Competitive equilibrium, investment runs, and renewable energy subsidies: a real options analysis

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Abstract

Foreseeable policy changes like a subsidy phaseout lead to competitive runs where a mass of investment suddenly occurs. We characterize this phenomenon, observed in several renewable energy markets, and provide a novel interpretation based on rational expectations of future entries. We show that the size of runs increases with the magnitude of the policy change and market uncertainty but decreases with policy uncertainty, and that runs are smaller when the policy change applies to both new and existing firms. Runs accelerate investment and lower welfare, with implications for the conduct of policy changes like subsidy phaseouts.

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1 Introduction

Economic policy uncertainty is a ubiquitous concern that can lead firms to reduce or delay investments (Hassett and Sullivan 2016). It is not uncommon either though for firms to face policy changes that are entirely predictable. As we highlight below for example, renewable energy investment subsidies in many countries have been revised according to pre-announced or foreseeable schedules. Competitive forces can take over in such situations, disrupting and accelerating the dynamics of industry investment. In the energy subsidy cases that motivate our study, the expectation of policy changes appears to have brought on significant spikes in investment. Such dynamics are suggestive of those arising in Bartolini (1993)'s pioneering work on competitive runs due to production caps. In his analysis, knowledge of a definite limit to an industry's expansion implies that incremental investment is ultimately disrupted by a massive rush of market entries, with adverse consequences for welfare. We show in this paper that a similar logic holds and leads to competitive runs under a much more general set of policies. Specifically, we show that if competitive equilibrium is accounted for, the anticipation of a subsidy reduction or equivalently a tax policy jump causes a run where a mass of investment suddenly occurs, resetting the market price process at a significantly lower level. We characterize such runs and their determinants, and discuss their impact on economic welfare.

To reach these results we develop a dynamic model of a perfectly competitive market following the standard assumptions of the literature on investment under uncertainty,[1] namely that profit flows evolve stochastically, that irreversible investment is required in order to start producing, and that firms time their investments optimally. We introduce a policy change consisting of a tax policy jump or subsidy reduction which happens once

[1]See Dixit and Pindyck (1994) and Schwartz and Trigeorgis (2004) for authoritative presentations of the fundamental insights, methodologies and results of this literature.

the market reaches a predetermined target, and establish that the equilibrium dynamics in our setup consist of the following three stages:

- first, far enough from the policy trigger, market quantity follows the gradual process described in the investment literature;

- second, at a quantity below the policy trigger, an investment run occurs which immediately brings the market up to the policy trigger;

- thereafter the gradual market quantity process resumes, at a slower pace than before due to the higher tax.

Our analysis helps to understand the reasons for the emergence of competitive runs. Previous analyses of runs have focused on policies involving caps on market quantity, leading to the impression that runs occur because firms are afraid that if they do not hurry into the market they will be unable to do so later. This interpretation is appealing because it is consistent with classic studies of runs like Krugman (1979), where a run occurs because a limited stock, e.g. of foreign exchange reserves, is expected to vanish. But our analysis reveals that competitive investment runs have a more subtle underlying logic: the policy change slows subsequent entries by lowering profitability, so firms that are active ahead of the change expect to benefit from a more favorable subsequent market process than without the policy. Such conditions would create supranormal profits if left unchecked, but potential entrants also anticipate the policy change and a mass of new firms is accordingly attracted into the market. This mass of investment resets the price process in the market at a lower level, so as to eliminate above normal profits. To underscore our point we show that similar equilibrium dynamics arise regardless of whether the policy change affects only new or both new and existing firms. The emergence of a competitive run is not tied therefore to any rewards that some firms secure by being lucky enough to enter before the policy change whereas others do not. Instead, the run is a dynamic equilibrium pattern which maintains equal value for all firms under competition.

Our analysis is relevant to understanding real-world policy applications, and in particular to rationalizing experiences policy makers have had phasing out renewable energy investment subsidies. First of all note that electricity price risk is known to represent a significant source of uncertainty, which makes investment in electricity generation capacity naturally amenable to real options analysis (Nadarajah and Secomandi 2023). In the specific case of renewables, subsidies have historically played an instrumental role in promoting investment. Such green subsidies are often phased out however as renewables catch up with established technologies, as in the following examples.

In 2006, the state of California enacted the California Solar Initiative (CSI), an up-front investment subsidy aimed at incentivizing 3000MW of additional solar power capacity within a decade and whose level followed a degressive rebate schedule with respect to cumulative capacity. The pre-announced rebate schedule stipulated, for example, a subsidy rate of \$2.50 per Watt up to 70MW of statewide installed capacity and \$2.20 subsequently, implying a subsidy drop of \$0.30 contingent upon hitting a pre-announced level of capital accumulation.[2]. In Belgium, De Groote and Verboven (2019) report that the Green Current Certificates (GCC) program enacted in 2006 committed to production subsidies for residential photovoltaic systems which would be revised at pre-announced dates. These revisions generally led to lower subsidies for new adopting households (from €450 per MWh initially to €90 per MWh in 2012, before being altogether abolished in 2014), impacting a significant portion of Belgium's electricity mix. Nor are such anticipated subsidy reductions limited to solar energy. In the United Sates, wind generation capacity investments in past decades were governed by the Wind Production Tax Credit

[2] The timing of each step was therefore unknown ahead of time (Reeves and Rai 2018, Burr 2016 Table A2.1).

(PTC), which was originally slated to phase out in 2013.[3]

From the standpoint of our analysis, the policies we have just described targeted essentially competitive actors and were scheduled to be phased out along a definite schedule that market participants could foresee. But also, and more importantly perhaps, the evidence suggests market participants anticipated such policy changes and reacted accordingly. With respect to the programmed phaseout of the PTC in 2013 for example, the magazine *The Economist* wrote:

> "[I]t took 25 years to get to 10 gigawatts (GW) of wind-power capacity but a mere five months, last year, to jump from 50GW to 60GW. The effervescence of the wind industry last year, however, was partly because the main federal tax credit for wind power was going to expire in December, and companies raced to qualify before the deadline" [4]

Similarly, De Groote and Verboven (2019) document significant spikes in new installations occurring just before each subsidy revision, followed by returns to gradual investment thereafter in the case of Belgium's GCCs, and Reeves and Rai (2018) report similar patterns for California's CSI.[5] The stylized model that we develop in this paper allows us to rationalize such phenomena as equilibrium behavior of forward-looking and profit-maximizing economic agents.

Our work relates broadly to the literature on investment and policy uncertainty surveyed by Hassett and Sullivan (2016). Following Hassett and Metcalf (1999), much of this work model policy changes as Poisson events which are imperfectly correlated with profitability. We focus instead on a policy change which is triggered directly by industry

[3] https://en.wikipedia.org/wiki/United_States_wind_energy_policy.
[4] https://www.economist.com/united-states/2013/06/08/blown-away.
[5] See their Figure 2 p. 2145.

expansion, and indirectly by an exogenous demand shock, making the timing of the policy change predictable for decision-makers. Our work also complements existing research on regime uncertainty where the calendar time of the policy change is known but there is uncertainty about the magnitude of policy changes (Nishide and Nomi 2009).

From a theoretical perspective our work also contributes to a stream of literature on competitive runs, starting with Bartolini (1993)'s study of the emergence of runs in a competitive industry facing a production cap, which represents a limiting case of our model as the size of the tax becomes arbitrarily high. Further work in this area includes Moretto and Vergalli (2010) who show uncertainty about the policy cap trigger can mitigate the run, and more recently Di Corato and Maoz (2019) who study the optimal cap for a given negative externality and show that it should either be set at the current quantity or infinite. Relative to these articles, our work contributes by providing insight into the role of future entries which is not apparent in the case of a cap and emerges clearly with a tax, and by incorporating a different form of policy uncertainty, with respect to the magnitude rather than timing of policy.

Finally our work contributes to studies of real options and investment in renewable energies. An important stream of this literature has studied how subsidy retraction affects investment, generally modeling policy risk as a stochastic process. A central finding is that subsidy withdrawal risk accelerates firm investment (Boomsma and Linnerud 2015, Nagy et al. 2021). Subsequent work has found as we do that incremental investment increases ahead of subsidy termination but slows thereafter, but in contrast to our analysis they focus on the case of a monopoly firm (Nagy et al. 2023). More recent work still has shown that anticipated policies lead to more investment, e.g. to "catch" a subsidy before its retraction (Hagspiel et al. 2025). Our study complements these findings, first by incorporating policy changes which are foreseeable as in the real-world cases highlighted above, and second by complementing the standard project-based analysis of individual expansion options with a competitive equilibrium involving free entry.[6] Finally, another stream of research has shown investment incentives are impacted by institutional and structural features of electricity markets, such as conditions governing how small producers connect to the grid (Castellini et al. 2021). Relative to the studies in this strand of the literature, our model focuses more on equilibrium effects and therefore relies on a more stylized representation.

The rest of the article is organized as follows: Section 2 presents the assumptions underlying our model, and Section 3 reviews some preliminary results regarding competitive investment which our analysis builds on. Section 4 develops our main results regarding the effect of a triggered tax policy on equilibrium investment. In Section 5 we extend this analysis to the case of a tax on operating costs which allows us to compare with more broadly targeted policies. Section 6 characterizes the equilibrium properties of the welfare that springs from the activity in the market. Section 7 offers some concluding remarks.

