life-time utilities of the representative consumer of country  $i$  in the cooperative solution and in case of *disagreement*, respectively. On BGP trajectories, both  $U^i$  and  $\bar{U}^i$  can be represented by the utilities  $u(c^i(0)) = \frac{c^i(0)^{1-\theta}}{1-\theta}$ , expressed as functions of consumption at  $t=0$  only, since consumption grows at a constant rate which is independent of initial capital endowments. Condition (15) follows from applying the BGP property  $k^S = k^N$  in (8).

We take the disagreement point in the model to be the autarky payoffs, for several reasons. First, we rely on the Shapley version of the Nash solution, where the disagreement point reflects the credible destructive power of each player, and therefore the disagreement point serves as a minimal payoff that each country can secure for itself on its own. Another reason for choosing autarky as the disagreement point is the endogenous linkage between foreign aid and trade policies. Consider the following scenario: The North and the South negotiate over agreeable trade policies and aid in the form of capital transfer from North to South. Both countries know that compared to autarky, agreement will improve their welfare. The North can *condition* the capital transfer on the bargaining outcome. If the bargaining process fails, the North will not give the capital transfer, and both countries will continue on their autarkic BGP.

Alternative disagreement points, such as the free-trade allocation with no aid, are not credible as threat points. A free trade unaccompanied by trade agreement can lead to each country imposing tariffs unilaterally in an attempt to extract welfare from the other

country. Kennan and Riezman (1988) showed how big countries can win such tariff wars. In Mayer (1981) and Riezman (1982) the disagreement points are based on Johnson (1953) Nash-Cournot tariff equilibrium as a possible threat point. However, this threat may not be robust when other commercial policies (like quotas) are allowed. In order to cover any unilateral policy that each country can undertake in case of a disagreement, we find the autarky to be the most suitable alternative.

Note that this choice for the disagreement point gives North an advantage, as its allocation under autarky is better than that of the South. But no ad-hoc assumptions on differential bargaining powers are exploited here.<sup>10</sup>

## 5.2 The Bargaining-Trade Outcome

*Proposition 1 Both countries are better off in equilibrium with trade and bargaining than in autarky, regardless of initial capital endowments.*

*Proof:* Equilibrium under a trade agreement and equilibrium under autarky are both on the BGP. Hence, by Lemma 1, the total production of consumption goods after the capital transfer is made is higher than in autarky. This implies that a pair of South and North consumption levels that are both above their autarky counterparts is feasible, and consequently the Nash product given by (14) is strictly positive at its maximum. A strictly positive product implies that either both factors are positive or that both are negative. The latter possibility can be ruled out since the utility functions are strictly

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<sup>10</sup> In the general form of the Nash Product  $U^N(c^N) - \bar{U}^N$  is raised by the power of  $\beta$ , where  $\beta$  represents bargaining power. Here we assume  $\beta=1$  implying that the superior bargaining power of the North springs merely from  $\bar{U}^N > \bar{U}^S$  and not also from  $\beta > 1$ .

increasing. Thus, both countries are better off at the solution  $(c^N, c^S)$  than they are in autarky. ■

The following *Proposition 2* shows that at least for some parameters North receives greater welfare from a trade agreement than from free trade.

*Proposition 2:* For some initial capital endowments, North is better off (and the South is worse off) under bargaining over trade and aid than under free-trade.

*Proof:* Using in (1) the BGP property that  $c^i(t) = c^i e^{g_c t}$  for a constant  $g_c$ , applying the resulting utility in (14) and maximizing the Nash product yields the following first order condition:

$$(16) \quad (c^N)^\theta \left[ (c^N)^{1-\theta} - (c_A^N)^{1-\theta} \right] - (c^S)^\theta \left[ (c^S)^{1-\theta} - (c_A^S)^{1-\theta} \right] = 0,$$

where  $c_A^i$  is the consumption level under autarky in country  $i$  at  $t=0$ .

Using (15) it is possible to define the LHS of (16) as the function:

$$(17) \quad H(c^N, k_0^N, k_0^S) \equiv (c^N)^\theta \left[ (c^N)^{1-\theta} - (c_A^N)^{1-\theta} \right] - \left( 2B(k^S)^\alpha - c^N \right)^\theta \left[ \left( 2B(k^S)^\alpha - c^N \right)^{1-\theta} - (c_A^S)^{1-\theta} \right]$$

Note that  $k_0^N$  and  $k_0^S$  appear as arguments of  $H$  because they directly affect  $c_A^N$  and  $c_A^S$  by (7), and  $k^S$  by (13). Evidently,  $H(c^N, k_0^N, k_0^S)$  is strictly increasing in  $c^N$ .

