Sectoral Heterogeneity, the Skill Premium and Productivity Dynamics

Abstract

An intricate dynamic pattern has been commonly observed in many developed countries during the past decades. This pattern contains a simultaneous rise in the following economic variables: (i) education premium, (ii) educated labor supply, (iii) total factor productivity, (iv) labor productivity, and (v) income inequality. Typical explanations for the different elements of this pattern assume a skill-biased technical change or capital-skill complementarity. However, none of the models in this literature has provided a unified explanation for all these elements. In this study we offer such a unified theory which is based on sectoral heterogeneity and endogenous factor mobility, rather than on a skill bias.

Keywords: Income Distribution, Economic Growth, Factor Productivity

1. Introduction

In the past few decades, most developed economies have experienced a dynamic pattern of physical and human capital accumulation, rising inequality, rising total factor productivity, rising labor productivity and rising skill premium.[1] The literature on economic growth has usually explained these dynamics by assuming a skill bias of the sources of economic growth – technical change or capital accumulation. However, while explaining some of the elements of the described dynamic pattern, none of the models in this literature has provided a unified explanation for all of them. In this study, we propose such a unified explanation which is based on sectoral heterogeneity and endogenous factor mobility, rather than on a skill bias or a technical change.

To that end, we construct a general equilibrium dynamic model with two sectors – one more productive than the other. In order to work in the more productive sector, an individual has to acquire education. Acquiring education is an individual choice based on expected future skill premium and on the cost of education. Firms choose their technology endogenously, as they have to choose between operating in the more advanced sector or in the less advanced one. The analysis of the resulting macroeconomic equilibrium highlights a feedback relationship between investment in education and investment of physical capital

[1] For a more detailed description of these trends, see Autor (2014).

in the advanced sector, as one promotes the marginal productivity of the other. This feedback mechanism leads to one of our main results, namely that the skill premium rises over time.

We also show that the rising skill premium may lead to a dynamic pattern of a rising income inequality. However, it is also possible that income inequality shall not be monotonically rising, but instead, shall exhibit Kuznets curve dynamics, in which it initially rises and from a certain point in time begins to decline, even though the skill premium continues to rise. The decline in inequality occurs when the number of high-skilled workers is sufficiently large to make the relative equality within the high-skilled workers dominate the rising inequality between the two groups of workers. This is a quite different mechanism from the one which generates the Kuznets curve dynamics in most models of the relevant literature. Usually, in these models a skill-biased shock raises inequality by raising the education premium, and then, in response to the rising education premium, the supply of educated labor increases, resulting a decline in the education premium and inequality. Thus, these mechanisms cannot generate falling unequality alongside a continuously rising skill-premuim.

The increasing share of physical capital allocated to the advanced sector, and the rise in the share of the population that chooses to become high-skilled, make both labor productivity and TFP rise over time. The rise of these two measures of productivity is not an outcome of a technical change (as technology is assumed constant in this model), but rather due to endogenous shift of investments in physical and human capital from the less productive sector to the advanced one, which increases the contribution of the advanced sector's TFP to the economy-wide TFP.[2]

Our study is related to several strands of the literature. First, in the past few decades, many studies have argued that differences in output per capita between countries stem from differences in productivities. Productivity differences between countries were explained by either technological differences (e.g. Romer (1993)), or other, non-technological, differences such as capital barriers (e.g. Restuccia (2004) and Parente et al. (2000)), or different institutional and governmental infrastructure (e.g. Hall and Jones (1999)).[3]

In dealing with productivity dynamics, with an emphasis of the role of sectoral heterogeneity, our study is particularly close in its nature to Acemoglu and Zilibotti (2001) and Caselli and Coleman (2006). Acemoglu and Zilibotti (2001) argue that the mismatch between technologies and human capital endowments yields productivity differences between countries. As in their model, our results spring from a mechanism in which with more high-skilled labor, the advanced sector attracts a larger magnitude of investment, either in physical capital (as in our model) or in R&D (as in Acemoglu and Zilibotti (2001)), which augments

[2]This result is close in its nature to the result in Zeira (2009). In his model, an increase of the stock of educated workers increases the profitability of adopting a new type of machines, and thus promotes economic growth indirectly.

[3]Another explanation for TFP differences relies on misallocation of production factors between heterogeneous firms. See the survey by Restuccia and Rogerson (2013).

the productivity of this sector. A major difference between our study and theirs is that in our model human capital is endogenous. Another important difference is that due to our simpler framework, we can analyze the transition towards the steady state, and not just focus on the steady state of a balanced growth path.

