Assessing Mathematical Problem Solving Behavior in Web-Based Environments Using Educational Data Mining – from Correspondence Scheme to Quantitative analysis

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Abstract

Problem solving can be described as composed of three dimensions: the problem, the process and the outcome. Over the years, mathematical problem solving research has focused on describing the process, as well as on understanding attributes affecting it, and assessing its outcomes. Most of the research in this field is qualitative, and this is understandable due to the fact that cognitive and meta-cognitive investigation involved in problems solving are complicated to be traced. Nowadays, when many problem solving environments are implemented using the web, innovative research methodologies may be applied for assessing problem solving behavior in large populations. These innovative research methodologies rely on log file records, which document (almost) every action taken by the student. This paper describes a part of a larger body of research. The core of this paper entails the (1) development of a correspondence scheme between the logged traces of the students and the observed problem solving behavior and the (2) behaviors of 230 students. Initial findings indicate that when students face a problem related to fractions, most of them don't exhibit conceptual knowledge in fractions and also those who possess this knowledge need several trials in order to solve. Intuitive knowledge interfere with the conceptual knowledge and cause (when strong) to quick response. Most of the students monitor their problem solving process (more than expected).

Keywords: Educational data mining, problem solving, web based learning.

Introduction

Problem solving can be regarded as a situation in which an individual is responding to a problem that he or she does not know how to solve with routine or familiar procedures. Problem solving can be described as composed of three dimensions: the problem, the process and the outcome.

Polya's (1957) seminal work suggested that solving a problem involves 4 phases (or episodes): (a) understanding the problem; (b) developing a plan; (c) carrying out the plan; and (d) looking back. Hence, the problem solving process is described as linear progression from one phase to the other. Schoenfeld (1985) observed that during problem solving, students display distinct categories of behavior, also called episodes. Crucial episodes are: analyzing the problem, selecting appropriate mathematical knowledge, making a plan, carrying it out, and checking the answer with relation to the question asked.

Schoenfeld (1985, 1987a, 1987b, 1989, 1992) contributed a framework of different factors (attributes) that affect students' abilities to solve problems. In his framework, four components comprise the major aspects of students' problem solving: (a) Resources: formal and informal knowledge about the content domain, including facts, definitions, algorithmic procedures, routine procedures, intuitive understandings of mathematics, and relevant competencies about rules of discourse; (b) Heuristics: strategies and techniques for approaching a problem; (c) Control: the ways in which students monitor their own problem solving process, use their observations of partial results to guide future problem solving actions, and decide how and when to use available resources and heuristics.

The outcome dimension consists of the assessment of the problem solving outcome, which involves the assessment of the outcome creativity. Research on creative thinking identified three key components of a creative product: fluency, flexibility and novelty (Silver, 1997). Fluency refers to the number of ideas generated in response to a prompt; flexibility refers to apparent shifts in approaches when generating responses to a prompt; and novelty – to the originality of the ideas generated in response to a prompt.

When engaging in Mathematical problem solving in web based learning environments, students leave traces of their activity in the form of log file records, which document every action taken by three basic parameters: what was the action taken, who took it and when (Pahl, 2004). Discovering and extracting educational information from these log files using data mining techniques is called Educational Data Mining (EDM).
Most research about online learners' activity on the web usually focuses on operational variables, (e.g., time patterns, pace, order of contents viewed), while higher-level cognitive variables, describing the characteristics of the learners' online learning, are less studied (e.g. Baker & Yacef, 2009). This is of no surprise: traditional research methodologies fail to cope with the complex gathering of information about the online learner.

