Teachers Uncover the Potential of 3D Printing Activities to Promote Analytical and Applied Mathematical Skills

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Abstract

Educators concur that schools should prepare students for the digital transformation era. 3D printing is a disruptive digital technology that can be integrated into education as a learning subject as well as educational technology. The potential of 3D design and printing (3DP) activities for learning different subjects and developing higher-order thinking skills is recognized. However, in practice, these activities fall short of fulfilling the potential. Our study was conducted in conjunction with a professional development program in which 38 teachers learned to integrate 3DP in education. Here we present a workshop focused on developing analytical thinking (AT) and applied mathematical skills (AMS). In the workshop, the teachers were assigned to design a spinning-top that could fit into a Kinder egg and analyze its geometry. We inquired whether and how the teachers employed AT and AMS during the activity by a post-workshop questionnaire. Nearly all teachers reported that they used analytical skills and specifically noted identifying the problem, breaking it into parts, and finding functional relationships. They also applied the mathematical skills of understanding and mathematizing real-world problems. The program helped teachers uncover the potential of 3DP technology to advance teaching mathematics and technology and foster higher-order thinking skills.

Keywords: Teacher education, 3D printing, analytical thinking, applied mathematical skills.

Introduction

The increased affordability and ease of use of 3D printers and the growing recognition of their potential for education have led to a rise in introducing 3D printing technologies into the classroom. There is growing evidence of the positive impact of integrating 3D printing practices in education (Ford & Minshall, 2019). Learning environments that incorporate 3D printing technologies enable students to make objects and in such a way to concretize the learned concepts. Learning mathematics through the design and creation of 3D-printed objects can positively impact students' learning outcomes and attitudes and foster creativity, problem-solving, reasoning, and understanding (Blikstein & Krannich, 2013; Ford & Minshall, 2019). Training students to cope with this advanced digital technology prepares them for life in the current era of digital transformation.

Educators and policymakers concur that school education should prepare students for the fast-paced changing world (Eisenberg & Selivansky, 2018). For that end, higher-order thinking skills and technological literacy are considered critical. This research focuses on developing two of these skills, namely, analytical thinking and applied mathematical skills.

Analytical thinking is defined as the mental ability to break an object or idea into parts, identify, and classify its different components, find causal relationships among the components and evaluate the usefulness of this analysis for decision-making and problem-solving (Sternberg & Spear-Swerling, 1996; Whimbey & Lochhead, 2013). Analytical thinking is one of the central skills necessary to comprehend the complexity of the world, reason, compare and organize information, and think critically (Powers & Enright, 1987; Valeeva & Bushmeleva, 2016).

Mathematical literacy is gaining a central role in mathematics curricula worldwide. Applied mathematical skills (AMS) are a synergistic combination of mathematical skills and the ability to solve practical problems by creating and exploring their mathematical models(Borromeo Ferri, 2018; Lesh & Doerr, 2003). They help to mediate between practical, real-world problems and the abstraction of pure mathematics and enable the application of mathematical methods to other disciplines, such as science, engineering, and economics. AMS includes the ability to analyze real-life situations, make abstractions, and synthesize events through a mathematical point of view (Schoenfeld, Conner, & Conner, 1992).

To promote students' thinking, teachers should provide students with ample opportunities to engage actively in thinking activities in different contexts (Liljedahl, 2016; Zohar & Schwartzer, 2005). In the context of 3DP, the learner's attention focuses on the created artifact as an "object-to-think-with" (Papert, 1980). Studies have shown that learning through 3DP improves students' mathematical and engineering understanding (Bull, Chiu, Berry, Lipson, & Xie, 2014). However, despite the perceived potential, some educators are concerned that in practice, such activities, while being fun and motivating, may not necessarily be intellectually demanding (Thornburg, Thornburg, & Armstrong, 2014). The lack of teaching methodologies and 3DP techno-pedagogical training hinders the integration of 3DP in school practice (Ford & Minshall, 2019). The research and development of such methodologies have only recently started.

In a previous study, we proposed and explored an approach to foster AT and AMS by engaging students in designing and creating 3D printed artifacts and their mathematical analysis (Levin & Verner, 2020). In this study, we examined teachers' perceptions regarding the proposed approach after participating in a workshop that implemented this approach. The workshop involved an abridged version of an activity developed for middle school students. In a post-workshop questionnaire, we inquired what skills teachers employed during the 3D design and printing activity. We were particularly interested in whether and how the teachers employed AT and AMS.