We now focus on the case where (12) holds, and show that in this case North finds the bargaining outcome better than the free trade outcome. To see that, note first that when (12) holds the free trade takes place along the BGP. Consequently, we can establish that North's utility under a trade agreement is larger than under free trade by comparing consumption at  $t=0$  in the bargaining agreement and under free trade. We do so by showing that  $H(c^N, k_0^N, k_0^S)$  is negative when evaluated at the free trade consumption at  $t=0$ . Since  $H(c^N, k_0^N, k_0^S)$  is increasing in  $c^N$ , this implies that the argument that maximizes the Nash product is larger than the consumption level of the North under free trade.

Suppose now that  $k_0^N$  and  $k_0^S$  do not satisfy (12) as an equality. By continuity of life-time utilities, if the proposition holds for the  $(k_0^N, k_0^S)$  combinations that satisfy (12), then it is also true for at least some neighborhood of these combinations.

From (6), (10), (11), (12) and the result that along the BGP  $k_C^N = k^S$ , it follows that the consumption levels satisfy:

$$(18) \quad c^N = B(k^S)^\alpha [1 + \alpha(1 - \gamma)],$$

and

$$(19) \quad c^S = B(k^S)^\alpha [1 - \alpha(1 - \gamma)].$$

Using (7), (18) and  $x \equiv 1-\gamma$  in (17) yields after simplifying:

$$(20) \quad H(c^N, k_0^N, k_0^S) = B(k^S)^\alpha \left[ 2\alpha x - (1 + \alpha x)^\theta (1 + x)^{\alpha(1-\theta)} + (1 - \alpha x)^\theta (1 - x)^{\alpha(1-\theta)} \right].$$

The RHS of (20) is negative if and only if the term in square brackets is negative, which we prove in appendix B.

Since in the case where (12) holds the world is on its BGP, then total world production is the same under both a trade agreement and free trade. Thus, under a trade agreement and relative to free trade, since North receives more – South gets less. ■

Proposition 2 shows that the bargaining outcome makes North better off compared to free trade when  $k_0^S$  is equal to  $\frac{\gamma}{2-\gamma} k_0^N$  and, by continuity, also on a certain neighborhood where  $k_0^S$  is smaller than  $\frac{\gamma}{2-\gamma} k_0^N$ . For smaller values of  $k_0^S$  outside such neighborhoods, the gap between what North gets under a trade agreement and what it gets under free trade is still positive, and in fact grows as  $k^S$  falls. To see that note that under free trade, equilibrium is determined by the intersection between demand and supply rather than in a cooperative manner. Consequently, the declining world supply as  $k_0^S$  falls, (holding  $k_0^N$  fixed), lowers North's welfare. In contrast, North's welfare under a trade agreement actually rises as  $k_0^S$  falls, because this strengthens North's threat point relative to South's. This is a rather interesting result because in this case too a lower  $k_0^S$

reduces world output. Yet, the decline in  $k_0^S$  also lowers South's welfare in the case of autarky and therefore lowers its bargaining power. *Proposition 3* formulizes and proves this result.

*Proposition 3.* For any given  $k_0^N$ , the smaller  $k_0^S$  the higher  $c^N$ .

*Proof:* See Appendix C ■

In summary, this section shows that North gains more from aid tied to trade, than from free trade, and that North's share of the surplus created by cooperative solution is higher the poorer is the other country.

## 6. Concluding Remarks

In this paper we construct a dynamic growth model that combines international trade and foreign aid. We evaluate welfare in the donor and the recipient countries, and argue that foreign aid need not affect growth rates in either country. We also argue that the consumption levels do change due to the foreign aid. The foreign aid in the paper is tied to international trade policies.

By endogenizing the tie rule of the foreign aid to international trade policies through a bargaining mechanism, welfare is transferred from the developing country to the developed one via trade agreements. While these trade agreements make both countries better off compared to autarky; they also make the developed country better off compared to free-trade. This implies, of course, that while the developing country prefers

free trade to a trade agreement, it would still be better off under the trade agreement than under autarky, and thus a trade agreement is still acceptable.

Although we do not model explicitly the trade policies over which countries bargain, we do show that there exist welfare transfers, which can emanate from direct resource transfers, or weakened subsidy or tariffs policies, which can be negotiated over along with foreign aid from the developed country to the developing one.