Caselli and Coleman (2006) find empirically that countries with higher human capital endowment choose more skill intensive technologies than countries where human capital is scarce do. Unlike our study, Caselli and Coleman (2006) do not focus on individuals choices and also do not focus on the dynamics of productivity and inequality.

Two studies, which provide a unified explanation to most of the elements of the dynamic pattern analyzed in this paper are Acemoglu (1998) and Galor and Moav (2000). In contrast to the current paper, both of these studies rely on technical change. Yet in Galor and Moav (2000) a strong emphasis is assigned to a mechanism by which initially the technical change decreases the productivity of all workers, and thus generates a significant, though temporary, productivity decline.

Our ability in the current study to provide a unified explanation for the phenomena mentioned above without technical change and by highlighting the very plausible assumption of sectoral heterogeneity may lower the importance that is attributed to technical change.

Our study also relates to the vast literature about the interrelation between human capital acquisition, the dynamics of the skill premium and income inequality trends. It is a well documented fact that the skill premium has risen in the past decades despite the large increase in the stock of educated workers.[4] It is also known that inequality has risen during these decades (See, for example, Autor et al. (2008) for evidence of the rising income inequality in the United States since 1980). These two phenomena took place while the supply of education rose as well.[5]

2. The Model

Consider a closed OLG economy with a constant population along time. Each generation lives three periods. In the first period of her or his life each individual chooses whether to acquire higher education or not; In the second period of life, all individuals work according to their educational level, consume, save and each one of them gives birth to one offspring; In the third period of life all individuals are retirees, and consume all their savings.

Production takes place according to two production processes: less advanced and more advanced. In order to work in the advanced sector, individuals have to acquire education, which is costly; Firms, too, have to decide in which sector

[4] Krusell et al. (2000) provide compelling evidence that the skill premium has risen dramatically from 1980.

[5] The rise in the supply of education is well documented as well. See, for example, Goldin & Katz (2007).

to invest. All markets are fully competitive, and therefore factor prices equal their marginal product.

2.1. Production and Factor Prices

Production takes place in a fully competitive environment. Aggregate output at period t, Y_t, is produced by two technologies, low-skilled intensive and high-skilled intensive, L and H, respectively:

$$Y_t = A_H \left(K_t^H\right)^{\alpha} H_t^{1-\alpha} + A_L \left(K_t^L\right)^{\alpha} L_t^{1-\alpha} = A_H H_t \left(k_t^H\right)^{\alpha} + A_L L_t \left(k_t^L\right)^{\alpha}, \quad (1)$$

where $A_H > A_L$ are sector specific technology parameters, K_t^O is the capital employed in sector $O \in \{H, L\}$ at period t; H_t and L_t are the stocks of high-skilled- and low-skilled- labor that are employed in production respectively; and $k_t^O \equiv \frac{K_t^O}{O_t}$.

2.1.1. Factor Prices

Factor markets are competitive, and therefore factor prices equal their marginal product:

$$R_t = \alpha A_H \left(k_t^H\right)^{\alpha-1} = \alpha A_L \left(k_t^L\right)^{\alpha-1}, \quad (2)$$

and the inverse demand for each type of workers is given by:

$$w_t^H = (1-\alpha) A_H \left(k_t^H\right)^{\alpha}, \quad (3)$$

and

$$w_t^L = (1-\alpha) A_L \left(k_t^L\right)^{\alpha}, \quad (4)$$

where R_t is the rental rate of physical capital and w_t^O is the wage paid at period t to a worker in sector O.

2.2. Individuals

Individuals derive utility from consumption in their second and third periods of life. For simplicity, we assume that the utility function of each individual i that is born at period t-1 takes the following form:

$$u^i(c_t^i, c_{t+1}^i) = (1-\beta)\ln(c_t^i) + \beta \ln(c_{t+1}^i), \quad (5)$$

where $\beta \in (0,1)$ and c_t^i is the consumption of individual i at period t. Each such individual faces a budget constraint:

$$c_t^i + \frac{c_{t+1}^i}{R_{t+1}} \leq W_t^i, \quad (6)$$

where W_t^i is the wealth of individual i at period t. The individual's wealth depends on the individual's educational level and the individual's educational cost (if higher education is acquired).