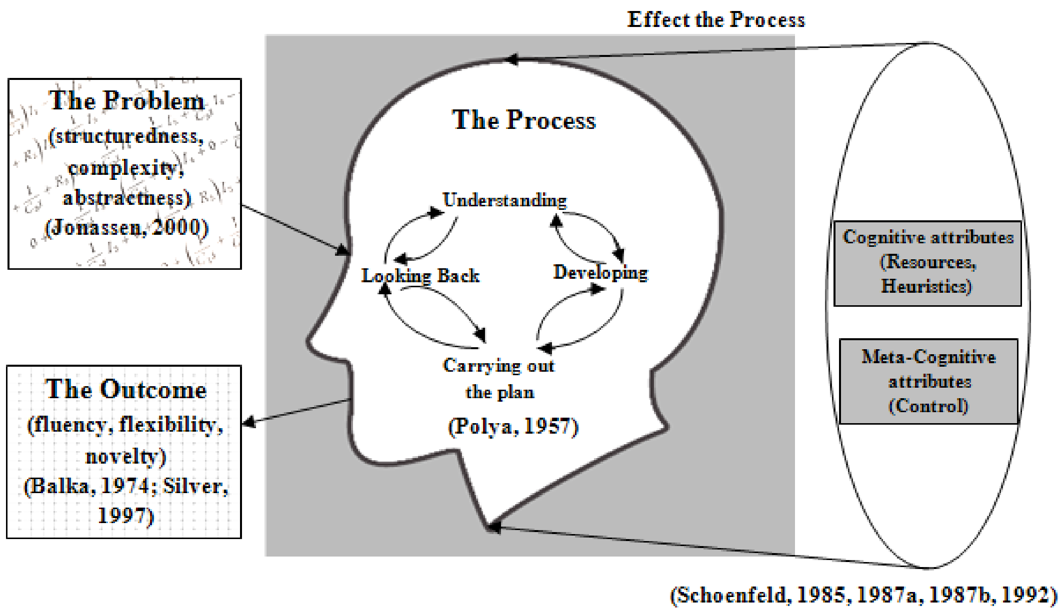

Figure 1. The conceptual framework of the research

Building on the existing body of literature (Fig. 1), this research focuses on gaining new information about the problem solving process, using an emerging methodology, Educational Data Mining.

Research Objectives

Two objectives have been defined for this research:

1. Developing a conceptual framework, a computational mechanism and a correspondence scheme between them for assessing mathematical problem solving behavior in Web-based environments by means of logged data.
2. Identifying different patterns of online problem solving behavior using quantitative analysis for larger population.

Methodology

A Mixed Method (Creswell, 2003) involving both qualitative and quantitative analysis, was chosen for this research. Problem solving behavior is assessed by means of qualitative research (using think-aloud protocols), as well as actual learning behavior in the online learning environment using Learnograms; patterns of problem solving processes and factors affecting them are investigated using quantitative methods using log files analysis.

Research Population

Participants in the research include 10 fifth and sixth grade students for the construction of the correspondence scheme. In addition, log files from a larger population (N=230) of students of the same grade levels were analyzed for the large-scale assessment of problem solving behavior.

Research Field

An online learning environment in Mathematics was chosen (developed by CET – Center for Educational Technology), using a grid on which students can construct geometry objects (e.g., dots, angles, lines, polygons), measure them (length, area, angle), and transform them (move, resize, delete). In addition, student can color squares formed by the grid.

Research Instruments

Three sets of instruments serve this research:

a. Think-aloud protocols –a method for describing and analyzing thinking processes, during which the student is being asked to verbally describe thoughts and feelings simultaneously during a task operation (Newell & Simon, 1972).
b. Learnogram – visualization tool for presenting log-based learning variables over time (Nachmias & Hershkovitz, 2007).
c. An online task – In this paper one task was analyzed. The task given to the students was to mark $\frac{6}{10}$ of a given grid (out of 120 squares).

Results

This section describes the preliminary results in two steps. The qualitative analysis (N=1) and the quantitative analysis (N=230).

The correspondence scheme

In order to construct the Correspondence Scheme, data from both students and log files were triangulated, aiming to reflect the students' problem solving behavior in the log files. To demonstrate how the correspondence scheme was developed, a log file and a think aloud protocol of one task is analyzed. Fig. 2 is a small part of student 7 (out of 10) cleaned log file. As seen in Fig. 2 the student started the solution by filling 10 squares (out of 120) – each data line represents an action (square fill). This action took 0.8 seconds (by dragging the brush in 2 steps).

Variables examines were creativity and solution strategy. From the thinking aloud protocols and the log files, variables identified were number of actions, order of actions, number of solutions and total time for reaching the solution.