2 Model setup

We model a competitive market which evolves stochastically over time. Firms produce a homogeneous product whose demand at any time $t \in R_+$ is described by an inverse demand function

$$P_t = X_t f\left(Q_t\right) \tag{1}$$

where Q_t is the market quantity, P_t is the output price, and X_t is an exogenous shock. The function $f(Q)$ is downward-sloping and differentiable, with $\lim_{Q\to\infty} f(Q) = 0$. The

[6] Real options analyses of energy investments focus most often on individual investment decisions in a price-taking environment. Boomsma et al. (2012) for example state, p. 230: "we disregard any equilibrium considerations, and assume that the investment is sufficiently small not to affect either electricity and certificate prices (through the portfolio standards or quotas obligations)."

shock X_t follows a geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t dZ_t, \tag{2}$$

where μ is the drift parameter, $\sigma > 0$ the volatility parameter, and dZ_t is the increment of a standard Wiener process, uncorrelated across time and satisfying $E[dZ_t] = 0$ and $E\left[(dZ_t)^2\right] = dt$ at any t.

The demand (1) is met by a competitive industry comprised of a large number of symmetric, price-taking firms.[7] These firms produce using a constant returns to scale technology with a single input, capital. We normalize so one unit of capital produces one unit of output, at a constant operating cost which we denote c. At any time, a firm can acquire additional capital at a constant cost of k per unit, i.e. paying Yk yields ownership of Y additional units of capital that produce Y units of output at a flow cost Yc. Because of constant returns to scale, we can treat each infinitesimally small parcel of capital as a separate firm and thus view an infinitesimal increase in the capital stock as the entry of a new firm into the market. We assume that firms cannot suspend operations or exit once they become active.[8] The quantity process Q_t is therefore non-decreasing. All firms have the same constant discount rate r. To focus on the case where firm value is finite, we suppose $r > \mu$.

The industry is regulated by a government body which has the ability to alter one or more industry cost parameters. We suppose the regulator intervenes in the market when the industry's size reaches a predetermined trigger level $\overline{Q}$, similarly to the CSI policy discussed in the introduction. The reasoning would be entirely similar if the policy change were instead triggered by a critical level of the exogenous shock. At the trigger,

[7] We abstract away therefore from situations where there are frictions in the electricity market, e.g. if a grid operator exerts market power over small producers (see Castellini et al. 2021).

[8] These simplifications allow us to get closed-form solutions and are not essential to our results.

the regulator increases the capacity cost from k to k'. Such an increase can result from the withdrawal of an investment subsidy s, in which case the true capacity cost is k' and $k = k' - s$, or from the introduction of a tax on investment at a constant rate τ, in which case $k' = (1 + \tau)k$ and k is the true capacity cost. At any time t, the private cost of new capital is therefore

$$k_t = \begin{cases} k, & \text{if } Q_t \leq \overline{Q} \\ k', & \text{if } Q_t > \overline{Q}. \end{cases} \tag{3}$$

We take the effect of the policy change to be deterministic for simplicity throughout most of our discussion, but our main insights also hold if the policy change is uncertain so k' is a random variable, with $E[k'] > k$ so as to reflect an expected cost increase. Finally, we also discuss policies involving operating cost and contrast those that apply to new entrants only with those that apply to all firms.

3 Industry equilibrium without policy

To lay the groundwork for our analysis we begin by describing the industry equilibrium without any policy, i.e. if k and c remain at their initial levels forever. This corresponds to the situation in Leahy (1993), whose analysis we use to present the optimal investment policy and competitive equilibrium (readers familiar with this analysis may wish to skip to Section 4).

We start by characterizing the entry decision of an inactive firm. Firms are small so the current market quantity enters into this decision as a parameter from the perspective of an individual firm. Individual entry is driven by expected profitability over the lifetime of the asset, and therefore occurs at sufficiently large values of the exogenous shock. This leads us to conjecture that an inactive firm enters only if X_t hits a threshold, which we denote $X^*(Q)$.

To find this entry threshold, we let $V(Q, X)$ denote the normalized or unit value of an active firm, i.e. of a firm owning one unit of capital if the current demand state is X. A Bellman equation argument (see Appendix A.1) shows that for given Q, $V(Q, X)$ is a continuous and differentiable function of X with the generic form

$$V(Q, X) = Y(Q)X^\beta + \frac{X}{r-\mu} f(Q) - \frac{c}{r} \tag{4}$$

where the coefficient term $Y(Q)$ is determined further below and $\beta > 1$ is the upper root of the quadratic $\frac{1}{2}\sigma^2 x(x-1) + \mu x - r = 0$. Standard properties of geometric Brownian motion imply[9]

$$E_{X_0=X}\left[\int_0^\infty (X_t f(Q) - c)\, e^{-rt} dt\right] = \frac{X}{r-\mu} f(Q) - \frac{c}{r}, \tag{5}$$

so the last two terms of (4) represent the expected present value of the profit stream the firm would obtain if the total output were to stay forever at its current level. The first term in (4), $Y(Q)X^\beta$, is therefore the effect of future market entries on the expected present value of an active firm.

When the exogenous shock reaches the entry threshold, investment is bound to occur. At that moment, the value of an inactive firm satisfies two conditions. The first is the value matching condition, which is due to instantaneous competition between inactive firms:[10]

$$V(Q, X^*(Q)) = k. \tag{6}$$

[9] See e.g. Dixit and Pindyck (1994) p. 72.

[10] Since $V(Q, X^*(Q))$ is normalized to represent the value of owning one unit of capital, the value of a single firm (whose size is ΔQ) is $\Delta Q V(Q, X^*(Q))$. The basic form of the firm's value matching condition is accordingly $\Delta Q V(Q, X^*(Q)) = \Delta Q k$, which is equivalent to (6).

(6) states that the firm's net value from entering (i.e. the value of becoming an active for given Q) is zero. The second condition is the smooth pasting condition, which captures optimal investment timing:

$$V_X(Q, X^*(Q)) = 0. \tag{7}$$

(7) requires the values of active and inactive firms to have the same slope with respect to the exogenous shock when investment occurs (for given Q). Substituting the specific form (4) into the value matching and smooth pasting conditions yields a unique investment threshold,

$$X^*(Q) = \hat{\beta}\,(r - \mu)\,\frac{k + \frac{c}{r}}{f(Q)}, \tag{8}$$

where $\hat{\beta} = \frac{\beta}{\beta - 1} > 1$ is the uncertainty wedge which scales up the investment threshold relative to the net present value rule to account for the presence of uncertainty and irreversibility (Dixit and Pindyck 1994, Section 5.2). Because $f(Q)$ is downward-sloping, the threshold is a monotonically increasing function of market size.

Having characterized the individual entry decision, we next turn to equilibrium. The increasing threshold function suggests that the market quantity process Q_t is the outcome of a sequence of standard single-firm decision problems, each parametrized by the market quantity, whose thresholds increase gradually with successive entries. Figure 1 depicts the corresponding evolution of market quantity in (Q, X)-space. The solid curve plots the threshold function $X^*(Q)$ for all possible values of Q. This threshold function represents an upper reflecting barrier for the joint process (Q_t, X_t). At a point like A that lies below the barrier, small movements of the demand shock X_t shift the state up or down vertically but do not provoke a change in market size. As soon as the demand shock hits the current entry threshold $X^*(Q_A)$ however, investment occurs which increases market size and moves the state to the right. Greater market size raises the investment threshold,

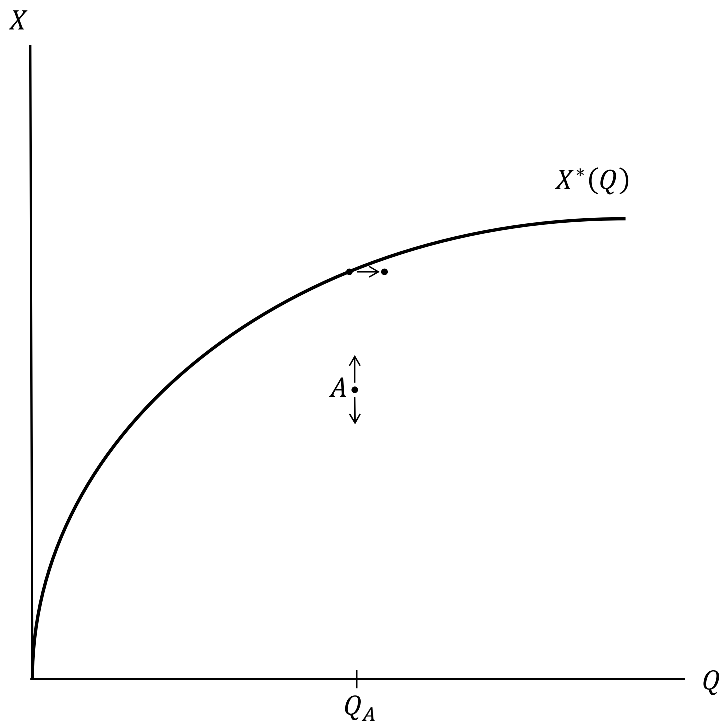


Figure 1: Entry process in the absence of policy. Below the threshold function $X^*(Q)$ the state (Q, X) evolves vertically according to the demand shock (point A), but whenever X_t hits $X^*(Q)$ the state moves rightwards.

so the industry lies again below the barrier and further investment is postponed until the next moment at which X_t hits the barrier. Market dynamics are therefore characterized by a gradual process of incremental steps.

We therefore conclude that (6) holds for any market size Q. Substituting the specific forms of $V(Q, X)$ and $X^*(Q)$ (equations (4) and (8)) and solving for the remaining unknown term $Y(Q)$ yields

$$Y^*(Q) = -\frac{k + \frac{c}{r}}{\beta - 1} \left(X^*(Q)\right)^{-\beta}. \tag{9}$$

This term is negative because future entries reduce the expected profit stream of active firms. Moreover, the higher is current industry capacity, the smaller is the effect of future entries. Intuitively this is because demand (specifically $f(Q)$) is downward-sloping, so higher capacity implies a longer expected time until the entry threshold is reached and new entries reduce the profit stream. Increases in the cost terms k and c have a similar effect.

