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Dynamics of Voluntary Contributions to Capacity and Operation of Public Goods

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Abstract

We study the dynamics of the private provision of a public good that requires both a capacity build-up and ongoing operation costs. We show cases of over-optimal provision of capacity to the public good. We show that over-optimal provision of capacity is more likely to occur when the subjective discount rate is higher, the group-size of contributors is smaller and the length of the first stage is longer. We also show that by setting a proper time limit for collecting contributions to the capacity build-up of the public good, the utility loss in the Nash equilibrium is minimized while avoiding excess burden of taxation.

Keywords: public good, capacity, ongoing operation, over-provision, under-provision, Markov-Perfect-Equilibrium

JEL classification: C73, H41.

1. Introduction

Since the seminal contribution of Olson (1965), the study of private voluntary provision of public goods has produced extensive literature.[3] The classical literature mainly used a static setup[4] to show the existence of a free-riding problem and the resulting sub-optimal provision

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3 For a comprehensive review of early literature, see Bergstrom, Blume, and Varian (1986).
4 See, for example, Olson (1965), Chamberlin (1974), McGuire (1974), Cornes and Sandler (1983), Bergstrom, Blume, and Varian (1986), Gradstein and Nitzan (1989).

of public goods in Nash equilibrium. Dynamic models were also developed, usually framed as repeated or differential games with finite or infinite time horizons.[5] The vast literature on voluntary provision of public goods yielded a consensus saying that private provision is always sub-optimal and over-provision is unlikely.

Nevertheless, in many sites around the world, there are gigantic monuments like shrines, worship houses as well as university campuses, medical compounds, auditoriums, opera houses and more; some of them noticeably magnificent in their poor surroundings and some of them also notably poorly maintained or operated. In many cases, construction costs and maintenance of these monuments were funded through private donations. These are, therefore, examples of voluntary over-contribution of public infrastructure followed by sub-optimal maintenance[6]. This phenomenon is almost ignored in economic literature, although it has long prevailed in history[7].

The theoretical literature on public and club goods generally assumes a dichotomy between "discrete" and "continuous" public goods. However, in the real world, there are many public goods which are neither purely continuous nor purely discrete, as their production requires a capacity build-up in addition to ongoing operating costs. For example, an initial capacity build-up cost is required to erect a hospital, a pool or a worship house, while its output depends on variable costs as well. Nevertheless, this kind of public good is almost neglected in the literature,[8] although economies to scale in club goods have been studied extensively (c.f. Kennedy (1990)).

We analyze the dynamics of the voluntary private provision of a public good, which requires an initial investment to build up a capacity as well as variable costs for its ongoing operation. We set up a two-stage game of voluntary contribution to a public good. In the first stage, whose length is T, individuals continuously contribute to a capacity build-up of the public

5 See, for example, McMillan (1979), Fershtmann and Nitzan (1991) and Kessing (2007).

6 Another example is a community that decides to build and operate public facilities like swimming pools.

7 The Biblical narrative on the erection of the Tabernacle in the desert of Sinai after the Exodus from Egypt (Exodus 25) is probably the first documented case. According to the Bible, the erection of the sanctuary and its ritual equipment (i.e. the Holy Ark, the Altar etc.) was financed by voluntary contributions and the enormous over-contribution led Moses to command: *"Let neither man nor woman make any more work for the offering of the sanctuary. So the people were restrained from bringing."* On the other hand, the variable costs were financed through a mandatory poll-tax (Exodus 30), which indicates that voluntary contributions were insufficient.

8 Baumol and Bowen (1968) and Hansmann (1981) are among the few exceptions.

good; in the second stage, individuals contribute repeatedly and infinitely to the ongoing operation of the public good. We show a case for over-optimal provision of capacity in the *Markov-Perfect-Equilibrium*, followed by sub-optimal operation of the public good. The dynamic setting of the contributions game allows us to study the effect of the observed accumulated contributions on an individual's contribution; moreover, it allows us to consider the effect of a time limit on the capacity build-up as well as the effect of subsidies on social utility.

The remainder of this paper is organized as follows: Section 2 contains a brief literature survey. In Section 3, we present our theoretical model. We start with a benchmark case in which after the capacity is accumulated, the public good yields a perpetual utility that positively depends on capacity. We follow with the capacity build-up and operation costs case. In Section 0, we consider policy tools aimed at minimizing utility loss which is an inevitable result of voluntary contribution. Section 5 contains a brief summary of our theoretical results and some empirical support.

2. Related Literature

The rapid growth of the literature on public goods provision during the last five decades[9] precludes any honest attempt for a comprehensive survey within the limits of one section. Earlier studies of the private provision of public goods mainly considered static models and assumed continuous public goods, an assumption that also prevailed in early dynamic models like McMilan (1979) who shows that in a repeated game with trigger strategies, a non-cooperative equilibrium can be efficient. Palfrey and Rosenthal (1984; 1988) analyzed the binary voluntary contribution game with complete information, and Bagnoli and Lipman (1989) studied the analogous continuous case and show that the core is implementable in undominated perfect equilibria. The efficiency result is counterintuitive, but the real problem is that no prediction can be made because the number of undominated equilibria is generally infinite. Later studies analyzed the voluntary contributions game assuming various sorts of uncertainty.[10]

9 see Zelmer (2003).