Method

Participants and setting

The study was conducted through two professional development programs focused on 3D printing in education. 38 in-service teachers participated in the programs, held in Israel in the spring of 2020, during the COVID-19 pandemic outbreak, 27 females and 11 males. The teachers' background was diverse; 20 were high school teachers, 6 middle school, and 12 elementary schools. The high school teachers were predominantly engineering or technology-related teachers. Middle and elementary school teachers were evenly distributed between math, science, art, and language arts subjects. This study is based on one workshop during the professional development program, in which 30 teachers participated and answered a subsequent questionnaire.

The professional development program and the AT and AMS activity

The 30-hour professional development program consisted of 6 synchronous online sessions conducted using the Zoom platform, and offline activities conducted by the participants independently. The program content covered 3D printing, its history, and actual uses. Special attention was devoted to learning theories such as constructionism and experiential learning and teaching strategies to integrate 3DP in the classroom. The participants learned to design objects for personal and educational use, using Tinkercad software and printing them remotely on a MakerBot 3D printer.

The fourth online session started with a discussion about 21st-century skills (Eisenberg & Selivansky, 2018), and an example of a STEM activity using 3DP was presented and analyzed in terms of these skills. AT and AMS were defined. The session concluded with a hands-on activity, "Making a spinning-top." The activity followed an approach developed and evaluated in a previous study (Levin & Verner, 2020). Accordingly, we assigned ill-defined problems that required from the learners to apply intuition, analytical thinking, and mathematical modeling. The learners needed to mathematize real-world situations, build a mathematical model, and validate the mathematical solution in the real-world context. We asked the participants to act as toy designers and design and 3D print a spinning-top toy that can fit into a Kinder Surprise egg and spins as long as possible. The surprise egg dimensions, and the spinning-top geometry were not specified; therefore, participants had to determine a mathematical model of the spinning-top and calculate its dimensions by themselves.

The teachers worked in groups of three or four, designing spinning-tops and discussing their solutions. The activity was guided by a worksheet that prompted the participants to think about the shapes that formed the spinning-top, calculate or estimate its biggest possible size, and calculate the surface area. Figure 1 exhibits two of the designed spinning-tops in Tinkercad 3D software.

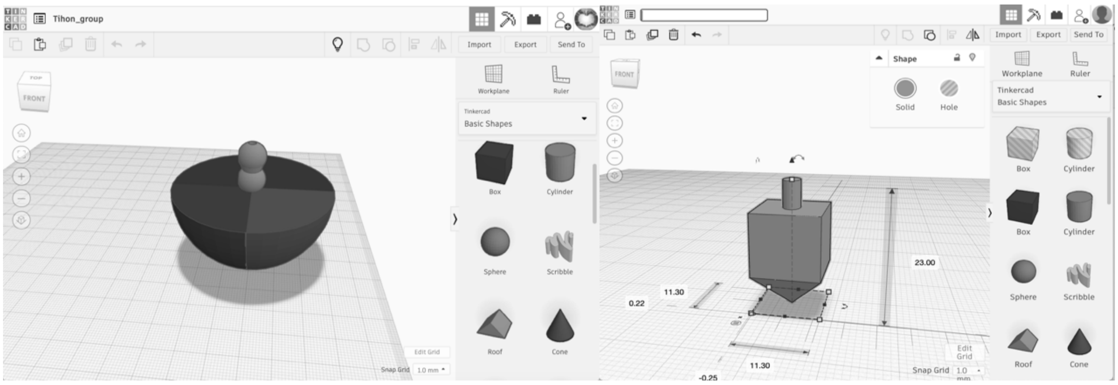

Figure 1. Spinning-tops designed by teachers

Data source and analysis

To examine teachers' perceptions regarding the activity as a way to foster AT and AMS, we administered an open-ended questionnaire at the end of the session. All 30 participating teachers answered the questionnaire. Two questions asked the teachers to identify and describe skills practiced during the activity in their perception. We coded these questions following the list of skills described by van Laar et al. (van Laar, van Deursen, van Dijk, & de Haan, 2017) and other skills that emerged from participant responses, as assessed by the authors of this paper. The categories and their definitions are presented in Table 1.