By (1), the price process P_t follows a geometric Brownian motion with the same parameters as X_t at times where quantity is not changing. However the pattern of competitive investment we have described implies the price process has a cap, given by

$$P^* \equiv X^*(Q)f(Q) = \hat{\beta}\,(r-\mu)\left(k+\frac{c}{r}\right). \tag{10}$$

where the second equality follows from (8). Because $\hat{\beta} > 1$, the price cap (10) implies that whenever entry occurs, the output price exceeds long term unit cost.[11] Entering firms need this markup because even though the exogenous shock may vary favorably in the future, upward price changes are limited by further firm entries. The term $Y^*(Q)$ therefore corrects the perpetual revenue flow of active firms for the asymmetric price risk that they face due to the price process cap.

Figure 2 illustrates this endogenous truncation of the price process due to entry. The dashed line shows an untruncated path of P_t in the hypothetical case where there is no additional entry, so $Q_t = Q$ is constant. The solid line shows the actual (capped) price process. Due to the fluctuations in the shock X_t, this process moves up or down at each instant just like the untruncated process, *except* when it is about to exceed P^*. At those instants, firm entry raises Q and prevents P_t from crossing the price barrier P^*.

[11] More precisely the expected discounted revenue stream exceeds that the capitalized unit cost, i.e. $\frac{P^*}{r-\mu} > k + \frac{c}{r}$.

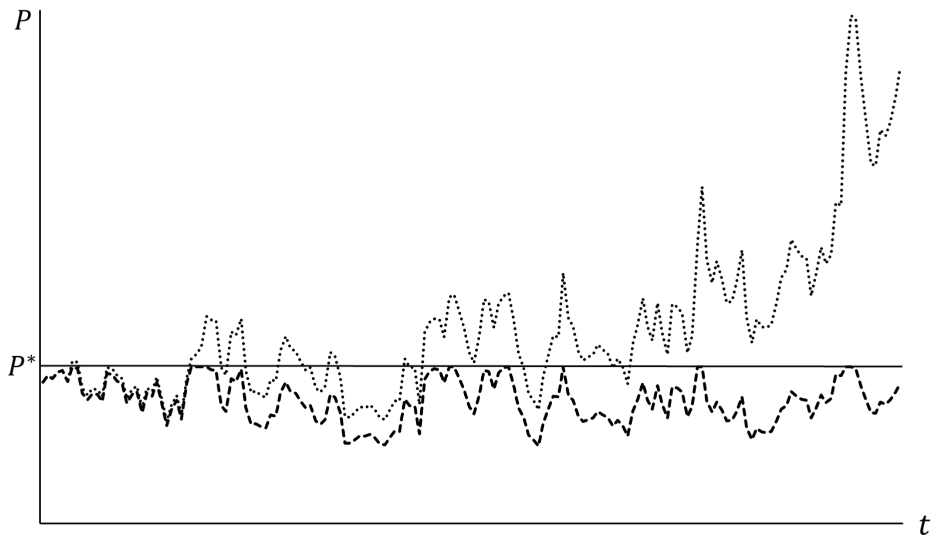


Figure 2: Price process truncation. Firm entry occurs whenever P_t hits P^* resulting in a capped price process (dashed plot). The dotted plot is the unconstrained price process, i.e. without accounting for equilibrium entry.

The interpretation of industry equilibrium as a succession of individual investment problems in this section applies because the equilibrium changes in market quantity are incremental, so inactive firms can reason at any point in time as if Q were constant (and in particular when determining the timing of entry individually). But the equilibrium dynamics are more involved if firms anticipate a policy change, as we show in the next section.

4 Industry equilibrium with a triggered policy

Having described the competitive equilibrium benchmark in the previous section we now incorporate triggered policy into the analysis. The policy intervention we study in this section is an increase in unit capacity cost to $k' > k$ which takes place once industry capacity reaches a predetermined level $\overline{Q}$, so the capacity cost follows (3). The magnitude of the cost increase and its timing are both known to firms. We will show in this section that equilibrium investment exhibits the following pattern:

- as long as the market quantity is smaller than a critical level $\widetilde{Q}$ which we characterize below, firm entry involves a gradual process regulated by the threshold function $X^*(Q)$ in (8);
- when the market quantity reaches $\widetilde{Q}$, there is a run – a massive entry of firms – which takes quantity immediately up to $\overline{Q}$;
- from then on, firm entry again involves a gradual process regulated by a higher threshold function $X^{**}(Q) > X^*(Q)$ which is based on the higher capacity cost k' rather than on k.

Figure 3 illustrates this pattern in the state space. For market quantities below $\widetilde{Q}$ or above $\overline{Q}$ (like Q_A and $Q_{A'}$), the state moves up or down vertically for small changes in the exogenous shock (points A and A'). When the exogenous shock reaches the threshold function (at $X^*(Q_A)$ and $X^{**}(Q_{A'})$ respectively), the state moves to the right incrementally. At $\widetilde{Q}$ however, the state moves up or down vertically for small changes in the exogenous shock (e.g., point $\widetilde{A}$) but when the exogenous shock reaches $\overline{X}$, a mass of investment $\overline{Q} - \widetilde{Q}$ occurs immediately, indicated by the black arrow.

The rest of the section is organized as follows. In Subsection 4.1, we derive the investment policy of firms and find its main parameters, in particular the threshold $\overline{X}$ that triggers the competitive run. In Subsection 4.2, we explain the nature and causes of the run. In Subsection 4.3, we explore the determinants of the size and timing of the run. In subsection 4.4 we show results for the case where the policy change refers to a policy change impacting production cost and therefore affects all firms, including those active before the policy change.

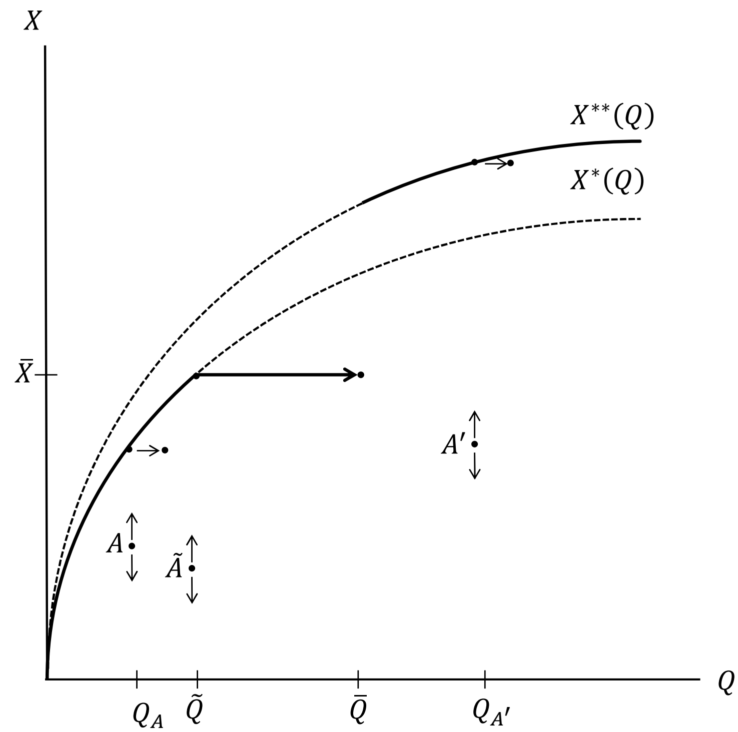


Figure 3: Entry process with an anticipated fixed cost increase at $\overline{Q}$: the barrier is $X^*(Q)$ up to a quantity $\widetilde{Q} < \overline{Q}$ but once the exogenous shock reaches $X^*(\widetilde{Q})$ there is a mass of investment $\overline{Q} - \widetilde{Q}$, after which the barrier shifts up to $X^{**}(Q)$.

4.1 Equilibrium investment

To establish the pattern described above we analyze the industry equilibrium in two steps, starting with market quantity where the policy change has already taken place and then addressing market quantity where the policy change has not yet occurred.

For $Q > \overline{Q}$, no further policy changes are expected. The analysis is therefore identical to the preceding section, with k' replacing k. The active firm value has the same generic form (4), and the value matching and smooth pasting conditions (6) and (7) imply an entry threshold

$$X^{**}(Q) = \hat{\beta}\,(r-\mu)\,\frac{k' + \frac{c}{r}}{f(Q)} \tag{11}$$

and coefficient term

$$Y^{**}(Q) = -\frac{k' + \frac{c}{r}}{\beta - 1}\,\left(X^{**}(Q)\right)^{-\beta}. \tag{12}$$

For $Q \leq \overline{Q}$, the fixed cost is expected to increase once Q_t reaches $\overline{Q}$. Active firm value still has the generic form (4) because the effect of future changes is embedded within the term $Y(Q)$. Competitive entry still prevents expected profits from exceeding their normal level, so the value matching condition (6) continues to hold (this is also because the policy change is known in advance). However the smooth pasting condition must be replaced by a more general condition, to which we turn next.

Consider market sizes at the rightmost end of the relevant range, i.e. where the quantity is "just below" $\overline{Q}$. At such points, idle firms know that the policy change will take place immediately if they enter. The value of the future entries term is accordingly

$$Y\,(Q) = Y^{**}(\overline{Q}). \tag{13}$$

Thus, at $\overline{Q}$, the value matching condition has the specific form

$$Y^{**}(\overline{Q})\overline{X}^{\beta} + \frac{\overline{X}}{r-\mu} f(\overline{Q}) - \frac{c}{r} = k \tag{14}$$

where $\overline{X}$ denotes the value of the entry threshold (which need not be the value of the threshold function) and $Y^{**}(\overline{Q})$ is based on (12). (14) is an equation with a single unknown, $\overline{X}$, and we prove in Appendix A.2 that this equation admits a unique root in the relevant range $\left(0, X^{*}(\overline{Q})\right)$. (14) therefore provides us with the entry threshold for quantities lying "just below" $\overline{Q}$.