10 For example, see Nitzan and Romano (1990), Keenan, Kim and Warren (2006), McBride (2006) and Markis (2008).

Fershtmann and Nitzan (1991) compared the dynamics of private provision of public goods under symmetric open-loop Nash equilibrium and feedback Nash equilibrium, where contributions are accumulated over time in an infinite dynamic game with continuous contributions and a flow of benefits, and show that both are characterized by under-provision of the public good. They also show that the free-riding problem is aggravated when players' contributions are conditional on the observable collective contributions.

The study of "discrete" public good voluntary provision began with "binary" participation models, in which agents can contribute either a predetermined fixed sum or nothing (c.f. Palfrey and Rosenthal (1984)). Gradstein and Nitzan (1989) extended the binary model and analyzed voluntary binary participation in a full information setting, where marginal product of participation is positive but decreases with the number of participants. It also shows that (as in the continuous setting), Nash equilibria are inefficient and the public good is underprovided in a pure strategies equilibrium, but can be overprovided in a mixed strategies equilibrium. Admati and Perry (1991) studied the pattern of contributions to a joint project where partners alternate in contributing to the project until the project is completed. They showed that socially desirable projects may not be completed, but when the costs of contributions are borne only when the project is completed, the outcome is efficient. Kessing (2007) showed that in a dynamic private provision game, voluntary contributions to a discrete public good are strategic complements in contrast to continuous public good private provision strategies, which are usually strategic substitutes.

Various studies on voluntary contributions to a discrete public good analyzed the effect of private information about the costs associated with contributions to the public good (Gradstein 1994), the threshold point level (McBride 2006) or group size (Markis 2008). Marx and Mathew (2000) studied the dynamics of private provision of public goods with imperfect information and show that if the horizon is sufficiently long, players' preferences are similar and they are patient or if the period length is short, perfect Bayesian equilibrium exists that essentially completes the project, and in certain circumstances, efficiency is achieved in the limit. However, they impose a constraint that precludes over-contributions.

The role of public policy was also extensively studied. Warr (1983) showed that the private provision of a single public good is unaffected by redistribution of income. However, this result is valid if and only if redistribution does not alter the composition of the contributing set.[11] Steinberg (1987) showed that the sign as well as the magnitude of crowding out responses to a cut in federal expenditures on social services by private and social non-profit organizations are ambiguous. Itaya and Schweinberger (2006) examined an economy consisting of two types of individuals, contributors and non-contributors, and provide necessary and sufficient conditions for raising the total provision of the public good when the public good is financed by voluntary contributions as well as by distortionary income tax[12].

3. The Model

Consider an economy consisting of $n \geq 2$ identical individuals,[13] who derive utility from a certain public good that is financed by voluntary contributions. We will divide our discussion into two cases: first, we will introduce a benchmark case where contributions to a capacity build-up of a public good are continuously accumulated from time $t = 0$ until time $t = T$. After time $t = T$, the public good yields an infinite stream of utilities that increase with capacity. In the next subsection, we will introduce the second case where the finance of the public good is divided into two stages: in the first stage, whose length T is exogenously given, contributions are continuously accumulated to build up capacity for the public good; in the second stage, whose length is infinite, contributions are continuously raised to finance the ongoing operation of the public good.[14] We start in subsection 3.1.1 by analyzing the optimal path and in subsection 3.1.2, we analyze the Markov-Perfect-Equilibrium.

3.1. Case I: The perpetual utility

A group of $n \geq 2$ individuals contributes to a public good. The time span for collecting contributions is of length *T*. At time t $\left(t \in [0, T]\right)$ each individual contributes $x(t)$ to the capacity of the public good, henceforth time *T* the public good yields a perpetual stream of utilities, $Zk(T)$ where $k(T)$ is the capacity of the public good and Z is some positive constant.

11 For further discussion of this result, see for example, Bergstrom, Blume, and Varian (1986) and Bernheim (1986).

12 See also Bruner and Falkinger (1999).

13 Kessing (2007) showed that no asymmetric MPE in linear strategies exists, therefore we consider only the symmetric case.

14 Unlike Admati and Perry (1991), there is no minimum capacity requirement to operate the public good, that is, by the end of the first stage, the public good is completed and ready for operation.

3.1.1. The Optimal Path

For analytical tractability[15], we assume a quadratic cost function $C\left(x(t)\right)=\frac{b}{2}x^2(t)$. The benevolent central planner therefore faces the following optimization problem:

$$\begin{aligned}
&Max\ \left\{-\int_0^T \frac{b}{2}x^2(t)e^{-\rho t}dt+\frac{Z}{\rho e^{\rho T}}k(T)\right\}\\
&s.t.\\
&\quad \dot{k}=nx(t)\\
&\quad x(t)\geq 0\\
&\quad k(0)=0\\
&\quad \lambda(T)=\frac{Z}{\rho e^{\rho T}}
\end{aligned} \tag{1}$$

Proposition 1:

The optimal contribution path is:

$$x_1^*(t)=\frac{Zn}{b\rho}e^{\rho(t-T)} \tag{2}$$

The optimal capacity level, $k_1^*(T)$ *is:*

$$k_1^*(T)=\frac{Zn^2\left(1-e^{-\rho T}\right)}{b\rho^2} \tag{3}$$

Proof: See Appendix.

