Intergenerational Transfers: An Integrative Approach

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Abstract

The empirical literature was unable to conclude whether intergenerational transfers are motivated mainly by altruistic or strategic motivations. I suggest that both may be plausible, namely that people are neither pure altruists who derive utility simply from being good to others, nor pure egoists who consider only strategic selfish considerations, but are actually driven by a combination of incentives. I show that assuming a combination of motives changes, sometimes dramatically, the results obtained in the traditional models and resolves the puzzle concerning the empirical results.

1. Introduction

Donations, contributions, and volunteering are three of many types of manifested altruism, namely the sacrifice of current or future felicity by one individual for the sake of others. Some of the transfers are made directly between persons or households, and some by contributing to non-profit organizations like charity organizations, hospitals, education institutions and more. The share of non-profit industry in civil labor force was estimated by 6.8% in the USA and 3.4% by average of European civil labor force. The share of private giving in the non-profit industry’s revenues was estimated by 19% in the USA and by 10% average in Europe (Rose-Ackerman, 1996). Estimations of intra-family intergenerational transfers in the United
State yielded a variety of numbers ranging from 80% of accumulated wealth (Kotlikoff & Summers, 1981) down to 20% only (Modigliani, 1986, 1988a, 1988b)\(^1\).

This paper concentrates on intergenerational transfers that take place within the family, namely sacrifices of donor’s current or future felicity in favor of a recipient who is a member of a different generation (or “cohort”)\(^2\) within the same family. There are many types of within-the-family sacrifices. Some are measurable (like inter-vivos gifts or post-mortem bequests) and some are non-measurable (like refraining from divorce for the sake of the children, selling or giving children to adoption by poor families to wealthier families, etc.). The analysis in this paper confines mainly to inter-vivos gifts and bequests, but can be easily generalized to all kinds of transfers.

There are many potential motives for private transfers. Three main approaches emerged in the economic literature that copes with the apparent altruism phenomenon: the normative, the altruistic and the strategic approaches (Shmueli, 1992). Samuelson’s (1958) normative approach was abandoned\(^3\), and the most studied motivations were altruism and exchange\(^4\). Although egoistic and altruistic motives are not necessarily mutually exclusive, these two remaining approaches were considered rival, and the general attitude of the prevailing literature was that egoistic and altruistic motives are indeed mutually exclusive. Therefore, the theoretical literature concentrated on one of the extreme edges of the continuum between pure egoism and

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1 Kotlikoff and Summers (1986) amended their estimation later, due to Modigliani’s criticism, (see also Barro, 1989).
2 In fact, intergenerational streams of resources are also a macroeconomic phenomenon and could stem, for instance, from a governmental intervention (like taxing young people wages to finance old people pensions in a pay-as-you-go social security system). In this paper we shall confine ourselves mainly to the microeconomic aspects of intergenerational transfers.
3 In Samuelson’s model the Pareto efficient equilibrium is obtained by the existence of social “norms”, based on a sort of a Hobbes-Rousseau “social convention” or legislation (like a social security or compulsory pension scheme). On the family level, the existence of norms is based on Kant’s moral imperative. There is no real reason for a child to be nice to his parents beyond these norms. But explaining irrational economic behavior on the basis of exogenous norms is actually assuming the results. Thus the normative approach seems more appropriate for ethical, philosophical or sociological discussion than for economic analysis. For instance, David Hume noted that human society is composed of overlapping generations of citizens, and that individuals cannot feel obliged to a “social convention” agreed upon before they were born and in whose formulation they took no part. Such analyses can be found even in more ancient era. (See for instance: Leviticus 19, 18, The Jerusalem Talmud Nedarim 9, 4, Babylonian Talmud Shabat 98a. For an ancient discussion reminding Samuelson’s consumption loans, see Jerusalem Talmud Kidushin 1, 4). In addition, Samuelson’s model prediction that introducing social security and transferring resources from young to old generations would stimulate economic growth and increase aggregate saving, is definitely rejected by the data (Feldstein, 1974, Gokhale et al. 1996).
pure altruism. According to this radical but prevailing attitude, if someone visits his old parents once a week, for instance, it is either because he really cares about their welfare and expect no return for his time and efforts, or because he tries to influence them to write their estate for him in their will (probably on the expense of his brothers and sisters). In short, the classical literature assumed that people are either angels or “devils” (or at least – hypocrites).

In this paper I argue that this radical simplifying attitude is actually an oversimplification which distorts the qualitative results of the theoretical studies as well as the quantitative results of empirical studies, implying both theoretical and empirical perplexity in the understanding of intergenerational transfers.

A possible conclusion from the insufficiency of each motive as a unique explanation for intergenerational transfers is indeed a rejection of both altruistic and strategic conjectures looking for other possible motivations. This paper’s approach is different. The basic assumption underlies this approach is that people are indeed neither angels nor devils. They are simply humans, driven by mixed incentives of altruism and egoism simultaneously. The mixed incentives assumption may be useful also in understanding why the empirical tests failed in confirming or rejecting any of the competing radical conjectures, but pose some difficulties in identifying the dominant motivation for a certain transfer.

