Investment under Uncertainty and the Recipient of the Entry Cost

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Abstract

A typical model of investment under uncertainty, where firms pay an irreversible cost in order to produce, is studied. The analysis has a novel focus on the recipient of this payment, which is modeled as a firm or government that sells a resource (or a right) necessary for the production of the final good. Our main finding is that the resource owner may choose to set the price of the resource at a level high enough so as to cause the producers of the final good to delay their purchase of the resource and withhold production. The resource owner does so because it expects increased demand for the final good in the future. Another important result is that the price of the resource is a decreasing function of the elasticity of demand for the final good.

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1. Introduction

Usually, the models on investment under uncertainty deal with the decisions of a firm that has to pay a certain irreversible exogenous cost in order to start producing and make profits.[1] In this article we remain within the traditional framework used in those models but shift the focus of attention elsewhere - to the recipient of this cost. We model this recipient as a firm or a government that sells a certain resource which is necessary for the production of a final durable good. The producers of the final good face the typical investment under uncertainty problem studied in the literature, as the cost of each unit of the resource is an exogenously given irreversible cost from their point of view. The sellers of the resource face a problem not yet studied - they must decide at what level to set the price of their resource.

For simplicity, we assume that the resource is sold by a monopolist. Two cases are studied: In the first case the resource owner is a firm interested in maximizing the value of its sales; in the second case the resource owner is a government interested in enhancing social welfare. The government case is of particular relevance to this setting because the government is indeed often an owner and provider of resources such as land, broadcast frequencies and franchises. Specifically, we assume that the government uses its income from the sales of the resource to finance its welfare enhancing activities in other markets. We show that since the government is also concerned about welfare in the market of the relevant final good, it sells the resource below the price that a profit maximizer would charge.

The common result in the literature on investment under uncertainty is that the optimal policy for the firm is to delay investment until profits from the investment are sufficiently large. In particular it has been found that a positive Net Present Value is

[1] For detailed surveys of this literature see Pindyck (1991) or Dixit and Pindyck (1994).

not enough to trigger investment, as the firm seeks to cover not only its direct investment cost but also the opportunity cost of the forgone option to delay investment. We find that these results may not survive the endogenization of the investment cost that we do here, as the resource owner may set the price of its resource low enough to induce the producers of the final good to invest immediately. Abstracting from full endogeniztion and assuming that there are some exogenous components to the investment costs revives the delay results.

Trivially, if the resource owner can change the price of its resource at any time and with no cost then it changes its price continuously in response to swings in the demand for the final good. By doing so, the resource owner strips the producers of the final good from any profit and keeps them at constant indifference as to whether to invest immediately or delay investment. Avoiding this redundant case, we assume therefore that the resource owner faces limitations on changing the price of its resource. For simplicity, we take this assumption to the extreme in which once the resource owner sets the price of the resource it cannot change it anymore. Consequently, in setting its price the resource owner faces the following dilemma: Pushing the price up yields, on the one hand, more upon selling, but on the other hand it may delay the timing of these sales because it may induce the producers of the final good to delay their purchases until the demand they face sufficiently rises.

We find that if buying this resource is the only cost for the producers of the final good then wishing to receive payments early is the overriding consideration and the resource owner sets its price low enough to induce immediate investment. This occurs for all levels of the demand for the final good. In particular, when this demand is very low the resource owner sets an accordingly low price for its resource rather than set a higher price which it could enjoy later when demand would eventually rise.

This happens because when the demand is low the probability of a large surge in it is accordingly small under the standard assumption of a geometric process taken here for the demand dynamics.

A situation where the producers of the final good delay their investments is possible therefore only later on in time if the demand for the final good falls sufficiently below its initial level. Note that this possibility hinges on the assumption that the resource owner is limited in its ability to change its price over time.

Assuming that there are some exogenous components to the investment costs alters the relative force of the two factors in the dilemma described above, restoring the possibility of delay of investment. Specifically, we find that in that case if demand is sufficiently low then the resource owner indeed sets a price that sends the producers of the final good to a period in which they delay their investments until demand is sufficiently high. Only if demand is sufficiently high does the resource owner set a price that induces immediate investment.

Another important assumption in the model is that the resource owner not only charges the producers of the final good for their purchases of the resource but also receives a share of their profits. This assumption fits the case of a government resource owner very well, as the government taxes firms' profits. Thus, in selling land, cellular frequencies or other resources, the government considers not just the direct payment it gets for selling the resource but also the future tax proceeds from the resulting economic activity. Such contracts exist nonetheless in the private sector as well.[2] This taxation assigns an important role to the elasticity of the demand for the

[2] In the pharmaceutical industry developers of new drugs usually have to pay the owners of the protected technologies they use not just a single payment but also a share of their sales revenues. Mall owners often rent out their stores for a percentage of the sales revenue. One final example is that of writers, musicians, as well as television production companies, who often have a sales dependent component in the payment they receive from the publisher (a book publisher, a record label or a broadcasting network) that sells their output to the general public.

final good in the determination of the price of the resource. Specifically, we find that if this demand is sufficiently inelastic then the resource owner will opt to avoid lowered tax revenues by preventing any increase in the quantity of the final good, setting therefore an infinite price for the resource.[3] At the other extremity we find that if demand is sufficiently elastic then the resource owner may set a negative price for its resource, subsidizing thus the production of the final good in order to raise its tax revenues.

The market for the final good in our model is the same market studied by Leahy (1993). This is true even though we introduce an endogenous determination of the investment cost. This endogenous determination concerns only the firm/government resource owner, and the producers of the final good take the cost of the resource as given, just as in Leahy's model. Since it is the same model, all relevant results already proven by Leahy are used here without proof.

As stated above, the subject at the focus of this paper has never to our knowledge been studied. The study closest to our paper is Yu et al. (2007) who examine the case where the irreversible investment cost of the firms is subject to the endogenous decisions of a government. Yet, it does not share the key element of our study, namely that the irreversible cost of the producers of the final good is the cost of a necessary resource or right determined endogenously by its owner. Specifically, they examine the effect of fiscal incentives aimed at accelerating foreign direct investment from the perspective of real option theory. They compare two policy alternatives for a host country wishing to draw in FDI: entry cost subsidy (which may take the form of a cash grant covering part of the investment cost, subsidized labor training, provision of free or cheap land, etc.) and tax rate reduction. Their conclusion

[3] Since the final good is a durable good, sales of the resource imply additions to the quantity of the final good that already exists in the market.

is that entry cost subsidy is preferable from the host government's point of view to tax rate reduction: when the two alternatives have an equal net present value, an entry cost subsidy will lower the threshold for market entry of the prospective multinational firm by more than a tax rate reduction (alternatively, a lower net present value is needed for entry cost subsidy than for tax reduction so as to reach the same threshold). The basic explanation for this result is that entry cost subsidy provides the prospective multinational firm greater flexibility.