3.1.2. The Markov-Perfect-Equilibrium

The game spans from $t=0$ to $t=T$, each player i chooses at time t $\left(t\in(0,T)\right)$ his contribution to the capacity build-up of the public good, based on the sum of all contributions made up to that time, $\phi_i\left(k(t)\right)$. Henceforth time $t=T$ each individuals receives a perpetual stream of utilities $Zk(T)$ where $k(T)$ is the capacity that had been accumulated until time T and Z is some positive constant. An individual i ‘s, maximization problem is:

15 See Starr and Ho (1969), Fershtman and Nitzan (1991), Kessing (2007).

$$
\begin{aligned}
&\underset{\phi_i}{Max}\, J^i = -\int_0^T e^{-\rho t}\frac{b\phi_i^2}{2}dt + \frac{Zk(T)}{\rho e^{\rho T}} \\
&s.t. \\
&\quad \dot{k} = \phi_i + \sum_{j\neq i}\phi_j \\
&\quad k(0) = 0 \\
&\quad \phi_i \geq 0 \\
&\quad \phi_{j\neq i} \ \ is\ given\ \forall j
\end{aligned}
\tag{4}
$$

Solving (4) yields the following proposition:

Proposition 2:

The symmetric Markov-Perfect equilibrium contribution is:

$$
\phi_1(t) = \frac{\beta_1 e^{\frac{n\gamma_1 t}{b}}}{b} \tag{5}
$$

Thus,

$$
\hat{k}_1(T) = \frac{\beta_1}{\gamma_1}\left(e^{\frac{n\gamma_1 T}{b}} - 1\right) \tag{6}
$$

Where,

$$
\begin{aligned}
\alpha_1 &= \frac{2Z^2\left(1 - e^{\frac{n\rho T}{2n-1}}\right)^2 e^{\frac{4n\rho T}{1-2n}}(2n-1)}{\rho^3 b}, \\
\beta_1 &= \frac{2Z\left(e^{\frac{n\rho T}{2n-1}} - 1\right)e^{\frac{2n\rho T}{1-2n}}}{\rho} \\
\gamma_1 &= \frac{\rho b}{2n-1}
\end{aligned}
\tag{7}
$$

Proof: See Appendix.

3.1.3. Global Optimum and Over-Provision of capacity

We first find the optimal time limit required to achieve maximal social welfare. By inserting (2) and (3) into the objective function and maximizing over T, the following proposition is easily obtained:

Proposition 3:

The optimal time limit for the capacity build-up is

$$T^{*}=\frac{\ln(2)}{\rho}, \tag{8}$$

the optimal capacity level is

$$k_1^{*}\left(T^{*}\right)=\frac{n^2 Z}{2\rho^2 b} \tag{9}$$

and the maximal attainable utility level is

$$u_1^{o}=\frac{Z^2 n^2}{8\rho^3 b} \tag{10}$$

Notice that T^{*} is independent of the number of individuals, n. Define the function $D_1 \equiv k_1^{*}\left(T^{*}\right)-\hat{k}_1\left(T\right)$ as the difference function between the optimal and the MPE levels of $k\left(T\right)$. Subtracting (9) from (6) evaluated at T^{*} yields:

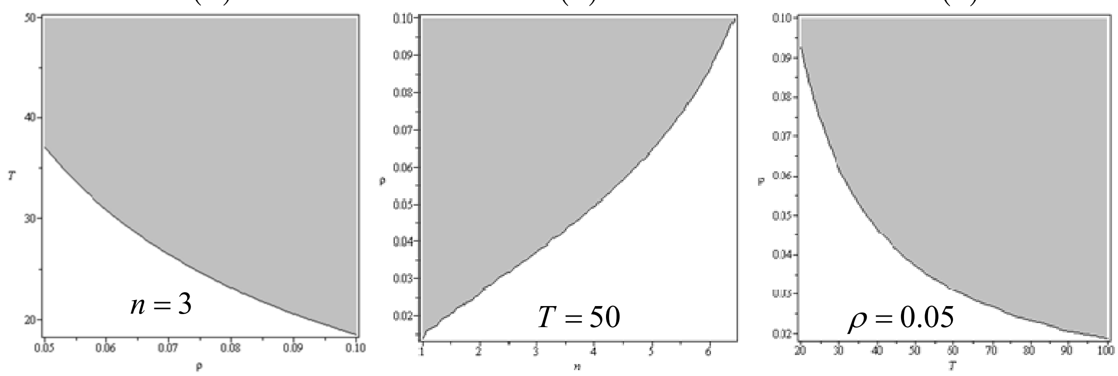
$$D_1 \equiv k_1^{*}\left(T^{*}\right)-\hat{k}_1\left(T\right)=\frac{n^2 Z}{2\rho^2 b}-\frac{\beta_1}{\gamma_1}\left(e^{\frac{n\gamma_1 T}{b}}-1\right) \tag{11}$$

where β and γ are defined by (7). Note that it is easily verified that the sign of D_1 is independent of b and Z, but depends on T,ρ and n. Figure 1 presents three simulations of D_1, calibrated by $a=b=c=Z=1$. The solid lines in the three panels of Figure 1 are the loci of two variables (given the third) satisfying $D_1=0$. The shaded area shows combinations of these variables satisfying $D_1<0$, which means over-provision of capacity. Apparently, although the effects of T,ρ and n on D_1 are ambiguous, over-contribution to the capacity of the public good is most likely to occur when:

1) The subjective discount factor, ρ, is sufficiently high: a decline in the importance of future benefits results in lower contributions to the capacity build-up at the optimum and at the MPE, but also to a decline in the optimal time limit for collecting contributions to the capacity build-up.
2) The group of contributors, n, is relatively small, that is, when the free riding effect is sufficiently small.
3) The time limit for collecting contributions to the capacity build-up is sufficiently long.

