

GREEK VERSUS TALMUDIC FAIRNESS AND ECONOMIC APPROACHES TOWARDS
PRIORITY-LESS BANKRUPTCIES: AN INTERDISCIPLINARY ANALYSIS

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Version: November 29, 2015

Bankruptcy problems are commonly associated with economic disasters, but can also emerge due to extraordinary economic performance. The choice of a sharing rule has a significant potential effect on the economy's general equilibrium. The economic literature hitherto neglected the search for the economically optimal bankruptcy solution and concentrated mainly in normative axiomatizations of sharing rules, but its findings did not attract much attention of legal scholars. The purpose of this article is to create a symposium between the economic and legal literatures on bankruptcy based on our interdisciplinary analysis of a fascinating polemic conducted by Jewish Law scholars over the course of 15 centuries about the appropriate bankruptcy solution.

JEL classification: K35

Keywords: Bankruptcy, Priority-less bankruptcy, sharing rules, fairness approach, economic approach.

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This article was written while we both were visiting scholars at the Berkeley Institute for Jewish Law and Israeli Law, Economy and Society, University of California, Berkeley.

Acknowledgments: We are gratefully indebted to Robert J. Aumann, Ronen Avraham, Shachar Kariv and Hervé J. Moulin for very helpful discussions and comments.

I. Introduction

Consider a contingent joint venture project. With probability p the project succeeds and produces profits, and with probability $(1-p)$ it fails and yields losses. Defining *bankruptcy* as excess of claims over available assets implies that if the project's production function exhibits increasing returns to scale, partners may face a bankruptcy problem even in case of outstanding success. For example, suppose the each partner demands his marginal contribution to the project's output in case of success, and his investment back in case of a failure¹. It is well known that in case of increasing returns to scale the sum of marginal contributions exceeds output (a numerical example is given in section III), therefore while increasing returns to scale is a good reason to engage in a joint venture project, partners of such joint ventures are expected to face a bankruptcy problem regardless of the project's performance.

A bankruptcy solution (or a *sharing rule*) is a distribution scheme that allocates all available assets such that no claimant is awarded more than his claim². There are plenty of distribution schemes that satisfy these conditions (see section II) which their normative characteristics were extensively studied in the economic literature. The contemporary legal distribution scheme for both profits and losses is the proportional *pro-rata* sharing rule, but suppose that partners are allowed to choose their favorite sharing rule for the project's output ex-ante. That is, they may allocate the project's output *pro-rata* if they wish, but they may adopt an egalitarian approach, apply the Shapley value formula or choose any other sharing rule. Suppose that all joint ventures in the economy differ only by their applied sharing

¹ We do not argue that these claims are legally valid or even "just". This is merely a behavioral *self-serving bias* assumption ("success has many fathers, failure is an orphan"). Therefore, in case of success partners claim at least their marginal contribution but in case of success they tend to blame others and claim back at least their investment.

² The formal definitions of a bankruptcy problem and its solution are given below in section II.

rule and consider the following three hypothetical cases:

- a. You are a potential investor. In what joint venture would you prefer to invest your money?
- b. You are the (hired) manager of the project. What sharing rule would you recommend to the partners (your employers)?
- c. You are a legislator. What sharing rule (if any) would you support to be imposed by law?

Naturally, the answer to all questions depends on the responder's perceptions of fairness³, but fairness is unlikely to be the sole parameter. The answer to question *a* would probably be influenced also by responder's attitude towards risk, evaluation of each sharing rule properties (see section IV), the potential effect on responder's expected payoff, the negotiability of the responder's share, his rank among other investors and so on. The answer to question *b* would probably be influenced by the responder's attitude towards risk and considerations regarding the sharing rules' characteristics and potential effect on investors' behavior, the competitiveness of the project in capital markets, responder's salary and so on. The answer to question *c* would probably be influenced from the responder's favorite economic paradigm as well as political considerations regarding the interests of the responder's constituency, the effect of his vote on his chances for being reelected and so on. It follows that it is quite natural to expect a variety of answers from different responders, and consequently intensive lobbying activity by various pressure groups seeking to promote legislation which adopts their favorite sharing rule.

The striking fact is that all western legislations adopted the Aristotelian fairness approach and ruled that profits and losses be distributed *pro-rata*. We

³ The evaluation of a sharing rule's fairness relates to the individual's ranking of its normative characteristics (see section IV). The interrelation between individuals' perceptions of fairness and their self-interests was mentioned in the Bible (Exodus 23, 8), pointed out by the Sages (see for example bKethuboth 105a) and demonstrated empirically by Kahneman, Knetsch and Thaler (1990).

are unaware of a lobbying activity in any western country in attempt to adopt a different approach. Moreover, the academic treatment of priority-less bankruptcy is odd. On the one hand, the economic literature concentrated mainly in normative axiomatizations of sharing rules⁴ and finding bijections between sharing rules and game theoretic solution concepts⁵ or between game theoretic solution concepts and maximization of certain social welfare functions⁶, neglecting classical economic issues like finding the *optimal* sharing rule – namely, the sharing rule that maximizes a certain target function (e.g. creditors’ ex-post aggregate value, ex-ante investments etc.)⁷. On the other hand, while legal scholars have wondered about this lacuna in the economic literature⁸, the fruits of economic analyses of bankruptcy solutions gained little echo in the legal literature. Modern legislations apply proportional *pro-rata* division scheme as standard solution for priority-less bankruptcies, but we are unaware of a real debate in the academic legal literature about justifications for choosing this specific division scheme, except for Jackson (1986). Comparative legal analyses of alternative schemes are absent in modern legal literature which repeatedly states that the *pro-rata* scheme meets the basic requirement of *equal treatment of equals*⁹, ignoring the fact that other division schemes also treat

⁴ Originally, the axiomatic normative approach was developed and applied for social welfare functions evaluation (Sahota, 1978), (Yaari, 1981), but the methodology was later applied for normative axiomatic analysis of priority-less bankruptcy solutions. See for example Nash (1950), Shapley (1953), Schmeidler (1969), Schummer and Thomson (1997), Moulin (2000), (2002), and Chun, Schummer and Thomson (2001). For a survey see Thomson, (2003) and Moulin (2004).

⁵ See for example Littlechild & Owen (1973), O'Neill (1982), Curiel, Maschler and Tijs (1987), O'Neill (1982),

⁶ See for example Keane (1969), Legros (1987), Dagan and Volij (1993), Serrano (1995), Thomson (2003), Dutta and Ray (1989) and Arin (2007).

⁷ For pioneering *economic* analyses of sharing rules see Karagözoğlu (2008), Ashlagi et. Al. (2012) and Kıbrıs and Kıbrıs (2013). These studies refer only to problems of loss allocation and ignore the problem of profit allocation. For pioneering optimal design of sharing rules see Steinhaus (1948), and Moulin (1984).

⁸ See for example Rasmussen (1994) and Jackson (1986).

⁹ Jackson (1986) p. 30 quotes a long-standing slogan of bankruptcy law: “The most common and uncontroversial of bankruptcy’s policies, the pro rata treatment of general unsecured creditors.... It is justified by a legal homily such as ‘bankruptcy courts are court of equity and equality is equity’”. See also *Canright v. General Finance Corp.*, 35 F. Supp. 841, 844 (E.D. Ill. 1940).

equals equally while satisfying additional desired normative requirements.

Surprisingly, this lacuna was filled during the last 15 centuries by generations Jewish Law scholars who conducted an amazingly rich, fascinating, profound and stormy polemic comparative discussion on several alternative sharing rules (defined and explained in section II). Those ancient discussions triggered the extensive modern economic study of bankruptcy solutions¹⁰, but missed modern legal literature.

The survival of the *pro-rata* scheme through millennia apparently indicates its superiority over all alternative sharing rules. This impression is, however, misleading. The history of science is abundant with theories and beliefs that prevailed over eons and were finally refuted. Moreover, during the last two millennia there is no record of a real competition among various sharing rules in western thought which adopted Aristotle's vague argumentation in favor of *pro-rata* distribution, although definitely not undisputable (see section IIB). Critical normative comparison of sharing rules became possible in the fall of the 20th century due to extensive economic study which proved that this is not an esoteric issue.

We believe that the fruits of three decades of economic analyses of sharing rules deserve interdisciplinary attention. Therefore, our purpose in this article is not to recommend any alternative sharing rule but to challenge the rigidity of thought expressed by the undisputed reign of the *pro-rata* scheme in legal academic literature, build a bridge between the economic and legal literatures and promote symposium and mutual interdisciplinary fertilization in the various aspects of academic research of priority-less bankruptcies. As a platform, we present an interdisciplinary analysis of the Jewish Law paradigms towards priority-less bankruptcies, based on Rasmussen (1994) discussion of *fairness* and the *economic* approaches.

¹⁰ The seminal papers of O'Neill (1982) and Aumann and Maschler (1985) which founded the fertile branch of economic literature on bankruptcy, were inspired by the 12th century Rabbi Abraham Ibn-Ezra quoted by Rabinovitch (1973), and a puzzling Mishnah, respectively.

Our analysis demonstrates the importance of the interdisciplinary symposium. First, we show that the picture that emerges from the legal literature is narrow, partial and does not reflect the richness of available sharing rules whose normative characteristics were extensively studied in economic literature. In particular, we show that the polemic among Jewish Law scholars stems from the unique characteristics of priority-less bankruptcy, and the differences among them regarding the appropriate sharing rule reflect the controversy between the *fairness* and the *economic* approaches towards bankruptcies categorized by Rasmussen (1994) (discussed in section IV). Contrary to the prevailing consensus in modern legal literature, all Jewish Law scholars perceived the proportional *pro-rata* sharing rule as incompatible with the fairness approach. We argue that this position, and the differences between proponents of economic approach among these scholars worth modern scholarly attention.

Second, our analysis highlights that mathematical duality does not imply legal equivalence. Jewish Law scholars recognized that the relevant social target function in primal bankruptcy problem may differ from its counterpart in the dual problem, implying that the legal treatment of primal and dual bankruptcy problem is not supposed to be necessarily the same, but rather guided by the relevant social target function. An intriguing application of this insight is demonstrated by the consensus among medieval Jewish Law scholars that in absence of social consensus regarding the appropriate social welfare function, there is no universal “fair” tax function. We argue that this position also worth modern interdisciplinary discussion.

Third, as shown in section III, bankruptcy problems are not confined to economic failures but may arise even due to outstanding business successes. The choice of output sharing rule may have enormous affect on the economy general equilibrium. Therefore, we argue, the superiority of the *pro-rata* over alternative allocation schemes implicitly assumed by contemporary legislations should be reconsidered, and further economic

research is required in search for the *economically optimal* sharing rule.

Ancient and modern legislations classified creditors into *priority classes*. We view this classification as a sort of trick aimed at the escape from coping with the more challenging problem of *priority-less* bankruptcies. Namely, bankruptcies in which no creditor has legal priority over another creditor and all claims are equally valid. Our attention in this article is confined to this type bankruptcy problem because the analysis of priority-less bankruptcies sharing rules reveals the implicit justice perceptions and/or economic paradigms of the legislator. Moreover, the intra-distribution of available asset among members of a certain priority class is actually a priority-less bankruptcy problem, implying that this problem can not be really avoided. But above all, we believe that technical local tricks like priority classifications can not obviate serious academic discussion of the root problem.

Modern business environments' sophistication strengthens our feeling that the interdisciplinary symposium on priority-less bankruptcy, which we seek to promote by this article, is important and on time. We believe that although the above three-point list is apparently not exhaustive, it is sufficient to establish that our analysis of Jewish Law approach towards priority-less bankruptcy is a good starting point for this interdisciplinary symposium, and hope that it will trigger further research of alternative sharing rules which may better fit sophisticated modern economies.

The article proceeds as follows. Section II sets out the conceptual framework for economic analyses of priority-less bankruptcies, presents 6 solutions recommended by Jewish Law scholars and their most important normative characteristics. Section III briefly demonstrates the potential economic impact of a sharing rule choice on the economy. Section IV presents Rasmussen's (1994) distinction between the fairness and economic approaches and section V classifies Jewish Law decisors accordingly. Section VI presents the controversy among medieval decisors regarding the interpretation of the economic approach. Section VII discusses the Jewish Law treatment of the

dual bankruptcy problem. Section VIII summarizes and concludes.