This result sheds some light over current negotiations between developed and developing countries, (in the context of the Doha Round), and the present stalemate in these talks. According to its proponents, the last round of negotiations aims to make trade fairer for the developing countries,<sup>11</sup> and it is frequently referred as “The Doha Developing Round”. This round and its failure in Cancun, Mexico (2003), and later again in Geneva (2008) was partly attributed to the wide gaps between the developed and developing countries. Furthermore, most computable general equilibrium measures of the forecasted outcomes of the Doha Round show not only low gains on the aggregate, but also skewed outcomes towards developed countries (Ackerman, 2005). We can forecast in light of our analysis, that if an agreement is eventually obtained, it will favor the developed countries rather than the developing ones, in contrast to the declared goals of these talks.

## **Appendix A**

The lifetime budget constraint of the representative agent in each economy is given by (2). Since  $k_C^N(t) = k^S(t)$  along the BGP, wages in both countries are equal and given by

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<sup>11</sup> For more details, see [http://en.wikipedia.org/wiki/Doha\\_Round#cite\\_note-7](http://en.wikipedia.org/wiki/Doha_Round#cite_note-7).



$w(t) = (1 - \alpha)B[k^S(t)]^\alpha$ . Thus, along the BGP wages grow at a rate of  $\alpha g_k$ . As Felbermayer (2007) shows, consumption too grows along the BGP at a rate of  $g_c = \alpha g_k$ . Hence, the lifetime budget constraint in each country can be written as:

$$(A1) \quad \int_0^{\infty} c_{FT}^i(0) \cdot e^{(\alpha g_k - r)t} dt = P_q(0) \cdot k^i(0) + \int_0^{\infty} (1 - \alpha)B[k^S(0)]^\alpha \cdot e^{(\alpha g_k - r)t} dt.$$

As Rebelo (1991, p. 504) shows, the first order condition derived by the *Current Value Hamiltonian* leads to  $\alpha g_k = \frac{r - \rho}{\theta}$  and  $r = A - \delta - (1 - \alpha)g_k$ . Solving these two equations for  $r$  and  $g_k$  yields:

$$(A2) \quad \alpha g_k - r = -A\gamma.$$

Applying (A2) and (6) in (A1) and simplifying yields (10) and (11).

## Appendix B

In this appendix we complete the proof of *Proposition 2* by showing that the RHS of (20) is negative. For that matter we define the RHS of (20) as the following function:

$$(B1) \quad F(x, \alpha, \theta) \equiv 2\alpha x - (1 + \alpha x)^\theta (1 + x)^{\alpha(1-\theta)} + (1 - \alpha x)^\theta (1 - x)^{\alpha(1-\theta)}.$$

We shall now show that  $F(x, \alpha, \theta) < 0$  for any set of values for  $x$ ,  $\alpha$ , and  $\theta$  satisfying  $0 < x < 1$ ,  $0 < \alpha < 1$  and  $0 < \theta < 1$ . We do so by looking, without loss of generality, at

the pair  $(\alpha_0, \theta_0)$ , where  $0 < \alpha_0 < 1$  and  $0 < \theta_0 < 1$  and showing that  $F(0, \alpha_0, \theta_0) = 0$ ,  $F_x(0, \alpha_0, \theta_0) = 0$ , and  $F_{xx}(x, \alpha_0, \theta_0) < 0$  for all  $0 < x < 1$ .

$F(0, \alpha_0, \theta_0) = 0$  follows directly from (B1). Partial differentiation of  $F(x, \alpha, \theta)$  yields:

$$(B2) \quad \frac{F_x(x, \alpha, \theta)}{\alpha} = 2 - \theta(1 + \alpha x)^{\theta-1}(1+x)^{\alpha(1-\theta)} - (1-\theta)(1 + \alpha x)^\theta(1+x)^{\alpha(1-\theta)-1} \\ - \theta(1 - \alpha x)^{\theta-1}(1-x)^{\alpha(1-\theta)} - (1-\theta)(1 - \alpha x)^\theta(1-x)^{\alpha(1-\theta)-1}.$$