The article is organized as follows. In section 2 the model is presented and the value of the resource to its owner is analyzed. In section 3 the resource owner's choice of the price of the resource and the resulting immediate market situation - sales or inaction - is analyzed for the case where the resource owner maximizes its profits. In section 4 the same choice is analyzed, this time for the case of a government concerned about welfare. Section 5 offers some concluding remarks.

2. The Model

Consider the market for the durable good X. Production of X requires the resource N. The seller of N is a monopoly that sets the price k per each unit of N. In addition, in each point in time the producers of X have to pay a fraction t of their revenues to the seller of N. Once k, the price of N, is set - it cannot be changed anymore. We assume that the X producers can buy N any time they choose and that when such a firm purchases N it must transform it to X immediately. All X producers are risk-neutral and have the same production process: a unit of N is transformed to a unit of X at a cost w, i.e., the production function of firm i is $Q_i \leq N_i$, where Q_i is firm i's output and N_i is the amount of N that firm i has. Thus, the supply of good X is $Q = N$, where Q is the aggregate amount of good X supplied in the market and $N = \sum_{i=1}^{\infty} N_i$.

The demand for X is given by:

(1) $$P = \frac{A}{Q^{\alpha}},$$

where P is the price of X. A is a geometric Brownian motion and its dynamics are described by the following rule:

(2) $$dA = \mu A dt + \sigma A dZ,$$

where Z is the standard Wiener process satisfying at each point in time:

(3) $$E(dZ) = 0, \qquad E[(dZ)^2] = 1.$$

μ and σ are constants and $\sigma > 0$. By Itô's lemma and (1), when Q is unchanged the evolution of P is governed by:

(4) $$dP = \mu P dt + \sigma P dZ,$$

which means that P is also a geometric Brownian motion. By Itô's lemma, when Q is unchanged the after-tax price, $\tilde{P} = (1 - t)P$, is also a geometric Brownian motion, evolving according to:

(4') $d\tilde{P} = \mu\tilde{P}dt + \sigma\tilde{P}dZ$

We denote the discount rate relevant to the X producers and to the resource owner by r. Following Dixit (1989) we assume that $r > \mu$, an assumption that makes the expected rate of growth of $\tilde{P}$, the instantaneous profit, smaller than the discount rate, preventing thus the value of the firms that produce X from going to infinity.

Under this modeling, the X market is the same market studied by Leahy (1993). As Leahy (1993) shows, under this setup there is a threshold price, $\tilde{P}_H$, that characterizes the optimal policy of each single X producer: when $\tilde{P} < \tilde{P}_H$ the X producer does nothing, when $\tilde{P}$ hits $\tilde{P}_H$ the X producer buys some N and produces X from it. This optimal policy is the same for all X producers since they are identical. The firms' purchases of N increase the supply of X and prevent $\tilde{P}$ from rising above $\tilde{P}_H$. As Leahy (1993) shows, the value of $\tilde{P}_H$ is[4]:

(5) $$\tilde{P}_H = \frac{\beta}{\beta - 1}(r - \mu)(k + w),$$

where β is the positive root of the quadratic:

(6) $$\tfrac{1}{2}\sigma^2 Y^2 + \left(\mu - \tfrac{1}{2}\sigma^2\right)Y - r = 0\,.$$

[4]Throughout most of the paper, Leahy (1993) studies a more general case than the one presented here. In page 1119, though, the analysis takes several assumptions that make it entirely equivalent to model presented in the current paper. The second equation in p.1199 is equation (5) of the current paper. Some notational differences should be mentioned: the instantaneous profit is denoted by $\tilde{P}$ here and by P there; the investment threshold is denoted here by $\tilde{P}_H$ and by $\overline{P}$ there; the irreversible cost of producing a unit is denoted by k in Leahy (1993) while the notations here make it $k + w$; the positive root of equation (6) here is denoted by Leahy as α. All the other relevant notations are identical.

Applying $Y = 0$ and then $Y = 1$ and using the assumption that $r > \mu$ shows that one root of this quadratic (denoted γ) is negative and the other one, denoted β, exceeds unity.

Dividing by $1-t$ yields the corresponding threshold level of P, the pre-tax price:

(7) $$P_H = \frac{\beta}{\beta-1} \cdot \frac{r-\mu}{1-t} \cdot (k+w).$$

Given the initial values of A and Q the resource owner sets a value of k optimally. We start with the case where this value of k is sufficiently high to make the producers of X delay their purchases of N, i.e., the case where $A/Q^{\alpha} \le P_H(k)$. By (7), in this case k is in the range:

(8) $$k \ge \frac{(\beta-1)(1-t)A}{\beta(r-\mu)Q^{\alpha}} - w \equiv k^{*}$$

Next we analyze the case where the resource owner sets a value of k in the range $-w < k < k^{*}$. This choice leads to $A/Q^{\alpha} > P_H(k)$ and therefore induces immediate purchase of N by the X producers, purchases that increase Q until $A/Q^{\alpha} = P_H(k)$.

2.1 Delaying purchases of N

In this case, the X producers delay their purchases of N because the resource owner sets a value of k that makes the market price $P = A/Q^{\alpha}$ lower than the threshold P_H.

Throughout the article we use the term "the value of the resource" for the present value of the stream of revenues that the resource owner extracts from the X market, revenues that spring both from selling N and from taxing the sales of X. Let $V(A, Q, k)$ denote this value in the range defined by (8) given the current levels of A and Q and given a value of k. By Itô's lemma,

(9) $$dV(A, Q, k) = \left[V_A(A, Q, k)\mu A + \tfrac{1}{2} V_{AA}(A, Q, k)\sigma^2 A^2\right] dt + V_A(A, Q, k)\sigma A dZ$$

and due to (3):

(10) $$\frac{E[dV(A, Q, k)]}{dt} = V_A(A, Q, k)\mu A + \tfrac{1}{2} V_{AA}(A, Q, k)\sigma^2 A^2$$

Equation (10) captures the resource owner's expected capital gain due to the change in A over time. The no-arbitrage condition implies that this expected capital gain, together with the instantaneous revenue from taxing the X market, should equal the normal return to the resource. This implies:

(11) $$\frac{E[dV(A, Q, k)]}{dt} + tPQ = rV(A, Q, k)$$

Applying (10) and (1) in (11) and rearranging yields:

(12) $$V_A(A, Q, k)\mu A + \tfrac{1}{2} V_{AA}(A, Q, k)\sigma^2 A^2 - rV(A, Q, k) + tAQ^{1-\alpha} = 0$$

(12) is a second-order non-homogenous differential equation. Trying a solution of the

form $V(A, Q, k) = C(Q, k)A^{Y}$ to its homogenous part yields the quadratic captured by (6). Recall that the two roots of this quadratic satisfy $\gamma < 0$ and $\beta > 1$.