C	D	E	F	G	H	I	J	K
sUserN	nSessio	nStep	dtCreated	sCurrer	nAction	sAction	nX1	nY1
Student 7	1	1	09:06:15.7	fill	1	Square fill	3	1
Student 7	1	1	09:06:15.7	fill	2	Square fill	4	1
Student 7	1	1	09:06:15.7	fill	3	Square fill	5	1
Student 7	1	1	09:06:15.7	fill	4	Square fill	5	2
Student 7	1	1	09:06:15.7	fill	5	Square fill	6	2
Student 7	1	1	09:06:15.7	fill	6	Square fill	7	2
Student 7	1	1	09:06:15.7	fill	7	Square fill	7	3
Student 7	1	2	09:06:16.5	fill	1	Square fill	5	5
Student 7	1	2	09:06:16.5	fill	2	Square fill	5	6
Student 7	1	2	09:06:16.5	fill	3	Square fill	4	6

Figure 2. Part of student 7 log file representing filling 10 squares

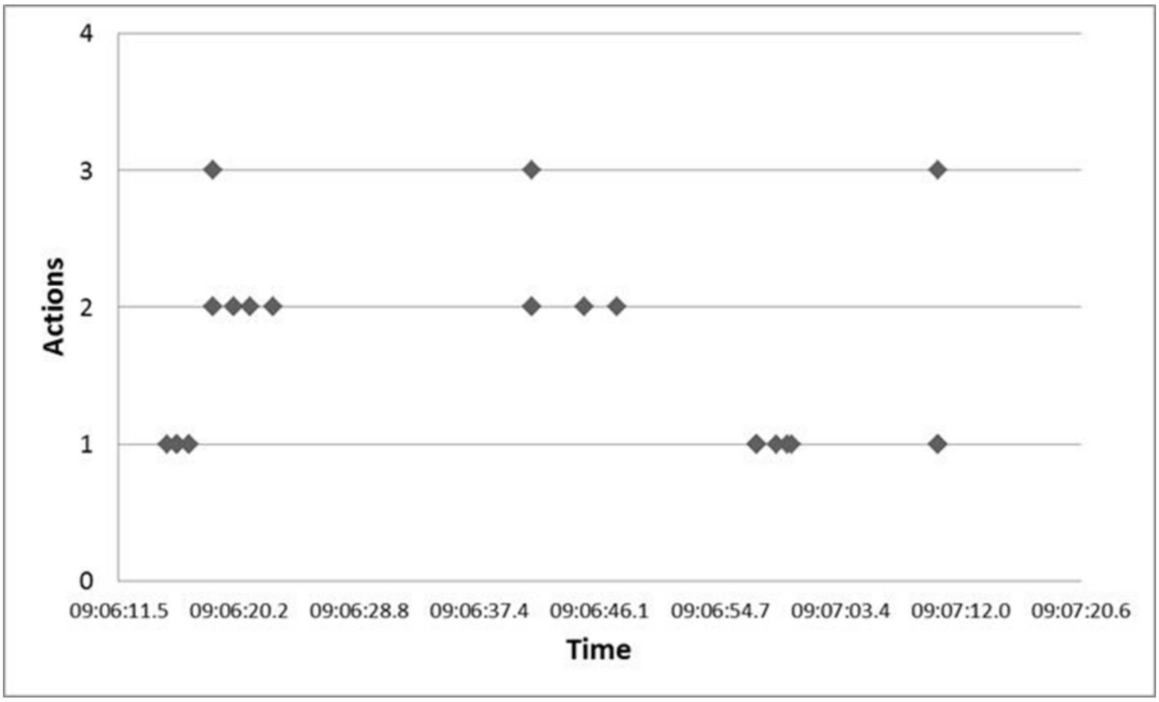

Figure 3. Student 7 Learnogram – actions sequence and solution strategies (1 – square fill; 2 – drawing a shape; 3 – cleaning the grid)

Fig. 3 represents the actions sequence and strategies taken in order to solve the problem. As seen in Fig. 3 the student started the solution by filling 10 squares (out of 120). Then, the student cleaned the grid and started over again this time trying to draw a shape that is $\frac{6}{10}$ of a given grid (for 29.1 seconds) – a shape built of 20 squares. At the student's 3rd attempt (13 seconds), he returned to filling squares (filled 17) and then (4th attempt) cleaned the grid and filled 6. His final answer was 6 squares.