Figure 1

(a) (b) (c)



3.2. Case II: The Capacity Build-Up and Ongoing Operating Case

In this section, we add a second stage to the game presented in Section 3.1. Assume that after the capacity was built in the first stage of the game, the public good must be operated and therefore operating costs must be collected at each instant in order to benefit from the public good.

3.2.1. The Optimal Path

The optimal path is calculated using backwards induction; therefore, we first derive the optimal contribution of an agent in the second stage of the game:

The second stage:

Individuals derive utility at time t from the operating level of the public good, $G(t)$. The utility is defined by $v(G(t)) = f(k(T))G(t)$ where $k(T)$ is the capacity of the public good that was determined in the first stage whose length is T. We assume that $f(\cdot)$ is monotonically increasing and $v(G(t))$ is increasing in $f(k(T))$. The operating costs of the public good at time t are defined by $C(G(t))$. For the sake of tractability and solvability, we assume that the individual's convex cost functions are quadratic.[16] More specifically, we assume the following instantaneous functions:

$$v(G(t)) = a\sqrt{k(T)}G(t), \quad C(G(t)) = \frac{c}{2}\left(\frac{G(t)}{n}\right)^2$$

Where a and c are positive parameters.

16 See footnote 15.

The net utility at time t $\left(t \in \left(T, \infty\right)\right)$ of a representative individual in the second stage of the game is defined by

$$u\left(G(t)\right) = v\left(G(t)\right) - C\left(G(t)\right) = a\sqrt{k(T)}G(t) - \frac{c}{2}\left(\frac{G(t)}{n}\right)^2 \tag{12}$$

Note that we assume that contributions to the public good must be collected at each instant and the benefit from the public good is not carried to the next instant.

Differentiating (12) with respect to $G(t)$ yields the first order condition:

$$\frac{\partial u\left(G(t)\right)}{\partial G(t)} = a\sqrt{k(T)} - \frac{cG(t)}{n^2} = 0 \tag{13}$$

The optimal contribution to the ongoing operation of the public good at time t is

$$G^* = \frac{an^2\sqrt{k(T)}}{c} \tag{14}$$

From equation (14) we see that the optimal contribution is time invariant. By inserting equation (14) into equation (12) we obtain the discounted to $t = 0$ sum of net utilities of the representative individual:

$$u\left(G^*\right) = \frac{(an)^2}{2c\rho e^{\rho T}} k(T) \equiv \frac{A}{\rho e^{\rho T}} k(T) \tag{15}$$

where $A \equiv \frac{(an)^2}{2c}$ and ρ is the subjective discount rate.

The first stage:

The length of the first stage is T, at time t $\left(t \in [0, T]\right)$ each individual contributes $x(t)$ to the capacity of the public good. Again, for analytical tractability, we assume a quadratic cost function $C\left(x(t)\right) = \frac{b}{2}x^2(t)$. The benevolent central planner maximizes the utility of the representative individual:

$$\begin{aligned}
&Max\ \left\{-\int_0^T \frac{b}{2}x^2(t)e^{-\rho t}dt + Ak(T)\right\} \\
&s.t. \\
&\quad \dot{k} = nx(t) \\
&\quad x(t) \geq 0 \\
&\quad k(0) = 0 \\
&\quad \lambda(T) = \frac{a^2n^2}{2cre^{\rho T}}
\end{aligned} \tag{16}$$

Solving (16) yields the following proposition:

Proposition 4:

The optimal contribution path is:

$$x_2^*(t) = \frac{a^2n^3}{2bc\rho}e^{\rho(t-T)} \tag{17}$$

The optimal capacity level, $k_2^*(T)$ *is:*

$$k_2^*(T) = \frac{a^2n^4\left(1-e^{-\rho T}\right)}{2bc\rho^2} \tag{18}$$

Proof: See Appendix.

3.2.1. The Markov-Perfect-Equilibrium (MPE)

The MPE is also calculated using backwards induction. Suppose that a capacity of $k(T)$ was accumulated in the first stage. At the second stage of the game, each individual i chooses at time t $\left(t \in (\mathrm{T},\infty)\right)$ the optimal contribution path to the ongoing operation of the public good, $g_i(t)$ to maximize his net utility:

$$\underset{g_i(t)}{Max}\ u^i\left(g_i(t)\right) = a\sqrt{k(T)}\left(g_i(t) + \sum_{j\neq i} g_j(t)\right) - \frac{cg_i^2(t)}{2} \tag{19}$$

Differentiating (19) with respect to g_i yields the first order condition:

$$\frac{\partial u^i\left(g_i(t)\right)}{\partial g_i(t)} = a\sqrt{k(T)} - cg_i(t) = 0 \tag{20}$$

Since all individuals are assumed to be identical and the individual's problem is time invariant, we obtain from (20) that, at the Nash equilibrium, the individual's contribution, $\hat{g}$, and the total contributions to the public good $\hat{G}$ at time t are:

$$\hat{g} = \frac{a\sqrt{k(T)}}{c} \Rightarrow \hat{G} = \frac{an\sqrt{k(T)}}{c} \tag{21}$$

Comparing (21) to (14) reveals that

$$G^* = n\hat{G} \tag{22}$$

implying that at the Nash equilibrium, total contributions to the ongoing operation of the public good are sub-optimal.