We now turn to finding a particular solution for (12). Trying a solution of the form $V(A, Q, k) = L(Q, k)A$ yields:

(13) $$L(Q, k) = \frac{tQ^{1-\alpha}}{r-\mu}$$

Combining the solution to the homogenous part of (12) and the particular solution to (12) yields:

(14) $$V(A, Q, k) = H(Q, k)A^{\gamma} + B(Q, k)A^{\beta} + \frac{tAQ^{1-\alpha}}{r-\mu},$$

where $H(Q, k)$ and $B(Q, k)$ will now be determined using two benchmark requirements. To that end, notice that by the standard properties of Brownian motions:

(15) $$E\left[\int_0^\infty e^{-rt} AQ^{1-\alpha} dt\right] = \frac{AQ^{1-\alpha}}{r-\mu}$$

Thus, the last addendum in the RHS of (14) is the expected value of tax revenues in the case that Q never changes, given the initial levels of A and Q. The two addendums preceding it in the RHS of (14) therefore capture the value of future sales of Q that occur each time A is sufficiently high so that the price of X hits the investment threshold P_H. However, if A is close to 0 then the probability of A ever rising so high is zero as well. In that case, therefore, the value of the resource is merely the expected value of the tax revenue as generated by the current quantity, Q. Formally:

(16) $$\lim_{A\to 0} V(A, Q, k) = \frac{tAQ^{1-\alpha}}{r-\mu}.$$

Since γ is negative, (16) implies that $H(Q, k) \equiv 0$.

We now turn to the determination of $B(Q, k)$. As appendix A shows, the condition for a no-arbitrage evaluation of the value of the resource in the time instants when there are changes in Q, i.e., when $A/Q^{\alpha} = P_H$, is:

(17) $$V_Q(A, Q, k) = - k.$$

Thus, by (14), (17) and $H(Q, k) = 0$, when $A/Q^{\alpha} = P_H$, :

(18) $$B_Q(Q, k)A^{\beta} + \frac{(1-\alpha)tAQ^{-\alpha}}{r-\mu} = - k.$$

Applying $A/Q^{\alpha} = P_H$ in (18) and rearranging it yields that when $A/Q^{\alpha} = P_H$:

(18') $$B_Q(Q, k) = -\frac{\frac{(1-\alpha)tP_H}{r-\mu} + k}{Q^{\alpha\beta} {P_H}^{\beta}}.$$

Straightforward integration of $B_Q(Q, k)$ leads to:

(19) $$B(Q,k) = \frac{\frac{(1-\alpha)tP_H}{r-\mu} + k}{(\alpha\beta-1)Q^{\alpha\beta-1}{P_H}^{\beta}} + C$$

Applying (7) in (19) and simplifying yields:

(19') $$B(Q,k) = \frac{(1-\alpha)\beta tw + (\beta-1-\alpha t\beta+t)k}{(1-t)(\beta-1)(\alpha\beta-1)Q^{\alpha\beta-1}{P_H}^{\beta}} + C$$

As Q goes to infinity P goes to 0 and the probability of P ever reaching P_H goes to zero as well. This implies that the resource owner is not going to sell any N in the future and its value should therefore spring merely from future tax revenues, i.e.:

(20) $$Lim_{Q\to\infty} B(Q,k) = 0.$$

This benchmark dictates a distinction between two cases based on the value of α. We start with a case in which $\alpha < 1/\beta$. In this case, Q in the denominator of the first term at the RHS of (19') is raised by a negative power and as it goes to infinity the entire term goes to either ∞ or $-\infty$. This, taken together with (20), implies that C goes to either ∞ or $-\infty$, and therefore that so are $B(Q, k)$ and $V(A, Q, k)$ for each finite level of Q, a case that is not at the focus of this paper.[5]

The economic logic underlying the infinite value of the resource in this case is based on the relation between α and the elasticity of demand which is $-1/\alpha$. The smaller α, the larger the demand elasticity and therefore the larger the increase in Q

[5] $\alpha<1/\beta$ implies $\alpha<1$ and therefore the numerator in the first term at the RHS of (19') is negative if k is below the negative value of $-\frac{(1-\alpha)\beta t}{(1-\alpha)\beta t+(\beta-1)(1-t)}w$.

each time that P hits P_H. Thus, the smaller α, the faster the process of sales of the resource N and the less heavily discounted are its revenues. In addition, the larger elasticity due to a smaller α also makes the tax revenues increase by more as Q grows. The two positive effects of a smaller α (faster sales of N and greater tax revenues) drive the value of future sales of N to infinity when α is sufficiently small, namely - below $1/\beta$.[6] Since this case is not in the focus of this paper, the rest of the paper refers to the case where $\alpha > 1/\beta$.

Returning to (19') and (20), now with $\alpha > 1/\beta$, the first term at the RHS of (19') goes to zero as Q goes to infinity, implying that $C=0$. Applying (7), $C=0$, (19') and $H(Q, k)=0$ in (14) yields:

$$(21) \qquad V(A, Q, k) = D\frac{(\beta - 1 - \alpha\beta t + t)k + (1-\alpha)\beta t w}{(k+w)^{\beta}} + \frac{tAQ^{-\alpha}}{(r-\mu)},$$

where:

$$(22) \qquad D \equiv \frac{(\beta - 1)^{\beta - 1}(1-t)^{\beta - 1} A^{\beta}}{(\alpha\beta - 1)\beta^{\beta}(r-\mu)^{\beta} Q^{\alpha\beta - 1}},$$

and $D>0$. The following *Proposition 1* shows some of the properties of $V(A, Q, k)$.

Proposition 1:

(a) If $\alpha > (\beta - 1 + t)/\beta t \equiv \alpha^{*}$ α, then $V_k(A, Q, k) > 0$ throughout the range $k > -w$.

[6] Based on this logic, in the case discussed in footnote 4 the value of the resource owner is -∞ because in that case the resource owner is subsidizing the X producers' intensive acquiring of the resource N.

(b) If $\alpha < \alpha^*$ then there exists a single value of k, denoted k_1, that maximizes $V(A, Q, k)$.

(c) k_1 is in the range where $V(A, Q, k)$ represents the value of the resource (the range $k > k^*$) iff A/Q^α is sufficiently small.

Proof: By (21):

$$V_k(A, Q, k) = D(\beta - 1)\frac{(1-t)\beta w + [(\alpha\beta - 1)t - (\beta - 1)](k + w)}{(k + w)^{\beta+1}}. \tag{23}$$

If $\alpha > \alpha^*$ then the term in the squares brackets is positive and therefore so is $V_k(A, Q, k)$ throughout the range $k > -w$. This proves part (a).