Table 1. Skills categories

Skill	Definition
Critical thinking	The skills to make informed judgments and choices about obtained information and communication using reflective reasoning and sufficient evidence to support the claims.
Creativity	The skills to generate new or previously unknown ideas or treat familiar ideas in a new way and transform such ideas into a product, service, or process recognized as novel within a particular domain.
Problem-solving	The skills to cognitively process and understand a problem situation in combination with the active use of knowledge to find a solution to a problem.
Information management	The skills to efficiently search, select, and organize information to make informed decisions about the most suitable information sources for a given task.
Technical	The skills to use devices and applications to accomplish practical tasks.
Collaboration	The skills to work in a team to exchange information, negotiate agreements, and make decisions with mutual respect towards achieving a common goal.
Communication	The skills to transmit information to others, ensuring that the meaning is expressed effectively.

Skill	Definition
Self-direction	The skills to set goals for yourself and manage progression toward reaching those goals to assess your own progress.
Adaptability	The skills to adapt one's thinking, attitude, or behavior to changing environments or situations.
Analytical thinking	The skills to break an object or idea into parts, identify and classify its different components, find causal relationships among the components and evaluate the usefulness of this analysis for decision-making and problem-solving (Sternberg & Spear-Swerling, 1996; Whimbey & Lochhead, 2013).
Mathematical thinking	The skills to analyze, make an abstraction, and synthesize the events through a mathematical point of view, the ability to use numbers to help solve real-world problems, the ability to use the language of math (Schoenfeld et al., 1992).
Engineering thinking	The skills to apply scientific knowledge and mathematical analysis to the solution of practical problems.
Spatial thinking	The skills to use space concepts, tools of representation like maps and graphs, and reasoning processes to organize and solve problems.
Time management	The skills to organize and prioritize tasks and plan how long to spend on specific activities.
Design thinking	The skills to analyze and frame problems and ideate design concepts (e.g., products, systems) to solve these problems.
Metacognition	The skills to organize, guide, and control one's own thinking, actions, and learning processes.
Inquiry/research skills	The skills to gather and analyze information to find answers to a question or a solution to a problem.
Intuitive thinking	The skills to make fast, effective decisions based on automatic cognitive processes.

The questionnaire also included six questions regarding analytical and mathematical thinking applied during the activity. The teachers were asked whether they used analytical and mathematical thinking during the activity and to describe a case or problem in which they applied the skills. Based on the literature (Goriely, 2018; Krathwohl, 2002; Lesh & Doerr, 2003; Sternberg & Spear-Swerling, 1996; Whimbey & Lochhead, 2013), we defined components of AT and AMS as presented in Table 2. Although separate questions were devoted to AT and AMS skills, the answers were tightly coupled; therefore, we analyzed the responses together.

Table 2. Analytical thinking and applied mathematical skills components

Skill category	Component	Definition
Analytical thinking	Problem identification	Identifying and defining what the problem is
	Decomposition	Breaking the problem into parts, identifying attributes and components: finding and labeling the pieces of the whole, classifying different components on an object
	Relationships and pattern recognition	Finding causal or hierarchical connections among the components, determining rules of relationships and patterns
	Process selection	Evaluating the usefulness and reliability of information for making decisions and solving problems, using relevant forms to structure information
	Strategy formation	Systematic, step by step sequencing process, allocating resources
	Evaluation	Verifying and judging outcomes by comparison to established criteria
Applied mathematical skills	Understanding real-world problems	Understanding, making sense, and building a model of the situation
	Mathematizing and working mathematically	Simplifying the problem, using mathematical language to define problems, communicate ideas, and solutions
	Interpreting numerical data	Organizing, analyzing, and interpreting numerical data to determine relationships among factors, validating and mapping back to the real-world situation

Findings

This study aimed to assess teachers' perceptions of the developed approach for fostering AT and AMS according to their experiences in the "Making a spinning-top" activity. In this section, we presented the results of the questionnaire.

Teachers' perceptions of the employed skills

Considering that the questions were open-ended, it is interesting that nearly all the teachers identified the application of analytical and mathematical skills during the activity (97% and 93%, respectively), by a wide margin compared to other skills, as shown in Figure 2. Although it was expected that teachers would identify these skills as they were reviewed before the activity, it is noteworthy that such a high percentage had been recorded. The next highly denoted skills were creativity (50%), collaboration (47%), and technical skills (37%). Other frequently noted skills were critical thinking, problem-solving (23% each), and spatial thinking (20%).