II. Economic Analysis of Priority-Less Bankruptcy

A. Mathematical Framework

A *priority-less bankruptcy problem* is a triple $B = \langle N, w, \mathbf{c} \rangle$ where $N \in \mathbb{N}$ denotes the set of creditors (henceforth assumed fixed), w is the available sum, $\mathbf{c} = \{c_i\}_{i=1}^n$ is the creditors' claims vector where $\sum_{i \in N} c_i > w$. The set of all bankruptcy problems is denoted by $\mathbf{B}$. A bankruptcy *solution* or *sharing rule* is a mapping $f : \mathbf{B} \rightarrow \mathbb{R}^n$ satisfying the *definitive constraints*:

$$a. \quad 0 \leq f_i(B) \leq c_i, \quad \forall i \in N.$$

$$b. \quad \sum_{i \in N} f_i(B) = w.$$

A *dual bankruptcy problem* is a triple $B^* = \langle N, \theta, \mathbf{b} \rangle$ where $N \in \mathbb{N}$ denotes the set of agents, θ is a sum to be raised from N , and $\mathbf{b} = \{b_i\}_{i \in N}$ is a vector of agents' maximum liabilities where $\sum_{i \in N} b_i > \theta$. A dual bankruptcy problem solution or *tax function* is a mapping $\tau : \mathbf{B}^* \rightarrow \mathbb{R}^n$ satisfying:

$$a^*. \quad 0 \leq \tau_i(B^*) \leq b_i, \quad \forall i \in N$$

$$b^*. \quad \sum_{i \in N} \tau_i(B^*) = \theta$$

Denote the collection of all bankruptcy (and dual bankruptcy) solutions by $\mathbf{F}$ ¹¹. The mathematical bankruptcy model is very broad and encompasses numerous real world situations like social welfare problems, fair taxation, allocation of profits among share holders or damage compensations among multiple tortfeasors and other situations which may differ substantially by their legal and judicial characteristics or informational structure.

¹¹ Dual bankruptcy problems should be carefully distinguished from dual sharing rules. $f^*(B)$ is the dual of $f(B)$ if $f^*(B) = \mathbf{c} - f(w - \sum_{i=1}^n c_i, \mathbf{c})$, $\forall B \in \mathbb{B}$. That is, $f^*(B)$ allocates awards the same way $f(B)$ allocates losses. A sharing rule satisfies *self-duality* if $f(B) = f^*(B)$.

The *Major Litigation Rule* of civil Jewish Law postulates that “*the onus of proof is on the claimant*”¹². This rule is irrelevant, of course, if the validity of all claims is commonly acknowledged or if the court is intrinsically unable to reach an evidence based verdict. In both cases the bankruptcy problem is not about proving a creditor’s claim, but about the appropriate allocation scheme of available but insufficient assets among claimants. When all claims are certainly valid, partition is a first best solution and the question relates to *what* partition. When some claims’ validity is doubtful, partition is at most a second best solution¹³.

B. Halakhic Sharing Rules

Numerous sharing rules satisfy the *definitive constraints*, implying that the mathematical definition of a bankruptcy solution is too broad. Characterization of a unique sharing rule requires additional normative restrictions. As mentioned above, the economic literature produced normative axiomatizations for numerous bankruptcy solutions¹⁴. This article is confined to sharing rules suggested by Jewish Law scholars.

The Basic 2-Agent Rule (BR): For $B = \langle 2, w, \{c_1, c_2\} \rangle$ define creditor i ’s *minimal share* $m_i = \max(w - c_j, 0)$, as the portion of w conceded for creditor i by creditor j . The *disputed portion* of w is $d = w - (m_1 + m_2)$.

$$(1) \quad BR_i = m_i + \frac{1}{2}d, \quad i, j = 1, 2.$$

Constrained Equal Award (CEA): For all $B \in \mathbf{B}^N$ and all $i \in N$

$$(2) \quad CEA_i(B) = \min(c_i, \lambda) \text{ such that } \sum_{i \in N} \min(c_i, \lambda) = w.$$

Constrained Equal Loss (CEL): For all $B \in \mathbf{B}^N$ and all $i \in N$

¹² mBaba-Batra Ch. 9 §6, mBechorot Ch. 2 §6, 7 and 8, mBaba-Kama Ch. 3 §11, bBaba-Kama 46a and more. Actually, the fundamental rule is controversial in the Talmud. See bBaba-Kama 35a, 46a, bBaba-Metzia 3a and more.

¹³ In case of a severe risk for false division, the preferred verdict would be deposition of the disputed assets under the custody of the court (mBaba-Metzia Ch. 3 §4). Tosfot Baba-Metzia 2a (starting at *veyachloku*) ruled that partition is applied only in full symmetric information bankruptcies with significant positive probability for true result, while cases with partial and asymmetric information require mechanism design.

¹⁴ See footnote 4 above.

$$(3) \quad CEL_i(B) = \max(0, c_i - \mu) \text{ such that } \sum_{i \in N} \max(0, c_i - \mu) = w$$

Proportional Allocation (P): For all $B \in \mathbf{B}^N$

$$(4) \quad P_i(B) = s_i c_i \text{ where } s_i = c_i / \sum_{i=1}^n c_i$$

The Talmud Rule (T)¹⁵: For all $B \in \mathbf{B}^N$ and all $i \in N$,

$$(5) \quad T_i(B) = \begin{cases} CEA_i(N, w, \frac{1}{2}\mathbf{c}) & w < \frac{1}{2} \sum_{i \in N} c_i \\ P_i(N, w, \frac{1}{2}\mathbf{c}) & w = \frac{1}{2} \sum_{i \in N} c_i \\ CEL_i(N, w, \frac{1}{2}\mathbf{c}) & w > \frac{1}{2} \sum_{i \in N} c_i. \end{cases}$$

Recursive Incremental Allocation (RI): For all $B \in \mathbf{B}^N$

$$(6) \quad RI_i(B) = RI_{i-1} + \sum_{j=1}^i \frac{c_j - c_{j-1}}{n-j+1}, \text{ where } RI_0 = 0 \text{ and } RI_1 = \frac{1}{n} c_1.$$

III. The Impact of the Sharing Rule Choice

The economic study of bankruptcy initiated with the seminal paper of O’neill (1982), who analyzed a problem introduced by R. Abraham Ibn-Ezra (1089-1164)¹⁶:

Jacob died and his son, Reuben, produced a deed dully witnessed that Jacob willed to him his entire estate on his death. Son Simeon also produced a deed that his father willed to him half of the estate. Levi produced a deed giving him one third and Judah brought forth a deed giving one quarter. All of them bear the same date.

In Ibn-Ezra’s numerical example, $B = \langle 4, 120, \{30, 40, 60, 120\} \rangle$. Table 1 presents the heirs’ shares according to the above defined sharing rules.

TABLE 1: SOLUTIONS TO IBN-EZRA’S BANKRUPTCY PROBLEM $\langle 4, 120, \{30, 20, 60, 120\} \rangle$
Sharing Rule

	CEA	CEL	P	T	RI
30	30	0	$14\frac{22}{55}$	15	$7\frac{1}{2}$
40	30	$6\frac{2}{3}$	$18\frac{3}{5}$	20	$10\frac{5}{6}$
60	30	$26\frac{2}{3}$	$28\frac{4}{5}$	30	$20\frac{5}{6}$
120	30	$86\frac{2}{3}$	$57\frac{3}{5}$	55	$80\frac{5}{6}$

¹⁵ This is the prevailing name of the rule in the economic literature, although according to Aumann and Maschler’s (1985) interpretation it originally appeared in the Mishnah.

¹⁶ *Sefer Ha-Mispar (Book of the Number)*, Kaufmann Verlag, Frankfurt a. M. with German translation by M. Silberberg, 1895 p. 57. The translation here is quoted from Rabinovitch (1973).

Table 1 demonstrates that the choice of a sharing rule has a significant impact on each heir's share and on the distribution of the bequeathed wealth among the heirs. In Ibn-Ezra's problem the sharing rule is chosen and imposed by the court; the heirs are passive and unable to affect their payoffs. In a business environment, on the other hand, investors may respond to legislation that imposes a specific sharing rule, implying that imposing a sharing rule by law may affect general equilibrium. To see this let us return to the n -partner joint venture project that opened the introduction. Denote partner i 's investment by x_i and define $X = \sum_{i=1}^n x_i$. Recall that with probability p the project succeeds and produces $F(X) > X$ and with probability $(1-p)$ it fails and produces $G(X) < X$. Define $m_i = F(X) - F(X - x_i)$ as partner i 's marginal contribution to $F(X)$. By our assumptions,

$$(7) \quad c_i = \begin{cases} m_i & \text{if the project succeeds} \\ x_i & \text{if the project fails.} \end{cases}$$

As mentioned above, $\sum_{i=1}^n x_i > G(X)$, and if $F(X)$ exhibits increasing returns to scale $\sum_{i \in N} m_i > F(X)$. For illustration let $n=3$, $\mathbf{x} = \{1, 2, 3\}$, $F(X) = X^2$ and $G(X) = \frac{1}{2}X$, implying $\sum_{i=1}^3 m_i = 58 > F(6) = 36$, and $\sum_{i=1}^3 x_i = 6 > G(6) = 3$. Table 2 and Table 3 present Partners' shares of $F(X)$ and $G(X)$, respectively, according to the various sharing rules.

TABLE 2: ALLOCATION OF $F(6) = 36$

x_i	c_i	CEA	CEL	P	T	RI
1	11	11	3.67	6.83	5.5	6
2	20	12	12.67	12.41	10	12
3	27	13	19.67	16.76	20.5	18

TABLE 3: ALLOCATION OF $G(6) = 3$

x_i	c_i	CEA	CEL	P	T	RI
1	1	1	0	0.5	0.5	0.33
2	2	1	1	1	1	0.83
3	3	1	2	1.5	1.5	1.83

The expected value of the project is,

$$(8) \quad E[v(X)] = pF(X) + (1-p)G(X),$$

and the expected sum of claims is,

$$(9) \quad E\left[\sum_{i=1}^n c_i\right] = p \sum_{i=1}^n m_i + (1-p) \sum_{i=1}^n x_i.$$

Assuming $p=0.8$ implies $E[v(6)] = 29.4$ with standard deviation $\sigma[v(X)] = 13.2$ and $E\left[\sum_{i=1}^3 c_i\right] = 47.6$ with standard deviation $\sigma\left[\sum_{i=1}^3 c_i\right] = 20.8$. Table 4 presents partners' expected payoffs (the numbers in parenthesis are standard deviations). The corresponding plots are depicted in Figure 1.