If $\alpha < \alpha^*$ then $(\alpha\beta - 1)t - (\beta - 1)$ is negative, implying that $V_k(A, Q, k)$ is positive for sufficiently small values of k and vice versa. Thus, in this case there is a single value of k that maximizes $V(A, Q, k)$. This proves part (b). Solving the first order condition $V_k(A, Q, k) = 0$ yields that this value satisfies:

$$k = \frac{(1-t) - (1-\alpha)t\beta}{(\beta - 1)(1-t) + (1-\alpha)t\beta} w \equiv k_1. \tag{24}$$

Applying (24) in (8) shows that k_1 is in the range $k > k^*$, in which $V(A, Q, k)$ represents the value of the resource, iff the following condition holds:

$$\frac{A}{Q^\alpha} < \frac{\beta^2 (r - \mu) w}{(\beta - 1)^2 (1-t) + (1-\alpha)t\beta(\beta - 1)} \equiv P^*. \tag{25}$$

This proves part (c). □

(25) implies that the resource owner will set a value of k that is sufficiently large to make the X producers delay their purchases N if current demand in the X market is sufficiently low to make the market price fall below P^*.

Note that $\alpha^*>1$ which means that in the range $\alpha<\alpha^*$ the demand for X can be either elastic ($\alpha<1$) or inelastic ($\alpha>1$).

Also note that if $w=0$, i.e., if the X producers do not face costs except for the purchase of N, then condition (25) cannot hold. In that case, when $\alpha<\alpha^*$ the function $V(A, Q, k)$ is strictly decreasing and maximized at the lower boundary of its definition range, namely at $k=k^*$, implying that the resource owner does not set k high enough to send the market to an inaction period.

From continuity it follows, by applying (7) in (21), that at $A/Q^\alpha = P_H$:

(26) $$V(A,\ Q,\ k) = \frac{k(1-t+t\alpha\beta)+t\alpha\beta w}{(1-t)(\alpha\beta-1)}Q.$$

2.2 Inducing immediate purchases of N

The X producers immediately purchase N when the market price, $P = A/Q^\alpha$, exceeds the threshold P_H. Applying (7) in $A/Q^\alpha \geq P_H$, this range becomes:

(27) $$k < \frac{(\beta-1)(1-t)A}{\beta(r-\mu)Q^\alpha} - w \equiv k^*.$$

In this range immediate investment increases Q to Q_1 so that the price after the investment is made is $P = A/Q_1^{\alpha} = P_H$. The resource owner receives from this increase in Q:

$$(28) \qquad k(Q_1 - Q) = k\left[\left(\frac{A}{P_H}\right)^{\frac{1}{\alpha}} - Q\right].$$

Let $G(A, Q, k)$ denote the value of the resource in the range defined by (27) given the current levels of A and Q and also for a given value of k. Equation (29) below shows $G(A, Q, k)$ as the sum of two factors: First, the immediate proceeds described by (28); Second, the value of the resource after the quantity immediately becomes Q_1, as described by (26).

$$(29) \qquad G(A, Q, k) = k(Q_1 - Q) + \frac{k(1 - t + t\alpha\beta) + t\alpha\beta w}{(1-t)(\alpha\beta - 1)} Q,$$

which can be simplified to:

$$(29') \qquad G(A, Q, k) = -\,kQ + \alpha\beta \frac{k + wt}{(\alpha\beta - 1)(1 - t)} Q_1 .$$

Applying $Q_1 = (A/P_H)^{1/\alpha}$ and (7) in (29') yields:

$$(30) \qquad G(A, Q, k) = JA^{\frac{1}{\alpha}} f(k) - kQ .$$

where:

(31.a) $$J \equiv \frac{(1-t)^{\frac{1}{\alpha}-1}(\beta-1)^{\frac{1}{\alpha}}}{(\alpha\beta-1)\beta^{\frac{1}{\alpha}-1}(r-\mu)^{\frac{1}{\alpha}}} > 0,$$

(31.b) $$f(k) \equiv \alpha \frac{k+wt}{(k+w)^{\frac{1}{\alpha}}}.$$

The following *Proposition 2* shows some important properties of $G(A, Q, k)$

Proposition 2:

(a) There exists a single value of k that brings $G(A, Q, k)$ to a maximum;

(b) This value of k, denoted by k_2, is an increasing concave function of A/Q^{α};

(c) k_2 is in the range $k<k^*$, the range in which $G(A, Q, k)$ represents the value of the resource, iff $\alpha<\alpha^*$ and $A/Q^{\alpha}>P^*$.

Proof: In the appendix. □

3. The optimal *k* when the resource owner maximizes its profits

In this section we analyze how the optimal k is chosen in the case where the resource owner is a firm that maximizes its profits. Based on the analysis in the previous sections, the value of the resource as a function of A, Q and k can be defined and denoted by:

(32) $$VG(A,Q,k) \equiv \begin{cases} G(A,Q,k) & if \ -w < k < k^*(A,Q) \\ V(A,Q,k) & otherwise \end{cases}$$

Note that $V(A, Q, k^*) = G(A, Q, k^*)$ as follows from applying (8) in (21) and then in (30). Three cases should be analyzed now: The case where $\alpha > \alpha^*$; The case where $\alpha < \alpha^*$ and $A/Q^\alpha < P^*$; The case where $\alpha < \alpha^*$ and $A/Q^\alpha > P^*$.

3.1 When $\alpha > \alpha^*$

In this case $k_2 > k^*$ for each value of A/Q^α as part (c) of *Proposition 2* shows. Thus, in the range $k < k^*$, the resource firm's value, represented by $G(A, Q, k)$, is increasing in k. From part (a) of *Proposition 1* it follows that the resource firm's value, now represented by $V(A, Q, k)$, is increasing in k also in the range $k > k^*$. Thus, in this case the value of the resource, $VG(A, Q, k)$, is an increasing function of k and it is optimal for it to push the value of k to infinity. The economic logic in action here is that when α is sufficiently large, demand is sufficiently inelastic to make it optimal for the resource owner to prevent increases in quantity in order to keep tax revenues from falling. Note from (21) that in this case as k goes to infinity $VG(A, Q, k)$ approaches $tAQ^{1-\alpha}/(r-\mu)$, which is the expected value of the tax collection if Q is fixed over time at its current level.

3.2 When $\alpha < \alpha^*$ and $A/Q^\alpha < P^*$

In this case $k_2 > k^*$ as follows from part (c) of *Proposition 2*. Thus, in the range $k < k^*$, the value of the resource, represented by $G(A, Q, k)$, is increasing in k. From parts (b) and (c) of *Proposition 1* it follows that in the range $k > k^*$ the value of the resource, now represented by $V(A, Q, k)$, reaches a maximum at $k = k_1$. Thus, since $V(A, Q, k^*)$ $= G(A, Q, k^*)$, the value of the resource, $VG(A, Q, k)$, reaches its maximum in $k = k_1$.