We assume that individuals in each generation are heterogenous in ability. The heterogeneity is materialized in the cost of acquiring higher education, h_t^i:

the higher the ability the lower the cost. We assume that this cost is uniformly distributed in the range $(0, 1)$ and i.i.d. across generations. Hence, the wealth of individual i is given by:

$$W_t^i = \begin{cases} w_t^L & \text{if } i \text{ is a low-skilled worker} \\ w_t^H - h_{t-1}^i \cdot R_t & \text{otherwise.} \end{cases} \tag{7}$$

3. Savings and Capital Investment

3.1. Individual's Optimization Solution

Each individual lives three periods. In the first period each individual decides whether to acquire higher education and become a high-skilled worker in the second period of life, or give up education and become a low-skilled worker. In the second period each individual supplies inelastically his unique unit of time to the labor market, according to his educational level; he consumes, gives birth to one offspring and saves for his consumption in his retirement period. Hence, in the second period of life each individual has to divide his wealth between consumption in the second and third periods of life. Since the utility function is separable, we can first analyze this last decision, and then, based on this decision we analyze backwards the educational decision.

3.1.1. Consumption–Savings Decision

Given his educational level, in his second period of life, individual i chooses c_t^i and s_t^i so as to maximize his utility as given by (5), under the budget constraint as given by (6) and $s_t^i = \frac{c_{t+1}^i}{R_{t+1}}$. It is straightforward that $c_t^{i*} = (1-\beta)W_t^i$, and $s_t^{i*} = \beta W_t^i$. Thus, due to (7), the individual's educational level uniquely determines the consumption level in both second and third periods of life.

3.1.2. Educational Decision

At the first period of life, individual i, born at period t, decides whether to acquire education or not. Clearly, individual i acquires education if his utility is higher as a skilled worker. The indirect utility of individual i from his wealth is given by:

$$V^i(W_{t+1}) = \ln[(1-\beta)W_{t+1}^i] + \ln(\beta W_{t+1}^i R_{t+2}). \tag{8}$$

It is straightforward that the higher the wealth the higher the (indirect) utility. As a result, individual i acquires education if (and only if):

$$w_{t+1}^H - R_{t+1} h_t^i \geq w_{t+1}^L$$

This in turn yields a cost threshold, $\overline{h}_t$:

$$\overline{h}_t = \frac{w_{t+1}^H - w_{t+1}^L}{R_{t+1}}, \tag{9}$$

below which all the individuals that were born at period t acquire education, and above which the individuals of that generation do not acquire education.

3.2. Physical Capital Allocation Decision

From (2) it follows that:

$$k_t^H = \gamma k_t^L, \tag{10}$$

where $\gamma \equiv (A_H/A_L)^{\frac{1}{1-\alpha}}$. Equation (10) implies that in equilibrium, the higher the ratio of productivities in the two sectors, the higher the ratio of capital per worker in the two sectors. This equation also implies that the higher the ratio of high-skilled- to low-skilled-labor, the higher the ratio of physical capital allocated in the skilled sector to the unskilled sector (K_t^H/K_t^L).

4. The Dynamical System

Recall that ability is uniformly distributed in the range $(0,1)$. Recall also that at each period t all individuals with ability lower than $\overline{h}_t$ acquire higher education, whereas the rest of the generation forms the low-skilled labor force. This in turn implies that the supply of skilled labor at period $t+1$ is given by:

$$H_{t+1} = \overline{h}_t, \tag{11}$$

and the supply of unskilled labor is given by:

$$L_{t+1} = 1 - \overline{h}_t. \tag{12}$$

Applying (9) in(11) and then applying (2), (3),(4), and (10) in it yields:

$$k_{t+1}^L = \frac{\alpha}{(1-\alpha)(\gamma-1)} H_{t+1}. \tag{13}$$

This last equation implies that output can be represented as a function of H_t alone. Specifically, applying (10), (11), (12), and (13) in (1) yields:

$$Y_t = \frac{\alpha^\alpha}{(1-\alpha)^\alpha(\gamma-1)^\alpha} A_L \left[(\gamma-1)H_t^{1+\alpha} + H_t^\alpha\right]. \tag{14}$$