While trying to solve the problem faced, the student had 4 trials using 2 different strategies. From his first trial, lasting for less than 1 second, the student filled 10 squares (similar to the denominator). From the thinking aloud protocol we can assume that this solution relied mainly on intuitive knowledge as described by Fischbein (1993). Intuitive knowledge is characterized as the type of knowledge that we tend to accept directly and confidently as being obvious, with a feeling that it needs no proof. This type of knowledge has an imperative power; that is, it tends to eliminate alternative representations, interpretations or solutions.

As the student progressed he returned to another misconception, filling 6 squares (similar to the numerator). Regarding the strategies, the student tried 2 different strategies to solve the problem: filling squares and drawing shapes. In between alternating, the student exhibited control (monitoring his problem solving process) by clearing 3 times the grid and starting to solve the problem again as described by Schoenfeld (1985, 1987a, 1987b, 1989, 1992).

Problem solving processes

After analyzing the think aloud protocols and log files of 10 students, a set of computational algorithms were written in order to transform the raw data (behavioral data) to higher order variables which represent cognitive and meta-cognitive aspects.

The analysis of the log file's extracted variables provided a summary of the students' behaviors. Table 1 demonstrates the distribution of students' actions (N=196) while trying to solve the fraction task (mark $\frac{6}{10}$ of a given 120 squares grid). A student used 168.41 actions on average (SD=224.29) to solve the problem with an average of 4.79 trails (SD=7.71), taking him (or her) an average of 200.31 seconds (SD=225.92) while performing 2.35 monitoring actions (SD=3.79).

Table 1. Distribution of students' actions and time

	N	Mean (SD)	Median
Number of Actions	196	168.41(224.29)	111.5
Number of Trails	196	4.79 (7.71)	2
Time on Task (Sec)	196	200.31 (225.92)	112.5
Monitoring Actions	196	2.35 (3.79)	1

As shown is Table 2, only 33.2% of the students exhibited conceptual knowledge and 17.3% exhibited intuitive knowledge which was strong enough to "disturb" the process. Nearly 43% of the students exhibited control (monitoring) behavior while trying to solve the problem presented to them.

Table 2. Frequencies of students' cognitive and meta-cognitive behaviors

<table>
<tr><th></th><th></th><th>N</th><th>Frequency</th><th>%</th></tr>
<tr><td rowspan="2">Conceptual Knowledge</td><td>Didn't Exhibit</td><td rowspan="2">196</td><td>131</td><td>66.8</td></tr>
<tr><td>Exhibited</td><td>65</td><td>33.2</td></tr>
<tr><td rowspan="2">Intuitive Knowledge</td><td>Didn't Exhibit</td><td rowspan="2">196</td><td>162</td><td>82.7</td></tr>
<tr><td>Exhibited</td><td>34</td><td>17.3</td></tr>
<tr><td rowspan="2">Control</td><td>Didn't Exhibit</td><td rowspan="2">196</td><td>84</td><td>42.9</td></tr>
<tr><td>Exhibited</td><td>112</td><td>57.1</td></tr>
</table>

Most of the students (nearly 51%) used three or less trails to try and solve the problem (Fig. 4) while only 19.9% solved it correctly (Table 3), shedding some light on their motivation.