The line marked with circles in Figure 1 below presents $VG(A, Q, k)$ in this case. The thin line shows $V(A, Q, k)$ and the thick line shows $G(A, Q, k)$.

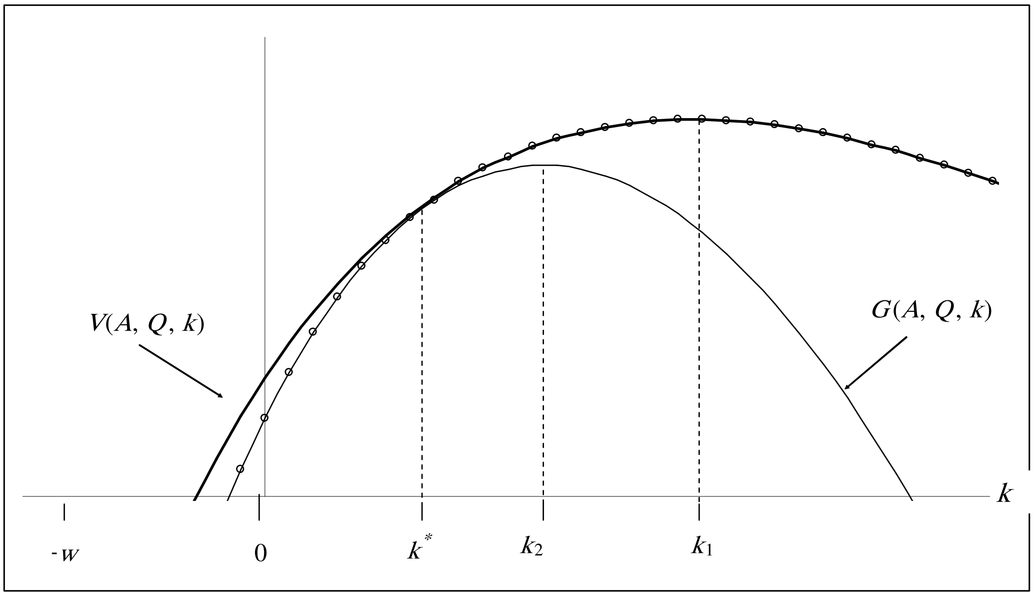

Figure 1: The resource firm's value, $VG(A, Q, k)$, when $\alpha < \alpha^*$ and $A/Q^\alpha < P^*$. The thick line shows $V(A, Q, k)$, the thin line shows $G(A, Q, k)$ and the circles indicate $VG(A, Q, k)$. In this case $VG(A, Q, k)$ is maximized at $k = k_1 > k^*$ implying that the resource firm sets a value of k sufficiently high to delay immediate purchases of N by the X producers.

3.3 When $\alpha < \alpha^*$ and $A/Q^\alpha > P^*$

In this case, in the range $k < k^*$, the value of the resource, which is represented by $G(A, Q, k)$, reaches a maximum at $k = k_2$ as follows from parts (a) and (c) of *Proposition 2.* Also, $k_1 < k^*$, as follows from parts (b) and (c) of *Proposition 1.* Thus, in the range $k > k^*$ the value of the resource, represented now by $V(A, Q, k)$, decreases in k. Therefore, since $V(A, Q, k^*) = G(A, Q, k^*)$, the value of the resource reaches its maximum at $k = k_2$.

The line marked with circles in Figure 2 below presents $VG(A, Q, k)$ in this case. The thin line show $V(A, Q, k)$ and the thick line shows $G(A, Q, k)$.

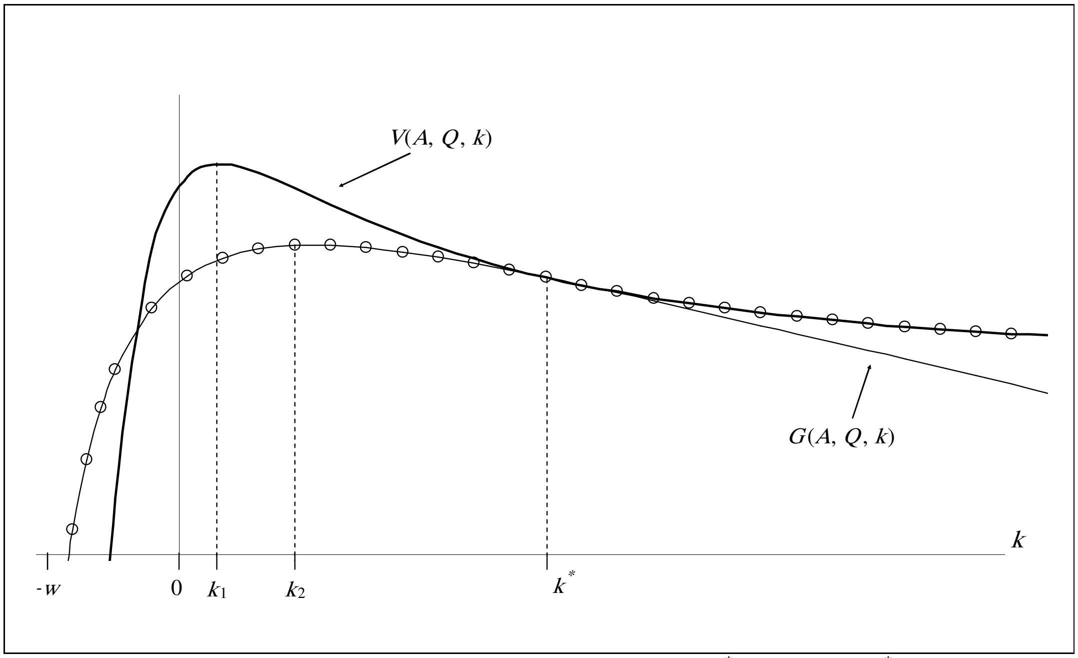

Figure 2: The resource firm's value, $VG(A, Q, k)$, when $\alpha < \alpha^*$ and $A/Q^\alpha > P^*$. The thick line shows $V(A, Q, k)$, the thin line shows $G(A, Q, k)$ and the circles indicate $VG(A, Q, k)$. In this case $VG(A, Q, k)$ is maximized at $k = k_2 < k^*$ implying that the resource owner sets a value of k sufficiently low to induce immediate purchases of N by the X producers.

Based on the analysis of the two previous sub-sections, Figure 3 below shows the optimal k as a function of A/Q^α for the case when $\alpha < \alpha^*$.