Physical capital for period $t+1$ is formed during period t, and satisfies (10). Note that the funds for financing the formation of physical capital stem from the aggregate savings in the economy, which are given by:

$$\begin{aligned} S_t = \int_i s_t^i di = \beta \left[(1-\alpha)Y_t - R_t \int_0^{\overline{h}_{t-1}} h_{t-1}^i \cdot f(h_{t-1}^i) dh^i \right] \\ = \beta \left[(1-\alpha)Y_t - \frac{1}{2} R_t \overline{h}_{t-1}^2 \right]. \end{aligned} \tag{15}$$

This implies that the aggregate amount of capital at period $t+1$ which is allocated in the two sectors accompanied by the investment in human capital, which equals $\int_0^{\overline{h}_t} h_t^i f(h_t^i) di = \frac{1}{2}\overline{h}_t^2$, must equal the savings of period t:

$$K_{t+1}^H + K_{t+1}^L + \frac{1}{2}\overline{h}_t^2 = \beta \left[(1-\alpha)Y_t - \frac{1}{2} R_t \overline{h}_{t-1}^2 \right] \tag{16}$$

Applying (2), (10), (11), (12), (13) and (14) yields the following autonomous first-order dynamic equation:

$$H_{t+1} = \frac{\sqrt{\alpha^2 + \Gamma\left[(\gamma-1)H_t^{1+\alpha} + 2H_t^{\alpha}\right]} - \alpha}{(1+\alpha)(\gamma-1)} \equiv f(H_t), \tag{17}$$

where $\Gamma \equiv \beta A_L \alpha^{\alpha}(1+\alpha)(1-\alpha)^{2-\alpha}(\gamma-1)^{2-\alpha}$.

It is straightforward that the initial stocks H_0, K_0^H and K_0^L uniquely determine H_1 via (1), (2) and (16), and this sets the economy's equilibrium path for all subsequent periods via (17).

As will become apparent in Lemma 1, the following parametrical assumption is a sufficient condition to assure the existence of a unique steady state in which the stock of high skilled workers, denoted $\overline{H}$, is smaller than unity:

Assumption 1. $\gamma < \frac{3-\alpha}{1+\alpha}$

Lemma 1. *$f(H_t)$ satisfies the following properties:*
(a) $f(0) = 0$
(b) $f'(H_t) \geq 0$*, for all* $H_t \geq 0$
(c) $f''(H_t) \leq 0$ *for all* $0 \leq H_t \leq 1$
(d) $\lim_{H_t \to 0} f'(H_t) = \infty$
(e) $\lim_{H_t \to \infty} f'(H_t) = 0$

The proof of Lemma 1 is in appendix A. From the lemma it follows that the dynamical system H_t has a single and stable steady state equilibrium with $\overline{H} < 1$.

Assumption 1 is a sufficient condition for the existence of a steady state with $\overline{H} < 1$, yet it is not a necessary one, and Appendix A provides less binding parametrical conditions for that. We choose this one for two reasons. First, its magnitude fits well the data: given the common evaluation of $\alpha = 0.3$, the RHS of this assumption equals 2, while equation (10) and the estimates for its observables in **?**, chapter???, show that γ should be around 1.4-1.6. Second, this parametrical assumption simplifies greatly the proof the Lemma.

5. Productivity Differences

In this section we analyze the transitional dynamics and show that as human capital accumulates, both TFP and labor productivity increase. Throughout the following sections we assume that $H_1 < \overline{H}$, i.e., that the economy is in transitional dynamics characterized by a rising H_t.

5.1. TFP Dynamics

A well known fact is that countries with higher human capital endowment tend to have a higher level of TFP. Explanations for this phenomenon have varied from technological differences (e.g. Romer (1993)), to non-technological differences such as social infrastructure differences (Hall and Jones, 1999) or barriers to physical capital (Restuccia, 2004). Acemoglu and Zilibotti (2001) argue that in the steady state, countries with different human capital endowments have different productivities, because of a mismatch between machines and human capital. Their paper, however, relies on the premise that human capital is exogenous and constant over time. The following proposition shows, however, that the same mechanism in which human capital attracts investment in physical capital (R&D in Acemoglu and Zilibotti (2001)) yields differences in TFP. However, since human capital is endogenous in our model, these differences exist only during these transitional dynamics.