**Main Findings and their Implications**

The model presented in this paper is an overlapping generations’ model relating to three periods, of which the third period is uncertain. Intergenerational transfers stem from a combination of altruistic and strategic motives. The strategic motive is based on the desire to expand life expectancy (raise the probability to live the third period). The model allows for two types of intergenerational transfers: bequests and inter-vivos gifts. It is shown that although people are motivated by a mix of motives, altruism and exchange, usually one of these motives is dominant. Furthermore, the dominancy of peoples’ behavioral motivations varies over time and it is sometimes

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4 More accurately, the normative approach was developed to the “Law and Economics” approach (see Becker and Murphy 1988, Elster, 1989, Posner, 2000). Different views on sacrifice, like those based on the theory of clubs (Iannaccone, 1992, 1998, Berman, 2000) are beyond the scope of this paper.
(but not always) possible to distinguish strategic transfers from altruistic transfers by their timing.

The model allows for three kinds of equilibria. In the first, no intergenerational transfers occur at all (a “corner solution”). In the second, only one type of transfers (bequests or inter-vivos gifts) exists but not both (a “non-satiated” solution). And in the third, all kinds of intergenerational transfers exist in the economy (an “interior solution”). Actually, even if a corner solution evolves in an economy, the conclusion that individuals have no motives for transfers whatsoever, altruistic and strategic, is inaccurate. It may be that both motives exist but some disturbances prevent their manifestations and more sophisticated empirical methods than hitherto applied are required. It is shown that a non-satiated solution is possible only if individuals are either pure altruists or nasty egoists. In this case there would be either bequests or inter-vivos transfers, but not both of them. An “interior solution” is therefore inconsistent with the existence of only one motive. However, in case of interior solution bequests are mainly altruistic, but the reason for transferring wealth posthumously is that even altruistic parents do not trust their children’s altruism towards them. So they prefer to hold the altruistically motivated bequest using it as a security for the “good behavior” of their children. Inter-vivos transfers from old parents to adult kids are usually a compensation for heirs for waiting until the transfer of wealth will happen. On the other hand, young parents’ investments in their kids are usually altruistic.

Within this analytical framework, it is certainly unsurprising that the empirical literature failed to confirm or to reject any of the competing hypotheses about the motives for transfers. Since both types of intergenerational transfers, bequests and inter-vivos gifts, exist in real world data, it follows according to this paper’s analysis that no motive can explain these transfers alone. This is true especially in empirical studies that analyze aggregate data which are not distributed on the basis of the timing of transfers, i.e., little attention is paid to the stage of life in which the transfers are made, implying that the same individual may be identified as egoistic in one empirical study, and as altruistic in another, depending on the individual’s life-cycle stage at which his behavior was observed and recorded. The empirical implication of this analysis is straightforward: intergenerational data should be sorted by their timing. However, this paper concentrates mainly on the theoretic aspect of the question, leaving aside the challenge of coping with the econometric problems and difficulties.
Although the main focus of this paper is microeconomic, the analysis presented here has for at least one macroeconomic implication. It is shown that the strict dichotomy between “Keynesian” and “Ricardian” economies assumed in the vast literature around this old polemic (Seater 1993) is inappropriate, and there is actually a continuum between these two extreme contingencies. The location of a certain economy on this continuum depends on a variety of cross influences.

The remainder of the paper is organized as follows: The next section provides a brief description of explanations proposed in the literature regarding intergenerational transfers and their motives as well as some of the empirical studies that were conducted to test them, and this literature’s deficiencies. Then, in section 3, we introduce an integrative model of intergenerational transfers and analyze the possible types of steady states and also the steady state under the traditional assumptions of radical human characteristics, namely perfect altruism or perfect egoism. In section five we return to the polemic surrounding Ricardian Equivalence and the real effects of governmental intervention and assert that the discussion so far has been partial and sometimes irrelevant. The final section is a brief summary followed by a technical appendix containing the proofs of the propositions presented in the text.

2. The Motivations for Intergenerational Transfers

Abel, (1985) counts five explanations that were mentioned by various authors for the existence of bequests. Except from early death of selfish individuals before consuming all their wealth, all other four reasons for bequeathing mentioned in the literature reflect either altruism or strategic motive of exchange.

The basic assumption underlying the altruistic approach, as introduced by Becker’s (1974, 1981) seminal papers, is that the utility of individual A creates positive externalities on the utility of individual B. Becker argued that as long as the behavior of the head of the family towards the rest of the members of the family

\footnote{For more references about existing literature see Bhaumik (2001)}

\footnote{The altruistic motive was sometimes formulated as parental paternalism towards their children, expressed by introducing the consumption of the next generation in the parents’ utility function (see Abel, 1988). This approach has yielded many papers because it was perceived to be related to concepts such as “love”, enabling the formulations of models describing idyllic families with one member (“father”) who takes care of the needs of all the others.}
reflects altruism, the “Rotten Kid Theorem” would apply, but this theorem is not
generally valid with more than one product, except for certain types of utility
functions (Bergstrom, 1989). Furthermore, even one-sided altruism does not
guarantee raising the sum of utilities in the economy and may even cause the opposite
(Becker and Murphy 1988, Stark, 1995), and the existence of an altruistic motive in
the utility function does not even guarantee intergenerational transfers unless the
altruistic coefficient exceeds a certain critical value (Abel, 1988)\(^7\). In the
macroeconomic context, the altruistic approach yielded Barro’s (1974) famous
Ricardian Equivalence theorem. Barro viewed bequests as the family’s device for
neutralization of fiscal policy effects’ intergenerational distribution of resources
within the family, implying that this kind of transfer is mainly altruistic. However, the
motives for making transfers to individuals do not necessarily reflect the market role
of transfers\(^8\).