The discounted to $t = 0$ sum of net utilities of the representative individual at the Nash equilibrium is

$$\frac{a^2(2n-1)}{2\rho c e^{\rho T}} k(T) \equiv Bk(T) \tag{23}$$

where $B \equiv \frac{a^2(2n-1)}{2\rho c e^{\rho T}}$. It is easily verified that $A > B$ for $n \geq 2$.

The first stage:

In the first stage of the game that spans from $t = 0$ to $t = T$, each individual, i, chooses at time t his contribution to the capacity build-up for the public good, based on the sum of all contributions made up to that time, $\phi_i(k(t))$. The individual i 's, maximization problem in the first stage of the game is:

$$\begin{aligned}
&\underset{\phi_i}{Max}\, J^i = -\int_0^T e^{-\rho t} \frac{b\phi_i^2}{2} dt + Bk \\
&s.t. \\
&\quad \dot{k} = \phi_i + \sum_{j \neq i} \phi_j \\
&\quad k(0) = 0 \\
&\quad \phi_i \geq 0 \\
&\quad \phi_{j \neq i} \ \ is\ given\ \forall j
\end{aligned} \tag{24}$$

Solving (24) yields the following proposition:

Proposition 5:

The symmetric Markov-Perfect equilibrium contribution is:

$$\phi_2(t) = \frac{\beta_2 e^{\frac{n\gamma_2 t}{b}}}{b} \tag{25}$$

Thus,

$$\hat{k}_2(T) = \frac{\beta_2}{\gamma_2}\left(e^{\frac{n\gamma_2 T}{b}} - 1\right) \tag{26}$$

Where,

$$\begin{aligned} \alpha_2 &= \frac{a^4\left[\left(1-e^{\frac{n\rho T}{2n-1}}\right)(1-2n)\right]^2 e^{-\frac{4n\rho T}{2n-1}}(2n-1)}{2bc^2\rho^3}, \\ \beta_2 &= \frac{a^2\left[\left(e^{\frac{n\rho T}{2n-1}}-1\right)(2n-1)\right]e^{\frac{2n\rho T}{1-2n}}}{c\rho} \\ \gamma_2 &= \frac{\rho b}{2n-1} \end{aligned} \tag{27}$$

Proof: See Appendix.

3.2.2. Global Optimum and Over-Provision of capacity

To obtain the global optimum, we find the optimal time limit for the capacity build-up. By inserting equations (17) and (18) into the objective function and maximizing over T, we obtain the following proposition:

Proposition 6:

The optimal time limit is

$$T^* = \frac{\ln(2)}{\rho}, \tag{28}$$

the optimal capacity level is

$$k_2^*(T^*) = \frac{a^2 n^4}{4bc\rho^2} \tag{29}$$

and maximal attainable utility of the representative individual is thus

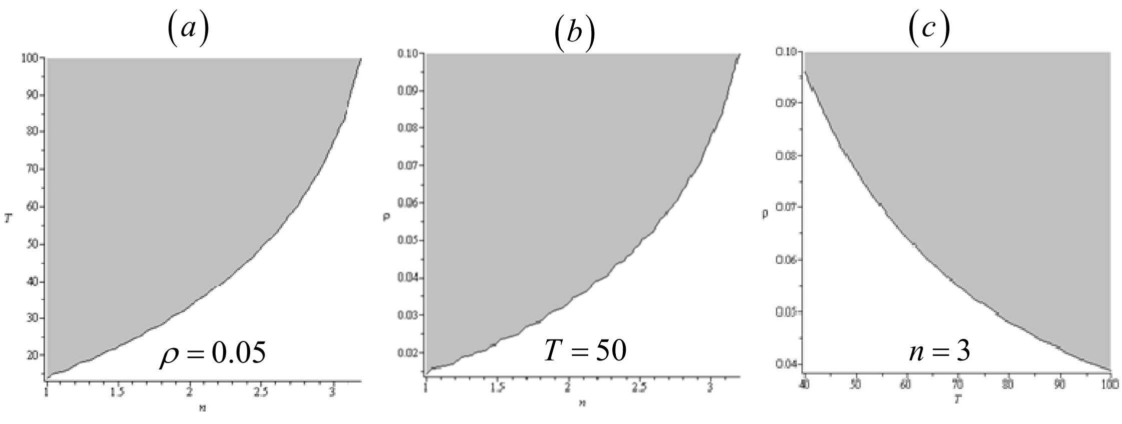
$$u_2^o = \frac{n^6 a^4}{32\rho^3 bc^2} \tag{30}$$

Define the function $D_2 \equiv k_2^*\left(T^*\right) - \hat{k}_2\left(T\right)$ as the difference function between the optimal and the MPE levels of $k\left(T\right)$. Subtracting (29) from (26) yields:

$$D_2 \equiv k_2^*\left(T^*\right) - \hat{k}_2\left(T\right) = \frac{a^2 n^4}{4bc\rho^2} - \frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}} - 1\right) \tag{31}$$

where β and γ are defined by (27). Again, it is easily verified that the sign of (31) is independent on the calibration of the constants, a, b and c. Figure 1 presents three simulations of D_2 assuming $a = b = c = 1$. The solid lines in the three panels of Figure 2 are the loci of two variables (given the third) satisfying $D_2 = 0$, and the shaded area shows combinations of these variables satisfying $D_2 < 0$, which means over-provision of capacity to the public good. The simulation shows that the effects of T, ρ and n on D_2 are ambiguous. As in the case I, over-provision of capacity to the public good is most likely to occur for a sufficiently high subjective discount factor, ρ, when the number of contributors, n, is relatively small, and when the time limit for collecting contributions to the capacity build-up is sufficiently long.