To determine the entire threshold function to the left of $\overline{Q}$, observe first that at an entry threshold $X^*(Q)$ the value function must satisfy

$$V\left(Q, X^*\left(Q+\Delta Q\right)\right) = V\left(Q+\Delta Q, X^*\left(Q+\Delta Q\right)\right). \tag{15}$$

(15) asserts that firm value must remain unchanged when the shock crosses the relevant investment threshold, where quantity increases by an increment ΔQ with probability one. Dividing by ΔQ and taking the limit as $\Delta Q \to 0$ yields

$$V_Q\left(Q, X^*\left(Q\right)\right) = 0. \tag{16}$$

Recall that due to free entry, the value matching condition holds for all levels of Q. Differentiating (6) totally therefore gives

$$\frac{dV\left(Q, X^*\left(Q\right)\right)}{dQ} = 0. \tag{17}$$

Together, (16) and (17) imply the following general optimality condition

$$V_X\left(Q, X^*\left(Q\right)\right)\frac{dX^*(Q)}{dQ} = 0. \tag{18}$$

(18) holds if either $V_X\left(Q, X^*\left(Q\right)\right) = 0$ or $\frac{dX^*(Q)}{dQ} = 0$. The first of these alternatives is the familiar smooth pasting condition (7) which leads to the gradual process of quantity increases described in Section 3. The second alternative corresponds to a situation where the entry threshold $X^*\left(Q\right)$ does not increase with Q. In this situation, when the process X_t reaches the threshold, firms enter without raising $X^*\left(Q\right)$ on the margin, implying that a mass of investment occurs.

We next argue that for quantities "just below" $\overline{Q}$ it is the second of these alternatives which prevails. The argument is by contradiction. Suppose that the smooth pasting condition held in this range. Then, by (4), (6), and (7), the value for $Y\left(\overline{Q}\right)$ would be obtained by evaluating (9) at $\overline{Q}$. But this value contradicts (13). The smooth pasting condition therefore does not hold, implying that $V_X\left(Q, X^*\left(Q\right)\right)$ is positive. It follows that the threshold is constant just to the left of $\overline{Q}$, and by continuity it is also constant over a neighborhood $\left(\widetilde{Q}, \overline{Q}\right)$. Over this interval the threshold is therefore given by the solution $\overline{X}$ to (14). Finally the smooth pasting condition holds to the left of $\widetilde{Q}$, implying that $\widetilde{Q}$ can be found by

$$X^*\left(\widetilde{Q}\right) = \overline{X}. \tag{19}$$

The following proposition summarizes the main findings in this section:

Proposition 1 (Policy-triggered competitive run) *The equilibrium path of industry capacity involves an incremental investment process along the threshold function $X^*\left(Q\right)$ for $Q < \widetilde{Q}$, a mass of investment $\overline{Q} - \widetilde{Q}$ at the trigger $\overline{X} \in \left(0, X^*\left(\overline{Q}\right)\right)$, and an incremental investment process along the threshold function $X^{**}\left(Q\right)$ for $Q > \overline{Q}$.*

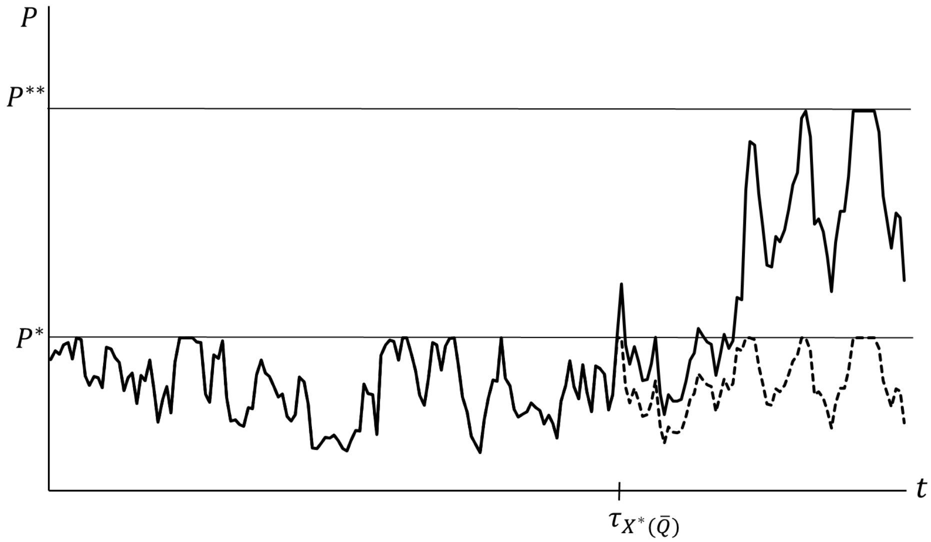
A distinctive feature in our model is that a second phase of gradual investment takes place after the policy change. This feature represents a key difference with situations like caps on industry capacity that have been studied in the literature, where investment invariably ceases after $\overline{Q}$. Because of this subsequent investment, firms entering before the policy shock must account for ongoing entries even after the policy change, whose effect on their value through the $Y^{**}(Q)$ coefficient is nonzero. This consideration leads to additional insights which we next turn to.

4.2 Explaining the run

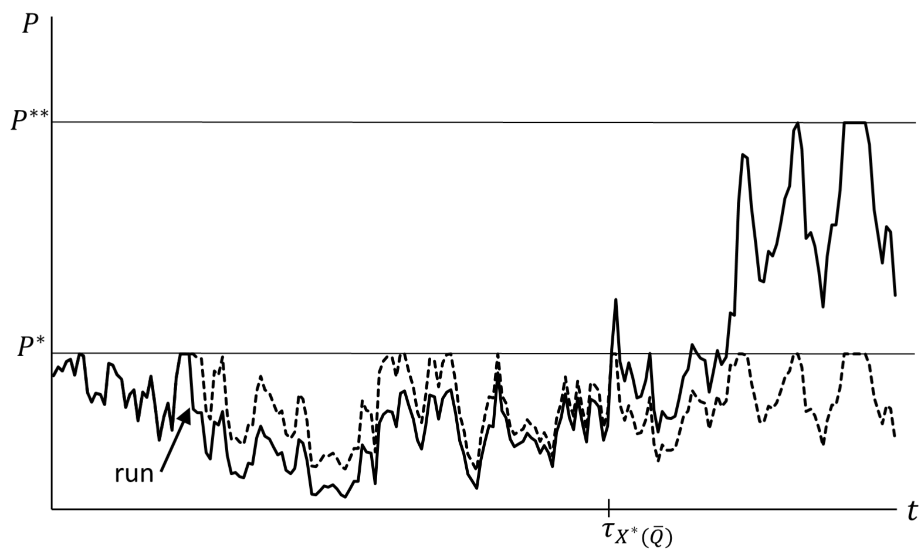
To put it briefly, the emergence of the run in a competitive equilibrium is due to the following logic: the increase in entry cost from k to k' implies that the entry threshold shifts up after the policy change so future entries are expected to occur later, which raises the profit stream of currently active firms *ex ante*.

To better appreciate the role that future entries play in generating a run, the first panel of Figure 4 illustrates the price process in an industry where firms do not expect any policy change. There is therefore policy surprise when Q_t attains $\overline{Q}$, at the hitting time $\tau_{X^*(\overline{Q})} = \inf\left\{t \geq 0, X_t \geq X^*(\overline{Q})\right\}$. The dashed plot shows what the price process would be with competitive entry if there were no policy change. The solid plot shows the price process with the policy surprise. Up to $\tau_{X^*(\overline{Q})}$, the price process is truncated at P^*. After the fixed cost increases, the truncation shifts up to P^{**} which has the same form as (10) but is premised on the higher entry cost k' instead of k. All the while, the price process lies between these two extremes, i.e. the unconstrained process (see preceding figure) and the process without the policy change given by the dashed plot.

If, as in our model, firms anticipate the policy change, the equilibrium dynamic has a very different pattern which is illustrated in the second panel of Figure 4. Firms in this case expect the increase in fixed cost to improve the price process by slowing down future entries. Left unchecked, this mechanism would lead to above-normal profits. Ahead of the policy change therefore, inactive firms lower their entry threshold, generating the run, and Q_t first attains $\overline{Q}$ at a smaller hitting time $\tau_{\overline{X}} < \tau_{X^*(\overline{Q})}$, where $\tau_{\overline{X}} = \inf\left\{t \geq 0, X_t \geq \overline{X}\right\}$. The mass of entry brings about a sharp price drop which resets the truncated process so that it starts from a lower level (relative to the case of a policy surprise), eliminating the potential for above normal profits. The solid plot in the figure shows the price process which is first truncated at P^*. Relative to the policy surprise case, the truncation shifts



(a) Unexpected policy change



(b) Anticipated policy change

Figure 4: The price process with unexpected vs. anticipated policy change. In both panels the dashed line shows the price process without the policy change and the continuous line shows the price process with the policy change; the latter two lines overlap up until the policy change and separated from then on as the competitive truncation rises to P^{**}. In panel 4a the policy change occurs at $\tau_{X^*(\overline{Q})}$ and the surprisingly improved price process from then on makes the existing firms enjoy supernormal profits. In panel 4b the anticipated policy change creates a run prior to $\tau_{X^*(\overline{Q})}$, the price drops significantly, making (for a while) the price process lower than it would have been if not for the run, and thus preventing the supernormal profits that spring from the improved price process.

up to P^{**} earlier, though the bound is first reached at the same hitting time $\tau_{X^{**}(\overline{Q})} = \inf\left\{t \geq 0, X_t \geq X^{**}(\overline{Q})\right\}$in both cases.