The strategic approach was developed during the 80s, and was mainly based
on game theoretic models (Estaban & Sákovics, 1993, Bernheim & Bagwell, 1988,
Cox, 1992, Cigno, 1993)\(^9\). This approach views intergenerational interaction as
motivated by expectations regarding the responses of other players in the game.
Transfers are strategies that aim to maximize the donors’ utility, not because this
utility depends on the recipient’s utility (as in an altruistic model), but as a strategy in
an exchange game, taking place over time. According to this attitude, people are
egoists, motivated by the assessment that other players will act similarly in
equilibrium. Altruistic behavior is not a consequence of the “Rotten Kid Theorem”
but part of an exchange game. Another possible strategic motivation was studied by
Stark (1995) who assumed that altruistic behavior towards parents is motivated by the
will to educate children and show them the “proper” way of dealing with elderly

\(^7\) The prevailing analytical frameworks underlying most of the altruistic approach studies (both
empirical and theoretical) were Diamond’s (1965) overlapping generations’ model, combined with
Modigliani’s (1957) life cycle hypothesis. Basically, these papers postulate that bequests result mainly
from two contingencies: unexpected earlier death (“accidental bequests”), or deliberate sacrifice of
income by individuals for the welfare of others (usually their descendants).

\(^8\) Although the literature dealt mainly with post-mortem transfers, most intergenerational transfers are
inter-vivos (Cox, 1987). Most authors who studied inter-vivos transfers viewed them mainly as
liquidity constraint neutralizing devices or as substitutes for a missing annuities market (Kotlikoff and
Spivak 1981, Cox 1990, Altig and Davis, 1992, Cigno 1993). If this argument is true, it may provide
some explanation for the fact that most observed altruistic behavior is towards family members.

\(^9\) Bernheim & Bagwell’s and Cox concludes that if there is an altruistic element in agents’ behavior
and familiar ties tie most of the population, there would be no actual liquidity constraint and no way for
the government to influence any real variable in the economy. This conclusion seems too radical. (See
people in the hope that they will imitate this behavior when the time comes (what Stark terms the “demonstration effect”).” However, the origin of the imitation tendency is a mystery, or an exogenous psychological fact implying that the most important results of the model are actually based on an exogenous factor. It is unclear what rational reason there could be for an egoist child to imitate his parents’ behavior towards his grandparents\textsuperscript{10}. This assumption is not very different from Samuelson’s normative approach, meaning that it is more suitable for sociological or psychological research than for economic analysis.

Barro’s (1974) Ricardian Equivalence theorem yielded numerous papers about the macroeconomic aspects of altruism. Much of the theoretical papers attempted to explain the incapability of the empirical literature to decide over this question, namely to supply explanations why there is no firm basis in the data for Ricardian Equivalence that comply with the altruistic hypothesis from one side, or to supply explanations for the opposite view, from the other side.

The empirical studies were conducted under the (sometimes implicit) assumption that was mentioned in the introduction. Namely those altruistic and strategic motivations are mutually exclusive and cannot coexist simultaneously. Thus, it is unsurprising that none of the alternative hypotheses was unambiguously confirmed by the data\textsuperscript{11}.

For example, it was found in one empirical study that there is a tendency to allocate bequests equally among heirs and this finding was interpreted as contradicting the altruistic hypothesis, since altruistic motive would lead to allocating bequests in negative correlation to recipients’ income (Menchick, 1980)\textsuperscript{12}. However, other study found that bequests are indeed negatively correlated to recipients’ income, in compliance with the altruism hypothesis (Tomes, 1981)\textsuperscript{13}.

\textsuperscript{10} In game theory jargon, Samuelson’s “norms” and Stark’s “demonstration effect” are not subgame-perfect equilibriums. Cigno (1993) claimed that a social norm of one-generation demonstration effect is a subgame perfect strategy, but his explanation of how such a strategy would evolve is vague.

\textsuperscript{11} The same is true for the vast macroeconomic empirical studies about Ricardian Equivalence (see Seater, 1993).

\textsuperscript{12} In fact, I was not convinced that this is the only valid interpretation for this empirical finding. For instance, it may reflect parents’ desire to prevent “inheritance wars” after their death.

\textsuperscript{13} Actually, we must be wary of such conclusions before we have an accurate definition of altruism. If altruism is defined by “doing good to others” then the polemic makes sense. But if altruism is defined as “doing good things” with no distinction – then these empirical findings are inconclusive since the altruistic motives are also related to various variables such as culture, social philosophy, etc., that were not estimated in these studies.
A positive correlation between the intensity of the connections between children and their parents and the wealth of the parents, led some researchers to conclude that parents use bequests as a mean of exchange for services that they extract from their children (Bernheim et al., 1985)\(^{14}\). But this finding can tell us something about the attitude of children to their parents, but not necessarily about the nature of the parents’ altruism towards their children, especially if the income effect of the parents was not taken into account in empirical studies that support the strategic hypothesis\(^{15}\). Rich children usually have rich parents with a tendency for larger transfers. When controlling for this income effect, the results are more compatible with the altruistic model (McGarry, 2000). Other researchers, who examined the altruism hypothesis among American families, concluded that it is definitely rejected by the data (Altonji, Hayashi and Kotlikoff, 1992)\(^{16}\). These empirical findings raised serious doubts regarding the validity of the assumption that individuals’ behavior is characterized by infinite dynasties and the validity of the “Rotten Kid Theorem”.