TABLE 4: ALLOCATION OF EXPECTED VALUE $E[v(6)] = 24.4$

x_i	$E(c_i)$	CEA	CEL	P	T	RI
1	9 (4.00)	9.00 (4.00)	2.93 (1.47)	5.56 (2.53)	4.5 (2.00)	4.87 (2.27)
2	16.4 (7.20)	9.80 (4.40)	10.33 (4.67)	10.13 (4.57)	8.2 (3.60)	9.77 (4.47)
3	22.2 (9.60)	10.60 (4.80)	16.13 (7.07)	13.71 (6.10)	16.7 (7.60)	14.77 (6.47)

FIGURE 1: EXPECTED PAYOFFS

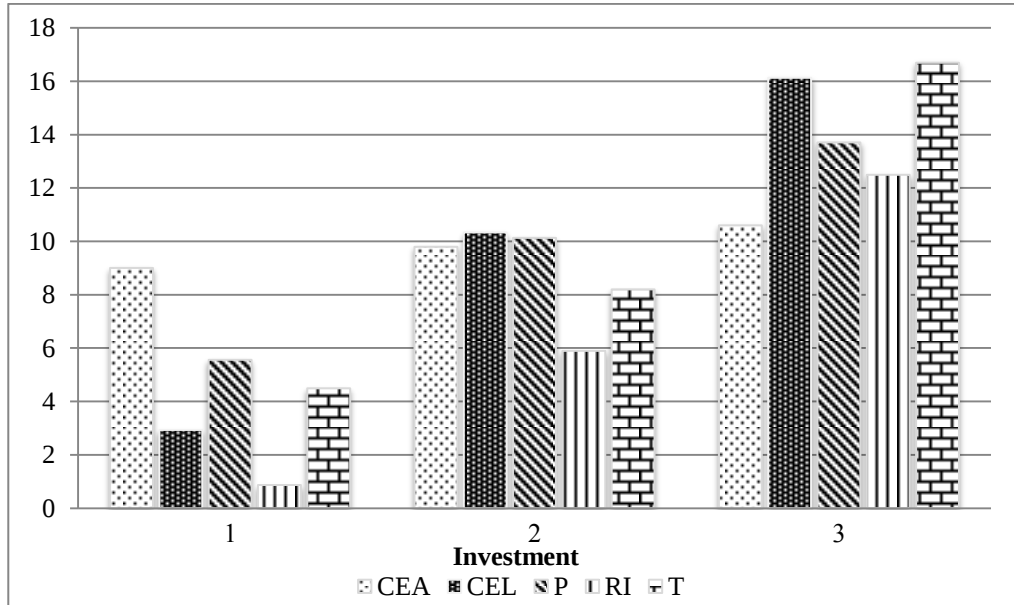


Table 5 presents the fitted regression equations of payoffs on investments, based on the data of Table 4 (assuming $v_i(X_{-i}, 0) = 0$). Figure 2 plots the curves corresponding to these fitted regression equations.

TABLE 5: REGRESSION EQUATIONS FITTED TO TABLE 4

Sharing Rule	Fitted Regression Equation	R^2
CEA	$E_i^{CEA}[v_i(X_{-i}, x_i)] = 8.9609x_i^{0.1461}$	0.9841
CEL	$E_i^{CEL}[v(X_{-i}, x_i)] = 0.7965x_i^2 + 3.0947x_i$	0.9779
P	$E_i^P[v(X_{-i}, x_i)] = 0.4966x_i^2 + 6.0586x_i$	1
T	$E_i^T[v(X_{-i}, x_i)] = 0.9263x_i^2 + 2.689x_i$	0.9789
RI	$E_i^{RI}[v(X_{-i}, x_i)] = 0.0325x_i^2 + 4.8237x_i$	1

FIGURE 2: REGRESSION LINES FITTED TO TABLE 4

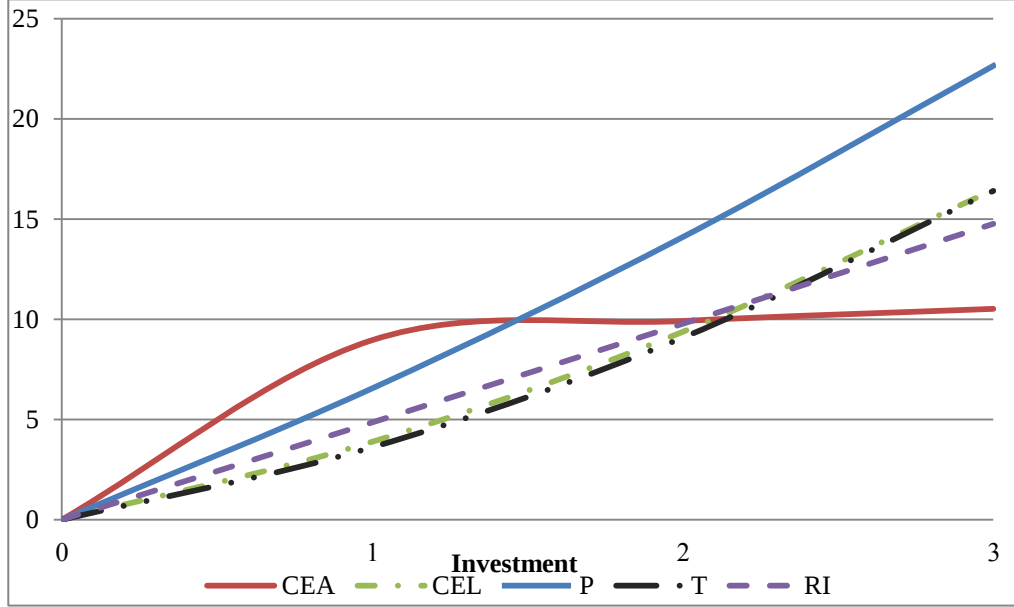


Table 6 and Figure 2 show that while under all sharing rules $E_i[v_i(X_{-i}, 0)]$ is monotonically increasing in x_i , the fitted curves differ significantly. The most exceptional is the CEA schedule which exhibits diminishing incremental increase, implying that the CEA is probably the most egalitarian while the P rule is the most differential.

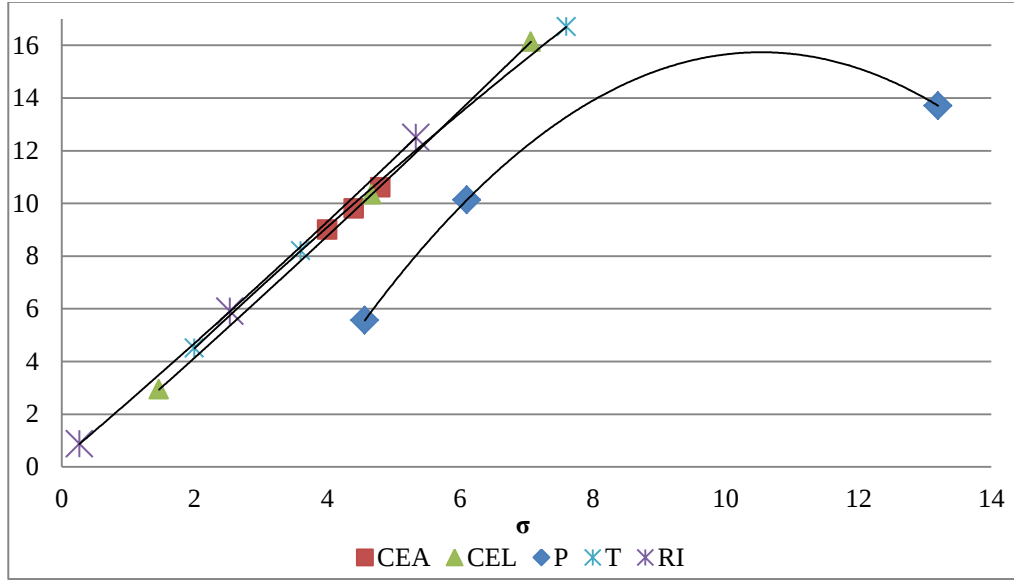
The real surprise, however, appears by regressing expected payoffs on risk, measured by the standard deviation, which under our assumptions is $\sigma_i[v_i(X)] = \sqrt{p(1-p)(\beta X - X^\alpha)^2}$. Table 6 presents the fitted regression

equations of $E[v_i(X)]$ on $\sigma_i[v_i(X)]$ (and their R^2 values) and Figure 3 presents their corresponding scatter diagrams and fitted regression lines.

TABLE 6: FITTED REGRESSION EQUATIONS OF $E[v_i(X)]$ ON $\sigma_i[v_i(X)]$.

Sharing Rule	Fitted Regression Equation	R^2
CEA	$E_i^{CEA}[v_i(X_{-i}, x_i)] = 2\sigma_i + 1$	1
CEL	$E_i^{CEL}[v_i(X_{-i}, x_i)] = 0.0186\sigma_i^2 + 2.1984\sigma_i + 0.331$	1
P	$E_i^P[v_i(X_{-i}, x_i)] = -0.2857\sigma_i^2 + 6.0191\sigma_i - 15.963$	1
RI	$E_i^{RI}[v_i(X_{-i}, x_i)] = 0.0649\sigma_i^2 + 1.79\sigma_i + 0.4756$	1
T	$E_i^T[v_i(X_{-i}, x_i)] = -0.0335\sigma_i^2 + 2.5\sigma_i - 0.3661$	1

FIGURE 3: RETURN TO RISK



The exceptional schedules in Table 6 and Figure 3 are T and P, which exhibit diminishing marginal compensation for risk (becoming even negative beyond a certain critical value of σ). Moreover, while the maximum point of T curve is *beyond* the data range, the P schedule lies *entirely* far beneath all others¹⁷. In other words, the prevailing *pro-rata* rule awards investors with the lowest compensation for risk and its marginal compensation is diminishing fastest.

This stylized example visualizes that the answers to questions *a*, *b* and *c*

¹⁷ It can be easily verified from Table 6 that $\partial E_i^T(v)/\partial \sigma = 0$ at $\sigma = 37.3$ and $\partial E_i^P(v)/\partial \sigma = 0$ at $\sigma = 10.5$.

of the introduction are nontrivial, and further research is required to determine the optimal allocation rule from each position's point of view. At the same time, this example justifies reconsideration of the superiority of *pro-rata* division over alternative allocation schemes in contemporary legislations.

IV. Legal Treatment of Bankruptcy

A. Normative Characterization of Sharing Rules

In the absence of a bankruptcy statute *might makes right*¹⁸; creditors race on the debtor's assets and grab as much as they could, subject to their claim ceiling (Jackson, 1986). In such a Hobbesian anarchy (or *jungle*), creditors are classified according to their relative power. The most powerful creditor's claim may be fully satisfied while weaker creditors' claims are satisfied partially or even not at all¹⁹. Formally, suppose that $\mathbf{c}$ is ordered according to creditors' *power measure* (e.g. quickness or strength), and consider Piccione and Rubinstein (2007) *jungle sharing rule* $J(B)$,

$$(9) \quad J_i(B) = \max\left(w - \sum_{j=1}^{i-1} c_j, c_i\right).$$

The allocation process according to (9) is as follows. First, creditor 1, (the most powerful), collects $\max(c_1, w)$. Then, creditor 2 (the most powerful among $N \setminus \{1\}$) collects $\max(c_2, w - c_1)$, and so forth. Notice that $J(B)$ satisfies the *definitive constraints*. Nevertheless, the $J(B)$ solution's fairness is questionable²⁰ and its efficiency even more doubtful as it deters weak potential investors and thus decreases economic growth. Civilized legal systems have replaced the *jungle power classification* of creditors by *priority classification* based on a variety of normative criteria²¹, and enacted

¹⁸ According to bBaba-Batra 34b.

¹⁹ For analyses of *jungle economy* equilibria see for example Bush & Mayer (1974), McGuire and Olson (1996), Olson, (2000), Piccione and Rubinstein (2007), Olson & Olson (2009) and Schwarz (2013).

²⁰ For a discussion of the jungle equilibrium solution fairness, see for example Schwarz (2013).

²¹ e.g 11 U.S.C § 507. Certain payments to employees are preferred over compensation for personal injury. For

bankruptcy laws in order to prevent jungle solutions and achieve fairer allocations. The distribution process, however, is basically the same. High-priority creditors are compensated first, and low priority creditors share the remainder which naturally does not meet their claims. The normative criteria for fair classification of creditors into priority classes are beyond the scope of this article. As mentioned in the introduction, our discussion is confined to the more challenging problem of *priority-less* bankruptcies. It turns out that a *fair* and efficient sharing rule should have some additional normative characteristics. For example:

Equal treatment of Equals: Creditors with equal claims should receive equal shares. Formally, $c_i = c_j \Rightarrow f_i = f_j \quad \forall B, \forall i, j \in N$.

Scale invariance: The sharing rule should be insensitive to measure units. Formally, the sharing rule should be homogenous of degree 1 in B . Namely, $f(\lambda w, \lambda c) = \lambda f(w, c) \quad \forall B \in \mathbf{B}^N, \forall \lambda > 0$.

Consistency: The sharing rule is invariant to application order on subsets of creditors. Formally, let $S \subset N$ and consider the problem $\hat{B} = \langle N \setminus S, w_S, c_S \rangle$ where $w_S = \sum_{i \in N \setminus S} f_i(B)$ and $c_S = \{c_i\}_{i \in S}$. A sharing rule f is consistent if $f(B) = f(\hat{B})$, $\forall S \subseteq N, \forall B \in \mathbf{B}$ and $\forall i \in S$.

