Tax, Stimuli of Investment and Firm Value

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Abstract

Pennings (2000) has shown that the government can speed-up investment by subsidizing the potential investing firm's entry cost while taxing the future proceeds from the investment, so as to render the net expected value of its subsidy program zero. This note argues that while speeding-up the investment timing this subsidy-tax program also lowers the value of the firm and therefore will be rejected by it.

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Introduction

The literature on irreversible investment under uncertainty has shown that a firm that contemplates investing in a certain project might delay its investment even when the expected net value of the project is positive.[1] The reason for that is that by entering the project the firm not only expends the direct entry cost, but also loses the option to delay this entry. Entry, therefore, only takes place when the value of the project exceeds the sum of the direct entry cost and the value of the delay option.

Pennings (2000) has studied a case in which the government subsidizes the potential investing firm's entry cost and, in addition, levies special taxes on the future rewards from this investment.[2] He shows that this tax-subsidy program can indeed speed-up investment by lowering the threshold value of the project that is required to trigger entry. Furthermore, it turns out that this objective can be reached even if the net present value of the government's expenditure in this program is zero.

Stimulating firms' investment at no expected cost to the government might seem at first blush as a Pareto improvement that points at some market failure taken care of by the government's interference. But no market failure can be detected in the model and we must therefore conclude that there is a hidden cost involved in this outcome. This cost springs from the fact that the tax-subsidy program lowers the entry trigger by bringing down the value of the option to delay entry - but this value, however, is a part of the firm's value. Thus, stimulating investment in this manner lowers the value of the potential investing firm and therefore this tax-subsidy program can only be enforced on the firm. The purpose of this note is to establish this point. To do so I use Pennings' model, and expand the analysis there by exploring the value of

[1] See Dixit and Pindyck (1994) for a derivation of this result and for a survey of the related literature.

[2] These special taxes are levied only on the rewards from this particular investment. Assuming that these rewards are taxed anyway, as part of the regular taxes on the firms' profits, requires the additional assumption that the proceeds from the regular taxes on the investment do not exceed the subsidy the government intends to offer, creating thus a need for these additional specific taxes.

the firm as well as the exact means by which the subsidy-tax program speeds-up investment. The analysis clarifies the reason why this tax-subsidy program lowers the value of the firm: The program has zero value (and therefore no burden on the firm) only at the relatively low threshold level the government desires. However, at levels of project rewards that are above this threshold - the taxes exceed the subsidy because the taxes are proportional to the rewards whereas the subsidy is not. Thus, the program speeds-up entry merely because the firm is punished by taxes if it delays its entry as it is waiting for the rewards from the project to increase. Naturally, this taxation of potential rewards lowers the value of the firm.

Incorporating some sort of a market failure in the model could indeed open the door to a result where the government's tax-subsidy program speeds-up investment, its expected cost to the government is zero and it does no harm to the firm's value. An example of such a market failure would be a positive externality to the investment that the firm could not cash in on but the government could. Another example is a credit market imperfection that makes the interest rate to the government lower than that relevant to the firm. However, the result that the existence of such failures may lead to a situation where such a tax-subsidy program may be beneficial to all the involved parties is already well known and has nothing to do with the interactions among uncertainty, irreversibility and the option to delay the timing of investment.

The Model

This model is the Pennings (2000) version of McDonald and Siegel (1986). The analysis in this section first re-derives Pennings (2000) main results and then extracts from this model original results about the firm's value.

Consider a risk-neutral firm that contemplates investing in a certain project. In order to do so the firm must incur the irreversible cost X. Once the firm enters the project it receives a stream of profits that its expected present value is denoted by V. Time is continuous and the discount factor relevant to the firm is denoted by r. Both X and r are constants. The value of the project is a stochastic process given by:

(1) $dV = \mu V dt + \sigma V dz,$

where μ and σ are constants, $\sigma > 0$ and dz is the increment of the standard Wiener process, uncorrelated across time and at any instant satisfying:

(2) $E(dz) = 0, \qquad E[(dz)^2] = 1.$

It is also assumed that $r > \mu$. This assumption is necessary in preventing the value of the investing firm to go to infinity.