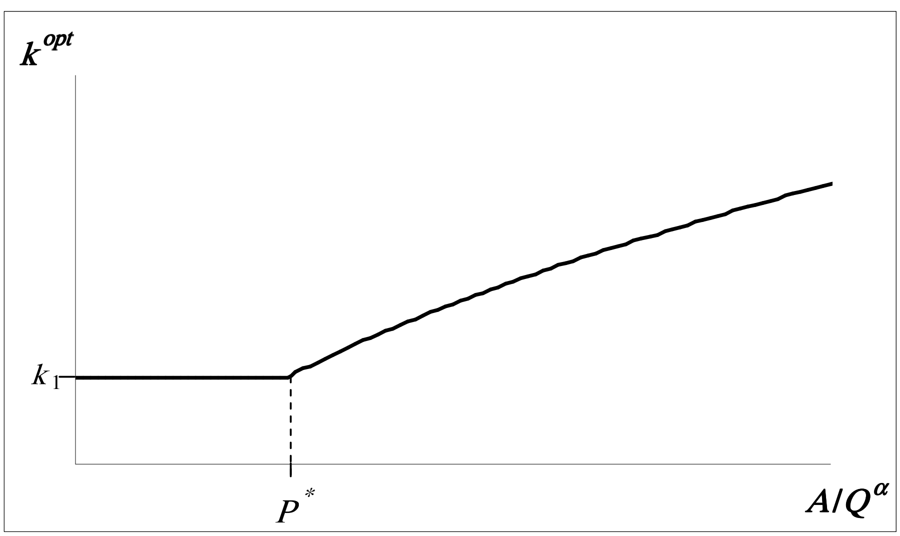

Figure 3: The optimal k as a function of A/Q^α for the case $\alpha < \alpha^*$.

The increasing part in this function is concave and the entire function may be below zero, as the following proposition establishes.

Proposition 3: If $\alpha < t$ then the optimal value of k is negative.

Proof: If $\alpha < t$ then $\alpha < \alpha^*$, implying that the resource firm's value is maximized either by k_1 or by k_2, depending on whether $A/Q^{\alpha} > P^*$ or not. As shall be shown now, in this case both k_1 and k_2 are negative.

To prove that $k_1 < 0$ in this case, note that $\alpha < t$ implies $\alpha < 1$ and therefore, by (24), the denominator of k_1 is positive. The numerator of k_1 in that case satisfies:

$$1 - t - (1 - \alpha)t\beta < 1 - \alpha - (1 - \alpha)\alpha\beta = (1 - \alpha)(1 - \alpha\beta) < 0,$$

where the first inequality springs from $\alpha < t$ taken together with the implied $\alpha < 1$ which makes the expression to the left of that inequality depend negatively on t. The second inequality springs from $\alpha\beta > 1$. Thus k_1 is negative in that case.

To prove that $k_2 < 0$ in this case, note from (30) that maximizing $G(A, Q, k)$ requires the first order condition:

$$-Q + JA^{\frac{1}{\alpha}} f'(k) = 0 .$$

Proposition 2 has established the existence and uniqueness of a root to this equation. It can also be noticed from the equation above that this root is also characterized by $f'(k) > 0$, since (31.a) shows that $J>0$. Differentiating (31.b) yields that this holds if and only if:

$$k_2(\alpha - 1) + w(\alpha - t) > 0 .$$

If $\alpha < t$ then α is also smaller then 1 and therefore this inequality holds only if k_2 is smaller than the negative term $-\frac{t-\alpha}{1-\alpha}w$. □

4. The optimal *k* in the case of a welfare objective

Assume now that the resource owner is a government that is not interested in maximizing the value of its potential sales of the resource *N*, namely $VG(A, Q, k)$. Instead, we now assume that the government is concerned about welfare in the *X* market, but also wishes to use it for financing its activities in other markets that could benefit from government intervention. Specifically, we assume that the government balances these two contradicting targets by setting an objective of bringing the value $VG(A, Q, k)$ to a certain level *M* which is below the maximal level of $VG(A, Q, k)$. As has been established, in the case where $\alpha < \alpha^*$ the function $VG(A, Q, k)$ has an inverse-U shape and therefore there are two values of *k* that yield the value *M* that the government seeks. We assume that the government, wishing to cause as little welfare loss as possible in the *X* market, chooses the lower of the two values of *k* that solve:

(33) $$VG(A, Q, k) = M$$

As in section 3, three cases will now be analyzed: The case where $\alpha > \alpha^*$; the case where $\alpha < \alpha^*$ and $A/Q^{\alpha} < P^*$; the case where $\alpha < \alpha^*$ and $A/Q^{\alpha} > P^*$.

4.1 When $\alpha > \alpha^*$

As established in section 3.1, in this case $VG(A, Q, k)$ is a monotonically increasing function of k that converges to the value $\frac{tAQ^{1-\alpha}}{r-\mu}$ as k approaches infinity. In addition, it approaches -∞ as k approaches $-w$, as follows from (30) and (31). Thus, there is single value that solves (33) for every level of M that is smaller than $\frac{tAQ^{1-\alpha}}{r-\mu}$.

Note that in contrast to the case where the resource owner is a profit maximizing firm, here the chosen value of k is not necessarily infinite. The reason is that in the current case the resource owner is a government that is not only interested in obtaining revenues from selling N and taxing the market for X, but is also concerned about welfare in the X market. The only possibility for the government to set an infinitely high level of k is if the M it wishes to extract from the X market is above $\frac{tAQ^{1-\alpha}}{r-\mu}$.

By (21), (32) and an implicit differentiation of (33):

(34) $$\frac{dk}{dM} = -\frac{-1}{V_K(A, Q, k)} > 0$$

where the inequality follows from the result that in this case $V_K(A, Q, k) < 0$, as established in the proof of part (a) of *Proposition 1.* Thus, the larger the value of the revenues that the government wishes to extract from the X market the larger the level of k it sets. In a similar manner it can be shown that in this case k is decreasing in A and increasing in Q.

We denote the value of $V(A, Q, k)$ at the end of its definition range:

(35) $$V^*(A, Q) \equiv V[A, Q, k^*(A, Q)] = G[A, Q, k^*(A, Q)]$$

where k^* is a function of A and Q by (8). Applying (8) in (21) yields $V^*(A, Q)$ explicitly. Based on the analysis of the properties of $VG(A, Q, k)$ in section 3.1, if M is smaller than $V^*(A, Q)$ then the k chosen by the government is below k^*, implying immediate purchases of N and production of X. Otherwise, the k that the government chooses is above k^*, a choice that sends the market to a period of inaction until A is sufficiently large so that $P = P_H$.