Composition: The sharing rule is invariant to piecemeal allocation of w . Formally, suppose that $w = w_1 + w_2$. A sharing rule $f(B)$ satisfies composition if $f(w, c) = f(w_1, c) + f[w_2, c - f(w_1, c)] \quad \forall B \in \mathbf{B}^N$.

Path independence: The sharing rule is invariant to any adjustments of w . Formally, define $w = w_1 \pm w_2$ and $\bar{c} = c - f(N, w_1, c)$. A sharing rule

creditors' classifications to priority-classes according to Jewish Law see for example *Shulchan Aruch Choshen Mishpat Ch. 97 §24*.

$f(B)$ is path independent if $f(N, w, c) = f(N, w_1, c) + f(N, w_2, \bar{c})$ ²².

Coalitional Manipulation Robustness: For a given set of agents, N , and any subset (coalition) $S \subseteq N$, $f(N^c, w^c, c^c)$ is invariant to any partition of S where $N^c = N \setminus S$ is the complement of N .

It should be emphasized that this list is far from exhausting all sharing rules' characteristics studied in the economic literature²³, and no sharing rule satisfies them all²⁴. It follows that the choice of a sharing rule is equivalent to selection of a set of normative axioms, while relinquishing others.

B. The Fairness Approach

Western fairness approach is associated with the writings of Aristotle who postulated that “both the unjust man and the unjust act are unfair or unequal” thus “it is clear that there is also an intermediate between the two unequals involved in either case”, implying that “[i]f, then, the unjust is unequal, just is equal, as all men suppose it to be, even apart from argument”. Aristotle argued that “equality implies at least two things. The just, then, must be both intermediate and equal and relative (i.e. for certain persons)”, as he explained:

The just, therefore, involves at least four terms; for the persons for whom it is in fact just are two, and the things in which it is manifested, the objects distributed, are two. And the same equality will exist between the persons and between the things concerned; for as the latter the things concerned-are related, so are the former; if they are not equal, they will not have what is equal, but this is the origin of quarrels and complaints-when either equals have and are awarded unequal shares, or unequals equal shares. Further, this is plain from the fact that awards should be ‘according to merit’²⁵.

This led Aristotle to conclude:

The just, then, is a species of the proportionate (proportion being

²² Path independence is the dual property of composition (Herrero & Villar, 2001).

²³ For surveys see for example Herrero and Villar (2001), Moulin (2002) and Dominguez and Thomson (2006).

²⁴ Apparently, $J(B)$ fails to satisfy any additional normative axiom.

²⁵ Aristotle, *Nicomachean Ethics* Book V Ch. 3, (1999).

not a property only of the kind of number which consists of abstract units, but of number in general).

That is, according to the Aristotelian fairness approach, “the just” is *pro-rata* distribution, in proportion to “merit”. Aristotle recognized that,

for all men agree that what is just in distribution must be according to merit in some sense, though they do not all specify the same sort of merit, but democrats identify it with the status of freeman, supporters of oligarchy with wealth (or with noble birth), and supporters of aristocracy with excellence.

It follows that if power or quickness are “merit in some sense” the jungle sharing rule is fair enough. Nevertheless, Aristotle argues that,

For the justice which distributes common possessions is always in accordance with the kind of proportion mentioned above (for in the case also in which the distribution is made from the common funds of a partnership it will be according to the same ratio which the funds put into the business by the partners bear to one another); and the injustice opposed to this kind of justice is that which violates the proportion²⁶.

That is, although people have various perceptions regarding *merit*, the *relevant* merit, according to Aristotle, is the *ratio which the funds put into business*. Apparently, the Aristotelian fairness approach rejects the jungle sharing rule because it classifies creditors to priority-classes according to irrelevant criteria (power, quickness, seniority etc.), while a civilized bankruptcy solution must comply with principles of *justice* and *fairness*. Aristotle’s arguments may be relevant for partnerships and joint ventures, but is irrelevant to other bankruptcy situations in which available assets are unrelated to creditors’ investments, as in a distributive justice problem of natural common resources nicely described in a famous Midrash²⁷:

The Lord trieth the righteous (Psalm 21, 5). By what does He try him? By tending flocks. He tried David trough sheep and found him to be a good shepherd, as it is said: He chose David also His servant and took him from the sheepfolds (Ib. 78 70). Why ‘from the sheepfolds’, when the word is the same as ‘and

²⁶ Aristotle, *ibid* Ch. 4.

²⁷ Exodus Rabbah, Ch. II, 2. Translated by Rabbi Dr. S. M. Lehrman, *The Soncino Press*, London, 1961.

the rain ... was restrained’?²⁸ (Gen 8, 2). Because he used to stop the bigger sheep from going out before the smaller ones, and bring smaller ones out first, so they should graze upon the tender grass, and afterwards he allowed the old sheep to feed from the ordinary grass, and lastly, he brought forth the young, lusty sheep to eat the tougher grass. Whereupon God said: ‘He who knows how to look after sheep, bestowing upon each the care it deserves, shall come and tend my people’, as it says, *From following the ewes that give suck He brought him, to be shepherd over Jacob His People (Psalms 78, 71)*.

David agreed with Aristotle that the jungle sharing rule is unfair, but holds that since the grass was created by the Creator *the ratio which funds put into business* is an irrelevant factor for allocation of common natural resources. Instead of proportional allocation, David applied a *reverse jungle rule* in order to classify his flock to priority classes fairly.

The traditional fairness approach views bankruptcy as a regular litigation, an adversary dispute between creditors, characterized by *judicial rivalry*. That is, claims and mutual denial of claims on the one hand, and concessions on the other hand (Rasmussen, 1994)²⁹. A fair sharing rule is a verdict between contradicting creditors’ rights, and the fairness of a sharing rule is evaluated according to its normative characteristics.

C. The Economic Approach

The *economic approach* towards priority-less bankruptcies views the fairness approach as parochial, and consequently inefficient and socially detrimental. In the above cited Midrash’s flock example, the real problem with the jungle sharing rule is its inefficiency due to the *collective action* problem, namely the conflict between self and social interests. David’s arrangement is fairer because it is *more efficient*; it ensures optimal nutrition for all sheep and maximizes the aggregate value of the entire flock. The economic approach seeks for sharing rules that neutralize collective action problems and

²⁸ An unusual word derived from a root נָחַ meaning ‘to be confined’. (Translator’s note).

²⁹ By claiming c_i , agent i concedes $w - c_i$ to $N \setminus \{i\}$. Mutual denial can relate either to *factual* or *judicial* claims.

incentivize all agents to act in harmony as a unified entity for the sake of collective welfare (Jackson, 1986).

The economic approach distinguishes bankruptcies from litigations which are mainly about evidence. Verdicts in regular litigations are based on testimonies and evidence, and then *let the law pierce the mountain*³⁰. In bankruptcy, on the other hand, there is no *judicial* rivalry between the creditors, because all claims' validity is commonly acknowledged and there is no mutual denial of claims. The rivalry among creditors is *economic* and stems from the scarcity of resources. It follows that priority-less bankruptcy is not litigation but a *social welfare problem* in which the task of the central social planner (the court) is not to decide between conflicting factual or judicial claims, but to maximize *social welfare*. Philosophical differences relate to the appropriate set of social norms and consequently to the derived specification of the social welfare function.

To summarize: the economic approach holds that a sharing rule is a solution of a social welfare problem (namely, a *social choice function*). The choice of the appropriate sharing rule, $f \in F$, is a *mechanism design* aimed at neutralization of creditors' collective action problem and incentivizing them to act in harmony in order to maximize their aggregate value.

V. The Sages' Approaches towards Bankruptcy

A. The Tanaitic Controversy³¹

The famous Mishnah³² states:

If a man who was married to three wives died, and the *kethubah*

³⁰ bSanhedrin 6b. Literally, justice should be done at all costs.

³¹ *Tannaim* (lit. *repeaters* or *teachers*) is an Alias General of Jewish Law Sages in the first two centuries CE. Their rulings and opinions were recorded in the Mishnah, which is the Magnum Opus codex of Jewish Oral Law, and in supplementary codices like the *Tosefta* and many *Baraitas*. *Amoraim*, (lit. *speakers* or *interpreters*) is the Alias General of Jewish Law Sages who lived in 200-500 CE and their discussions and interpretations of the Tanaitic sources are recorded in the Jerusalem and the Babylonian *Talmudim*. Jewish Law scholars who lived in 500-600 CE are known as *Savoraim*, followed by *Geonim* (600-1000 CE) and *Rishonim* (1000-1500 CE). Jewish Law scholars who lived since 1500 CE up to nowadays are known as *Acharonim*.

³² Kethuboth 10 §4.

of one was a mane³³, of the other two hundred *zuz* and of the third three hundred *zuz*, and the estate [was worth] only one *mane*, [the sum] is divided equally. If the estate [was worth] two hundred *zuz* [the claimant of the mane receives fifty *zuz* [and the claimants respectively] of the two hundred and the three hundred *zuz* [receive each] three gold dinars³⁴. If the estate [was worth] three hundred *zuz* [the claimant] of the mane receives fifty *zuz* and [the claimant] of the two hundred *zuz* [receives] a mane while [the claimant] of the three hundred *zuz* [receives] six gold dinars³⁵.

This Mishnah is summarized in Table 7.

TABLE 7: THE MISHNAH SOLUTIONS FOR $\mathbf{B} = \langle 3, \mathbf{w}, \{100, 200, 300\} \rangle$

Estate ($\mathbf{w}$)	Claims ($\mathbf{c}$)		
	100	200	300
100	$33\frac{1}{3}$	$33\frac{1}{3}$	$33\frac{1}{3}$
200	50	75	75
300	50	100	150

This Mishnah is puzzling. The three rows of Table 7 seem contradictory. The first row seems compatible with the CEA rule, the second row is vague and the third row seems as proportional allocation. This puzzling Mishnah challenged rabbinical authorities throughout generations. For example, the Babylonian and Jerusalem Talmuds suggested that the three rows of Table 7 relate to three legally different cases³⁶, but these explanations are unsatisfactory because there is not a single clue in the Mishnah that it discusses three legally different cases³⁷. After some feeble attempts to explain the Mishnah, the Babylonian Talmud finally concludes³⁸:

It was taught: This is the teaching of R. Nathan. Rabbe, however, said, ‘I do not approve of R. Nathan’s views in these [cases] for the division should be equal’.

³³ A *mane* is 100 *zuz*.

³⁴ A gold dinar worth 25 silver dinars or *zuz*, implying that each claimant takes 75 *zuz*.

³⁵ Namely, 150 *zuz* (see footnote 34).

³⁶ See bKethuboth 93a and further, and jKethuboth 10,4.

³⁷ R. Menachem HaMeiri (1249-1315) argued that usually when Amoraim suggest this kind of explanations they do not necessarily mean to revise the Mishnah text, but to reject its ruling. See Meiri, *Seder Hakabala, Machon Ofek edition* (1993) p. 103. See also Tosfot Berachot 15b starting at *Dilma*, Tosfot Shabat 37a starting at *Leolam*. R. Israel of Shklov, (*Peat Hashulchan*) quotes a similar argument by R. Eliyahu of Vilnius (the Gra).

³⁸ bKethuboth 93a.

Based on this quotation, mainstream Jewish Law decisors have rejected the Mishnah and ruled according to Rabbe. Nevertheless, this puzzling Mishnah challenged generations of Talmudic scholars who tried to decipher its mysterious sharing rule³⁹. These attempts reached a dead-end, as reported by R. Itzhak Alfasi (Rif, 1013-113)⁴⁰:

This Mishnah and its related Talmudic topic were extensively studied by our late predecessors, who have reached to a dead-end. Recognizing their failure, they started from scratch and ruled proportional allocation, based on Rabbe's saying 'I do not approve of R. Nathan's views in these [cases], for the division should be equal'⁴¹.