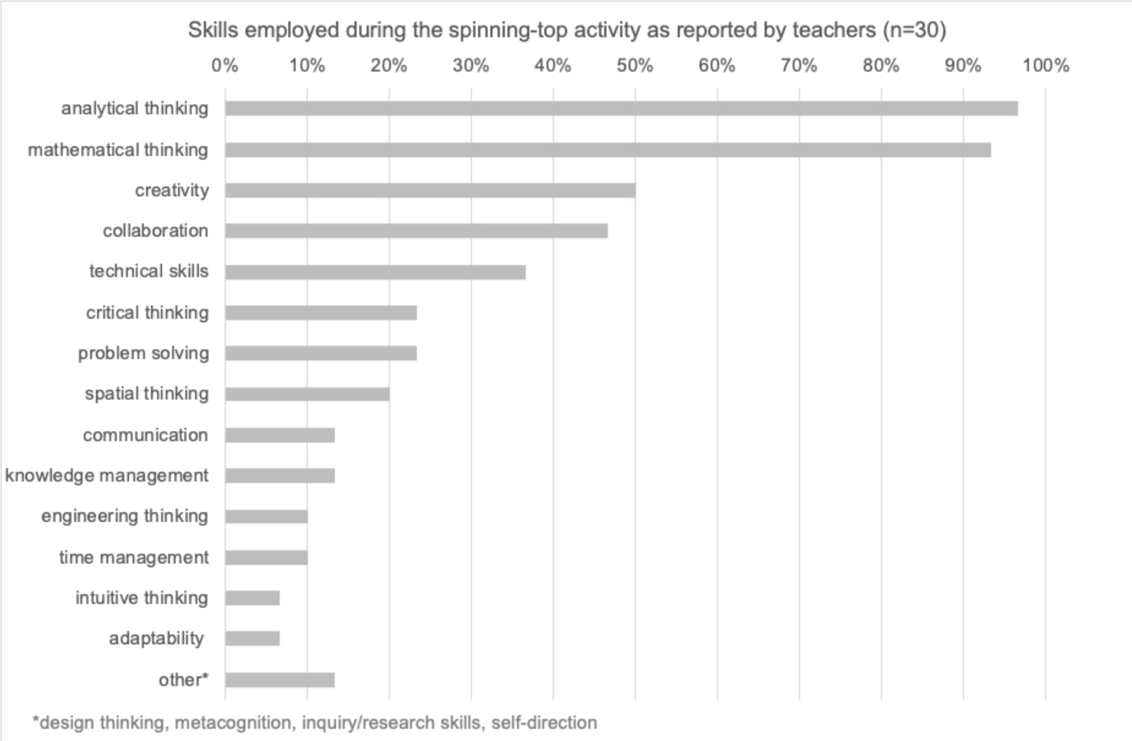

Figure 2. Skills employed during the spinning-top activity as reported by teachers

Analytical thinking components identified in teachers' responses

This section presents the frequency of different AT components identified by the participants, as shown in Figure 3 in lightly shaded bars, followed by sample answers.

Decomposition was the most identified AT component, noted by 87% of the teachers. A typical response identifying this component was:

> *"Breaking the problem of designing a spinning-top into size constraints, mass distribution considerations…"*

Problem identification was the second most noted component, by 80% of respondents. For example, two teachers responded:

> *"We wondered what is the size of the Kinder egg container, I assessed the dimensions using a real egg and tell my peers what has to be the size of spinning-top."*

> *"I tried to think how to design the spinning-top to spin smoothly and to be 3D printable."*

Another frequently identified component was relationship and pattern recognition (70%). For example, one of the teachers responded:

> *"Finding the relationship between the dimensions of the spinning-top body, its surface area, and the size of the sticker to be wrapped around."*