Proposition 1. *Total factor productivity increases along time.*

Proof. TFP is given by:

$$TFP_t = \frac{Y_t}{K_t^{\alpha} \cdot \left(\frac{A_H}{A_L} H_t + L_t\right)^{1-\alpha}}, \tag{18}$$

where K_t is the total amount of capital in the economy in period t. Applying (1) and rearranging yields:

$$TFP_t = \frac{A_H \left(\frac{K_t^H}{K_t}\right)^{\alpha} H_t^{1-\alpha} + A_L \left(\frac{K_t^L}{K_t}\right)^{\alpha} L_t^{1-\alpha}}{\left(\frac{A_H}{A_L} H_t + L_t\right)^{1-\alpha}}. \tag{19}$$

The following relations: $K_t^H = k_t^H \cdot H_t$ and $K_t^L = k_t^L \cdot (1 - H_t)$, together with (10) lead to:

$$\frac{K_t^H}{K_t} = \frac{\gamma H_t}{1 + H_t(\gamma - 1)}$$

and

$$\frac{K_t^L}{K_t} = \frac{1 - H_t}{1 + H_t(\gamma - 1)}$$

Plugging these two expressions into (19) and noting that $A_H/A_L = \gamma^{1-\alpha}$ yields:

$$TFP_t = A_L \left[\frac{(\gamma - 1)H_t + 1}{(\gamma^{1-\alpha} - 1)H_t + 1}\right]^{1-\alpha}. \tag{20}$$

We define the expression within the squared brackets by $\phi(H_t)$. It follows from straightforward differentiation that $\phi'(H_t) > 0$ and therefore that $TFP_t'(H_t) > 0$. Thus, since H_t increases along time, so does the TFP. □

From Proposition 1 and equations (10) and (13), it follows that during the transitional dynamics, as human capital accumulates, more physical capital is allocated to the more advanced sector, a mechanism that increases the total factor productivity. This is in line with Zeira (2009), who shows that human capital increases the profitability of adopting a new type of machines, and thus promotes economic growth.

5.2. Labor Productivity Dynamics

Another productivity measure that is common in the literature is labor productivity. The following proposition states that along the transitional dynamics, as human capital accumulates, labor productivity increases:

Proposition 2. *Labor productivity increases along time.*

Proof. Following Acemoglu and Zilibotti (2001), labor productivity is given by

$$\tilde{y}_t \equiv \frac{Y_t}{A_H H_t + A_L L_t}. \tag{21}$$

Applying (14) and (13) in the expression for $\tilde{y}_t$, output per efficiency unit becomes:

$$\tilde{y}_t = \frac{A_L \left(k_t^L\right)^{\alpha} [(\gamma - 1)H_t + 1]}{A_H H_t + A_L(1 - H_t)} = \left(k_t^L\right)^{\alpha} \cdot \phi(H_t) \tag{22}$$

k_t^L is rising in H_t by (13), and $\phi'(H_t) > 0$, as was established in the proof of Proposition 1. Thus, $\tilde{y}_t$ is also rising in H_t. □

Proposition 2 shows how the feedback effect between human capital and physical capital affects labor productivity along the transitional dynamics. In particular, it shows that investment in human capital affects the allocation of physical capital, as more physical capital is allocated in the advanced sector. This mechanism makes raw labor more productive, since skilled labor works with more physical capital, which increases the marginal productivity of the skilled labor. This implies that countries with more human capital than others invest more in high-skilled labor intensive sectors, and this shift of physical capital from low-skilled intensive sectors to high-skilled intensive sectors increases the productivity of labor. This proposition, therefore, may explain why countries with different human capital endowments have different labor productivity levels, and not only TFP differences, even when human capital is taken into account.

6. Skill Premium and Inequality Dynamics

It is a well known fact that in the last few decades many economies have experienced both a rise in the skill premium and a rise in income inequality accompanied with a rise in the educated labor force. The main two explanations for the coincidence of the three phenomena were skill-biased technological change (Galor and Moav, 2000; Acemoglu, 1998) and capital-skill complementarity (Krusell et al., 2000). In this section we provide another possible explanation for these phenomena. In particular, we show that along the transitional dynamics the skill premium increases, and that income inequality increases in the beginning of the development process, but may exhibit a Kuznets curve pattern.