The striking fact is that a few lines later Rif himself reports that R. Hai Gaon (939-1038) have already pointed out that the Mishnah sharing rule is related to the BR-principle⁴². But this path of thinking was heavily criticized by later authorities and abandoned. The critics⁴³ pointed out numerous Talmudic topics which unambiguously prove that basically Rabbe accepts the BR-principle⁴⁴, implying that the Mishnah is inevitably based on a different principle. The critics also pointed out that the BR-principle was introduced by the Mishnah as a regular *indecisive litigation* solution⁴⁵ (defined below), emphasized the judicial differences between bankruptcy and regular litigations and argued that it is unlikely that the BR-principle underlies both Mishnaic rulings. None of the critics could provide a

³⁹ These scholars include R. Seadia Gaon (882 – 942, quoted in Responsa *Sha'arei Zedeq* part 4 gate 4, see also Rema di Pano *Responsa*, Ch. 128), R. Hai Gaon (939 – 1038, Quoted by the Rif, Kethuboth 93a), Piniles Z. M (1806-1870, *Darka Shel Tora* Buchdruckerei von Bendiner, Vienna 1863 p. 64), R. Y. L. Diskin (1817- 1898, *Torat Ha'Ohel*, vol. 1 page 2b) and more.

⁴⁰ Rif on Kethuboth 93a (folio 51a in the prevailing Vilnius edition).

⁴¹ For another example of rejection of distinguished authority incomprehensible ruling see bBaba-Batra 107b. See also Lipschits and Schwarz (forthcoming).

⁴² Other medieval scholars reached this conclusion independently. See R. Yehonathan and a commentary attributed to *mahadura kama* (earlier edition) of Rashi (both quoted in *Shita Mekubetzet* to Kethuboth 93a), *Mordechai* and *Pnei Yehoshua*, (op. cit). It should be noted that *mahadura kama* of Rashi is not necessarily Rashi's, but one of the *Magentza commentaries* (see footnote 90).

⁴³ Like Rif, R. Yeshai'a D'trani and others. See *Shita Mekubetzet*, Kethuboth 93a.

⁴⁴ See below footnote 53.

⁴⁵ The Contested Garment case, see mBaba-Metzia Ch. 1 §1.

satisfactory explanation to Table 7, but this fact is, of course, unsurprising since medieval scholars were unfamiliar with Aumann and Maschler (1985). Nevertheless, the fact that some medieval scholars linked the BR-principle with the Mishnah proves that this intuition was not unperceived in ancient epoch. A possible explanation for generations of scholars' perplexity regarding this Mishnah is that they were influenced by the Talmudic attempts to reconcile this Mishnah with Rabbe's opinion⁴⁶. Although the Babylonian Talmud abandoned these attempts explicitly, its classical commentators remained subordinated to Rabbe's doctrine⁴⁷, and consequently missed the core of the Tanaitic controversy. It took almost two millennia until Aumann and Maschler (1985) proved the following theorem.

Theorem 1 (Aumann & Maschler, 1985):

The Talmud rule is the unique BR-consistent n -creditor bankruptcy solution.

Aumann and Maschler explained the legal rationale of (5) based on the Halakhic Majority Principle: *the majority is treated like the whole (rubo k'khulo)*⁴⁸. From Halakhic point of view if $w < \frac{1}{2} \sum_{i \in N} c_i$, then w is considered as equal zero and the “nonexistent” available sum is divided according to the CEA rule. If $w > \frac{1}{2} \sum_{i \in N} c_i$, then from Halakhic point of view the aggregate loss is “nonexistent” and divided according to the CEL rule. When $w = \frac{1}{2} \sum_{i \in N} c_i$, both rules coincide with the Proportional rule⁴⁹.

⁴⁶ See bKethuboth 93a and jKethuboth 10,4. Interestingly, although those medieval scholars could not know Theorem 1, they refrained from claiming that there exists another CG-consistent solution, but rather that the Kethuboth Mishnah is based on a different sharing rule which occasionally seems CG-consistent.

⁴⁷ Rif quoted rumors that R. Hai Gaon recanted in his later years. In a meeting with us on August 10th, 2014 Aumann conjectured that the reason was that R. Hai could not prove his brilliant intuition. According to our thesis, it might be that R. Hai Gaon accepted Rabbe's doctrine. Another possibility is that R. Hai Gaon recantation relates to ruling but not to the interpretation of the Mishnah.

⁴⁸ Hulin 27a. Another formulation of this principle states that “*the minority is nonexistent*” (Tosfot, Baba-Kama 27b starting at *Ka Mashma Lan*).

⁴⁹ Aumann and Maschler (1985) indicated that (5) is related to “Piniles Rule”: For all $B \in \mathbb{B}$ and all $i \in N$,

We assert that it is unlikely that Rabbe, R. Nathan's disciple⁵⁰, did not understand his mentor's ruling, and even less likely that as the editor of the Mishnah he would include a mysterious incomprehensible ruling⁵¹. we argue that their controversy reflects fundamental differences regarding bankruptcy. Unfortunately, neither the Talmud nor any of its classical commentators have provided any explanation for this Tanaitic controversy⁵². Nevertheless, a careful examination of all topics pointed out by critics of R. Hai Gaon to prove the consensus around the BR-principle⁵³ reveals all these cases share the following three common characteristics:

- a. Claims and mutual denial of claims.
- b. The disputed object is not under the possession of any litigant⁵⁴.
- c. No party submitted sufficient evidence.

These characteristics define *indecisive litigations*, namely litigations in which evidence based verdict is unattainable. According to Rabbe, the Kethuboth Mishnah's case is different because bankruptcy is not a regular

$$PIN_i(B) = \begin{cases} \frac{1}{2}c_i + CEA_i\left(w - \frac{1}{2}\sum_{i \in N} e_i, \frac{1}{2}\mathbf{c}\right), & \sum_{i \in N} c_i \leq 2w \\ CEA_i\left(w, \frac{1}{2}\mathbf{c}\right), & \sum_{i \in N} c_i \geq 2w \end{cases}$$

Piniles (1806-1870) suggested his rule in an attempt to solve the Kethuboth Mishnah puzzle. Piniles rule coincides with the Talmud rule when $\sum_{i \in N} c_i \leq 2w$. See Piniles Z. M. *Darka Shel Tora* (The way of Torah), Buchdruckerei von Bendiner, Viena 1863 p. 64.

⁵⁰ jKethuboth 4, 11.

⁵¹ Nevertheless, inclusion of rulings against his own opinion in the Mishnah is not exceptional. See for example bShavuot 5a and footnote 41.

⁵² It is really astonishing that the first attempt to explain the Tanaitic controversy, suggested by Aumann in a series of article published in Talmudic studies journals, refrained from using Theorem 1. In a nutshell, Aumann suggested that both Tanaim apply a legal paradigm which he termed *shiabud* (lien). According to the *shiabud* paradigm, each creditor takes his pledge share, and the remainder is equally divided between all creditors. That is, the CG-principle is applied not on *disputed portion* but on *equally pledged portion* of the available asset. The Tanaitic controversy, according to Aumann, relates to the application of the *shiabud* paradigm in certain circumstances. But this explanation is unacceptable. First, the claim that the controversy is technical, about the application of a consensual paradigm in certain circumstances (for instance, coalition formation), is unconvincing in light Rabbe's unequivocal language "*I do not approve of R. Nathan's views in these [cases]*". Second, the *shiabud* paradigm is based on "coloring of assets" and creates artificial priorities with no clear legal basis (See Rashi Kethuboth 93a starting at *Ein Ani Roe*). Moreover, this paradigm seems contradictory to R. Nathan's famous and consensually accepted *lien principle* (bKethuboth 82a).

⁵³ For example: The Contested Garment (mBaba-Metzia Ch. 1 §1); The Collapsed House (mBaba-Metzia Ch. 10 §3); (bBaba-Metzia 117b, see there Rashi starting at *kama mafsid* and R. Shmuel Shtrashon (Rashash) on bBaba-Metzia 7b); The Oxen and the Pit (bBaba-Kama 53a), The Doubtful Son and the Sons of the Levir (bYebamoth 338a); The Contested bill (bBaba-Metzia 7b, Tosefta Baba-Metzia Ch. 1 §15). For a description of these cases see Aumann (2010). See also the Two Husbands, Wife and a Package (bKidushin 65b and Ramban's commentary there), which is the only case (except the Kethuboth Mishnah) which involves three litigants.

⁵⁴ Therefore, the *major litigation rule* is inapplicable.

indecisive litigation. Among all topics pointed out by critics of R. Hai Gaon, this case is the only for which Rasmussen (1994) distinction is relevant.

The consensus over the BR-principle as indecisive litigation solution implies that all Sages viewed this principle as a unique application of the *fairness* approach. It follows that the application of the Talmud rule by R. Nathan in the Kethuboth case stems from a fairness approach and reveals his view of priority-less bankruptcy as a regular indecisive litigation. By the same token, Rabbe's *equal division* ruling stems from an economic approach and reveals his view of bankruptcy as a social welfare problem. It should be emphasized that while R. Hananel and others suggested the proportional *pro-rata* rule as an application of Rabbe's *equal division*, no commentator attempted to reconcile proportional division with the Mishnah although the third row of Table 7 is compatible with proportional allocation. The inevitable conclusion from this discussion is that all Jewish Law scholars throughout ages consensually agreed that:

- (a) Contrary to contemporary prevailing view⁵⁵, the proportional division may reflect an economic approach but it is incompatible with the Talmudic fairness approach.
- (b) The unique sharing rule compatible with the Talmudic fairness approach is the Talmud rule. Hence, this is the consensual Talmudic solution for indecisive litigations.
- (c) The Talmud rule is incompatible with the Talmudic economic approach and thus inappropriate as a social welfare problem solution.

B. The Talmud Rule as Indecisive Litigations Solution

The consensus among the Sages regarding the Talmud rule as the unique indecisive litigations solution stems from the Categorical Imperative of Jewish

⁵⁵ See Rasmussen (1994) and Jackson (1986).

Law as preached by Prophet Zechariah⁵⁶: “*Execute the judgment of truth and peace in your gates*”. Zechariah’s preachment seems self-contradictory. *Judgment of truth* implies that justice should be done at all costs, while requires compromises and occasional deviations from *judgment of truth*. Sages have suggested several settlements to the contradiction⁵⁷, one based the teaching of R. Shimon ben Gamliel⁵⁸: *The world rests on three things: Justice, Truth and Peace*. The Jerusalem Talmud explains⁵⁹:

...And the three of them are really one thing. If justice is carried out, truth is realized. If truth is realized, peace is made. Said R. Mana, All three of them derive from a single verse of Scripture: *Execute the judgment of truth and peace in your gates*.

That is, judgment of truth and peace are not substitutes, but complements. Judgment of truth is a necessary and sufficient condition for peace. With the absence of either *judgment of truth* or *peace*, the world would deteriorate to a Hobbesian anarchy⁶⁰. Execution of judgment of truth is relatively easy when reliable evidence is available, but challenging in indecisive litigations in which the *Major Litigation Rule* of civil Jewish Law⁶¹ is inapplicable. The BR manifests seek to combine both judgment of truth and peace under indecisive litigation conditions⁶²; each claimant receives what is conceded for him by other claimants, (*truth*), and the remaining disputed sum is

⁵⁶ Zechariah 8, 16.

⁵⁷ For example, some Sages have suggested that the prophet alludes to compromise. See Tosefta Sanhedrin Ch. 1 §1, bSanhedrin 6b and more. On compromise in Jewish Law see Lipschits (2004).

⁵⁸ mAvot Ch. 1 §18.

⁵⁹ jMegilah 3, 6, jTa’anit 4,2. This translation is taken from Neusner edition (1982) p. 134. See also tractate Derech Eretz Ch. 2 §2, *Psikta d’Rav Kahana* 19, R. Bahye commentary on Deuteronomy Ch. 16 and more.

⁶⁰ The description of a decentralized society without a central enforcement authority as a Hobbesian chaos is common in ancient and classical thought. See for example Psalms Ch. 124, mAvot Ch. 3 §2, bAvoda-Zara 4a, Plato (1981) *Crito* and *The Republic* and compare with Marcius’ words in Shakespeare’s *Coriolanus* Act 1, Sc. 1. The common denominator of all social contract theories suggested by Hobbes (1651), Locke (1689), Rousseau (1762) as well as modern philosophers like Rawls (1971) and Gauthier (1986) is the inferiority of *the state of nature*, namely lawless society, although the philosophers differed in their descriptions of the state of nature, its flaws and the optimal formulation of the social contract.