Actually, the perplexity about the real motivations of manifested altruism applies more generally than the family context. Giving charity to poor people or contributing to charity organizations may indeed be motivated by altruism, but may stem from pure selfish motivations as well, (like seeking prestige or tax deductions). Rose-Ackerman’s (1996) comprehensive survey’s conclusion is that although the motivations for non-profit organizations activities as well of the motives of individuals in contributing to these organizations were extensively studied, “empirical work has not succeeded in providing hard evidence on the motivations for charitable giving”.

The failure of empirical studies to identify the more appropriate of the two rival approaches, both in the microeconomic as well as the macroeconomic fields, suggests that while no single approach can provide a sufficient explanation for the phenomenon of intergenerational transfers, perhaps an integrative approach can

\(^{14}\) It was also found that parents tend to be more altruistic towards their parents than childless people, or than people who do not live with their own children, and it was found that women are more altruistic toward their parents than men. Stark, (1995) relates this finding to the well-known fact that women’s life expectancy is higher than men’s, encouraging them to enforce the “demonstration effect” within their children.

\(^{15}\) An alternative explanation raised by Becker and Murphy (1988) is that the power of altruism is a function of the intensity of intra-family connections. This argument resembles an ancient idea raised in the Talmud and developed by Jewish medieval philosophers.
succeed where they failed. The intuitive feeling that people are driven by both altruistic and egoistic incentives is in fact supported by the findings of Lucas and Stark (1985), who studied the motives of remittances from immigrants to their families in their countries of origin and showed that they could be explained by a combination of altruism and exchange considerations (see also Stark 1995). Cox (1987) presents a composite econometric model and concludes that the exchange motive explains the empirical data better than the altruistic motive, but does not negate the existence of an altruistic motive.  

3. The Integrative Approach to Intergenerational Transfers

The model introduced here encompasses both altruistic and strategic motivations as well as both types of intergenerational transfers. We assume that people derive utility from their own consumption and from the utility of their descendants. We also assume that intergenerational transfers can be both bequests and inter-vivos gifts. The latter form of transfer may be, for instance, a kind of compensation to the next generation for postponing the bequest due to increased life expectancy of the current generation. This increase may also reduce the bequest since the elderly parents will probably consume some of their wealth during their extended life period. In return, parents receive services that extend their longevity from their children. Thus transfers are driven by the two types of incentives at the same time. To finance these transfers, additional saving is sometimes required.

It should be emphasized that the integrative approach does not rule out the possibility of pure altruism in intergenerational transfers. On the contrary, it is explicitly assumed (though by a different formulation) that it is certainly possible that

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16 The same methodology was applied in Japan, and yielded the same conclusions (Hayashi, 1995). See also Cox (1987), Cox and Rank (1992) who found that empirical data are better explained under the exchange hypothesis than under altruism.

17 Ando and Kennikel, (1986) found that elderly Japanese use to live with their descendants and pool their wealth together with that of their children. Namely, they transfer their wealth to their children in return for common tenancy and other services that have a positive impact on their life expectancy. Thus, the accumulated wealth finances not only expenses in old age but also represent intergenerational transfers for buying longevity. However, I believe that living together within an extended family's home may indicate that this habit indicates a mix of altruism and exchange motives.

18 So even if the altruistic motive is lower than the critical value of Abel (1988) it can be conjectured that a “weak” altruistic motive can be combined with a “weak” strategic motive and together stimulate intergenerational transfers, as discussed below.
the utility of parents (children) depends on the utility of their children (parents), but at the same time, along with pure altruism, there are also egoistic considerations behind transfer decisions. In other words, pure egoism and pure altruism are the extreme cases of all possible situations in which the individual takes both types of considerations (in different dosages, of course) into account.\textsuperscript{19}

\textbf{The Model}

Assume an overlapping generations’ model. Each individual in this model can live for either two or three periods\textsuperscript{20}. The probability of living for at least two periods is 1. The probability of living for only 2 periods is \( p \), so the probability of living for 3 periods is \( 1-p \). The life expectancy of the individual is therefore \( 3-p \). Following Stark (1995), we assume that \( p = p(l) \) is a decreasing function of the services that the individual receives from his children, like “keeping in touch”, “love”, etc., denoted by \( l \). That is, \( p'(l) < 0 \). We also assume that \( p''(l) > 0 \).

“Generation \( t \)” refers to the generation born at time \( t \), and the size of the total population at time \( t \) is denoted by \( N_t \). This implies that the proportion of the old men (living in their third period) in the entire population (denoted by \( N \)) in generation \( t \) is \( (1-p)N_{t-2}/N_t+N_{t-1} \). Substituting \( N_t=(1+n)N_{t-1} \) into this gives the expected fixed proportion of old people at time \( t \), \( (1-p)[(1-p)+(2+n)(1+n)] \). Note that if \( p=n=0 \), then this proportion is simply \( 1/3 \).

During his first two periods the individual works and earns \( w_1 \) and \( w_2 \), respectively. In his third period, the individual (if he is still alive) collects his pension and does not work. The pension in this model is DC and funded, implying that the workers contribute a certain portion of their income during their two periods of work and the contributions earn a certain rate of return.