A striking feature in Figure 4 is that the price process with an anticipated increase first reaches P^* at the same time $\tau_{X^*(\overline{Q})}$ as it does with the unexpected policy chage. To understand why, recall first that the demand shock X_t is an exogenous process which has the same trajectory in both cases, so the time at which it hits $X^*(\overline{Q})$ for the first time is the same regardless of whether the policy is expected or not. At that instant, the quantity is also the same in the two cases, i.e. $\overline{Q}$. The difference lies in the underlying quantity process ahead of the change. With an unanticipated policy change the quantity results from an incremental process of quantity additions whenever the threshold function is hit. In the anticipated policy case, it is reached when the run occurs, i.e. when the demand shock hits $\overline{X} < X^*(\overline{Q})$. The run pushes the market price down below P^*, enough so that from that instant on and up until $\tau_{X^*(\overline{Q})}$ the quantity remains at $\overline{Q}$. Thus, the competitive run must "reset" the price process just enough so that it meets up again with the price process from the unannounced policy at $\tau_{X^*(\overline{Q})}$.

We now are in a position to defend our earlier claim that the main insights of our analysis carry over to situations of uncertainty with respect to the magnitude of the policy change. Suppose that instead of the specification (3) when the market reaches $\overline{Q}$ the fixed cost increases to k' with probability ρ and remains the same with probability $1-\rho$. Then we can reason exactly as we have up to now, except that the effect of future entries on active firms is now replaced by an expected effect term, $E\left[Y\left(\overline{Q}\right)\right] = (1-\rho)Y^*\left(\overline{Q}\right) + \rho Y^{**}\left(\overline{Q}\right)$. For any positive probability $\rho > 0$, this expected effect term lies above the level without the policy, $Y^*\left(\overline{Q}\right)$. A policy which is known to occur at $\overline{Q}$ but which has uncertain magnitude therefore still improves the future entry process for active firms and leads to a competitive run by similar logic.

4.3 Comparative statics

To better apprehend the properties of competitive runs, we present several comparative static results regarding their timing and size. The first result is about how the size of the policy change affects the run. The underlying logic here is similar to what we explained above: *ceteris paribus*, the greater is the increase in entry cost, the less frequent are future entries and the higher is the potential positive profit, which attracts firms to enter market earlier.

Proposition 2 (Magnitude of policy change) *An increase in the magnitude of the policy change ($k' - k$) hastens the run by lowering the trigger $\overline{X}$ at which the run occurs, and also increases the magnitude of the run ($\overline{Q} - \widetilde{Q}$).*

Proof. Differentiating (14) (for the effect of k', the reasoning for k is similar) and rearranging gives

$$\frac{d\overline{X}}{dk'} = -\frac{\overline{X}^{\beta+1}\frac{dY^{**}}{dk'}}{\beta Y^{**}\left(\overline{Q}\right)\overline{X}^{\beta} + \frac{\overline{X}}{r-\mu}f\left(\overline{Q}\right)}. \tag{20}$$

Because $\frac{dY^{**}}{dk'} > 0$ this expression has the opposite sign of its denominator. Using (14) to substitute for $Y^{**}\left(\overline{Q}\right)\overline{X}^{\beta}$ and then (8) to substitute for $k + \frac{c}{r}$, the denominator can be expressed as

$$\begin{aligned} \beta Y^{**}\left(\overline{Q}\right)\overline{X}^{\beta} + \frac{\overline{X}}{r-\mu}f\left(\overline{Q}\right) &= \beta\left(k + \frac{c}{r}\right) - (\beta - 1)\frac{\overline{X}}{r-\mu}f\left(\overline{Q}\right) \\ &= (\beta - 1)\frac{f\left(\overline{Q}\right)}{r-\mu}\left(X^{*}\left(\overline{Q}\right) - \overline{X}\right), \end{aligned} \tag{21}$$

which is positive because $\overline{X} < X^{*}\left(\overline{Q}\right)$. This establishes that $\overline{X}$ decreases with k'. The increase in $\overline{Q} - \widetilde{Q}$ follows from $\frac{d\overline{X}}{dk'} < 0$ and (19). □

As the fixed cost k' becomes arbitrarily large, the coefficient $Y^{**}\left(\overline{Q}\right)$ representing the effect of future entries accordingly becomes arbitrarily small. In the limit, as $k' \to \infty$ the entry costbecomes completely prohibitive and there are no further entries, implying $Y^{**}\left(\overline{Q}\right) = 0$. The threshold $\overline{X}$ at which the run occurs therefore has a lower bound which, by (14), has an explicit expression $\overline{X} = \frac{(r-\mu)\left(k+\frac{c}{r}\right)}{f(\overline{Q})}$. In fact, this limiting case of an infinite entry cost is just the threshold for runs with a production cap studied elsewhere in the literature.

The effect of greater uncertainty on the run is more involved (see Appendix A.3). Greater volatility tends to increase option values and lower the difference between active and inactive firm values, so the threshold of the run, $\overline{X}$, increases. This increase in $\overline{X}$ raises the right hand side of (19) and tends to increase $\widetilde{Q}$ because, by (8), the threshold function is increasing. Yet, on the other hand, rising volatility shifts up the entire entire threshold function, which tends to decrease $\widetilde{Q}$ all things equal. It turns out that it is the second of these effects which dominates, as the following proposition establishes.

Proposition 3 (Greater market uncertainty) *An increase in volatility delays the run by raising the trigger $\overline{X}$ at which the run occurs and increases the magnitude of the run $(\overline{Q} - \widetilde{Q})$.*

Finally, if there is uncertainty about the magnitude of the policy change so k' is random, the effect of greater policy uncertainty on equilibrium investment can be captured by considering a mean-preserving spread, as described by the next proposition.

Proposition 4 (Policy magnitude uncertainty) *An mean-preserving increase in the dispersion of k' delays the run by raising the trigger $\overline{X}$ at which the run occurs and lowers the magnitude of the run $(\overline{Q} - \widetilde{Q})$.*

Proof. First, if k' is random the analysis follows the same lines as with a deterministic policy, leading to an analogous condition to (14) defining $\overline{X}$,

$$E\left[Y^{**}(\overline{Q})\right]\overline{X}^{\beta}+\frac{\overline{X}}{r-\mu}f(\overline{Q})-\frac{c}{r}=k, \tag{22}$$

where $E\left[Y^{**}(\overline{Q})\right]$ replaces $Y^{**}(\overline{Q})$.

Next observe that by (12) $Y^{**}(\overline{Q})$ is a concave function of $k'+\frac{c}{r}$. If k' is random, it follows from Jensen's inequality that $E\left[Y^{**}(\overline{Q})\right] < Y^{**}(\overline{Q})\big|_{E[k']}$ where the last term denotes $Y^{**}(\overline{Q})$ evaluated at $E[k']$. A mean-preserving spread therefore decreases $E\left[Y^{**}(\overline{Q})\right]$.

Differentiating (22) and rearranging gives

$$\frac{d\overline{X}}{dE\left[Y^{**}(\overline{Q})\right]}=-\frac{\overline{X}^{\beta+1}}{\beta E\left[Y^{**}(\overline{Q})\right]\overline{X}^{\beta}+\frac{\overline{X}}{r-\mu}f\left(\overline{Q}\right)}. \tag{23}$$

We have established in the proof of Proposition 2 that the denominator is positive, so $\frac{d\overline{X}}{dE\left[Y^{**}(\overline{Q})\right]}<0$. Because $E\left[Y^{**}(\overline{Q})\right]$ decreases with a mean-preserving spread, it follows that $\overline{X}$ increases with the dispersion of k'. Finally the increase in $\overline{Q}-\widetilde{Q}$ follows from (19). □

4.4 Variable cost policy

To pave the way for the next section, observe that our analysis need not be limited to an increase in the entry cost. For example, the tax increase could also be levied on the flow of operating cost c. In this case equations (11) to (17) would be the same, with c' replacing c and k instead of k'. More generally, any policy change (and for that matter any technological change) which causes the threshold function of entering firms to jump upward can affect equilibrium dynamics the same way. I.e., if entering firms faced an additional tax at rate τ so their effective price is $(1-\tau)Xf(Q)$, the industry would face a similar upward jump in the investment threshold function at the market size $\overline{Q}$, slowing the process of future entries and sparking a run ahead of the policy change. In the next section we study one such variation which leads to a slightly different argument regarding active and inactive firms.

5 Industry equilibrium with policy affecting all firms

In this section we study an alternative policy intervention which involves an increase of the operating cost to $c' > c$. The increase in cost is triggered once industry capacity reaches a predetermined level $\overline{Q}$. The policy and its timing are both known to firms. In contrast with Section 4 we suppose that the cost increase applies not just to new firms but universally, including to those firms which were active before the change. An example of such a policy in renewable energy markets is a feed-in tariff, i.e. a measure where firms receive an additional fixed payment per unit of output. In our framework such a measure can be construed as a negative operating cost, so that its removal effectively represents an increase in operating cost.

Although the formal expressions differ slightly, our analysis in this section establishes that the equilibrium pattern of investment under these assumptions consists of the same three phases represented in Figure 3, with initial investment based upon an entry threshold function $X^*(Q)$ reflecting a low operating cost, a competitive run which is triggered at a quantity $\widetilde{Q} < \overline{Q}$ ahead of the policy change and takes quantity immediately to $\overline{Q}$, and from then on investment based on an entry threshold function $X^{**}(Q)$ which reflects the higher operating cost of c' instead of c. The fact that we get the same pattern of equilibrium investment serves to underscore the insight in Section 4.2, as the run here cannot be attributed to any attempt by some firms to secure more favorable operating conditions than those firms that invest later. Instead, as explained above, the run stems from the less competitive future entry process induced by the higher cost.