Figure 2



To compare the two cases of over-provision of capacity to the public good, we set Z to be equal to A, that is, we set the utility per unit of capacity in case I equal to the utility per unit of capacity at the optimum in case II. Since $\beta_1 > \beta_2$ we may deduce that when individuals must contribute to the ongoing operation of the public good, they will reduce the contribution to the capacity build-up.

4. Public Policy

Suppose that the policymaker's goal is to minimize the deviation from the maximal attainable utility caused due to the voluntary contribution.

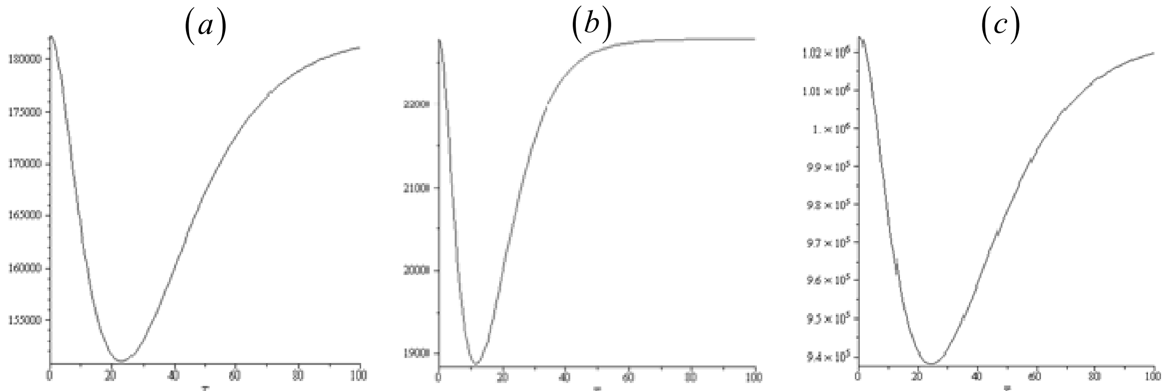
Almost all studies of public goods have considered two main policy tools: taxes and subsidies. It is easily verified that within the context of our model subsidizing the contributions to the public good can yield a socially optimal result; nevertheless, in reality, subsidies involve excess burden. In this model, a third policy tool is available to the regulator: setting a time limit T to the first stage of the game. The government or the regulator can limit the time horizon for accumulating k through legislation. As Figure 3 shows, by setting a time limit to the first stage of the game, the regulator can minimize the utility loss while avoiding the excess burden of taxation.

By inserting equations (25)-(27) into the objective function, we obtain the utility at the Nash equilibrium. The maximal attainable utility is given by equation (30). We define ΔW as the difference between the utility of the representative individual at the optimum and at the Nash equilibrium.

Figure 3 presents simulations for ΔW assuming $a = b = c = 1$ for various calibrations of n, T and ρ.

The simulations show that by allowing the contributions to the capacity of the public good to be collected during a longer than the optimal period, the regulator can minimize the utility loss that is an inevitable result of voluntary contributions.

Figure 3



$n = 3,\ \rho = 0.05$ $\quad$ $n = 3,\ \rho = 0.1$ $\quad$ $n = 4,\ \rho = 0.05$

5. Summary and Discussion

We constructed a two-stage game of voluntary contributions to a public good which requires both initial investment to build up a capacity for the public good and also variable costs to finance its ongoing operation. Contributions to finance the initial investment (capacity) are collected from time $t = 0$ to time $t = T$ at the first-stage, and contributions to finance variable costs are collected continuously in the infinitely lasting second stage.

We showed that the MPE second-stage provision of variable costs is unambiguously sub-optimal, and also that both suboptimal and over-contribution to the capacity of the public good are contingent. Over-provision is most likely to occur when the subjective discount factor is sufficiently high, when the group of contributors is relatively small and when the time limit of the first stage is sufficiently long. Moreover, we showed that in comparison to the case where the public good yields a perpetual utility stream, the contributions to the capacity are lower.

We also considered the use of several policy tools: taxes, subsidies and the setting of a time limit on the capacity build-up, to achieve a social optimum. Our results imply that in the case of voluntary provision of a public good that requires both an initial investment as well as ongoing operation costs, setting a time limit on the collection of contributions to the capacity build-up serves as a useful policy tool to minimize the utility loss that results from the voluntary contributions while avoiding the deadweight loss of taxation.