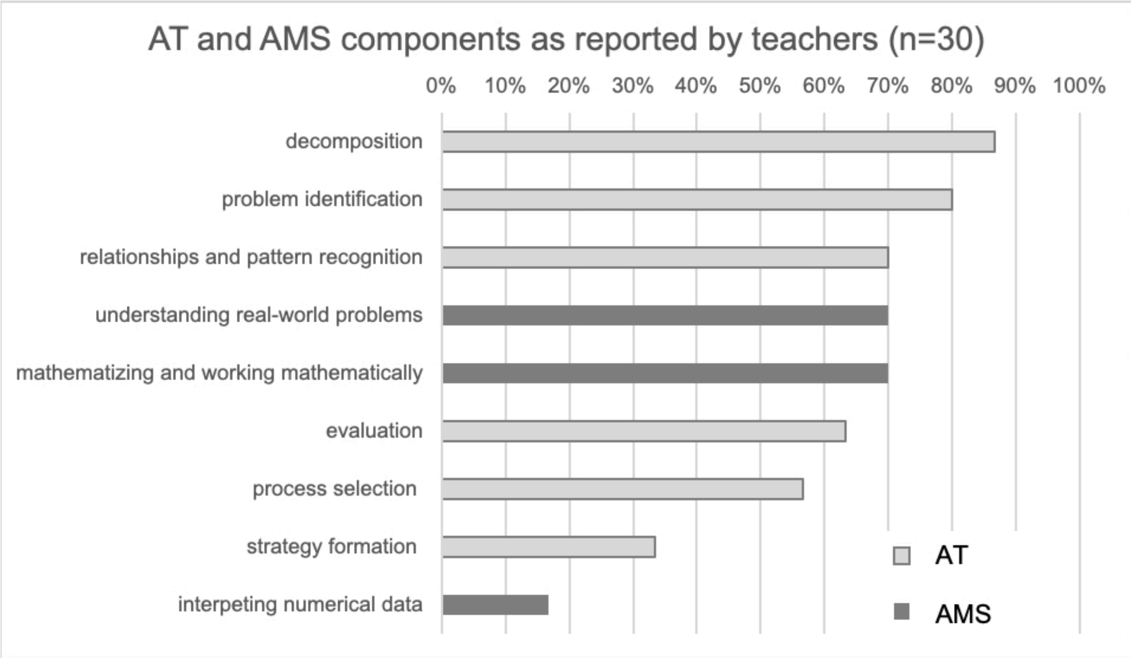

Figure 3. Analytical thinking and applied mathematical skills components as reported by teachers

The components evaluation and process selection were also identified by 63% and 57% of the respondents. The following quotes exemplify these components:

> *"At first, we used a cylinder as the body of the spinning-top, and it was quite narrow, but after my peers pointed out that a sticker has to be wrapped around it, I understood that the surface area was not sufficient and we must to enlarge it."*
>
> *"Breaking the assignment into small tasks in order to organize what has to be found, choosing the predominant factor for the spinning-top size and checking its size to fit with other [constraints]."*

The least mentioned component, strategy formation, was also frequently identified, in 10 cases, 33% of the respondent.

It is important to note that the teachers identified the skills components, but this is not to conclude that they correctly implemented the identified skills.

Applied mathematical skills identified in teachers' responses

This section presents the frequency of different AMS identified by the teachers in their responses, as shown in Figure 3 in darkly shaded bars, followed by sample answers.

Two out of 30 respondents stated that they did not know what problems they solve mathematically or what mathematical skills they employed.

One of the most identified skills, understanding real-world problems, was mentioned by 21 teachers, 70%. It seems to be related to the AT component of problem identification, as is exemplified in the corresponding example quoted above regarding the Kinder egg size assessment. Other examples of real-world problem understanding referred to the need for the spinning-top to be 3D printable.

The skill mathematizing and working mathematically was also frequently identified in 70% of the teacher's responses. This skill can be analyzed in a more granular way, differentiating between mathematical modeling and the use of mathematical language to define a problem and communicate ideas and solutions. However, the data collected in this study does not allow a higher level of detail. The following quote illustrates a mathematical modeling example, and Figure 4 presents the frequency of mathematical concepts found in teachers' responses.

> *"I wanted to set maximum dimensions for the spinning-top cube within the Kinder cylinder dimensions constraint. To do this, I drew the cross-section of the cylinder as a circle. I drew the cross-section of the spinning-top cube as a square inscribed in the circle. I represented the square's length using the radius of the cylinder from geometry calculations (use of geometric theorems, Pythagorean theorem, etc.). When I got the dimensions of the Kinder, I placed them in the general expression and easily calculated the length of the edge of the cube."*

Regarding the use of mathematical language, in 11 utterances, concepts such as measures, shapes, and proportion were used as colloquial expressions; therefore, they were not counted as mathematical terms and excluded from the analysis. Not surprisingly, the most common concepts found were measures, used by 16 teachers, and 2D and 3D geometrical shapes, such as cube, cylinder, circle, recorded by 12 respondents. We note the relatively high frequency of perimeter, 5 cases, since it was not a relevant concept for the problem solution.