$F_x(0, \alpha_0, \theta_0) = 0$  follows directly from (B2). Differentiating the LHS of (B2) and simplifying, yields after tedious, yet straightforward, arithmetics:

$$(B3) \quad \frac{F_{xx}(x, \alpha, \theta)}{\alpha(1-\alpha)(1-\theta)} = \frac{\alpha\theta(1-\alpha)x^2 + (1+\alpha x)^2}{(1+\alpha x)^{2-\theta}(1+x)^{2-\alpha(1-\theta)}} - \frac{\alpha\theta(1-\alpha)x^2 + (1-\alpha x)^2}{(1-\alpha x)^{2-\theta}(1-x)^{2-\alpha(1-\theta)}},$$

which implies that:

$$\frac{F_{xx}(x, \alpha_0, \theta_0)}{\alpha(1-\alpha_0)(1-\theta_0)} < \frac{(1+\alpha_0 x)^2}{(1+\alpha_0 x)^{2-\theta_0}(1+x)^{2-\alpha_0(1-\theta_0)}} - \frac{(1-\alpha_0 x)^2}{(1-\alpha_0 x)^{2-\theta_0}(1-x)^{2-\alpha_0(1-\theta_0)}} \\ < \frac{(1+\alpha_0 x)^2}{(1+\alpha_0 x)^{2-\theta_0}(1+\alpha_0 x)^{2-\alpha_0(1-\theta_0)}} - \frac{(1-\alpha_0 x)^2}{(1-\alpha_0 x)^{2-\theta_0}(1-\alpha_0 x)^{2-\alpha_0(1-\theta_0)}}$$

$$= \frac{1}{(1 + \alpha_0 x)^{2 - \alpha_0(1 - \theta_0) - \theta_0}} - \frac{1}{(1 - \alpha_0 x)^{2 - \alpha_0(1 - \theta_0) - \theta_0}} < 0$$

where the three inequalities are based on  $\alpha_0$ ,  $\theta_0$  and  $x$  being within the interval  $(0, 1)$ .

This establishes  $F_{xx}(x, \alpha_0, \theta_0) < 0$  which completes the proof.

### Appendix C

In this appendix we show that  $c^N$ , the consumption of the North at  $t=0$  under a trade agreement, is a decreasing function of  $k_0^S$ .

Due to (16) and (17):

$$(C1) \quad H(c^N, k_0^N, k_0^S) = 0,$$

which, according to the Implicit Function Theorem, defines  $c^N$  as an implicit function of  $k_0^S$  and  $k_0^N$ . Simplifying (17) yields:

$$(C2) \quad H(c^N, k_0^N, k_0^S) = 2c^N - (c^N)^\theta (c_A^N)^{1-\theta} - 2B(k^S)^\alpha + [2B(k^S)^\alpha - c^N]^\theta (c_A^S)^{1-\theta}$$

Partial differentiation of (C2), bearing in mind the BGP property of  $k^S = \frac{\gamma}{2}(k_0^N + k_0^S)$  and also that  $c_A^S$  is a function of  $k_0^S$  through (7), yields:

$$(C3) \quad \frac{\partial H(c^N, k_0^N, k_0^S)}{\partial k_0^S} = -\alpha\gamma B(k^S)^{\alpha-1} + \theta [2B(k^S)^\alpha - c^N]^{\theta-1} B\alpha(k^S)^{\alpha-1} \gamma (c_A^S)^{1-\theta} \\ + [2B(k^S)^\alpha - c^N]^\theta (1-\theta)(c_A^S)^{-\theta} B\alpha(k_0^S)^{\alpha-1} \gamma$$

Substituting  $2B(k^S)^\alpha - c^N$  by  $c^S$ , which follows from (15), and noticing that

$k^S > k_0^S$  because the South imports capital from North, leads to:

$$(C4) \quad \frac{\partial H(c^N, k_0^N, k_0^S)}{\partial k_0^S} > \frac{\alpha\gamma B}{(k^S)^{1-\alpha}} \left[ -1 + \theta(c^S)^{\theta-1} (c_A^S)^{1-\theta} + (1-\theta)(c^S)^\theta (c_A^S)^{-\theta} \right] \\ > \frac{\alpha\gamma B}{(k^S)^{1-\alpha}} \left[ -1 + \theta(c^S)^{\theta-1} (c_A^S)^{1-\theta} + (1-\theta)\frac{c_A^S}{c^S} (c^S)^\theta (c_A^S)^{-\theta} \right] = 0.$$

The second inequality follows from the result that  $c_A^S < c^S$  shown in *Proposition 1*.

According to the Implicit Function Theorem, the derivative of  $c^N$  with respect to  $k_0^S$  is:

$$(C5) \quad \frac{\partial c^N}{\partial k_0^S} = -\frac{\frac{\partial H}{\partial k_0^S}}{\frac{\partial H}{\partial c^N}} < 0.$$

This derivative is negative because the numerator is positive by (C4) and the denominator is positive too, as follows immediately from (17).

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