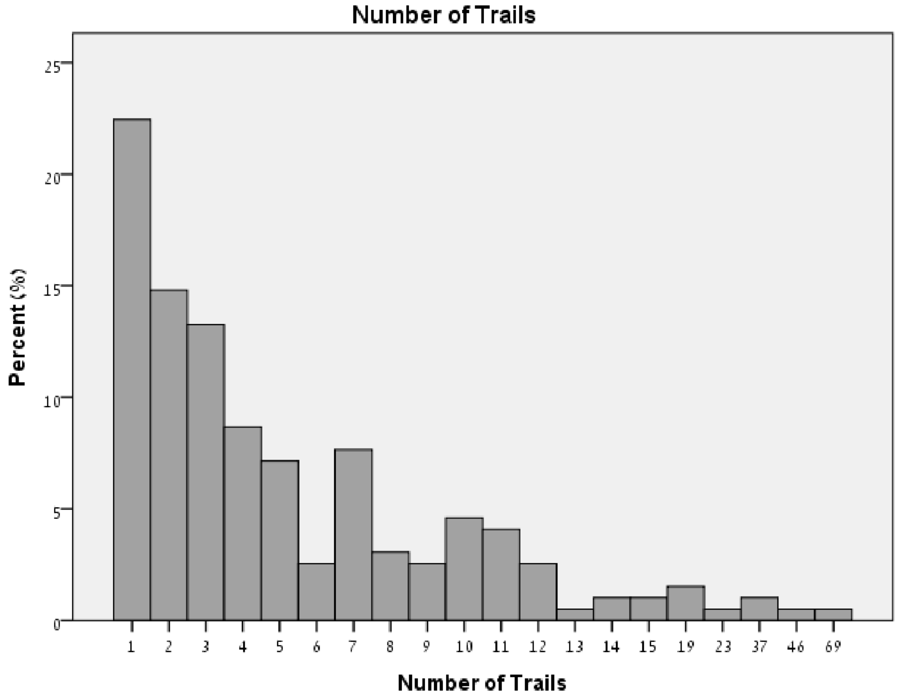

Figure 4. Distribution of students' trails

60 students (nearly 30%) spent only 40 seconds to solve the problem (M=200.31, SD=225.92) while 70 more spend between 40 and 200 seconds (Fig. 5).

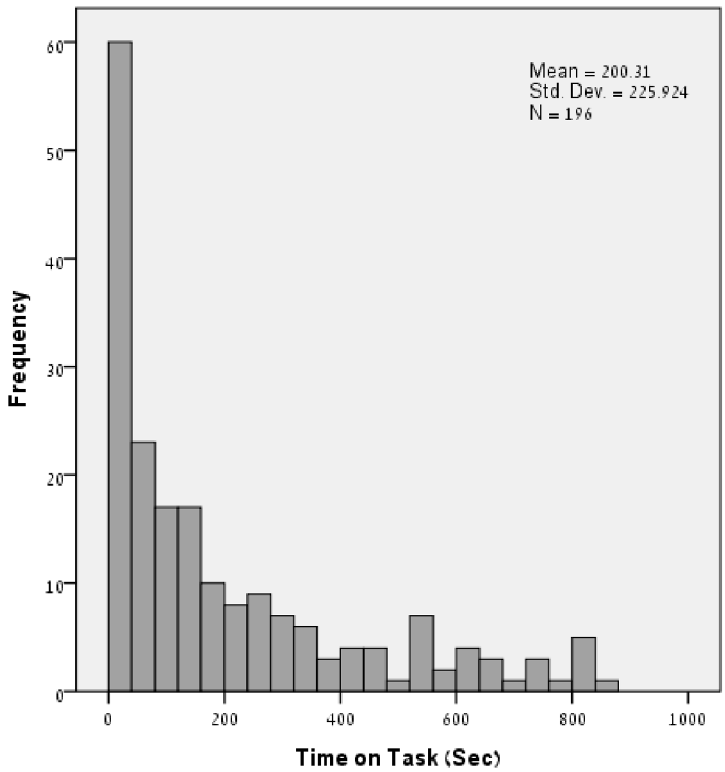

Figure 5. Distribution of students' time on task

When examining the types of responses the students gave and trying to categorize the different misconceptions (that were observed in the qualitative part), 4 major types were observed: a minor mistake (arithmetic or mis-counting), misconception related to the amount of squares in the grid (100 instead of 120), a misconception related to the numerator of the fraction (6) and a misconception related to the denominator of the fraction (10). One can see (Table 3) that only 19.9% of students reached the correct answer and 23% had the known misconceptions. Only 4 students (2%) had the misconceptions related to the numerator and denominator. 13.3% had arithmetic difficulties. The other 57.1% had mistakes that were not categorized.

Table 3. The misconceptions frequency in the students' responses

Response	N	Frequency	%
Correct	196	39	19.9
Minor mistake	196	26	13.3
Amount	196	15	7.7
Numerator	196	1	0.5
Denominator	196	3	1.5
Else	196	112	57.1

When trying to examine the relations between the number of trails and the different misconceptions (Fig. 6), one can observe that the average numbers of trails is similar in correct answers and wrong ones (M=4.18 vs. M=4.88) while students who had minor arithmetic mistakes tried "harder" (M=6.88). Those who had misconceptions related to the numerator and denominator had less than 1.67 trials (on average). This is expected due to the fact that these misconceptions are strongly associated with intuitive knowledge that interfere and conceptual one and thus students tend to solve the problem quickly.