4.2 When $\alpha < \alpha^*$ and $A/Q^\alpha < P^*$

Based on Figure 1 and the analysis in section 3.2, in this case $VG(A, Q, k)$ has an inverse-U shape maximized at $k = k_1 > k^*$. In the range $-w < k < k^*$ the function $VG(A, Q, k)$ is based on $G(A, Q, k)$ and for higher levels of k it is based on $V(A, Q, k)$. The value of $VG(A, Q, k)$ at its maximum satisfies:

$$(36) \qquad V^1(A, Q) \equiv V[A, Q, k_1(A, Q)].$$

Note that k_1 is a function of A and Q by (24). Applying (24) in (21) explicitly yields $V^1(A, Q)$, which is the maximal level of M that the government can extract from the market in this case. If the government is interested in a level of M that satisfies: $V^*(A, Q) < M \leq V^1(A, Q)$ then the level of k that the government chooses, based on (33), is above k^*, implying inaction until A is sufficiently large so that $P = P_H$. If, on the other hand, the level of M that the government seeks satisfies $M < V^*(A, Q)$, then the government chooses a value of k that is smaller than k^*, implying immediate purchases of N and production of X.

An important difference from the case where the resource owner maximizes the value of $VG(A, Q, k)$ is that in that case the value of k is constant at k_1, whereas here it is increasing in Q and decreasing in A.

4.3 When $\alpha < \alpha^*$ and $A/Q^\alpha \leq P^*$

Based on Figure 2 and the analysis in section 3.3, in this case $VG(A, Q, k)$ has an inverse-U shape maximized at $k = k_2 < k^*$. In the range $-w < k < k^*$ the function $VG(A, Q, k)$ is based on $G(A, Q, k)$ and for higher levels of k it is based on $V(A, Q, k)$. The value of $VG(A, Q, k)$ at its maximum satisfies:

$$(37) \qquad G^2(A, Q) \equiv G[A, Q, k_2(A, Q)]$$

where k_2 is function of A and Q by *Proposition 2.*

$G^2(A, Q)$ is the maximal level of M that the government can extract from the market in this case. If the level of M desired by the government is below $G^2(A, Q)$ then the level of k that the government chooses, based on (33), satisfies $k < k_2 < k^*$, implying immediate purchases of N and production of X.

As in the case where the resource owner maximizes the value of $VG(A, Q, k)$, the value of k that the government chooses is increasing in Q and decreasing in A.

5. Concluding Remarks

In this paper, we returned to the typical model of investment under uncertainty and examined it from a new angle – that of the recipient of the investment cost. We modeled the recipient of this cost as a firm or a government that sells a resource or a right that is necessary for the production of the final good. The focus of our study was on how the resource owner sets the price of its resource. A key assumption in the model was that the price of the resource cannot be changed on a continuous basis. For simplicity, we took this assumption to its extremity – i.e., once the price is set, the resource owner may not change it at all under any circumstances. Relaxing this extreme assumption should not change the qualitative results of the analysis, so long as the basic assumption – that there are indeed some technical barriers or costs to changing the price of the resource continuously – is maintained.

Our main finding is that when the demand for the final good is sufficiently low, the resource owner does not lower its price accordingly, and thus the producers of the final good delay their purchases of the resource and withhold their production of the final good until demand rises sufficiently. The resource owner does so because it expects demand to be higher in the future and does not want to be committed to providing the resource at a low price.

Another important result is that the price of the resource is a decreasing function of the elasticity of demand for the final good. This result is based on the assumption that the government or the firm that owns the resource does not only receive a one-time payment for the resource when it is sold, but also secures for itself a certain share of the profits made by the producers of the final good.

Finally, we compared the case where the resource is sold by a private firm that is interested only in profit maximization with the case where the resource is sold by a government that uses its proceeds for welfare enhancing activities in other markets, while also of course concerned about welfare in the market of the resource-based final good. We found that in the latter case the price charged for the resource will be lower, reflecting the government's interest in welfare rather than profit. In particular, it might be the case that under the same initial conditions the market for the final good would be lead either to immediate production, if the resource is owned by a government, or to a period of inaction, if the resource is owned by a private firm. This difference should be taken into account when the government privatizes a neutral resource.

Appendix

A. Establishing condition (17)

In this appendix we derive the benchmark condition (17) for the value of the resource at the time instants in which P hits P_H. For this end we use the discrete approximation of a Brownian Motion presented in Dixit (1991). Since it is more convenient to perform this approximation for a Brownian Motion rather than for a Geometric Brownian Motion, the analysis is based on the function:

(A.1) $$F(a, Q, k) \equiv V(A, Q, k)$$

where $a \equiv \ln A$. Due to this definition, by Itô's lemma, a is a Brownian Motion since A is a Geometric Brownian Motion. The drift and variance parameters of a are denoted here by μ_a and σ_a^2. To approximate the motion of a we divide time to small intervals of length τ and the variable a space into steps of size ξ. The variable a now ranges over a discrete set of values a_i such that:

(A.2) $$a_{i+1} - a_i = \xi \quad \text{for all } i.$$

Starting at state a_i, time τ later the variable a takes with probability p a step down to the value of a_{i-1}, or takes with probability q=1-p a step up to the value of a_{i+1}. Two conditions relating τ, ξ, p and q to μ_a and σ_a should be used in order to make this process an approximation of the original Brownian Motion. First:

(A.3) $$\mu\tau = q\xi + p(-\xi),$$

which leads to:

(A.4) $$q = \frac{1}{2}\left(1 + \frac{\mu\tau}{\xi}\right), \qquad p = \frac{1}{2}\left(1 - \frac{\mu\tau}{\xi}\right)$$

The condition regarding the variance of the process is:

(A.5) $$\sigma^2\tau = q(\xi - \mu\tau)^2 + p(-\xi - \mu\tau)^2 = \xi^2 + 2\mu\tau\xi(p - q) + (\tau\mu)^2$$
$$= \xi^2 - \mu^2\tau^2$$

Eliminating the term with τ^2 leaves:

(A.6) $$\sigma^2\tau = \xi^2$$

When a_i is such that $P = \frac{A}{Q^\alpha}$ is at the investment threshold P_H then, by (1):

(A.7) $$Q = \left(\frac{A}{P_H}\right)^{\frac{1}{\alpha}} = \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}}$$

If time τ later a takes a step up, the endogenous investment by the X producers raises

Q such that P remains at P_H. This implies that Q is raised to the level:

(A.8) $$\left(\frac{e^{a_i+\xi}}{P_H}\right)^{\frac{1}{\alpha}} = \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} + \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} \frac{1}{\alpha}\xi + \frac{\left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} \frac{1}{\alpha^2}\xi^2}{2} + \frac{\left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} \frac{1}{\alpha^3}\xi^3}{6} + \ldots$$

The change in Q during that time is therefore:

(A.9) $$\Delta Q = \left(\frac{e^{a_i+\xi}}{P_H}\right)^{\frac{1}{\alpha}} - \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} = \left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} \frac{\xi}{\alpha} + \mathrm{o}(\xi),$$

where $\mathrm{o}(\xi)$ collects all the terms that go to zero faster than ξ, such that $\mathrm{o}(\xi)/\xi \to 0$ as $\xi \to 0$. Note from (A.6) that τ too falls under the category of $\mathrm{o}(\xi)$.