\textsuperscript{19} The recognition that egoistic considerations include good deeds even within the relationship between parents and children has actually very ancient roots in the Talmudic and Midrashic literature, and also in the Jewish medieval philosophic literature. For a brief survey of this literature, see Schwarz (2000, Hebrew).

\textsuperscript{20} Three periods’ models as well as two periods’ models are widespread in the literature since the seminal papers of Samuelson (1958) and Diamond (1965). The third period is needed here to encompass the possibility of buying longevity by transfers, which is modeled here as buying higher probability to live the third period. Another possible way is to assume that individuals live only two
The second period begins with having children. Thus, if the individual is going to live only two periods, his children will receive the bequest at the end of their first period, (when they have their own children). If the individual lives for three periods, he will transfer the bequest to his heirs when they are in their second period. This means that if someone lives two periods and his father lives three periods he will not have a bequest at all.\footnote{We assume that there is no correlation between life expectancy of fathers and the life expectancy of their sons. Of course, this assumption is unrealistic and made for the sake of simplicity.} In this case, the bequest will go to the grandchildren. Since the utility of the grandson appears as an argument in the utility function of the son, we can view the population as composed of Barro-style “linear dynasties” with “chains” of utilities adding up in a geometrical sequence to infinity.

Suppose that the direct cost of $l$ for the individual is negligible. The real economic cost of $l$ increases the probability of postponing the timing of receiving a reduced bequest, because it is very likely that the old generation will consume part of it. There is no non-negativity constraint on $l$. In addition, suppose that the capital market is imperfect and young people cannot borrow against future income or future bequests.

We apply the following notation:

- $\beta$ – Time preference coefficient $0 < \beta < 1$.
- $a_t$ – Contribution to the pension fund, $(a_{3t} = 0)$.
- $V^m_t$ – Inter-vivos transfers that generation $t$ receives from his parents.
- $V^x_t$ – Inter-vivos transfers that generation $t$ makes for the next generation, $(V^m_t = -V^x_t)$.
- $B^t$ – Bequest that generation $t$ plans to leave his heirs after living three periods.
- $\delta$ – Intergenerational utility coefficient.
- $U^*_{t+1}$ – Maximum available utility for next generation, as a function of $V$ and $B$.

Inter-vivos transfers may be compensation to young people for the loss they suffer by postponing the transfer of the bequest. Strategic transfers are very likely made for the purpose of buying longer life expectancy. Thus, $l$ is a function of $V^x$ implying
that \( p = p(l(V^*)) = \hat{p}(V^*) \). Assume that \( l'(V^*) > 0 \) and \( l''(V^*) < 0 \) so \( \hat{p}'(V^*) < 0 \) and \( \hat{p}''(V^*) > 0 \), as was assumed earlier. The hidden assumption here is that \( l \) is a function of inter-vivos gifts \( (V^*) \) only, and not of bequests \( (B^*) \). This assumption relies only on intuition.\(^{22}\) Also, assume that \( V^*(l) \) satisfies \( V''^* > 0 \), \( V''''^* < 0 \).

We have to define a kind of index to measure the relative power of the strategic motive, as we have \( \delta \) to measure the relative power of the altruistic motive. This index, denoted by \( \psi \), will be defined as the ratio between the probability to live three periods, and the probability to enlarge this probability by intergenerational transfers. Namely:

\[
\psi = \frac{(1 - \hat{p}(V^*))}{\hat{p}'(V^*)}.
\]

And since this index is negative, we refer to the absolute value of \( \psi \).

Let \( u(C_{it}) \) be a utility function of a typical member of generation \( t \), from consumption in the \( i \)th period of his life. (From this point, index \( i \) denotes period of life and index \( t \) denotes generation). Assume that the utility function satisfied the Inada conditions, namely:

\[
u'(C_{it}) > 0, \quad u''(C_{it}) < 0
\]

\[
\lim_{C_{it} \to 0} u'(C_{it}) = \infty, \quad \lim_{C_{it} \to \infty} u'(C_{it}) = 0
\]

At the beginning of his life, the individual faces some contingent states of nature. One state of nature \( (I) \) is receiving a bequest during his second period, as a result of his parents having lived only two periods. A second state of nature \( (II) \) is receiving the bequest in his third period (if he live through this period himself) since his parents live three periods.

\(^{22}\) I believe (without any empirical basis) that if there is any functional linkage between bequest to \( l \), it must be negative, because the larger the bequest, the larger the loss caused by postponement in receiving it. But I find it difficult to believe that even the most egoistic people treat their parents in such a cynical and evil way, or put differently, that such people can be considered representative agents. Therefore, for the sake of simplicity, we assume that \( l \) is not a function of \( B \).
If the individual transfers income to his son during his second period, the latter receives the transfer in his first period. Transfers made in the third period are received during the recipient’s second period. Note the difference between $\hat{\rho}(l(V^x))$ and $p(l)$, namely, between life expectancy of parents as a function of $l$ and the life expectancy of a certain individual, as a function of $V^x$. $V^x$ is the parents’ control variable while $l$ is a control variable of the younger generation.