As far as we know, since the erection of the Tabernacle in the desert of Sinai[17], a direct experiment of the private provision of contributions to the capacity and ongoing operation of a public good has not yet been conducted.[18] Some supportive empirical indications were found in Baumol and Bowen (1968) who reported that almost all nonprofit performing art groups depend upon donations for a substantial fraction – commonly between one-third and one-half – of their income. [19] Hansmann (1981) quotes several explanations which were suggested for this financing pattern, but as he emphasized, the considerable costs of performing arts productions are essentially fixed costs, independent of audience size. On the other hand, marginal costs of performing arts are relatively low; once a performance has been staged, the costs of an additional performance or admitting an additional individual to a certain performance (provided that the theatre is not filled), are close to zero. As Hansmann indicates, the potential audience for high-culture live entertainment is limited even in large cities; consequently, for any given production, there are typically only a few performances over which to spread the fixed costs – often three or fewer for an orchestral program and only several times that for opera, ballet and many theatrical productions, implying that fixed costs represent a large fraction of total costs for each production. Therefore, as Hansman points out, the demand curve lies below the average cost curve at all points. Hansmann also points out that most of the audience that attends a performance are mostly the donors; nevertheless, the performing art groups found that it was easier to raise donations to finance the fixed costs (the very existence of the performing art group and the spectacular staging) rather than raise donations by charging higher tickets prices. Moreover, tickets are sold at a price that does not cover the variable costs of the performance.

Our model and simulations provide the rationale of this observance; with relatively small groups of donors, the MPE may be characterized by over-provision of fixed costs and under-provision of variable costs. This result explains why opera houses, for example, reside in

17 See footnote 7. Indeed, the group-size of contributors was very large, but so, probably, was ρ (see Exodus 32 1) and the perceived "second stage" utility.

18 Most experiments considered threshold point type public goods tested the effect of time on the contributions to threshold point type public goods. See, for example, Isaac et al. (1989), Suleiman and Rapoport (1992), Cadsby and Maynes (1999), Dorsey (1992), Coats, Gronberg, and Grosskopf (2009). For experiments which also tested the effect of various rebate rules on the contributions to threshold point type public goods, see for example Marks and Croson (1998), and also Falkinger (1996) and Falkinger, Fehr, Gächter, and Ebmer (2000).

19 McCarthy, Brooks, Lowel, and Zakaras (2001) indicated that individual contributions, rather than corporate donations, are the largest source of unearned revenues of these performing organizations, approximately twice the size of foundation grants and contributions from business.

magnificent buildings and why opera productions are characterized by expensive casts, large orchestras and luxurious costumes and sets. The same applies to the academic or religious sphere. Namely, it may be easier to raise donations for luxurious university campuses, research laboratories or worship houses than to cover their daily maintenance. In other words, according to our model and simulations, a high ratio between fixed and variable costs leads performing arts groups, academic institutions and other organizations to prefer the nonprofit form, which may cause over-provision of fixed costs (expressed in luxury productions or campuses) and under-provision of variable costs (reflected in relatively low ticket prices or sub-optimal operation level).

Our model also provides a rationale to the policy of Israeli municipalities towards allocation of land to charity and religious institutions. Many charity centers and worship houses in Israel are held by private associations or local communities. Municipalities donate land to these associations for the establishment of a building, conditional upon finishing the project within a specified time limit (between 3 and 5 years). If the community or the association fails to erect the building within the specified time limit, the land reverts back to the municipality. Probably the main rationale of the time limit is to prevent fraud extract of philanthropic donations; nevertheless, our model provides an additional rationale, namely using the time limit as a policy tool for enhancing efficiency without a deadweight loss.

Our difficulty in finding direct empirical support for our model emphasizes that the model presented in this paper provides fruitful ground for further theoretical and empirical research on a voluntary provision of more complex types of public goods.

Appendix

In this Appendix we prove propositions 1, 2, 4, 5 in the main text.

Proof of proposition 1:

Setting the Hamiltonian:

$$H(\cdot) = -\frac{b}{2}x(t)^2 e^{-\rho t} + \frac{Z}{re^{\rho T}}k(T) + \lambda(t)nx(t) \tag{32}$$

The first order conditions are:

$$H_x = -bx(t)e^{-\rho t} + \lambda(t)n = 0 \tag{33}$$

$$\lambda'(t) = -H_k = 0 \tag{34}$$

From (34) we obtain that $\lambda(t)$ is constant over time. From (33) we obtain that

$$\lambda(t) = \frac{x(t)}{n}be^{-\rho t} \tag{35}$$

From (34) and (35) we obtain that

$$\lambda'(t) = \left[\dot{x}(t) - \rho x(t)\right]e^{-\rho t} = 0 \tag{36}$$

By solving (36), we obtain that

$$x(t) = Ce^{\rho t} \tag{37}$$

Therefore we obtain that

$$k(T) = nC\int_0^T e^{\rho t} = \frac{nC\left(e^{\rho T} - 1\right)}{\rho} \tag{38}$$

From (35) and (37), and since that $\lambda(T) = \dfrac{Z}{\rho e^{\rho T}}$ we obtain that

$$\frac{Cb}{n} = \frac{Z}{\rho e^{\rho T}} \tag{39}$$

Therefore

$$C = \frac{Zn}{b\rho e^{\rho T}} \tag{40}$$

From (37) and (40) we obtain that,

$$x^*(t) = \frac{Zn}{b\rho}e^{\rho(t-T)} \tag{41}$$

From (38) and (40) we obtain that

$$k^*(T)=\frac{Zn^2\left(1-e^{-\rho T}\right)}{b\rho^2} \tag{42}$$

Proof of Proposition 2:

The Hemilton-Jacobi-Bellman (HJB) equation associated with (4) is:

$$-J_t^i=\max_{x_i}\left[-\frac{b}{2}x_i^2e^{-\rho t}+J_k^i\left[x_i+\sum_{j\neq i}x_j\right]\right], \tag{43}$$

where $x_i=\phi_i\left(k(t),t\right)$.