⁶¹ See above, around footnote 12.

⁶² bBaba-Metzia 2b, Maimonides Laws of Plaintiff and Defendant Ch. 9 §7, Shulchan Aruch Hoshen Mishpat Ch. 138 §1-2.

equally divided (*peace*)⁶³. The BR is *comprehensive*, as it takes both claims and (derived) concessions into account.

Alternatively, define agent i 's *minimal right* as $mr_i = \frac{1}{n} \min\{c_i, w\}$. A sharing rule $f \in F$ satisfies *securement* if $f_i(B) \geq mr_i(B)$, and *minimal rights first* if $f_i(B) = mr_i(B) + f\left(c - \mathbf{mr}, w - \sum_{i=1}^n mr_i\right)$. Securement implies that no creditor is left with nothing (*peace*), while minimal rights first means that the residual problem $\bar{B} = \left\langle N, c - \mathbf{mr}, w - \sum_{i=1}^n mr_i \right\rangle$ is treated consistently (*truth*).

Theorem 2 (Moreno-Ternerero & Villar, 2004):

The BR is the only 2-agent sharing rule satisfying securement and minimal rights first.

By Theorem 1, the Talmud rule is the unique consistent generalization of the BR-principle to n -agent litigation. The following result establishes that the Talmud rule preserves the BR *securement* and *minimal rights first* features.

Theorem 3 (Chun-Hsien, 2008):

The Talmud rule is the only BR-consistent rule satisfying securement and minimal rights first.

Assuming that judgment of peace and truth implies securement and minimal rights first, explains both the consensual adaptation of the BR-principle as standard indecisive litigation solution by all Talmudic Sages on the one hand, and their consensual rejection of the proportional division as incompatible with the fairness approach on the other hand.

Another aspect of the combination *judgment of truth* with *peace* embedded by the Talmud rule is related to its game theoretic interpretation. For every

⁶³ Schwarz (2013) showed that the Talmud rule allocation coincides with the expected equilibrium allocation of a Hobbesian anarchy. In other words, the Talmud rule peacefully substitutes the jungle's *Might Makes Right* law (see bBaba-Batra 34b).

$B \in \mathbf{B}$ and coalition $S \subseteq N$ define the *coalitional claim*, $c_S = \sum_{i \in S} c_i$, and the corresponding *coalitional value*,

$$(10) \quad v_{(B)}(S) = \max\left(w - \sum_{T \subseteq N \setminus S} c_T, 0\right)$$

where $v_{(B)}(\emptyset) = 0$ and $v_{(B)}(N) = w$. By (10), $v_{(B)}(S)$ is the *minimal* of coalition S . Denote the payoff of agent $i \in N$ in the cooperative game by x_i , and the payoff of coalition $S \subseteq N$ by $x(S) = \sum_{i \in S} x_i$. The *excess* of coalition $S \subseteq N$ with respect to $x(S)$ is $e(S, x) = v(S) - x(S)$, which is a measure for the aggregate coalitional dissatisfaction from $x(S)$. Positive excess means that coalition S could do better and obtain $v(S)$ independently. Put differently, positive excess implies that by deviation no member of S would be worse off and at least one member of S would be better off. Negative excess means that members of S should feel (aggregately) satisfied and cooperation is beneficial for all of them. Define the *excesses vector*, $\theta(x) = \{e(S_j, x)\}_{j=1}^{2^n}$, where $e(S_j, x) \geq e(S_{j+1}, x)$. The *nucleolus* of $v_{(B)}$ is defined as the imputation which *lexicographically* minimizes $\theta(x)$ (Schmeidler, 1969)⁶⁴. Formally,

$$(11) \quad \mathcal{N}(v_{(B)}) = \{x | \theta(x) \preceq_L \theta(y), \forall y \neq x\}.$$

The nucleolus allocation process can be described as follows. First, from the imputations set of $v_{(B)}$ pick those which minimize $e(S_1, x)$. From this subset, pick the imputations which minimize $e(S_2, x)$ and so on⁶⁵.

⁶⁴ Let $\theta, \theta' \in \mathbb{R}^n$ be two excesses vectors. Vector θ is *lexicographically* larger than θ' ($\theta \succ_L \theta'$) if and only if $\theta = \theta'$ or there is an integer $k \in \mathbb{N}$, $1 \leq k < n$, such that $\theta_k > \theta'_k$ and $\theta_i = \theta'_i, \forall i \in [1, k)$. Intuitively, the lexicographic order reflects the order of words in the dictionary.

⁶⁵ Under certain assumptions, every cooperative game has a unique nucleolus (Schmeidler, 1969).

Theorem 4 (Aumann & Maschler, 1985):

The Talmud rule coincides with the nucleolus of the corresponding cooperative game $v_{(B)}$.

By Theorem 4 the Talmud rule combines *judgment of truth* and *peace* also in terms of lexicographic minimization of coalitional dissatisfactions.

The *Halakhic Majority Principle* used by Aumann and Maschler (1985) to provide a legal rationale for (5) sheds light on additional aspect of the combination *judgment of truth* with *peace* embedded by the Talmud rule. Medieval commentators of the Talmud⁶⁶ based this principle on the Talmudic legal fiction *the minority is as if nonexistent*. In an indecisive litigation context this principle can be interpreted as a behavioral *framing bias* assumption. That is, individual's utility is affected by the "bigger half" of the glass implying that although $x_i < x_j$, if $x_i \geq \frac{1}{2}c_i$ and $x_j < \frac{1}{2}c_j$ then $u_i(x_i) \geq u_j(c_j)$ ⁶⁷.

These aspects of the nice combination of judgment of truth and peace which made the Talmud rule a perfect solution for indecisive litigations in the eyes of the Sages, are irrelevant for social welfare problems. With the absence of judicial rivalry, claims, mutual denials and concessions, judgment of truth is meaningless and the only task left for the benevolent social planner is to achieve peace under economic rivalry. In other words, the social planner's problem is mechanism design; creation of balanced incentives to neutralize agents' collective action problem and encourage them to act in harmony for the maximization of their aggregate value. As explained bellow, the Talmud rule performs poorly in neutralizing creditors' conflict of interests. Thus, while all Jewish Law scholars adopted the Talmud rule for indecisive litigations, they consensually rejected it as a social welfare problem solution.

VI. The Medieval Controversy over the Economic Approach

⁶⁶ Tosfot Baba-Kama 27b starting at *Ka Mashma Lan*. See also Meiri Eruvin 23b starting at *Karpaf* and Psachim 7a starting at *Maot* and also Ran's commentary on Rif Avoda Zara 17b starting at *Gm'*.

⁶⁷ See for example McKenzie and Nelson (2003).

Mainstream Jewish Law decisors adopted Rabbe's *equal division*, which according to our interpretation stems from economic approach towards priority-less bankruptcies⁶⁸, but differed regarding its interpretation. Recognizing that the first definitive constraint precludes literal interpretation of *equal division*⁶⁹, the majority among mainstream decisors interpreted it as referring to CEA⁷⁰. Others interpreted it as referring to *pro-rata* division⁷¹, and three commentators suggested that Rabbe actually meant the RI rule⁷². No decisor suggested that Rabbe might have referred to the CEL rule⁷³.

Legally, these differences are related to the definition of creditor's lien (*shiabud*). Proponents of CEA allocation hold that each creditor has an equal floating charge over each unit of available assets⁷⁴, proponents of the RI rule hold that a creditor's lien is constrained by his claim⁷⁵, while the most original argument was apparently raised by proponents of the proportional rule who claim that creditors are *partners*, thus no creditor possesses a lien over any portion of the available sum at all⁷⁶. *Equal division*, according to proportional division proponents, refers to *equal treatment of equal partners*⁷⁷. Deep discussion of Jewish Law lien paradigms goes far beyond the scope of this article. Our interest here is

⁶⁸ A separate medieval Talmudic school known as *Magenza Commentators* adopted R. Nathan fairness approach and recommended the Talmud rule as the standard priority-less bankruptcy solution. We discuss this school in section VII below.

⁶⁹ See Tosfot Kethubut 93a starting at *Rabbe Omer*.

⁷⁰ Rif on Kethuboth 93a (folio 51a in Vilnius edition), Maimonides, *Laws of matrimony* Ch. 17 §8 and *Laws of Arachin and Haramin* ch. 8 §4, *Shulchan Aruch Even Ha'ezer* Ch. 96 §18.

⁷¹ R. Hananel, quoted in Tosfot Kethuboth 93a, starting at *Rabbe Omer*, see also *Piskei Rid*, *Kethuboth* 93a.

⁷² Rashi on bArachin 27b starting at *Meshalshin*, Rabad Glosses on Rif Kethuboth 93a, Maimonides, *Laws of matrimony* Ch. 17 §8 and *Laws of Arachin and Haramin* ch. 8 §4, Abraham Ibn-Ezra, *Sefer Ha-Mispar*, (with German translation by M. Silberberg), Kaufmann Verlag, Frankfurt a. M. 1895 p. 57.

⁷³ For explanation see Schwarz (2013).

⁷⁴ See Rif, *Temim Deim*, (translation of a Rif response from Arabic), printed with the Rif on Kethuboth 93a.

⁷⁵ See Rabad, gloss on Rif Kethuboth 93a.

⁷⁶ R. Hananel, quoted in Tosfot Kethubut 93a starting at *Rabbe Omer*. This argument is based on a Talmudic topic in bBaba-Kama 36a.

⁷⁷ The originality of this argument, raised by a 10th century scholar, highlights the lack of serious discussion of proportional division rationale in modern legal literature.

confined to *economic* analysis of this controversy. We argue that the disputations between proponents of RI and CEA rules on the one hand, and proponents of CEA and proportional division on the other hand, reflect differences between *minimalist* and *maximalist* versions of the economic approach, as explained below.

A. The Minimalist Economic Approach

The minimalist economic approach holds that economic efficiency and general equilibrium considerations cannot expropriate the essence of bankruptcy as litigation and its solution as a verdict. As in any other litigation, litigation, the court should not be concerned neither with possible hidden evidence nor future contingent revelations of new evidence or unknown assets. A verdict should be based on *current* known facts and evidence only⁷⁸. Implicitly, the minimalist economic approach assumes that creditors' race also refers to current available assets and that creditors ignore contingent future changes in debtor's financial situation. The minimalist economic approach corresponds with early static models of litigation games⁷⁹, attributing creditors either static or adaptive expectations.

In a static framework creditors' race is a contest over current available assets. Creditor i 's *ex-post* payoff is given by (9), where creditors' index indicates the creditors *ex-post* position in the order of arrivals to the finish line. Denote the aggregate *ex-post* payoff of coalition $S \subseteq N$ by $J(S) = \sum_{i \in S} J_i(B)$, and the marginal *ex-post* contribution of creditor $i \in S$ to coalition S 's *ex-post* aggregate payoff by $J(S \cup \{i\}) - J(S)$. Assuming equal athletic skills, the *ex-ante* expected payoff of creditor i ', denoted by $Sh_i(N, J)$, is the average of his marginal *ex-ante* contributions to every

⁷⁸ See Taz and Shakh commentaries on Yore Dea, Ch. 188 and contradictory opinion of Bakh (ibid).

⁷⁹ C.f. Landes (1971).

contingent coalition. It follows that,

$$(12) \quad Sh_i(N, J) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} [J(S \cup \{i\}) - J(S)]$$

Where $s = \#S$ is the cardinality of $S \subseteq N$. By (12) $Sh_i(N, J)$ is actually creditor i 's *Shapley value* (1953). Theorem 5 confirms this impression.

Theorem 5 (Littlechild & Owen, 1973), (O'Neill, 1982):

The Recursive Incremental Rule coincides with the Shapley value of the corresponding cooperative game.