We can again analyze the industry equilibrium with policy intervention in two steps. For $Q > \overline{Q}$, no further policy changes are expected and the analysis is identical to Section 3 with c' replacing c, yielding the generic firm value expression

$$V(Q, X) = Y(Q)X^{\beta} + \frac{X}{r-\mu} f(Q) - \frac{c'}{r}, \tag{24}$$

and applying the value matching and smooth pasting conditions (6) and (7) gives the entry threshold

$$X^{**}(Q) = \hat{\beta}\,(r-\mu)\,\frac{k + \frac{c'}{r}}{f(Q)} \tag{25}$$

and coefficient term

$$Y^{**}(Q) = -\frac{k + \frac{c'}{r}}{\beta - 1}\,(X^{**}(Q))^{-\beta}\,. \tag{26}$$

Next we turn to the range $Q \leq \overline{Q}$. The general form of the value function is this range is still given by (4), with a term $Y(Q)X^{\beta}$ representing the effect of future entries on the active firm value. This term now has two parts, because the policy change affects active firms in two ways:

- future entries occur at a higher threshold, and
- operating cost increases by $c' - c$.

The sum of these two effects at the market quantity $\overline{Q}$ is

$$Y(\overline{Q})X^{\beta} = Y^{**}(\overline{Q})X^{\beta} - \frac{c'-c}{r}\left(\frac{X}{X^{**}(\overline{Q})}\right)^{\beta}. \tag{27}$$

In (27), the first term is the effect of the more favorable entry process in (26). The second term is the increase in operating cost that active firms expect to experience once market quantity exceeds $\overline{Q}$, which happens at the threshold $X^{**}(\overline{Q})$.[12] Applying (26) to (27) and simplifying gives

$$\hat{Y}^{**}(\overline{Q}) = -\frac{k + \beta\frac{c'}{r} - (\beta - 1)\frac{c}{r}}{\beta - 1}\left(X^{**}(\overline{Q})\right)^{-\beta}. \tag{28}$$

Similar reasoning to the preceding section establishes that a run occurs at a threshold $\overline{Q}$ which is the unique solution in $\left(0, X^{*}(\overline{Q})\right)$ to[13]

$$\hat{Y}^{**}(\overline{Q})\overline{X}^{\beta} + \frac{\overline{X}}{r - \mu}f(\overline{Q}) - \frac{c}{r} = k. \tag{29}$$

The industry capacity level at which the run occurs is then determined again by (19).

Comparing with the previous section allows us to determine how a broadly applied policy affects equilibrium investment relatively to one with more narrow scope. In the following proposition, we establish that the run is triggered at a higher threshold and has a smaller magnitude. To facilitate comparison, we adapt the analysis of the preceding section by replacing k' with k and c with c' in (11) and (12) to cover the case of a policy affecting operating cost rather than fixed cost, as discussed in Section 4.4. The intuition

[12] This term results from the following calculation. Letting $\tau_{X^{**}(\overline{Q})}$ denote the first time the exogenous shock reaches $X^{**}(\overline{Q})$, the present value of the operating cost increase is

$$E\left[\int_{\tau_{X^{**}(\overline{Q})}}^{\infty}(c' - c)\,e^{-rt}dt\right] = \frac{c' - c}{r}E_X\left[e^{-r\tau_{X^{**}(\overline{Q})}}\right] = \frac{c' - c}{r}\left(\frac{X}{X^{**}(\overline{Q})}\right)^{\beta}$$

(see footnote 9).

[13] Arguing by contradiction, the smooth pasting condition cannot hold at $\overline{Q}$. If it did, then (4), (6), and (7) would imply an expression (9) for $Y(\overline{Q})$, contradicting (28). To verify the difference between these expressions, observe that $Y(\overline{Q})$ converges to $Y^{*}(\overline{Q})$ as c' goes to c, but straightforward differentiation of (28) gives

$$\begin{aligned}\frac{dY(\overline{Q})}{dc'} &= -\frac{\hat{\beta}}{r}\left(X^{**}(\overline{Q})\right)^{-\beta} + \beta\frac{k + \beta\frac{c'}{r} - (\beta - 1)\frac{c}{r}}{\beta - 1}\frac{\hat{\beta}}{r}\frac{r - \mu}{f(\overline{Q})}\left(X^{**}(\overline{Q})\right)^{-\beta - 1} \\ &= \frac{\hat{\beta}^{2}}{r^{2}}\frac{r - \mu}{f(\overline{Q})}(\beta - 1)(c' - c)\left(X^{**}(\overline{Q})\right)^{-\beta - 1} > 0.\end{aligned}$$

behind the proposition is simply that the more favorable price process active firms enjoy as the result of the policy is moderated by the cost increase that they are now also subjected to, which lessens the attractiveness of entering ahead of the policy.

Proposition 5 (Universal policy) *Extending the cost increase so it applies to all firms delays the run by raising the trigger $\overline{X}$ at which the run occurs, and the magnitude of the run $(\overline{Q} - \widetilde{Q})$ is smaller than if the policy applies only to new entrants.*

Proof. Note from (28) that $\hat{Y}^{**}(\overline{Q}) < Y^{**}(\overline{Q})$, i.e. future entries lower profit more if the cost increase applies to all firms rather than only to new entrants. Implicit differentiation of (29) gives

$$\begin{aligned} \frac{d\overline{X}}{d\hat{Y}^{**}(\overline{Q})} &= -\frac{\overline{X}^{\beta}}{\beta\hat{Y}^{**}(\overline{Q})\overline{X}^{\beta-1} + \frac{f(\overline{Q})}{r-\mu}} \\ &= -\frac{\overline{X}^{\beta+1}}{\beta\left(k + \frac{c}{r}\right) - (\beta-1)\frac{f(\overline{Q})}{r-\mu}\overline{X}} = -\frac{(r-\mu)\overline{X}^{\beta}}{\left(\frac{X^*(\overline{Q})}{\overline{X}} - 1\right)(\beta-1)\frac{f(\overline{Q})}{r-\mu}\overline{X}} < 0 \quad (30) \end{aligned}$$

where the second equality follows from (29), the third equality from (8), and the inequality follows from $X^*(\overline{Q}) > \overline{X}$.

Now note from (28) that $\hat{Y}^{**}(\overline{Q}) < Y^{**}(\overline{Q})$, i.e. the effect of future entries is smaller if the cost is raised for all firms than if it is raised for new entrants only. Based on that, and on the negative sign of the derivative in (30), it follows that $\overline{X}$ is higher in the case where the cost is raised for all firms and the effect of new entries is given by $\hat{Y}^{**}(\overline{Q})$ rather than $Y^{**}(\overline{Q})$. Because the threshold function $X^*(Q)$ is unchanged, by (19) this implies that $\widetilde{Q}$ increases. $\square$

6 Normative analysis

We next turn to the welfare effects of competitive runs in order to address policy implications. For this we take the policy intervention studied in Section 4 where the fixed cost that firms face increases from k to k' when industry capacity reaches a threshold level $\overline{Q}$, either because a tax is introduced or because a subsidy is removed. For the sake of simplicity we limit our analysis to the direct welfare effect of the run, without accounting for any market failure that might have prompted the policy in the first place. Let k^* denote the social cost of a unit of capital. Then we have $k = k^* - s$ and $k' = k^*$ in the case of a subsidy withdrawal which we focus on in the example later in the section, or $k = k^*$ and $k' = (1 + \tau)k^*$ in the case of a new tax at rate τ. In either case, the subsidy s or unit tax τk^* just represent transfers between firms and the government, so they do not enter into the welfare calculation directly. We assume that the social discount rate is the same as for firms, r. We take a constant elasticity demand specification:

$$P_t = \frac{X_t}{Q_t^{\gamma}} \tag{31}$$

where $\gamma < 1$.

We start by looking at the range $Q > \overline{Q}$. In this range, social welfare is

$$W^{**}(Q, X) = E_X \left[\int_0^{\infty} \left(\int_0^{Q_t} \left(X_t q^{-\gamma} - c \right) dq \right) e^{-rt} dt - \int_0^{\infty} k^* e^{-rt} dQ_t \right]. \tag{32}$$

An analysis which is similar to the one in Section 3 for the value of the firm establishes that $W^{**}(Q, X)$ is a continuous and differentiable function of X over the inaction region with the following general form:

$$W^{**}(Q,X) = Z^{**}(Q)X^{\beta} + \frac{X}{r-\mu}\frac{Q^{1-\gamma}}{1-\gamma} - \frac{c}{r}Q, \tag{33}$$

where $Z^{**}(Q)$ is found below via boundary conditions. The second and third terms on the right-hand side represent the expected value of the welfare stream if industry capacity were to remain perpetually at its current level. Therefore $Z^{**}(Q)X^{\beta}$ represents the expected effect of future entries on welfare. To find $Z^{**}(Q)$, we use the following condition

$$W^{**}_{Q}(Q, X^{**}(Q)) = k^{*} \tag{34}$$

(see Dixit and Pindyck 1994, p. 286). (34) follows from the definition of $X^{**}(Q)$ as an entry threshold, so that whenever X crosses $X^{**}(Q)$ the market quantity immediately increases by an infinitesimally small amount dQ with probability 1, at a cost $k^{*}dQ$. Therefore,

$$W^{**}(Q, X^{**}(Q+dQ)) = W^{**}(Q+dQ, X^{**}(Q+dQ)) - dQk^{*}. \tag{35}$$

Dividing both sides by dQ and taking the limit as $dQ \to 0$ gives (34). Applying (11) and (33) to (34) yields:

$$Z^{**\prime}(Q) = -\frac{\hat{\beta}\left(k' + \frac{c}{r}\right) - k^{*} - \frac{c}{r}}{\left(\hat{\beta}(r-\mu)\left(k' + \frac{c}{r}\right)Q^{\gamma}\right)^{\beta}}. \tag{36}$$

Because $Z^{**}(Q)$ is the social value of future entries, it satisfies $\lim_{Q\longrightarrow\infty} Z^{**}(Q)$. For this condition to hold, i.e. in order for welfare to converge, we assume hereafter that $\beta > \frac{1}{\gamma}$.[14]

[14]This restriction stems from the isoelastic specification, see for example Dixit and Pindyck (1994), p. 365.