Proposition 3. *Along the transitional dynamics the skill premium increases.*

Proof. Let $w_t^H - w_t^L$ be the skill premium. Then it equals:

$$w_t^H - w_t^L = (1-\alpha)A_L(\gamma - 1)\left(k_t^L\right)^{\alpha}.$$

The only element that evolves along time is k_t^L, which increases along time as was shown before. Consequently the skill premium increases as well. □

Proposition 3 suggests that along the transitional dynamics the skill premium increases, despite the rise of the high-skilled labor force and the decline of the low-skilled labor force. The reason for this result is the above mentioned mechanism. During the transitional dynamics, both wages –of low-skilled- and high-skilled labor –increase. The increase in the wage of low-skilled labor is a consequence of a decrease in the supply of low-skilled labor and an increase in the demand for low-skilled labor, which is the result of allocating more physical capital (per worker) in this sector than in the previous period. The increase in the wage of the high-skilled labor is a consequence of an increase in the demand for high-skilled labor due to an increase in the physical capital allocated for this sector. The rise in the demand for high-skilled labor offsets the negative effect that the increase in the supply of high-skilled labor has on the wage of the high skilled workers, so their wage increases as well. Note that the increase in the physical capital per worker in sector H is larger than the increase in the capital per worker allocated for sector L, since $k_t^H = \gamma k_t^L$. This relatively large increase in the wage of high-skilled labor offsets the negative effect on the skill premium that the increase in the wages of the low-skilled labor has.

The result about the skill premium assists us to explore the dynamics of income inequality in the economy. The following proposition shows that income inequality increases for small values of H_t and declines for sufficiently high values of H_t. Since H_t increases along time we conclude that income inequality rises at the outset of development, and may decline at later stages of development. Hence, income inequality may increase along time, or exhibit a Kuznets curve pattern.

Proposition 4. *At the beginning of the development process, income inequality increases as H_t increases. At later stages of development, income inequality may decline.*

Proof. We measure income inequality by the variance of income. The average income at period t is given by:

$$\overline{w}_t = H_t \cdot w_t^H + (1 - H_t) \cdot w_t^L.$$

Therefore, the variance of income is given by:

$$\sigma_t^2 = H_t \cdot \left(w_t^H - \overline{w}_t\right)^2 + (1 - H_t)\left(\overline{w}_t - w_t^L\right)^2 = H_t(1 - H_t)(w_t^H - w_t^L)^2, \quad (23)$$

where the last expression is obtained after substituting into the first expression the wages as given by (3) and (4). Differentiating the last expression with respect to H_t yields:

$$\frac{\partial \sigma_t^2}{\partial H_t} = (1 - 2H_t)(w_t^H - w_t^L)^2 + 2H_t(1 - H_t)(w_t^H - w_t^L)\frac{\partial(w_t^H - w_t^L)}{\partial H_t}.$$

Using (23), this equation becomes:

$$\frac{\partial \sigma_t^2}{\partial H_t} = [(1 - \alpha)A_L(\gamma - 1)(k_t^L)^\alpha]^2(3 - 4H_t),$$

where the last expression is obtained by using (13). The derivative is positive as long as $0 < H_t < 0.75$, and negative as long as $0.75 < H_t < 1$. Since H_t increases along time, the variance in income increases at the outset of the development process, and may decline at later stages of the development process, if $\overline{H} > 0.75$. □

The same result, via a similar proof, is obtained by using the Gini coefficient and not the variance of income. A recent important example for the complete Kuznets curve pattern is Germany's income inequality, which after a long period of a stable increase, has been gradually declining since 2010 (Hutter and Webber, 2017).

Proposition 4 sheds light on the dynamics of income inequality in the economy. As the economy develops, two forces with opposite signs affect income inequality: wage inequality between the two groups of workers (the skill premium) and the relative abundance of high-skilled workers. As the economy develops the skill premium increases (as shown in Proposition 3), a force that increases inequality, but the relative abundance of high-skilled workers increases as well, which in turn decreases income inequality. According to Proposition 4, at the outset of the development process, the former is greater than the latter, while in later stages of development the opposite may occur. In later stages of development, it is the relative equality between high-skilled workers that may dominate the rising inequality between the two groups of workers. Note that this result is not a trivial outcome of the fact that the factor $H_t(1-H_t)$ has an inverse-U shape, as occurs in models where wages are exogenous. Here with the endogenous determination of wages, and the Inada condition of the production functions, the third element in RHS of (23) falls at an infinite rate at the vicinity of $H_t = 0$, which, in a single sector model, dominates the rise in $H_t(1-H_t)$ in that vicinity.[6]

7. Conclusions

We presented a general equilibrium dynamic model with two sectors –one more productive than the other. We used this model to analyze how the heterogeneity in sector productivity affects the dynamics of physical and human capital accumulation, skill premium, income inequality, labor productivity and total factor productivity.