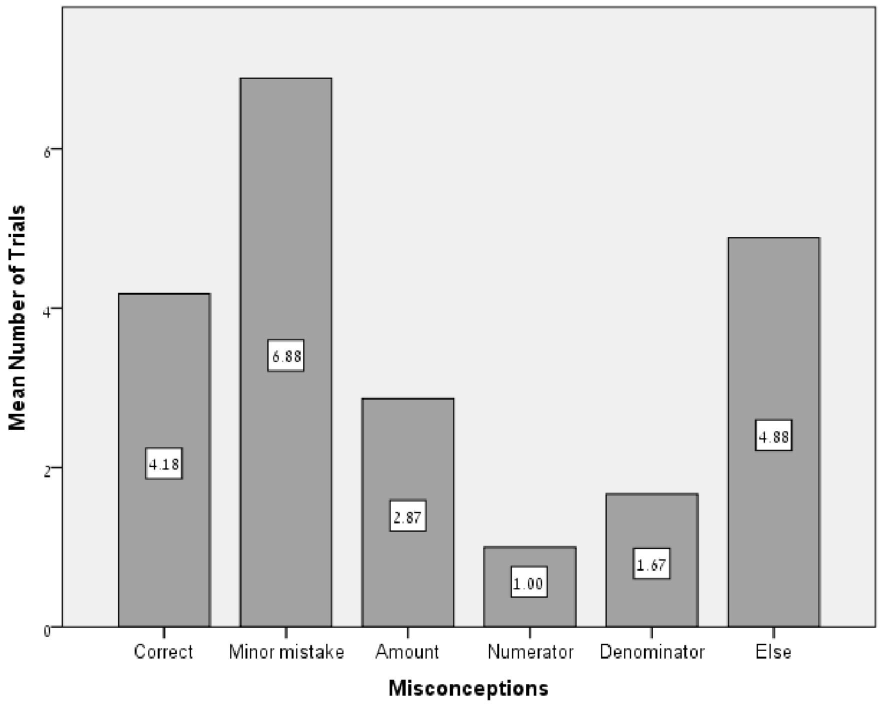

Figure 6. The relationship between number of trials and misconceptions

Fig. 7 demonstrated the relationship between the different misconception observed and the time on task. Students who answered correctly spent an average of 144.8 seconds while those who had mistakes took longer. Again, we can see that misconceptions related to numerator and denominator took much less time (intuitive knowledge) and also those who had used the grid of 100 (instead of 120) took less time due to the easy calculations needed to solve the problem.