The Bellman equation for the value of the resource when a_i and Q are such that $P = P_H$ is:

(A.10) $$F(a_i, Q, k) = tP_HQ\tau + e^{-r\tau}\,[pF(a_{i-1}, Q, k) + qF(a_{i+1}, Q + \Delta Q, k) + qk\Delta Q]$$

(A.10) shows the value of the resource in this situation as the sum of the immediate tax revenue and the time τ later value of the resource discounted by $e^{-r\tau}$. With probability p the variable a takes a step down and the value of the resource becomes $F(a_{i-1}, Q, k)$. With probability q the variable a takes a step up. In this case, endogenous investment by the producers of X raises Q by ΔQ and the value of the resource becomes $F(a_{i+1}, Q + \Delta Q, k)$. In addition, in this case the resource owner also gains $k\Delta Q$ from sales to the X producers.

Expanding $e^{-r\tau}$ to a Taylor series, bearing in mind that that τ is o(ξ), yields:

(A.11) $$e^{-r\tau} = 1 + (-r\tau) + \frac{(-r\tau)^2}{2} + \frac{(-r\tau)^3}{6} + \ldots = 1 + o(\xi)$$

Applying this in (A.10) and expanding terms of (A.10) to Taylor series yields:

(A.12) $$F(a_i, Q, k) = tP_H Q\tau + p[F(a_i, Q, k) + F_a(a_i, Q, k)(-\xi) + o(\xi)]$$
$$+q[F(a_i, Q, k) + F_a(a_i, Q, k)(\xi) + F_Q(a_i, Q, k)\Delta Q + o(\xi) + k\Delta Q]$$

Using $p + q = 1$ and the result that τ is o(ξ) by itself helps simplify (A.12) to:

(A.13) $$0 = (q - p)F_a(a_i, Q, k)\xi + q\, F_Q(a_i, Q, k)\Delta Q + qk\Delta Q + o(\xi)$$

By (A.4), $(q - p)\xi = \mu\tau = o(\xi)$ which simplifies (A.13) into:

(A.14) $$0 = F_Q(a_i, Q, k)\Delta Q + k\Delta Q + o(\xi)$$

Dividing by ΔQ and applying (A.9) yields:

(A.15) $$F_Q(a_i, Q, k) = -k - \frac{o(\xi)/\xi}{\frac{1}{\alpha}\left(\frac{e^{a_i}}{P_H}\right)^{\frac{1}{\alpha}} + o(\xi)/\xi}$$

By the definition of o(ξ), as $\xi \to 0$ the numerator and the second addendum on the

RHS of (A.15) approach 0 as well. This, together with $F_Q(a, Q, k) \equiv V_Q(A, Q, k)$, which follows from the definition of $F(a_i, Q, k)$ in (A.1), concludes establishing (17).

B. Proof of *Proposition 2*

By (30) the first order condition for a maximum is

(B.1) $$G_k(A, Q, k) = -Q + JA^{\frac{1}{\alpha}} f'(k) = 0,$$

Manipulating (B.1) and applying (1) in it, (B.1) becomes:

(B.2) $$f'(k) = \frac{1}{JP^{\frac{1}{\alpha}}},$$

where the RHS of (B.2) is positive. To establish existence of a root to (B.2) note that by (31.b):

(B.3) $$f'(k) = \frac{k(\alpha - 1) + w(\alpha - t)}{(k + w)^{\frac{1+\alpha}{\alpha}}}.$$

By (B.3), $f'(k)$ approaches infinity when k approaches $-w$ and approaches 0 when k goes to infinity. Thus, by continuity, there exists a level of k in the relevant range (namely $k>-w$) for which $f'(k)$ equals the positive term at the RHS of (B.2). To see that there is only one such level of k, note from (B.3) that:

(B.4) $$f''(k) = -\frac{k(\alpha-1)+w(\alpha-t)+w\alpha(1-t)}{\alpha(k+w)^{\frac{1+2\alpha}{\alpha}}} = \frac{-(k+w)^{\frac{1+\alpha}{\alpha}} f'(k) - w\alpha(1-t)}{\alpha(k+w)^{\frac{1+2\alpha}{\alpha}}}$$

where the second equality follows from (B.3). From (B.4), together with $f'(k)>0$, it follows that $f''(k)<0$, implying that there can only be a single value of k for which $f'(k)$ equals the positive term on the RHS of (B.2). Denoting this single root by k_2, the result that $f''(k_2)<0$ also asserts that k_2 brings $G(A, Q, k)$ to a maximum since:

(B.5) $$G_{kk}(A, Q, k_2) = CA^{\frac{1}{\alpha}} f''(k_2) < 0.$$

(B.2) presents k_2 as an implicit function of P. Differentiating it leads to:

(B.6) $$\frac{dk_2}{dP} = -\frac{f'(k_2)}{\alpha P f''(k_2)} > 0$$

where the inequality follows from $f'(k_2)>0$ and $f''(k_2)<0$.

Applying (8), (31.a) and (B.3) in (B.2) and simplifying yields that $k_2=k^*$ if and only if P equals either 0 or P^*. Applying (8), (31.a), (B.3), (B.4) and (25) in (B.6) yields that when $P=P^*$ and $k=k^*$:

(B.7) $$\frac{dk_2}{dP} = \frac{(\beta-1)(1-t)^2}{\beta(r-\mu)} \cdot \frac{\alpha\beta-1}{(\alpha\beta-1)(1-t)+\alpha(\beta-1+t-\alpha\beta t)}$$

$$= \frac{dk^*}{dP} \cdot \frac{(1-t)(\alpha\beta-1)}{(\alpha\beta-1)(1-t)+\alpha\beta t(\alpha^*-\alpha)} < \frac{dk^*}{dP}$$

The second equality follows from (8) and from the definition of α^* in *Proposition 1.* The inequality follows from the assumptions that $\alpha\beta>1$ and $\alpha<\alpha^*$.

k_2 and k^* are both increasing functions of P. They meat one another only when $P=0$ and when $P=P^*$ and in that second meeting point $\frac{dk_2}{dP}<\frac{dk^*}{dP}$. From these properties it follows that $k_2>k^*$ as long as $P<P^*$ and vice verse.

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