Integrating and rearranging gives:

$$Z^{**}(Q) = \frac{\left(\hat{\beta}\left(k' + \frac{c}{r}\right) - k^* - \frac{c}{r}\right) Q}{\left(\beta\gamma - 1\right)\left(X^{**}(Q)\right)^{\beta}}. \tag{37}$$

Then the constant of integration takes the value zero because no further entries occur as capacity becomes arbitrarily large, implying $\lim_{Q\to\infty} Z^{**}(Q) = 0$.

Next, in the range $Q \leq \overline{Q}$, a similar analysis shows that social welfare has a general form

$$W^*\left(Q, X\right) = Z^*(Q)X^{\beta} + \frac{X}{r-\mu}\frac{Q^{1-\gamma}}{1-\gamma} - \frac{c}{r}Q \tag{38}$$

with

$$Z^*(Q) = \frac{\left(\hat{\beta}\left(k + \frac{c}{r}\right) - k^* - \frac{c}{r}\right) Q}{\left(\beta\gamma - 1\right)\left(X^*(Q)\right)^{\beta}} + C^*, \tag{39}$$

where the integration constant C^* is nonzero and determined by the following boundary condition:

$$W^*\left(\widetilde{Q}, \overline{X}\right) = W^{**}\left(\overline{Q}, \overline{X}\right) - \left(\overline{Q} - \widetilde{Q}\right) k^*. \tag{40}$$

This boundary condition is based on the result that when the quantity is $\widetilde{Q}$ and the exogenous shock hits the value $\overline{X}$ defined in (14), a run occurs with probability 1 that immediately raises quantity at the instantaneous cost $\left(\overline{Q} - \widetilde{Q}\right)k^*$.

To illustrate the effect that a run can have in industry investment and welfare, we return to the case of subsidy withdrawals discussed in the introduction. We use the following set of parameter values for the discount rate and market process: $r = 0.04$, $\mu = 0$, and $\sigma = 0.08$ (low volatility) or 0.12 (high volatility). To reflect renewable energy operating costs which are typically low or negligible, we set $c = 0$. We set the baseline investment cost at $k^* = k' = 1,000$, and consider a subsidy of either 250 (small subsidy) or 500 (large subsidy), so $k = 750$ or 500 respectively. The capacity threshold at which

Table 1: Market dynamics and welfare with a competitive run versus unanticipated policy benchmark for different subsidy retraction and volatility levels.

(a) Low volatility case ($\sigma = 0.08$)

Subsidy (s)	250	500
Run size ($\overline{Q} - \widetilde{Q}$)	43.81	48.96
Run threshold ($\overline{X}$)	199.25	127.83
Benchmark threshold ($X^* \left(\overline{Q}\right)$)	250.93	167.29
Welfare with run ($W^* \left(\widetilde{Q}, \overline{X}\right)$)	$94,055$	$33,896$
Benchmark welfare, unannounced policy ($W^U \left(\widetilde{Q}, \overline{X}\right)$)	$99,236$	$47,976$

(b) High volatility case ($\sigma = 0.12$)

Subsidy (s)	250	500
Run size ($\overline{Q} - \widetilde{Q}$)	53.15	60.88
Run threshold ($\overline{X}$)	212.97	132.09
Benchmark threshold ($X^* \left(\overline{Q}\right)$)	288.42	192.28
Welfare with run ($W^* \left(\widetilde{Q}, \overline{X}\right)$)	$144,790$	$40,622$
Benchmark welfare, unannounced policy ($W^U \left(\widetilde{Q}, \overline{X}\right)$)	$151,305$	$55,839$

the subsidy is withdrawn is set at $\overline{Q} = 100$. We compare the results with an unanticipated policy benchmark where the fixed cost shifts up unexpectedly when $\overline{Q}$ is reached and no run occurs. Table 1 reports our results.

The first three rows in each case describe industry investment, i.e. the size of the run, the threshold at which the run occurs, and the threshold at which the policy trigger would have been reached in the unanticipated policy benchmark. The magnitude of the run is significant throughout, representing roughly half of the policy target capacity in the different subsidy and volatility scenarios we examine. With respect to investment timing, the run accelerates investments significantly relative to the threshold function the industry would otherwise follow. In fact, the run brings the threshold for investment very close to the NPV thresholds, which at the private capacity costs and for $\overline{Q} = 100$

amount to 189.29 in the small subsidy case and 126.19 in the large subsidy case. The effects of varying subsidy size and volatility on the run are apparent in the table. In line with Propositions 2 and 3, a larger subsidy hastens the run and increases its magnitude, whereas greater volatility delays the run while raising its magnitude.

We measure the welfare loss from the run by comparing the actual welfare, as captured by (32), (37), and (38), with the welfare level in a benchmark where the policy change is unanticipated so no run occurs, denoted $W^U(Q, X)$. In the latter case, if firms either do not anticipate the policy change or place a sufficiently low instantaneous probability of its occurring, industry investment follows the threshold policy $X^*(Q)$ up to $\overline{Q}$ and the updated threshold policy $X^{**}(Q)$ thereafter. Note that in the case of an unanticipated change welfare in the range $Q < \overline{Q}$ satisfies (38) and (**??**) but in the absence of a run the integration constant C^* is determined instead by the boundary condition

$$W^*\left(\overline{Q}, X^*\left(\overline{Q}\right)\right) = W^{**}\left(\overline{Q}, X^*\left(\overline{Q}\right)\right). \tag{41}$$

Both in the benchmark and the anticipated policy scenarios, the path of investment is the same up until $X^*(\widetilde{Q})$ is first reached and the run occurs, as well as after $X^*(\overline{Q})$ is first reached and where capacity in the benchmark scenario has caught up with the run. The welfare effect of a run is therefore due to the divergence in the two industry paths over the interval of time where industry capacity lies in $\left(\widetilde{Q}, \overline{Q}\right)$.

To compare welfare with the run and with the unanticipated policy benchmark, we measure welfare at the onset of the run, that is at the demand state $\overline{X}$ and with an industry capacity $\widetilde{Q}$. Because we only model the distortionary effect on investment without incorporating any externalities that investment might otherwise exert, welfare is higher in the small subsidy case. The effect of the run on welfare is significant, ranging from a 3%

to a 27% welfare loss. One implication of this analysis, which is reflected in the CSI and GCC policies we discussed above, is that policy makers might prefer to introduce policy changes in gradual steps rather than all at once, so as to reduce the disruption larger policy changes otherwise cause.

7 Conclusion

In this article, we have studied how the equilibrium path of investment in a competitive industry is affected by an anticipated tax or subsidy withdrawal and shown that a competitive run emerges, generalizing the analysis of quotas which had been the focus of the literature previously. Such a run causes a mass of firms to rush to take advantage of a transitory profitability increase ahead of the implementation of the policy. The run does not result from any coordination failure but is an inevitable consequence of an anticipated policy. The emergence of the run is best attributed to the slower intensive entry process after the policy change. This phenomenon occurs for a range of policy measures, whether these affect fixed or operating cost for example, and more generally so long as the policy change generates an upward jump in investment threshold function of inactive firms.

Our results run counter to a conventional economic wisdom that announcing policies ahead of their implementation generally benefits economic actors, by highlighting a drawback if firms have too precise a knowledge of policy timing. Moreover our results complement previous work on policy uncertainty showing that greater policy risk leads to investment delay by showing that, at the other end of the spectrum, precise knowledge of a policy change results in a mass of earlier investment. Finally we have highlighted several factors which affect the size of a run, such as the magnitude of the policy, uncertainty as to its magnitude, and whether the policy affects new entrants asymmetrically, which can help identify when these effects are liable to be relevant for policy makers, such as the case of renewable energy subsidy phaseouts we have discussed.

In addition, our study has focused on studying a very specific form of a policy change, namely a subsidy withdrawal. We now briefly discuss some alternative policy designs, and their possible effect on the equilibrium outcome of the policy change. One such alternative would be to examine the case where the policy change does not happen at once, but gradually. Ignoring the unrealistically extreme case where the subsidy is continuously lowered from the moment it was given, and maintaining the feature that it is removed only after policy goals were reached, is expected to make the run emerge as part of the dynamic equilibrium in that case too. The reason for that is that under this modeling too the subsidy withdrawal improves the future profitability process and thus creates the possibility of a supernormal profit, that leads to the run. Yet, with several stages for the subsidy withdrawal, a richer array of equilibrium patterns emerges. Depending on parameter values, the equilibrium may either display two runs, or just one bigger run which attaches the second withdrawal to the first one.

Another natural extension would be to look at a case where the policy change is not state-dependent, as in the current model, but time-dependent. This difference in the trigger is not expected to change the main results of the analysis, as it does not alter the main forces in action. In particular, under this modelling too the subsidy withdrawal improves the future profitability process and thus creates the possibility of a supernormal profit that leads to the run. It should be noted though that in that case the differential equation that leads to the value function has also a derivative with respect to time, unlike under the current modeling, rendering it unsolvable, and enabling only a numerical solution.

Another natural policy alternative would be to add uncertainty about whether the policy change will take place or not, in contrast to the case of uncertainty about the size of the change that we have analyzed in the article. Moretto and Vergalli (2010) have shown that this type of uncertainty could prevent the emergence of the run in the equilibrium of the market. Yet, in their model the policy implemented in the market was that of a cap on market size, so it can be of value to see if the policy uncertainty can eliminate the run in the case of subsidy withdrawal too.