In light of Theorem 5 the economic rationale of the RI rule is straightforward. A risk-averse creditor would prefer to settle on accepting $Sh_i(N, J)$ with certainty, over competing in a contest in which his *ex-post* payoff $J_i(B)$ is contingent and conditioned on his relative position in the arrivals order. Assuming that all creditors are risk-aversers, division of available assets according to the RI rule would prevent creditors' race.

B. The Maximalist Economic Approach

According to the maximalist economic approach, the distinction between judicial and economic rivalry is substantial. Priority-less bankruptcies should not be treated as litigations, but as social welfare problems. Moreover, bankruptcy is not a one-shot litigation but a dynamic multi-stage game, implying that the court should be concerned by broader considerations than current evidence and facts submitted by the parties⁸⁰. Static models are incompatible with the basic rationale of the economic approach, because cooperation in maximization of aggregate value can be creditors' common interest if and only if the sharing rule is *dynamically consistent*, namely *path-independent*. Put differently, cooperation can be an equilibrium strategy if and only if the sharing rule is *history robust* and

⁸⁰ Recent economic analyses of litigations also applied dynamic modeling. For example see Schwarz (2012).

indifferent to dynamic alterations of parameters or arbitrary split of the alternatives set⁸¹. A dynamically inconsistent sharing rule *intensifies* creditors' creditors' conflict of interests and consequently their collective action problem. In a bankruptcy context path-independence means that allocation of w at once or piecemealed should yield the same result. Theorem 6 reveals that dynamic consistency is precisely the Achilles heel of both the Talmud and RI rules.

Theorem 6 (Moulin H. J., 2000) :

There are three and only three rules on $\mathbf{B}$ satisfying simultaneously equal treatment of equals, scale invariance, composition, path independence and consistency: The Proportional rule, the Constrained Equal-Awards rule and the Constrained Equal-Loss rule.

Notice that the three rules listed in Theorem 6 are exactly those combined by the Talmud rule according to (5). Nevertheless, neither the Talmud nor the RI rules are listed in Theorem 6's list, because both are not path-independent.

Mainstream Jewish Law decisors adopted the CEA rule. We are unaware of any similar modern legislation, but it should be emphasized that the prevailing adaptation of proportional division in modern legislations is fairness motivated, according to the Aristotelian fairness approach. Indeed, the CEA rule seems incompatible with the Aristotelian perception of fairness because it disproportionately favors small creditors at the expense of larger creditors (and vice versa with the CEL rule). From *economic* point of view, however, this discrimination could be a good reason for consideration of either CEA or CEL schemes as standard priority-less bankruptcy solutions in modern economies. In many countries, regulators prohibits public corporations (like banks) from holding a single lender position to certain borrowers, and impose restrictions

⁸¹ See Plot (1973), (1976).

on creditors whose share in a debtor's debt exceeds the legal limit⁸². The existence of such regulations indicate that under the prevailing proportional scheme creditors are not sufficiently deterred by the risks associated with single or large lender position. An interesting question for further research is whether introduction of the CEA rule, for example, may do a better job and save regulatory enforcement costs.

VII. The Dual Bankruptcy Problem

It is very tempting to assume that consistency of legal systems implies similar treatment of primal and dual bankruptcy problems. But mathematical duality is not a synonym for legal or economic equivalence, and even pairs of apparently similar primal or dual bankruptcies may be treated differently by the law. The overriding consideration is the legislative target function. In this section we demonstrate that this is the guiding principle of Jewish Law differential treatment of two dual bankruptcies.

A. Bidders Retractions in Auctions

Bidding in a typical English auction is equivalent to writing a put option. That is, by bidding b_i bidder i undertakes to buy the auctioned item for b_i and no withdrawal is allowed. The option expires when bidder j bids $b_j > b_i$. Ancient bidding laws in auctions over redemption of sacred objects in the Temple of Jerusalem ordered on the one hand that bidder i 's obligation holds until the item is sold for no less than b_i even if a higher bid was submitted, but on the other hand bidders may withdraw their bids in return for opting out fee. If bidder i withdraws, bidder $i-1$ should buy the item for b_{i-1} and bidder i pays an opting out fee $\delta_i = b_i - b_{i-1}$. In the Mishnah's numerical example five bidders $(1,2,3,4,5)$ bade $\{10,20,30,40,50\}$

⁸² See for example 'Lending limits' 12 U.S.C § 32.3.

respectively⁸³. If bidder 5 withdraws, the sacred object is sold to bidder 4 for 40, and bidder 5 pays $\delta_5 = 10$. If bidder 4 withdraws too he pays $\delta_4 = 10$, the item is sold to bidder 3 for 30 and so on⁸⁴. The Talmud⁸⁵ discusses a case of *simultaneous* withdrawals where the item was finally sold for $r < b_1$ causing the auctioneer a loss of $\theta = b_n - r$. R. Hisda ruled that “*we divide it among them*”. That is, θ is burdened on all 5 bidders collectively. Unfortunately, the Talmud does not specify how, and does not explain why sequential and simultaneous withdrawals should be treated differently.

Mathematically, according to the ancient Temple Auction Law, the simultaneous retraction case is a dual bankruptcy problem $B^* = \langle N, \theta, \mathbf{b}^* \rangle$ where $b_i^* = b_i - r$, $b_i^* < b_{i+1}^*$, $\forall i \in N$ and $\theta = b_n - r$. Suppose that in the Mishnah’s example $r = 5$, thus $B^* = \langle 5, 45, \{5, 15, 25, 35, 45\} \rangle$.

Aumann and Maschler (1985) pointed out that all medieval decisors who adopted Rabbe’s economic approach in the Kethuboth case consistently applied their favorite sharing rule in this topic too⁸⁶, but this consistency requires an explanation because it is not crystal clear that R. Hisda’s wording “*we divide it among them*” cannot be interpreted according to the fairness approach⁸⁷. On the one hand, the ancient Temple Auction Law creates a mutual bail among the bidders. By their simultaneous withdrawal the bidders became multiple-tortfeasors⁸⁸ and the case apparently resembles a classical litigation with claims, denials and concessions, implying that the Talmud rule should be applied. On the other hand, it is very likely that the ancient Temple

⁸³ Arakhin Ch. 8 §3.

⁸⁴ If bidder 1 also retracts and the item was finally sold for r , his opting out fee is $\delta_1 = b_1 - r$.

⁸⁵ bArakhin 27b.

⁸⁶ That is, Maimonides (*Laws of Arakhin and Haramin* ch. 8 §4) applied the CEA rule on this problem, while Rabad (in a gloss on Maimonides op. cit.) and Rashi (Arakhin 27b starting at *meshalshin*) applied the RI rule.

⁸⁷ The Talmudic discussion in this topic does not mention the Tanaitic controversy at all.

⁸⁸ See the Talmudic discussion on the *oxen and the pit* case, bBaba-Kama 53a. See also footnote 53 above.

Auction Law is aimed at maximizing the auctioneer's revenue according to the economic approach⁸⁹. Obviously, according to R. Nathan who classified the Kethuboth case as litigation this case should be treated similarly. The application of (5) in this case, however, should be carried out carefully because the upper bound of bidder i 's liability is b_i , implying that the bidder n , (ab invito) admits sole responsibility for

$$(12) \quad \bar{\delta}_n = \begin{cases} \delta_n - r & r \leq \delta_n \\ \delta_n & r > \delta_n. \end{cases}$$

Thus, bidder n is burdened with $\bar{\delta}_n$ first, and (5) is applied on the *residual* problem $\bar{B}^* = \langle N, \bar{q}, \bar{\mathbf{b}}^* \rangle$ where $\bar{\theta} = \theta - \bar{\delta}_n$ and $\bar{\mathbf{b}}^* = \{\bar{\mathbf{b}}_{-n}^*, b_n - \bar{\delta}_n\}$. In the Mishnah's example $\bar{\delta}_n = 5$, $\bar{\theta} = 40$, and $\bar{B}^* = \langle 5, 40, \{5, 15, 25, 35, 40\} \rangle$. Thus, by (5) $T(\bar{B}^*) = \{2\frac{1}{2}, 7\frac{1}{2}, 10, 10, 10\}$. Adding $\bar{\delta}_n$ to $T(\bar{B}^*)$ yields the final payments vector $\{2\frac{1}{2}, 7\frac{1}{2}, 10, 10, 15\}$, which explicitly appears in an anonymous commentary affiliated with the *Magenza Commentaries*⁹⁰ and attributed in the Vilnius edition to R. Gershom (10th century)⁹¹.

It is very tempting to infer that this anonymous commentator adopted R. Nathan's ruling and applied the Talmud rule. The attractiveness of this conclusion is strengthened since although consensually "*the Halakha is according to Rabbe as against one of his fellow-scholars*"⁹², medieval decisors differed in case of Rabbe against one of his mentors, particularly R.

⁸⁹ See for example Asker (2000).

⁹⁰ A collection of commentaries on the Babylonian Talmud, probably written in *Magenza* (Mainz, Rhineland-Palatinate, Germany) during the 10th or the 11th centuries, by Talmudic scholars affiliated with the *Magenza School* founded by R. Gershom. The identification of each commentary's author is, of course, beyond the scope of this article. R. Gershom is explicitly declared as the author of a segment of the Nedarim commentary from folio 22b to folio 25b. The Hida holds that actually the entire commentary on Nedarim was authored by R. Gershom, at least from folio 22b to the end of the tractate. (Hida, *Shem Hagdolim, Ma'arechet Gedolim*, §300 [35]). R. Yossef Karo also determined undoubtedly that the commentary on Nedarim 31b was not authored by Rashi (*Beit Yosef Hoshen Mishpat* 186). The consensus among contemporary scholars is that this commentary is also one of the *Magenza commentaries*. For further discussions see Epstein (1986) and Ta-Shma (2005).

⁹¹ Commentary attributed to R. Gershom, Arachin 27b, starting at *Tania nami hachi*.

⁹² bEiruvim 46b.

Nathan. While many decisors ruled *Halakha according to Rabbe* even against R. Nathan⁹³, many others hold that in this case *Halakha is according to R. Nathan*⁹⁴ and this opinion is common among all commentaries affiliated with the *Magenza Commentaries*⁹⁵. Moreover, as quoted above⁹⁶, Rif explained that even mainstream decisors who ruled according to Rabbe in the Kethuboth case did so reluctantly, having reached a dead-end in their attempts to decipher R. Nathan's mysterious ruling. The natural conclusion is, therefore, that this anonymous commentator followed his *Magenza school* discipline and interpreted the topic according to R. Nathan. Nevertheless, this conclusion should be treated with caution, because the anonymous commentator also provided a *calculation algorithm* which generally produces different solutions than (5) (although fairly closed to). A full analysis of this interesting algorithm is beyond the scope of this article. Based on our careful textual examination of the anonymous commentary text we conjecture that its author was inspired by R. Nathan's logic and his algorithm is, indeed, a pioneering medieval attempt to generalize the BR-principle to n -agent dual bankruptcy problem. To summarize this subsection:

- (a) All decisors exhibited consistency and applied their favorite sharing rule also in the withdrawing bidders' case.
- (b) An anonymous commentator affiliated with the *Magenza School* who generally rule according to R. Nathan against Rabbe apparently attempted to apply the Talmud rule in this case too and provided an algorithm which according to our conjecture is a pioneering attempt to

⁹³ See for example Tosfot Baba-Metzia 50b starting at *Amar, Sefer Hamaor*, Rashba, Ran and Meiri on Kidushin 64b, *Temim Deim* Ch. 220 and more.

⁹⁴ See for example Tosfot on Menachot 38b starting at *Ela Le'gardomin*, Eiruvim 46b starting at *Rabbi Assi*, Rashi on Baba-Metzia 50b starting at *Umachzir Ona'a*, R. Yerucham Ch. 25, 3 in the name of *Ramah*, Rif and Rosh on bGitin 85, Rosh on Kethuboth Ch. 22 and on bBaba-Metzia Ch. 14, *Seder Tanaim Ve'amoraim* Ch. 22. *Gaonica* p. 68 in the name of R. Yaakov Gaon, R. Hananel on bShabat 19a, Ritba on bGitin 52b. See also *Yad Malachi* Ch. 238.