Trying to speed-up investment, the government launches a program under which the firm gets a fixed subsidy denoted by θ_2 when it enters the project, but the project's proceeds are taxed at the tax rate θ_1 satisfying $0 \le \theta_1 \le 1$. Let V_θ denote the level of V that triggers the firm to enter the project and let $F(V)$ denote the firm's value given the current value of the project, V. When $V < V_\theta$ the firm's value is merely the expected present value of the project at the entry time:

(3) $F(V) = E\left\{e^{-rT}\left[(1-\theta_1)V_\theta - (X-\theta_2)\right]\right\} = E\left(e^{-rT}\right)\left[(1-\theta_1)V_\theta - (X-\theta_2)\right]$

where T is the time in which the firm's value reaches the entry threshold V_θ for the first time, given that its current value is V. In the appendix it is shown that:

$$(4) \qquad F(V) = \left(\frac{V}{V_\theta}\right)^\beta \left[(1-\theta_1)V_\theta - (X-\theta_2)\right].$$

where β is the positive root of the quadratic:

$$0.5\sigma^2 Y^2 + (\mu - 0.5\sigma^2)Y - r = 0$$

It is also shown in the appendix that $\beta > 1$. By straightforward differentiation, the value of V_θ that maximizes $F(V)$ is:

$$(5) \qquad V_\theta = \frac{\beta}{\beta-1}\frac{X-\theta_2}{1-\theta_1}.$$

Now assume that the government wants the present value of its expenditure on this finance program to be zero. This means that:

$$(6) \qquad \theta_2 = \theta_1 V_\theta.$$

Applying V_θ, as captured by (5), in (6) yields:

$$(7) \qquad \theta_2 = \frac{\theta_1 \beta X}{\beta - 1 + \theta_1}.$$

Applying θ_2, as captured by (7), in (5) yields that in this case the entry threshold is:

$$(8) \qquad V_\theta^* = \frac{\beta X}{\beta - 1 + \theta_1},$$

Comparing the entry threshold in this case to the entry threshold in the case where the government does not exercise its tax-subsidy program, i.e., the case where $\theta_1 = \theta_2 = 0$, yields that the Government's program indeed stimulates investment by lowering the entry threshold. This is the main result in Penning (2000). However, applying (8) and (7) in (4) shows that given the optimal choice of V_θ, as captured by (8), the firm's value is:

$$(9) \qquad F^*(V) = \frac{(\beta - 1 + \theta_1)^{\beta-1}(1 - \theta_1)}{\beta^\beta X^{\beta-1}} V^\beta.$$

By straightforward differentiation:

$$(10) \qquad \frac{dF^*(V)}{d\theta_1} = -\frac{(\beta - 1 + \theta_1)^{\beta-2}\theta_1}{\beta^{\beta-1} X^{\beta-1}} V^\beta < 0,$$

where the inequality sign follows from $\beta > 1$ and $0 \le \theta_1 \le 1$. Thus, as (8) shows, the government's program indeed stimulates investment by lowering the entry threshold. Yet, as (10) shows, this program also lowers the firm's value.

Figure 1 helps understand why the program lowers the value of the firm. The values of θ_1 and θ_2 are chosen so that the tax proceeds are equal to the subsidy exactly

when V equals the threshold desired by the government, V_θ^*. This means that for all higher levels of V the taxes levied on the firm, $\theta_1 \cdot V$, are larger than the subsidy, implying that the program lowers the net value of the project at that range. With θ_1 sufficiently high - the net value at that range is sufficiently reduced so that its present value is less than the value of entry at $V = V_\theta^*$. Thus, the program speeds-up entry merely because the firm is punished by taxes if it delays its entry. Naturally, this punishment policy has an adverse effect on the value of the firm.