Assume that

$$J^i\left(k(t),t\right)=e^{-\rho t}\left(\alpha+\beta k(t)+\frac{\gamma}{2}k^2(t)\right) \tag{44}$$

The first order condition for (43) is

$$-b\phi_i\left(k(t),t\right)e^{-\rho t}+J_k^i=0 \tag{45}$$

From (45) using the fact that individuals are identical, we obtain that

$$\phi(\cdot)=\frac{J_k}{b}e^{\rho t}=\frac{\left(\beta+\gamma k(t)\right)}{b} \tag{46}$$

and therefore,

$$k'(t)=n\phi(\cdot)=\frac{n\left(\beta+\gamma k(t)\right)}{b} \tag{47}$$

By integrating (47) and using the boundary condition, we obtain,

$$k(t)=\frac{\beta}{\gamma}\left(e^{\frac{n\gamma t}{b}}-1\right) \tag{48}$$

Therefore at time T

$$\hat{k}(T)=\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right) \tag{49}$$

From (46) and (48), we obtain,

$$\phi(t)=\frac{\beta e^{\frac{n\gamma t}{b}}}{b} \tag{50}$$

Deriving the Coefficients

From (46) and by using the symmetry assumption, (43) can be written as:

$$\rho\left(\alpha+\beta k+\frac{\gamma}{2}k^{2}\right)e^{-\rho t}=\frac{J_{k}^{2}}{b}e^{\rho t}\left(n-\frac{1}{2}\right) \tag{51}$$

or

$$\rho\left(\alpha+\beta k+\frac{\gamma}{2}k^{2}\right)=\left[\frac{\beta^{2}\left(n-\frac{1}{2}\right)}{b}+\left(\frac{2\beta\gamma\left(n-\frac{1}{2}\right)}{b}\right)k+\frac{\gamma^{2}\left(n-\frac{1}{2}\right)}{b}k^{2}\right] \tag{52}$$

Therefore we obtain that

$$\frac{1}{2}\rho\gamma-\frac{\gamma^{2}\left(n-\frac{1}{2}\right)}{b}=0 \tag{53}$$

$$\rho\beta-\frac{\beta\gamma\left(2n-1\right)}{b}=0 \tag{54}$$

$$\rho\alpha-\frac{\beta^{2}\left(n-\frac{1}{2}\right)}{b}=0 \tag{55}$$

From (53) we obtain that either $\gamma=\frac{\rho b}{2n-1}$ or γ=0. If γ=0 then $\beta=0$ and $\alpha=0$, which correspond to a non-contributions equilibrium. Let us concentrate on the case in which $\gamma=\frac{\rho b}{2n-1}$. From the boundary condition, we obtain that

$$\begin{aligned}J\left(k\left(T\right),T\right)&=e^{-\rho T}\left\{\alpha+\frac{\beta^{2}}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)+\frac{\gamma}{2}\left[\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)\right]^{2}\right\}\\&=\frac{Z}{e^{\rho T}\rho}\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)\end{aligned} \tag{56}$$

From (55), we obtain that

$$\alpha=\frac{Z}{\rho}\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)-\frac{\beta^{2}}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)-\frac{\gamma}{2}\left[\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)\right]^{2} \tag{57}$$

By inserting (56) into (55), we obtain that

$$\beta = \frac{2Z\left(1-e^{\frac{n\rho T}{2n-1}}\right)e^{\frac{2n\rho T}{1-2n}}}{\rho} \tag{58}$$

and

$$\alpha = \frac{2Z^2\left(1-e^{\frac{n\rho T}{2n-1}}\right)^2 e^{\frac{4n\rho T}{1-2n}}\left(2n-1\right)}{\rho^3 b} \tag{59}$$

Proof of proposition 4:

The proof in analogous to the proof of proposition 1 with the difference that the Hamiltonian is

$$H\left(\cdot\right) = -\frac{b}{2}x\left(t\right)^2 e^{-\rho t} + \frac{\left(an\right)^2}{2c\rho e^{\rho T}}k\left(T\right) + \lambda\left(t\right)nx\left(t\right), \tag{60}$$

and the boundary condition is $\lambda\left(T\right) = \frac{a^2 n^2}{2c\rho e^{\rho T}}$.

Proof of Proposition 5:

The proof in analogous to the proof of proposition 2 with the difference that equation (56) becomes

$$\begin{aligned} J\left(k\left(T\right),T\right) &= e^{-\rho T}\left\{\alpha + \frac{\beta^2}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right) + \frac{\gamma}{2}\left[\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right)\right]^2\right\} \\ &= \frac{a^2\left(2n-1\right)}{2\rho c e^{\rho T}}\frac{\beta}{\gamma}\left(e^{\frac{n\gamma T}{b}}-1\right) \end{aligned} \tag{61}$$

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