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A Appendix

A.1 Value of an active firm

In this appendix we show that the value function $V(Q,X)$ has the general form (4) given in the text. For that, we use the Bellman equation analysis carried out by Dixit and Pindyck (1994), p. 122. We start with noting that, by definition, $V(Q,X)$ satisfies

$$V(Q_0, X_0) = E_{X_0}\left[\int_0^{\infty} (X_t f(Q_t) - c)\, e^{-rt} dt\right]. \tag{42}$$

(42) leads to the following Bellman equation for time instants in the inaction region:

$$V(Q_t, X_t) = (X_t f(Q_t) - c)\, dt + \frac{1}{1+rdt} E\left[V(Q_t, X_{t+dt})\right]. \tag{43}$$

Multiplying by $1 + rdt$ and rearranging yields

$$rV(Q_t, X_t)dt = (X_t f(Q_t) - c)\, dt\, (1 + rdt) + E\left[dV(Q_t, X_t)\right] \tag{44}$$

where $dV(Q_t, X_t) = V(Q_t, X_{t+dt}) - V(Q_t, X_t)$. By Itô's lemma,

$$E_X\left[dV(Q,X)\right] = \left(\mu X V_X(Q,X) + \frac{1}{2}\sigma^2 X^2 V_{XX}(Q,X)\right) dt, \tag{45}$$

where time indexes are omitted from here on for notational convenience. Substituting (45) into (44), dividing by dt, taking the limit $dt \to 0$, and rearranging yields

$$\frac{1}{2}\sigma^2 X^2 V_{XX}(Q,X) + \mu X V_X(Q,X) - rV(Q,X) + Xf(Q) - c = 0. \tag{46}$$

Trying a solution of the type X^b for the homogeneous part of (46) and a linear form as a particular solution to the entire equation gives

$$V(Q,X) = Z(Q)X^{\alpha} + Y(Q)X^{\beta} + \frac{X}{r-\mu}f(Q) - \frac{c}{r} = 0 \tag{47}$$

where $Z(Q)$ and $X(Q)$ are determined further below and α, β are roots of the characteristic equation

$$\frac{1}{2}\sigma^2 x(x-1) + \mu x - r = 0. \tag{48}$$

Because $\sigma > 0$, (48) is a convex quadratic in x which takes negative values at $x = 0$ and $x = 1$ (because $r > \mu$), implying that there exist two distinct roots, $\alpha < 0$ and $\beta > 1$.[15]

By (5) the last two terms $\frac{Xf(Q)}{r-\mu} - \frac{c}{r}$ in (47) represent the expected value of the profit stream if Q remains at its current level forever so the two other terms represent how expected future changes in Q affect the value of the firm. As X goes to zero, the probability of ever hitting $X^*(Q) > 0$ and thus of an increase in Q, tends to zero. Therefore $\lim_{X\to 0}\left(Z(Q)X^{\alpha} + Y(Q)X^{\beta}\right) = 0$, which implies $Z(Q) = 0$ since $\alpha < 0$. Substituting in (47) then gives (4) in the text. □

A.2 Run threshold

Denote the left-hand side of (14) by $G(X)$. Then:

1. $G(0) = -\frac{c}{r} < k$,

2. $G(X^*(\overline{Q})) > k$, and

3. $G''(X) < 0$,

[15] Moreover, for $x > 1$ an increase in σ shifts the left hand side of (48) upward, implying that the upper root β decreases so $d\beta/d\sigma < 0$.

which implies that $G(X)$ has a unique root $\overline{X}$ in $\left(0, X^*\left(\overline{Q}\right)\right)$. Note that 2. follows from

$$\begin{aligned} G(X^*\left(\overline{Q}\right)) &= Y^{**}(\overline{Q})\left(X^*\left(\overline{Q}\right)\right)^{\beta} + \frac{X^*\left(\overline{Q}\right)}{r-\mu} f(\overline{Q}) - \frac{c}{r} \\ &> Y^{*}(\overline{Q})\left(X^*\left(\overline{Q}\right)\right)^{\beta} + \frac{X^*\left(\overline{Q}\right)}{r-\mu} f(\overline{Q}) - \frac{c}{r} = k \end{aligned} \tag{49}$$

where the inequality follows from (9), (12), and $k' > k$, and the second equality uses (4) and (6). 3. follows from taking the second-order derivative

$$G''(X) = \beta\left(\beta-1\right) Y^{**}(\overline{Q})\overline{X}^{\beta-2}, \tag{50}$$

which is negative because $Y^{**}(\overline{Q}) < 0$. $\square$

A.3 Proof for Proposition 3

We start the proof by observing that changes in σ affect $\overline{X}$ and $\widetilde{Q}$ through β, and that $\frac{d\beta}{d\sigma} < 0$ (see footnote 15). As a preliminary step, note that differentiating (11) and (12) gives

$$\frac{dX^{**}(Q)}{d\beta} = -\frac{X^{**}(Q)}{\beta(\beta-1)} \tag{51}$$

and

$$\begin{aligned} \frac{dY^{**}(Q)}{d\beta} &= \frac{k' + \frac{c}{r}}{(\beta-1)^2\left(X^{**}(Q)\right)^{\beta}} + \frac{k' + \frac{c}{r}}{(\beta-1)\left(X^{**}(Q)\right)^{\beta}} \ln\left(X^{**}(Q)\right) + \frac{\beta(k' + \frac{c}{r})}{(\beta-1)\left(X^{**}(Q)\right)^{\beta+1}} \frac{dX^{**}(Q)}{d\beta} \\ &= -Y^{**}(Q)\ln\left(X^{**}(Q)\right). \end{aligned} \tag{52}$$

With this result we now turn to (14) which defined $\overline{X}$. Define the left-hand side as

$F\left(\overline{X},\beta\right)$ and differentiate to get

$$\begin{aligned} F_\beta\left(\overline{X},\beta\right) &= Y^{**}(\overline{Q})\left(\overline{X}\right)^\beta \ln\left(\overline{X}\right) - Y^{**}(\overline{Q})\ln\left(X^{**}(\overline{Q})\right)\left(\overline{X}\right)^\beta \\ &= -Y^{**}(\overline{Q})\left(\overline{X}\right)^\beta \ln\left(\frac{X^{**}(\overline{Q})}{\overline{X}}\right) > 0 \end{aligned} \tag{53}$$

where the inequality results from $X^{**}(\overline{Q}) > \overline{X}$ (see A.2) and $Y^{**}(\overline{Q}) < 0$, and

$$F_{\overline{X}}\left(\overline{X},\beta\right) = \beta Y^{**}(\overline{Q})\left(\overline{X}\right)^{\beta-1} + \frac{f\left(\overline{Q}\right)}{r-\mu} > \beta Y^{**}(\overline{Q})\left(X^{**}(\overline{Q})\right)^{\beta-1} + \frac{f\left(\overline{Q}\right)}{r-\mu} = 0 \tag{54}$$

where the last equality follows from (18) which $X^{**}(\overline{Q})$ solves so $V_X\left(\overline{Q}, X^{**}(\overline{Q})\right) = 0$. It follows that

$$\frac{d\overline{X}}{d\beta} = -\frac{F_\beta\left(\overline{X},\beta\right)}{F_{\overline{X}}\left(\overline{X},\beta\right)} < 0, \tag{55}$$

which proves the first part of the proposition.

Turning next to the effect on $\widetilde{Q}$, first develop (55) to get

$$\frac{d\overline{X}}{d\beta} = \frac{Y^{**}(\overline{Q})\left(\overline{X}\right)^\beta \ln\left(\frac{X^{**}(\overline{Q})}{\overline{X}}\right)}{\beta Y^{**}(\overline{Q})\left(\overline{X}\right)^{\beta-1} + \frac{f(\overline{Q})}{r-\mu}} = \frac{\overline{X}}{\beta}\frac{\left(\frac{\overline{X}}{X^{**}(\overline{Q})}\right)^\beta \ln\left(\frac{X^{**}(\overline{Q})}{\overline{X}}\right)}{\left(\frac{\overline{X}}{X^{**}(\overline{Q})}\right)^\beta - \frac{\overline{X}}{X^{**}(\overline{Q})}} = \frac{\overline{X}}{\beta}\frac{\ln(x)}{1-x^{\beta-1}} \tag{56}$$

where $x \equiv \frac{X^{**}(\overline{Q})}{\overline{X}} > 1$ and the second equality uses (12) to substitute for $Y^{**}(\overline{Q})$. From (19),

$$f\left(\widetilde{Q}\right) = (r-\mu)\left(k' + \frac{c}{r}\right)\frac{\hat{\beta}}{\overline{\overline{X}}}, \tag{57}$$

the sign of $\frac{d\widetilde{Q}}{d\beta}$ follows from differentiating the right-hand side which gives

$$\begin{aligned}
\frac{d\left(f\left(\widetilde{Q}\right)\right)}{d\beta} &= (r-\mu)\left(k' + \frac{c}{r}\right)\frac{-\frac{\overline{X}}{(\beta-1)^2} - \hat{\beta}\frac{d\overline{X}}{d\beta}}{\overline{X}^2} \\
&= -\frac{f\left(\widetilde{Q}\right)}{\beta}\left(\frac{1}{\beta-1} + \frac{\ln(x)}{1-x^{\beta-1}}\right) < 0. \qquad (58)
\end{aligned}$$

This last expression is negative because the expression $\frac{1}{\beta-1} + \frac{\ln(x)}{1-x^{\beta-1}}$ is positive:

1. it converges to zero as x approaches 1 by l'Hôpital's rule,
2. is increasing in x,
3. $x > 1$.

Because $f\left(\widetilde{Q}\right)$ is decreasing, it follows that $\frac{d\widetilde{Q}}{d\beta} > 0$ which establishes the second part of the proposition. □