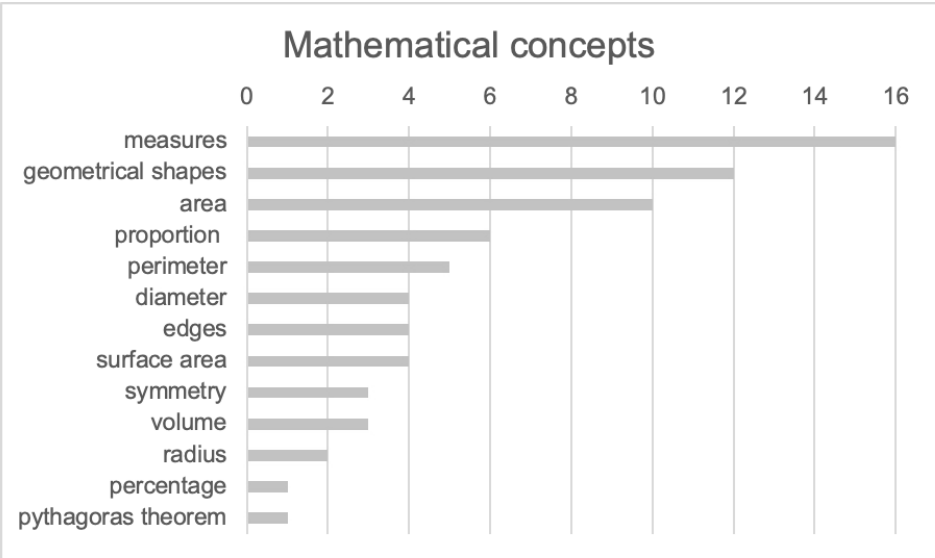

Figure 4. Mathematical concepts used by the teachers

Discussion

Our study inquired whether and how the teachers who participated in a professional development program employed analytical thinking and applied mathematical skills during an open-ended 3DP assignment requiring design and making a 3D printed artifact and its mathematical analysis. In data collection and analysis, based on teachers' responses, we identified the skills they mostly employed during the activity. The results of the study indicated that nearly all teachers employed AT and AMS skills. This finding supports our research focus on the development of these skills (Levin & Verner, 2020).

The literature points out the skills that students can develop through practice in 3DP projects. Among these skills, educators frequently noted problem-solving and critical thinking (Trust & Maloy, 2017). Studies have shown that learning through 3DP can improve students' understanding of mathematical and engineering concepts (Bull et al., 2014) and facilitate spatial reasoning (Corum & Garofalo, 2015; Verner & Merksamer, 2015). In line with these findings, the teachers also noted the workshop's contribution to the above skills. Although, our study contributes a new insight, enlightening AT and AMS skills that were not previously discussed in the context of 3DP activities.

Our study specified the components of AT and AMS skills that were involved in the 3DP activity. Our findings indicate that the most predominant AT components were identifying the problem, breaking it into parts, and finding functional relationships. The teachers employed AT skills to understand the ill-defined assignment and delineate the 3D design and printing procedure for solving the problem. To design with CAD software, the teachers had to break the design object into parts and perform the geometrical analysis while creating a three-dimensional model by combining shapes or modifying sketches. The relatively low frequency of the evaluation component could be explained by the short time allotted to the activity, which impeded a thorough evaluation of the solution.

Our findings related to AMS indicated their specific components: understanding real-world problems, their mathematizing, and interpreting numerical data. In many cases, understanding and mathematizing real-world problems came after the analytical identification of the problem. The design assignment was formulated to implicitly encourage teachers to apply mathematics to find quantitative solutions to the problems and satisfy the existing constraints.

In conclusion, our study showed that the program helped teachers uncover the potential of 3DP technology to advance teaching mathematics and technology and foster higher-order thinking skills. We plan to continue this research and investigate the development of analytical and applied mathematical thinking through the processes of performing mathematically meaningful assignments of 3D design and printing.

References

Blikstein, P., & Krannich, D. (2013). The Makers' Movement and FabLabs in Education : Experiences, Technologies and Research. In *Proceedings of the 12th International Conference on Interaction Design and Children* (pp. 613–616). New York, New York: ACM.

Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. New York: Springer. https://doi.org/10.1007/978-3-319-68072-9

Bull, G., Chiu, J., Berry, R., Lipson, H., & Xie, C. (2014). Advancing Children's Engineering Through Desktop Manufacturing. In J. M. Spector, M. D. Merrill, J. Elen, & M. J. Bishop (Eds.), *Handbook of research on educational communications and technology* (4th ed., pp. 675–688). New York: Springer. https://doi.org/10.1007/978-1-4614-3185-5

Corum, K., & Garofalo, J. (2015). Using Digital Fabrication to Support Student Learning. *3D Printing and Additive Manufacturing*, *2*(2), 50–55. https://doi.org/10.1089/3dp.2015.0008

Eisenberg, E., & Selivansky, O. (2018). *Hatamat Maharechet Hachinuch Lamea Ha-21 [Preparing and Adapting Israel's Education System for the Challenges of the 21st Century]*. The Israel Democracy Institute.

Ford, S., & Minshall, T. (2019). Where and how 3D printing is used in teaching and education. *Additive Manufacturing*, *25*, 131–150. https://doi.org/10.1016/j.addma.2018.10.028

Goriely, A. (2018). *Applied Mathematics: A Very Short Introduction*. Oxford University Press. https://doi.org/10.1093/actrade/9780198754046.001.0001

Krathwohl, D. R. (2002). A Revision of Bloom's Taxonomy: An Overview. *Theory Into Practice*, *41*(4), 212–218. https://doi.org/10.1207/s15430421tip4104

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3–33). Mahwah, NJ: Lawrence Erlbaum Associates. https://doi.org/10.4324/9781410607713

Levin, L., & Verner, I. M. (2020). Fostering students' analytical thinking and applied mathematical skills through 3D design and printing. In *IEEE Global Engineering Education Conference*, *EDUCON* (pp. 145–149). https://doi.org/10.1109/EDUCON45650.2020.912535

Liljedahl, P. (2016). Building Thinking Classrooms: Conditions for Problem-Solving. In P. Felmer, J. Kilpatrick, & E. Pekhonen (Eds.), *Posing and Solving Mathematical Problems: Advances and new perspectives* (pp. 361–386). New York: Springer. https://doi.org/10.1007/978-3-319-28023-3_21

Papert, S. (1980). Mindstorms: Computers, children, and powerful ideas. *NY: Basic Books*.

Powers, D. E., & Enright, M. K. (1987). Analytical Reasoning Skills in Graduate Study: Perceptions of Faculty in Six Fields. *The Journal of Higher Education*, *58*(6), 658–682. https://doi.org/10.2307/1981103

Schoenfeld, A. H., Conner, E., & Conner, E. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334–370). New York: MacMillan.

Sternberg, R. J., & Spear-Swerling, L. (1996). *Teaching for thinking. Teaching for thinking*. Washington, DC: American Psychological Association. https://doi.org/10.1037/10212-000

Trust, T., & Maloy, R. W. (2017). Why 3D Print? The 21 st -Century Skills Students Develop While Engaging in 3D Printing Projects. *Computers in the Schools*, *34*(4), 253–266. https://doi.org/10.1080/07380569.2017.1384684

Valeeva, R. A., & Bushmeleva, N. A. (2016). Forming analytical competency of higher school students. *Mathematics Education*, *11*(8), 3137–3148.

van Laar, E., van Deursen, A., van Dijk, J., & de Haan, J. (2017). The relation between 21st-century skills and digital skills: A systematic literature review. *Computers in Human Behavior*, *77*(577–588). https://doi.org/10.1016/j.chb.2017.03.010

Verner, I., & Merksamer, A. (2015). Digital design and 3D printing in technology teacher education. *Procedia CIRP*, *36*, 182–186. https://doi.org/10.1016/j.procir.2015.08.041

Whimbey, A., & Lochhead, J. (2013). *Problem solving & comprehension, sixth edition*. *Problem Solving and Comprehension, Sixth Edition*. Philadelphia: The Franklin Institute Press. https://doi.org/10.4324/9781410601391

Zohar, A., & Schwartzer, N. (2005). Assessing Teachers' Pedagogical Knowledge in the Context of Teaching Higher-order Thinking. *International Journal of Science Education*, *27*(13), 1595–1620. https://doi.org/10.1080/09500690500186592