The distinction between $\hat{\rho}(l(V^x))$ and $p(l)$ is important, because apparently the states of nature for the individual are much more complex. For instance, consider the case when the father of an individual lives two periods and his grandfather lives three periods. Thus, during his second period the child will receive two kinds of bequests: one from his parent and one from his grandparent. Also, in period 3 the individual may receive no bequest at all or transfer his bequest to his grandchild and not necessarily to his child. But all these fine-tunings of the model have no impact on the results since the question the individual faces in each case is how much to devote to bequests (either to the child or to the grandchild) and what is the optimal amount of $l$ to be given to his parent (or his grandparent). There may, of course, be differences in the size of $B^{t-1}$, depending on the exact circumstances of the receipt of the inheritance. But these differences have no impact on the optimization process since from the child’s point of view $B^{t-1}$ is a previous generation’s decision variable, and from the father’s point of view, $l$ is the child’s decision variable.

In sum, the individual faces two effective states of nature: receiving a bequest in the second period of life or receiving it in the third period of life. These two contingencies create two systems of constraints which the individual faces. In state of nature $I$ (with probability $p$) the constraints system is:

$$C_{1t} = w_1 - a_{1t} + V^m$$  \hspace{1cm} (1)
$$C_{2t} = w_2 - a_{2t} - V_{21}^x + B^{t-1}$$  \hspace{1cm} (2)(I)
$$C_{3t} = R^2a_{1t} + Ra_{2t} - V_{31}^x - B'$$.  \hspace{1cm} (3)(I)

On the other hand, under state of nature $II$ (with probability $1-p$) the individual faces a slightly different constraints system. The difference is in the timing of the bequest:
\[ C_{it} = w_t - a_{it} + V^m \]  
\[ C_{2t} = w_t - a_{2t} - V^r_{2t} \]  
\[ C_{3t} = R^2 a_{4t} + Ra_{2t} + B_{3t} - V^r_{3t} - B^r. \]

These constraints reflect the assumption that the capital market is not perfect and there are effective liquidity constraints that prevent young people from borrowing against future income, either from work or from intergenerational transfers. Otherwise, the timing of the bequests has no significance. We also assume that individuals can insure neither the bequest nor its timing.\(^{23}\)

The individual’s problem is:

\[
\begin{aligned}
&\max \left\{ u(C_{1t}) + \beta \left[ p(l)u(C_{2t}) + (1 - p(l))u(C_{2r}) \right] \right. \\
&\quad \quad + \left. (1 - \hat{p}(V^r))\beta^2 \left[ p(l)u(C_{3t}) + (1 - p(l))u(C_{3r}) \right] \right. \\
&\quad \quad \quad \left. \quad + \beta \delta U^*_{t+1} \right\} \\
&= \max \left\{ u(C_{1t}) + \beta Eu(C_{2t}) + (1 - \hat{p}(V^r))\beta^2 Eu(C_{3t}) + \beta \delta U^*_{t+1} \right\}
\end{aligned}
\]

The liquidity constraints imply that we must analyze the model under three alternatives: interior, non-satiated and corner solutions.

The first order conditions are:

\[ u'(C_{1t}) \leq \beta^2 (1 - \hat{p}(V^r))R^2 Eu'(C_{3t}) \]  

\(^{23}\) The natural basis for such an assumption is “reality”. However, in this model, which seeks to explore the economic interrelations between various generations in a society, this assumption is necessary since the timing of the bequest is unimportant if the bequest itself is ensured and the capital markets are perfect. Moreover, as Sheshinski and Weiss (1981) showed, annuity is the preferred insurance device against longevity risk while other saving devices are better for bequests (no matter what motivates them). In such a model, the individual may interpret the saving level of his parents, in both saving tracks, as a signal of their bequeathing motive, or the expected size of the bequest. Namely, if the parents invest more in annuity and less in other saving devices, this may signal a relatively weak bequeathing motive (and long life expectancy). Also, a relatively large investment of a young person in saving programs that are not annuities may be a cheating strategy to make the next generation believe that they will inherit a big bequest. A game theoretic analysis is more appropriate for this topic, which is beyond the scope of this paper.
\[ Eu'(C_{2i}) \leq \beta(1 - \hat{p}(V^x))REu'(C_{3i}) \] (6)

\[
\frac{\partial V^x}{\partial t}u'(C_{1i}) = p'(l)B^{-1}\left[u'(C_{2i}^l) - u'(C_{2i}^u) + \beta(1 - \hat{p})\left(u'(C_{3i}^l) - u'(C_{3i}^u)\right)\right] 
\] (7)

(7) holds as equality always, because of the assumption about no non-negativity constraint on \( l \). The decision on \( V_{1i}^x \) \( (i = 1, 2) \) and also on the magnitude of the planned bequest are made when the state of nature is known and the individual also knows how many years he is going to live himself. Consequently, the expectation operator \( E \) disappears in the Lagrangian derivative with respect to \( V_{1i}^x \). These derivatives, as well as the derivatives with respect to \( B \), can be calculated by using the envelope theorem, implying that:

\[
\delta u'(C_{1i+1}) \leq u'(C_{2i}) + \beta\hat{p}V^xu(C_{3i}) 
\] (8)

\[
\delta u'(C_{2i+1}) \leq \beta[(1+\hat{p}(V^x))u'(C_{3i}) + \hat{p}'V^xu(C_{3i})] 
\] (9)

\[
\delta u'(C_{3i+1}) \leq \beta(1-\hat{p}(V^x))u'(C_{3i}) 
\] (10)

(8) becomes equality if \( V_{2i}^x > 0 \), (9) becomes equality if \( V_{3i}^x > 0 \) and (10) becomes equality if \( B > 0 \). 24 A solution is a vector \( (a_i, V^x, B, l) \) satisfying the first order conditions.