The analysis of the resulting macroeconomic equilibrium highlights a feedback relationship between investment in education and investment of physical capital in the advanced sector, as one promotes the marginal productivity of the other. This feedback mechanism leads to one of our main results, namely that the skill premium rises over time. We also find that the rising skill premium may lead to a dynamic pattern of a rising income inequality. However, it is also possible that income inequality shall not be monotonically rising, but instead, shall exhibit Kuznets curve dynamics, in which it initially rises and from a certain point in time begins to decline, even though the skill premium continues to rise. The decline in inequality occurs when the number of high-skilled workers is sufficiently large to make the relative equality within the high-skilled workers dominate the rising inequality between the two groups of workers.

This feedback mechanism also highlighted another indirect channel through which human capital promotes economic growth. As human capital is accumulated, more physical capital is allocated to the more advanced sector, and thus output grows faster. This effect leaded to the rise in the TFP and labor productivity along time. We showed that in this sense our results are close in their nature to Acemoglu and Zilibotti (2001), Caselli and Coleman (2006) and Zeira (2009), only in our model we analyze the transitional dynamics and not merely the steady state equilibrium. We also added to their results and used the model for analyzing inequality pattern. These results about the rise in the skill premium, income inequality and productivity fit the empirical findings presented by a massive body of literature.

The model is of a closed Overlapping Generations framework with some specific assumptions. Hence, the issue of robustness should be discussed. First,

[6] See, for example, Maoz & Moav (1999), where in a model with a single sector, inequality is monotonically falling for all positive H_t, as one example of many for the dominance of the wage effect induced by the Inada conditions.

similar specific functional forms are widespread in this strand of the literature. Second, the functional forms are motivated strongly by empirical evidence. Using another production functions and a utility function that satisfy the usual assumptions will change the results quantitatively but not qualitatively.

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Appendix A

The following appendix provides the proof for Lemma 1. From (17):

$$H_{t+1} = f(H_t) = \frac{\sqrt{\alpha^2 + \Gamma g(H_t)} - \alpha}{(1+\alpha)\cdot(\gamma-1)}, \tag{A.1}$$

where

$$g(H_t) \equiv (\gamma-1)\cdot H_t^{1+\alpha} + 2\cdot H_t^{\alpha} \tag{A.2}$$

and

$$\Gamma \equiv \beta \cdot A_L \alpha^{\alpha} \cdot (1+\alpha)(1-\alpha)^{2-\alpha} \cdot (\gamma-1)^{2-\alpha} \tag{A.3}$$

It is immediate from (A.2) that $g(0) = 0$, and therefore by (A.1) that $f(0) = 0$, which proves (a). From (A.1), (A.2) and (A.3) it also follows that $f(0) > 0$ for all $H_t > 0$.

From (A.2) it also follows that:

$$g'(H_t) = (\gamma-1)(1+\alpha)\cdot h_t^{\alpha} + 2\alpha H_t^{\alpha-1} > 0, \tag{A.4}$$

and from (A.1) if follows that:

$$f'(H_t) = \frac{\Gamma g'(H_t)}{2\cdot(1+\alpha)(\gamma-1)\cdot\sqrt{\alpha^2+\Gamma g(H_t)}} > 0, \tag{A.5}$$

where the inequality follows from (A.4). This proves (b).

From (A.4) it follows that:

$$g''(H_t) = \alpha(\gamma-1)(1+\alpha)H_t^{\alpha-1} + 2\alpha\alpha - 1\cdot H_t^{\alpha-2}, \tag{A.6}$$

and from (A.6) it follows that $g''(H_t) < 0$ throughout the range:

$$0 < H_t < \frac{2 \cdot (1-\alpha)}{(\gamma - 1) \cdot (1+\alpha)} \equiv \tilde{H}. \tag{A.7}$$

If $\gamma < \frac{3-\alpha}{1+\alpha}$ then, by (A.7), $\tilde{H} < 1$, implying that both $g''(H_t) < 0$ and $f''(H_t) < 0$ throughout the range $0 < H_t < 1$. This proves (c). (d) follows from (A.4).

From this lemma it follows that the dynamical system $\{H_t\}_0^\infty$ converges to a unique stable steady state. We denote this steady state level of H_t by $\overline{H}$. If parameter values lead to $f(1) < 1$, then $0 < \overline{H} < 1$ and otherwise $\overline{h} = 1$.