⁹⁵ See for example, commentary attributed to Rashi, on Nedarim 90a starting at *man hakim*. *Yad-Mala'achi*, Rules of the Talmud (rule 591), quotes opinions that according to this commentator Halacha is according to R. Nathan even in *diney-nefashot* (criminal law involved in death penalty).

⁹⁶ See around footnote 40.

generalize the BR-principle.

(c) While the consistency of the Magenza School affiliated decisors is quite expected, there are three alternative explanations for mainstream decisors consistency in this case:

1. Mainstream decisors hold that *Halakha is according to Rabbe*, even against R. Nathan.
2. Mainstream decisors interpreted the ancient Temple Auction Law as aimed at maximizing auctioneer's revenue and consequently applied a maximalist economic approach.
3. Mainstream decisors agree that this case is a regular litigation. Nevertheless, they were forced to rule according to Rabbe either due to their inability to decipher R. Nathan's method or their inability to generalize the BR-principle to n -agent case⁹⁷.

B. Taxation, Social Welfare and Dual Bankruptcy Problem

The economic literature produced *correspondence theorems* which uniquely associate sharing rules with game theoretic solution concepts on the one hand, and game theoretic solution concepts with maximization of specific social welfare functions on the other hand. By these theorems a decisor's implicit social norms and social welfare function can be inferred from his choice of a priority-less solution and the mainstream choice apparently reflects the mainstream Jewish social justice philosophy.

For example, on the one hand Theorem 4 associates the Talmud rule with the nucleolus of the corresponding cooperative game, and on the other hand the nucleolus solves $\max_{x_i} \min_{i \in N} \{x_i\}_{i \in N}$ subject to $\sum_{i=1}^n x_i = w$ ⁹⁸. That is, the

⁹⁷ An interesting example of ruling explicitly against a superior authority due to inability to decipher his underlying rationale appears in bBaba-Batra 107b. For further discussion see Lipschits and Schwarz (forthcoming).

⁹⁸ See for example Serrano (1995), Legros (1987) and Arin (2007).

Talmud rule maximizes a Rawlsian social welfare function⁹⁹. Similarly, Theorem 5 associates the RI rule with the Shapley value, which is the imputation $\varphi(N, v)$ that solves $\min \sum_{\emptyset \neq S \subset N} \delta(s) [v(S) - \varphi(S, v)]^2$ subject to $\sum_{i \in N} \varphi_i(N, v) = w$ where $\delta(s) = \binom{n-2}{s-1}^{-1}$ are the *Shapley weights*¹⁰⁰. In other words, the RI rule maximizes a weighted Benthamite social utility function. Finally, let $u_i(x_i)$ denote agent i 's utility function. The *Nash bargaining solution* is defined as the imputation $\varphi(N, v)$ which solves $\max \prod_{i \in N} u_i^{\alpha_i}$ subject to $\sum_{i \in N} \varphi_i(N, v) = w$. The *weighted Nash bargaining solution* is defined as the imputation $\varphi(N, v)$ which solves $\max \prod_{i \in N} s_i u_i^{\alpha_i}$ subject to $s_i \geq 0, \forall i$ and $\sum_{i=1}^n s_i = 1$.

Theorem 7 (Dagan & Volij, 1993):

- a. *The constrained equal awards rule corresponds with the Nash (1950) bargaining solution.*
- b. *The proportional rule corresponds with the weighted Nash solution where $s_i = c_i / \sum_{i=1}^n c_i$.*

Combining these results implies that the CEA rule is compatible with maximization of a Nash social welfare function with $\alpha_i = 1 \forall i \in N$, and the Proportional rule is compatible with maximization of a Nash social welfare function with $\alpha_i = s_i \forall i \in N$.

Prima facie, these correspondences indicate that the controversy among medieval decisors regarding the interpretation of Rabbe's *equal division* is not a mere scholastic textual dispute, but deep philosophical difference regarding

⁹⁹ This is an illustrative example only, since according to our thesis the proponents of the Talmud rule among Jewish Law scholars were motivated by the fairness approach which disregards social welfare maximization.

¹⁰⁰ Keane (1969).

social justice. It would be natural to expect that the decisors who exhibited remarkable consistency in the withdrawing bidders case analyzed above, would exhibit similar consistency regarding the classical dual bankruptcy problem and the core issues of social welfare and distributive justice – taxation. Instead, Jewish Law literature is silent on taxation. This surprising silence was explained by Rashba¹⁰¹:

The foundation of tax laws is not in the holy mountains of the Talmud¹⁰², and you can find everywhere a variety of customary legislations approved by local Sages and defined by their ancestors. A community is allowed to enact legislation as they wish, even not with accordance to Halakha since this is a pecuniary issue. Hence, if a community has a known local customary law, follow the custom, because in such issues the custom prevails over the Law.

Rashba simply states there is no Talmudic tax function and no Talmudic canonical social justice paradigm. This statement and its consensual adaptation by later authorities¹⁰³ is puzzling, in light of the polemic over the appropriate application of the economic approach towards priority-less bankruptcies.

Jewish social and distributive justice paradigms are far beyond the scope of this article. Nevertheless, Rashba's statement should not be interpreted as if Jewish Law poses no limits to social legislation. Such a conclusion would be definitely wrong. The famous biblical stories of the deluge¹⁰⁴, Sodom and Gomora¹⁰⁵, the Seven Laws of Noah¹⁰⁶ and the preachments of Biblical prophets against corrupted lords and tyrant kings prove that according to Jewish Law every society ought to obey basic universal rules of honesty and

¹⁰¹ Rashba, Responsa vol. 4 Ch. 260. See also *ibid* vol. 2 Ch. 270.

¹⁰² Paraphrase on Psalms 87, 1.

¹⁰³ See for example R. Meir of Ruthenburg, Responsa (Prague edition) Ch. 106, 130 and 596, R. Israel Iserlein, Responsa *Trumat Hadeshen* Ch. 342, R. Shmuel di Modena, Responsa Ch. 369 and more.

¹⁰⁴ Genesis Ch. 6.

¹⁰⁵ Genesis Ch. 18-19.

¹⁰⁶ Genesis Ch. 9 1-7.

equity. In fact, Jewish Law is probably the most ancient legislation in the history which limited the king's authorities and duties within a constitutional framework, negated king's status as an almighty ruler¹⁰⁷ and imposed restrictions on society to prevent tyranny of the majority¹⁰⁸. However, as R. Yitzhak Abrabanel (1437–1508) and R. N. Z. Y. Berlin (*Natziv*, 1816–1893) argued¹⁰⁹, social norms and preferences are dynamic and variable over societies, cultures and historic epochs. The ambition to formulate a universal and eternal social norms set is futile¹¹⁰. Moreover, there are indications that Jewish scholars had an intuitive perception of Arrow's (1963) impossibility theorem that debunked the naïve belief that individual preferences can honestly be aggregated by a social ranking (Schwarz, 2013)¹¹¹.

Rashba's statement means that there is no unique canonical set of social norms or unique legitimate form of social and political arrangement. Societies are free to choose their preferred social and political arrangements according to their contemporary prevailing social norms as long as they comply with Seven Laws of Noah and basic natural rules of justice and equity. It turns out that Jewish Law maximalist economic approach is relatively modest and limited in its goals, and does not stem from a coherent social justice or distributive philosophy, but from economic efficiency considerations. Its aim is, as explained above, to eliminate creditors' conflict of interests and

¹⁰⁷ Deuteronomy Ch. 17 Vs. 14-20, Maimonides *Laws of Kings* and *Laws of Robbery and Lost* Ch. 8 § 12-18.

¹⁰⁸ See Exodus Ch. 23 Vs. 2 and the commentaries of Ibn-Ezra and R. Bahyeh there.

¹⁰⁹ In their commentaries to Deuteronomy Ch. 17 Vs. 14.

¹¹⁰ According to these commentators this is why the Torah instructions regarding social and political institutional arrangements are not mandatory, but this point lies in the core of a famous Tanaitic controversy (bSanhedrin 20b). Abrabanel's claim that his opinion represents also Maimonides is weird since Maimonides (*Laws of Kings*, Ch. 1 §1) explicitly ruled according to R. Yehuda that crowning a king is mandatory (see also R. Y. F. Perla commentary on R. Seadia's *book of commandments*, 65 portions, portion 7). See also R. A. Y. Kook, *Responsa Mishpat Kohan* Ch. 144 §15, and Meiri Sanhedrin 52b.

¹¹¹ For example, R. Yisrael Nagara (1555–1628) explained, using an intuition remarkably similar to Arrow's, why ranking Torah commandments and offences according to any reasonable criteria set is impossible. See R. Yisrael Nagara, *Sermon for Month Adar, Yeshurun*, Vol. 10 p. 134 (by a unique manuscript from the Budapest Rabbinical Seminary No. k30).

neutralize their collective action problem in order to maximize their aggregate value. However, unlike regular social welfare problems, creditors' aggregate value function is well defined and tangible, thus can be universal, constant over historic epochs and indifferent to cultural variations. Yet, the choice of a sharing rule is not straightforward but may depend on specific modeling and subject to legal lien and moral equity considerations, and this explains how decisors affiliated with the maximalist economic approach could recommend different sharing rules.

To summarize this section:

- (a) Mathematical duality does not imply judicial equivalence.
- (b) According to Jewish Law, there is no universal set of social justice norms or a universal social welfare function. Consequently, there is no canonical Talmudic taxation function.

VIII. Summary and Conclusion

Bankruptcy problems are commonly associated with economic disasters, but can emerge also due to extraordinary economic performance. The choice of a sharing rule is related to fairness and justice perceptions, but has a potential significant effect on the economy's general equilibrium. The economic literature hitherto neglected the search for the economically optimal bankruptcy solution and concentrated mainly in normative axiomatizations of sharing rules, but its findings did not attract much attention of legal scholars. Our purpose in this article was to bridge between the economic and legal literatures to create an interdisciplinary symposium on bankruptcy.

As a platform for our suggested interdisciplinary symposium we suggest our interdisciplinary analysis of a fascinating polemic conducted by Jewish Law scholars over the course of 15 centuries, based on Rasmussen (1994) discussion of the fairness and economic approaches towards bankruptcies. We showed that Jewish Law scholars considered alternative sharing rules: CEA, RI, P and T. This Talmudic polemic triggered finally the 20th century

normative economic research on sharing rules. The enormous potential affect of the sharing rule choice on the economy's general equilibrium and social welfare implies that the importance of further economic research of the optimal sharing rule and the interdisciplinary symposium cannot be exaggerated.

Contrary to the prevailing assumption in modern legal literature, the consensus in the Talmudic literature is that the Talmud rule is the only sharing rule compatible with the fairness approach, presumably because it is the only BR-consistent n -creditor sharing rule. The fairness appeal of the BR-principle in Talmudic thought stems from the various aspects of combining *judgment of truth* and *peace*.

Most Talmudic authorities adopted the economic approach to priority-less bankruptcies, and hold that a priority-less bankruptcy is not a regular litigation because the rivalry among creditors is economic, not judicial. The controversy within the Talmudic economic paradigm is between proponents of *minimalist* and *maximalist* economic approaches. Minimalists view bankruptcy as a static one-stage game, and attribute creditors with either static or adaptive expectations, and thus recommend the RI rule. Maximalists hold that a priority-less bankruptcy is a social welfare problem, modeled as a dynamic multi-stage game, and its solution must be dynamically consistent (*path-independent*). Mainstream proponents of the maximalist economic approach recommended the CEA rule, and a minority recommended proportional *pro-rata* division.

Talmudic scholars consensually agreed that mathematical duality is not legal equivalence. The guiding principle is the social target function to be maximized. It is reasonable to impose a sharing rule when the social target function is clearly defined and widely accepted, which is usually the case in bankruptcies where the social target function is maximization of creditors' aggregate pecuniary value.

Talmudic scholars realized the difficulties associated with honest

aggregation of individual preferences into a representative social ranking, and doubted the existence of a universal social welfare function is doubtful. Consequently, all Talmudic authorities agreed, for example, that there is no canonical Talmudic taxation law or canonical political regime. Political arrangements and taxation reflect contemporary social norms and social perceptions of social justice, but they have to obey basic Torah requirements of justice and equity.

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