**The Steady States**

Henceforth the discussion focuses on the analysis of stationary equilibria of Steady States, disregarding dynamics and the exact path of convergence to a stationary equilibrium. 25 To prove the existence of a steady state it is necessary to extend the model into a full equilibrium model, which is beyond the scope of this paper. However, since the model is based on standard assumptions about the individual’s preferences, and in particular their discount factor, it is reasonable to assume that with

---

24 In fact, one can argue that there is no non-negativity constraint on \( B \) since people might bequeath debts. We ignore this possibility, which is actually impossible according to Hebrew law, for instance.

25 Recall that the objective of this study is to analyze the individuals’ behavior in micro – partial intergenerational context.
a standard production function, the dynamic analysis would yield a saddle path leading to a (probably unique) stationary equilibrium. So let us assume that there is a steady state in this model, described by:

\begin{align*}
C_{it} &= C_{i,t+1} = C_i \\
B' &= B^{i-1} = B \\
V_{i,t-1}^+ &= -V_{i,t}^+ = V_i^+, \quad i = 2, 3
\end{align*}

Thus,

\begin{align*}
\mathcal{u}(C_{it}) &= \mathcal{u}(C_{i,t+1}), \quad \mathcal{u}'(C_{it}) = \mathcal{u}'(C_{i,t+1}).
\end{align*}

Assuming a steady state, we can rewrite equations (8), (9) and (10) as:

\begin{align*}
\left(1 - \hat{p}(V^+)\right)Ru'(C_{3,t})(\delta \beta R - 1) &\leq \hat{p}'(V^+)u(C_{3,t}) \quad (8') \\
\left[\delta(1 - p)R - (1 + p)\right]u'(C_{3,t}) &\leq p'(V^+)u(C_{3,t}) \quad (9') \\
\delta &\leq \beta \left(1 - \hat{p}(V^+)\right) \quad (10')
\end{align*}

This model has more than one solution satisfying first order conditions and more than one candidate point for steady state. It is necessary therefore to examine all the possible solutions.

When a solution of the model is not an interior one, it can be either a corner solution or a non-satiated solution. In a corner solution, at least one of the constrained variables is equal to zero in equilibrium (or, in a full corner solution, all of them). In a non-satiated solution, at least one of the constrained variables is positive and one of the constraints that hinges on that variable is satisfied as a strict inequality. Naturally, a combination of a corner solution with a non-satiated solution is possible.

---

26 Actually, this is not a simple conjecture at all, since we have to show that in a steady state the population grows at a rate necessary to ensure that the capital/labor ratio, as well as other relevant variables, remain constant. This is absolutely not self-evident in a model like this with endogenous life expectancy. The purpose here is to examine the stationary equilibrium, on the assumption that such equilibrium indeed exists.
Corner Solutions

In this case all first order conditions, except (7), are satisfied as strict inequalities, which means that people do not save, do not make inter-vivos transfers and leave no bequests for their descendants. Put differently: \( a_i = V_i^V = B = 0 \).

Proposition 1

The fulfillment of all of the following two conditions is necessary for the existence of a corner solution:

i. The altruistic motive is sufficiently weak to oppress any will to transfer income to the next generation.

ii. The strategic motive is sufficiently weak for making it not worthwhile to buy life expectancy in return for intergenerational transfers.\(^27\)

Proof: See appendix.

Proposition 1 simply states that contrary to Abel (1988), when people consider a mixture of altruistic and strategic considerations, it is not enough to hold the altruism coefficient, \( \delta \), lower than a certain critical value to ensure that there will be no intergenerational transfers. Apparently intergenerational transfers may stem from strategic motivations. In fact, they can exist even if \( \delta = 0 \).\(^28\) Therefore, the additional condition for no intergenerational transfers is that the strategic motive, \( \psi \), is also low enough, below a certain critical value.

Non-Satiated Solutions

\(^27\) The sum of all temporal utilities for an individual in this economy is given by \( U_i = u(w_1) + \beta u(w_2) + \beta \delta U_{r+1}(0,0) \).

\(^28\) The critical value of \( \delta \) that ensures no intergenerational transfers even with a relatively strong strategic motive should be negative, contrary to the basic assumption of the model. Negative coefficient may be interpreted as “jealousy” or “envy”, phenomena that are not discussed in this paper.
Assume that individuals manage to smooth their consumption although there are no intergenerational transfers. That is, (5), (6) and (7) hold as equalities while (8), (9) and (10) hold as inequalities. From the above discussion it is clear that the whole difference between the two types of corner solutions is that in partial corner solution the liquidity constraints are ineffective. Namely, individuals are able to decide on the saving rate for old age at every period in order to smooth their consumption path. Also, by proposition 1 we already know that in this case the altruistic and the strategic motives are both too weak to encourage people to give up part of their consumption for the sake of future generations. Now it is not very difficult to find the critical value of $\psi$ that ensures no intergenerational transfers at all. As we shall now see, this critical value depends on the stage in life of the individuals.

Consider first the case in which (8) holds as strict inequality. Denote utility function elasticity by $\eta_{u,C} = u'(C_{3,t})\frac{C_{3,t}}{u(C_{3,t})}$. Assuming that $\delta R < 1$, we can write (8) (after some algebraic manipulations) as:

$$\psi < \frac{C_{3,t}}{\eta_{u,C}} \frac{1}{R(\delta \beta R - 1)}$$

which is a sufficient condition for no intergenerational transfers during second period for any given value of $\delta$. (As expected, $\frac{\partial \psi}{\partial \delta} < 0$).