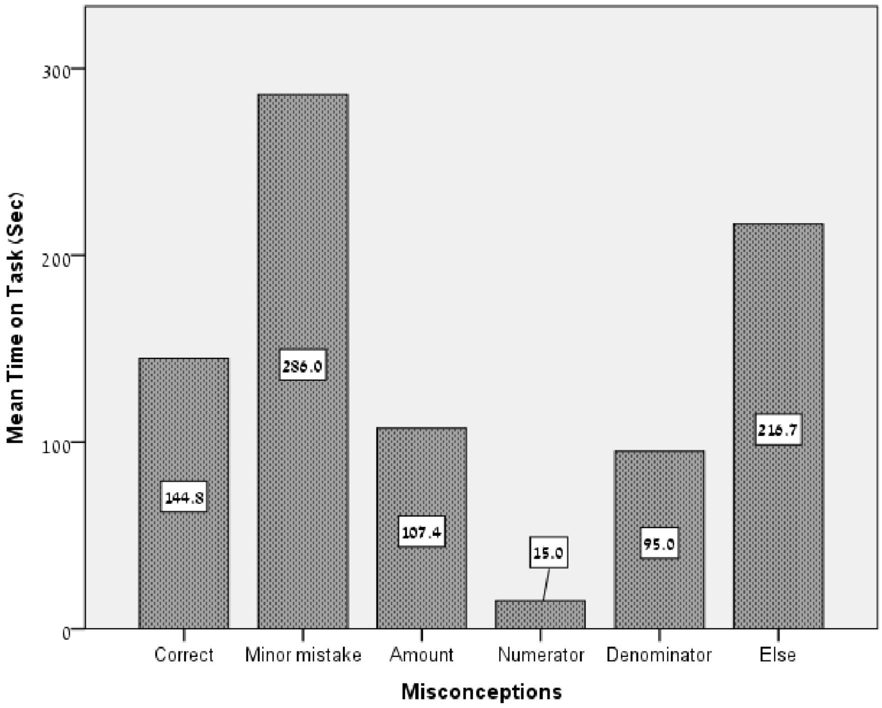

Figure 7. The relationship between time on task and misconceptions

Discussion and future work

One of the main difficulties of EDM is to determine which actions can be interrupted to meaningful cognitive, meta-cognitive and affective attributes. As shown, the logged data can be explained and "translated" by the thinking aloud protocols and the theoretical framework. While the correspondence scheme is comprised of many similar use cases found in the qualitative part of the research, this paper presented one use case and how raw data can be transformed to meaningful cognitive and meta-cognitive variables.

The second part of the research used the variables that were determined and computed in the correspondence phase and tried to examine the behaviors of 230 students. The analysis of the log file's extracted variables provided a summary of the students' behaviors. Our initial findings indicate that when students face a problem related to fractions, most of them don't exhibit conceptual knowledge in fractions and also those who possess this knowledge need several trials in order to solve (Eisenhart et al., 1993) Intuitive knowledge interfere with the conceptual knowledge and cause (when strong) to quick response (time and trial wise as suggested in the literature by Fischbein (1993)) although most of the students didn't exhibit this behavior. Most of the students monitor their problem solving process (more than expected), maybe due to the fact that the Israeli new Mathematics curriculum encourage students to do so.

On the practical level, results of this study will enable the automatic assessment of problem solving processes for large populations; this will aid both instructors and researchers to better understand students' behavior.

Our future work will focus on gathering more variables; examine more behaviors and employing data mining techniques in order to find problem solving patterns and attributes that affect them.

References

Baker, R. S.J.d., & Yacef, K., (2009). The State of Educational Data Mining in 2009: A Review and Future Visions. *Journal of Educational Data Mining, 1*(1), 3-17.

Creswell, J. W. (2003). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*, Thousand Oaks, CA: Sage Publications.

Eisenhart, M., Borko, H., Underhill, R. G., Brown, C. A., Jones, D., & Agard, P. C. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding, *Journal for Research in Mathematics Education*, *24*, 8-40.

Fischbein, E. (1993). The interaction between the formal, the algorithmic and the intuitive components in a mathematical activity. In R. Biehler, R. Scholz, R. Straser, & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline*, pp. 231–245. Dordrecht, The Netherlands: Kluwer.

Nachmias, R. & Hershkovitz, A. (2007). Using Web mining for understanding the behavior of the online learner, *Paper presented at The International Workshop on Applying Data Mining in e-Learning (ADML'07)*, Crete, Greece.

Newell, A., & Simon, H. (1972). *Human problem solving*, Englewood Cliffs, NJ: Prentice Hall.

Pahl, C. (2004). Data mining technology for the evaluation of learning content interaction, *International Journal of E-Learning*, *3*(4), 47-55.

Polya, G. (1957). *How to Solve It; a New Aspect of Mathematical Method (2nd ed.),* Doubleday, Garden City.

Schoenfeld, A.H. (1985). *Mathematical problem solving*, Orlando, FL: Academic Press.

Schoenfeld, A.H. (1987a). Cognitive science and mathematics education: An overview, in A.H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education*, pp. 1-31. Lawrence Erlbaum Associates, London.

Schoenfeld, A.H. (1987b). What's all the fuss about metacognition? In A.H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education*, pp. 189–215. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A.H. (1989). Explorations of students' mathematical beliefs and behavior, *Journal for Research in Mathematics Education*, *20*, 338–355.

Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics, In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning*, pp. 334–370. New York: Macmillan.

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing, *Zentralblatt fuer Didaktik der Mathematik*, 29(3), 75–80.