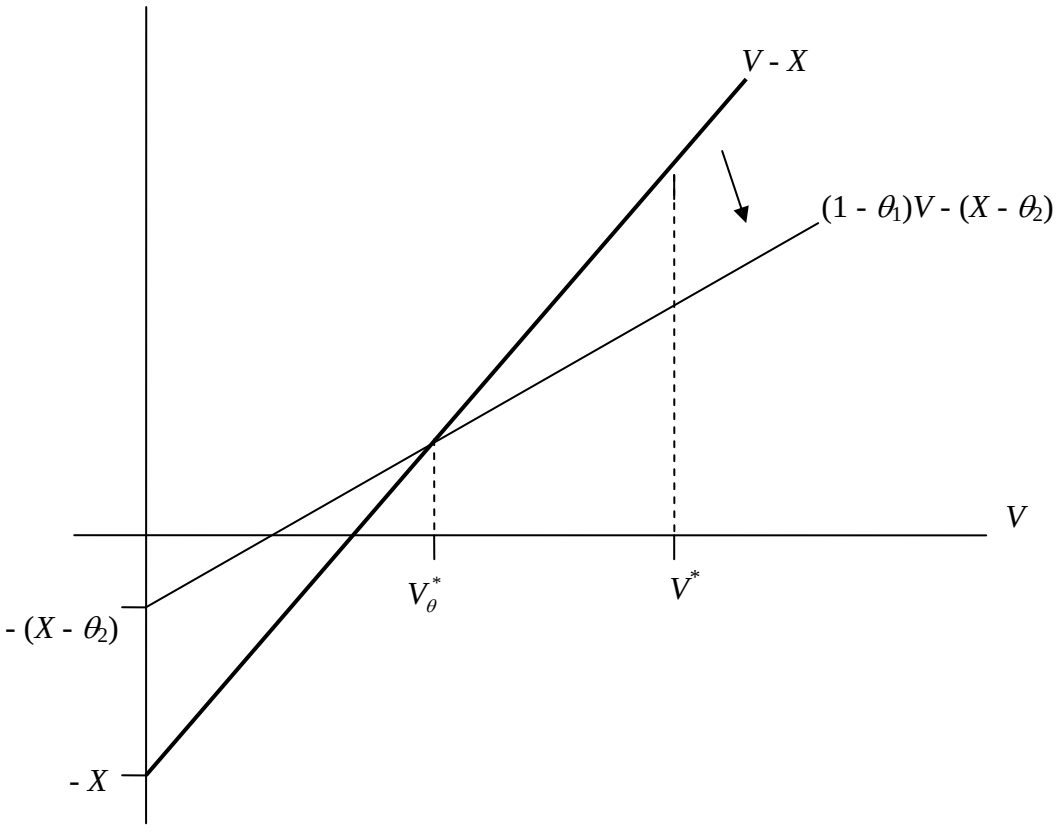

Figure 1: The thick line shows the net value of the project without the government's tax-subsidy program. The thin line shows the net value of the project under the government's tax-subsidy program. The program lowers the net value of the project for all $V > V_\theta^*$.

Concluding Remarks

In this note I have shown that the tax-subsidy program for the stimuli of investment that was studied in Pennings (2000) lowers the value of the potentially investing firm and therefore has to be enforced on the firm, rather than offered to it. Thus, this program is close in its nature to warranting a fine on firms that delay their investments. An example of such a fine is the tax rate on idle urban land, which is higher than the tax on developed urban land.

References

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Appendix

In this appendix (4) is established. Define $M(V) \equiv E(e^{-rT})$ as the present value of a bond the yields \$1 at time T, where T is the time where V reaches the value V_θ for the first time, and given the current value of V. Since V is a Geometric Brownian Motion, then, by Ito's lemma:

$$\text{(a1)} \qquad dM(V) = [\mu VM'(V) + 0.5\sigma^2V^2M''(V)]dt + \sigma VdZ$$

The instantaneous expected capital gain on this bond must be equal to the normal return in the market, thus:

(a2) $$\frac{E[dM(V)]}{dt} = \mu VM'(V) + 0.5\sigma^2V^2M''(V) = rM(V)$$

where the first equality follows from (2) and (a1). This differential equation has a solution of the form V^Y. Applying it in the (a2) yields that Y is the root of the quadratic:

(a3) $$0.5\sigma^2Y^2 + (\mu - 0.5\sigma^2)Y - r = 0$$

Denote the two roots of this equation by α and β. Applying $Y = 0$ and then $Y = 1$ and using the assumption that $r > \mu$, yields that one root of this quadratic, α, is negative and the other one, β, exceeds unity. These notations lead to:

(a4) $$M(V) = AV^{\alpha} + BV^{\beta}$$

Where A and B are constants that need to be determined yet. By the properties of Geometric Brownian Motion, if initially $V = 0$ than the probability that its value shall ever reach V_θ is zero and therefore so is the value of the bond that yields \$1 when V_θ is reached. This means that $M(0) = 0$ and since $\alpha < 0$ it also leads to $A = 0$. If, on the other hand, the current value of V is already V_θ then this bond yields \$1 immediately, implying that $M(V_\theta) = 1$, which leads to $B = 1/V_\theta{}^{\beta}$. This establishes $E(e^{-rT}) = \left(\frac{V}{V_\theta}\right)^{\beta}$ and therefore (4).