By the same token from (9) we have:

$$\psi < \frac{C_{3,t}}{\eta_{u,C}} \frac{1}{(\delta R - 1)}$$

which is the sufficient condition for no third period intergenerational transfers.

Assuming $|R(\delta \beta R - 1)| > |(\delta R - 1)|$, it is clear that the upper limit for $\psi$ that ensures no second period transfers is lower than the upper limit for third period transfers. This yields the following corollaries.
Corollary 1:

*The critical value of the strategic motive that ensures no intergenerational transfers rises over time. Namely, other things being equal, the relative power of the strategic motive that ensures no second period transfers is lower than the relative power of this motive that ensures no third period transfers.*

Put proposition 1 differently, if the strategic motive is too weak to encourage third period’s transfers, it not sufficient to prevent second period’s transfers. Prevention of second period’s transfers requires lowering the strategic motive relative power more. The intuitive explanation of this result is that extending life expectancy is relevant during the second period. As the individual gets older his strategic motive would weaken since life expectancy is revealed to be higher.

Another result that appears here is the negative correlation between $\psi$ and $\eta$. In other words, as the elasticity of the utility function rises, a relatively smaller strategic motive is needed for intergenerational transfers.

The fact that the critical value for no intergenerational transfers rises over time yield,

Corollary 2:

*There is only one possible form of a non-satiated solution in this model, namely when there are second period intergenerational transfers and no third period transfers.*

The explanation of this corollary is simple. If the strategic and altruistic motives are so weak to ensure no second period transfers, there would be no transfers at all. But this is actually a corner solution. Therefore, in a non-satiated solution inter-vivos transfers can take place only in second period. This result allows us to suggest that the timing of intergenerational transfers may sometimes identify their nature. Third period transfers are mainly altruistic in nature, while second period transfers come from a mix of motives. A person who transfers during his second period and refrains
from transfers during his third period is very likely to be motivated mainly by strategic motives.

Note that we have made no assumptions about the relative size of $w_1$ and $w_2$, implying,

**Corollary 3:**

*The effects of the strategic and altruistic motives of individuals on intergenerational transfers are independent of any assumption about the income profile and in particular whether $w_1 < w_2$ or $w_1 > w_2$.***

During the second period of an individual’s life, his son is living in his first period and probably needs financial support more than in his second period, but the result is valid for a general case as well. The explanation for this seems to be that the more relevant timing for strategic transfers is the second period, when the donor does not yet know whether he is going to live three periods or not. Therefore, for a given strategic motive, the individual needs to be more egoistic to refrain from transfers in this period. In other words, as one would intuitively expect, for a given altruistic motive there is less need for a strategic motive in the second period than in the third.

Recalling that the empirical literature has rejected both strategic and altruistic hypotheses as motives for intergenerational transfers, this result seems to be surprising, especially because no significant correlation was found between gifts and recipients’ income. However in our integrative model such a correlation is not expected at all, no matter what really motivates intergenerational transfers. Corollary 3 implies that much of the empirical literature that tried to distinguish strategic from altruistic motives of individuals, by estimating correlations between transfers and recipients’ is actually irrelevant. Finally, we have,

**Corollary 4:**

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29 Bhaumik (2001) reports (based on German data), that demographic and other events determine transfers to a significant extent. However, his research took the prevailing attitude that transfers are either altruistic or strategic and ignored the possibility raised in this paper of mixed motivations.
Even if individuals are driven by both strategic and altruistic motives, there is no endogenous mechanism ensuring that the steady state is not a corner-solution type or non-satiated type equilibrium. In our model, therefore, there is no reason to expect that an interior solution is more reasonable or more likely to emerge, than the alternative extreme type of solution.

**Interior Solutions**

In an interior solution all first order conditions hold as equalities. Under the assumption of a full interior solution, conditions (5) and (6) determine the ratio between marginal utilities of various periods. As expected, this ratio depends on the subjective rate of time preference, the interest rate and life expectancy. Let us first investigate the conditions for the existence of a full interior solution.

It is easily seen from (8') and (9') that they can be satisfied as equalities if either both sides are set to zero (case 1) or both sides are negative (case 2). Each of these cases has sub-cases that deserve careful examination.

Case 1 (when both sides of each equation are zeroed) is possible if the right hand side of (8') and of (9') equal zero. This happens if either $u(C_{3,t}) = 0$ or $\hat{\rho}'(V^+) = 0$.

Although a full interior solution with $u(C_{3,t}) = 0$ seems strange, it is by no means impossible. Therefore, in this case even if $\hat{\rho}'(V^+) < 0$ and it is apparently possible to extend life expectancy by intergenerational transfers, there is no reason for individuals to do so since there is no reason to live to the third period if consumption is such that the utility is zeroed. Hence, if there are intergenerational transfers in this

Furthermore, Bhaumik refers to single events in an individual’s life-cycle as “transfers inducing” (like marriage or illness). Our attention is paid to streams of permanent inter-households transfers.

30 Such a solution is sometimes even reasonable. For instance, consider the case of a logarithmic utility function $u(c) = \ln(c)$. In this case, it could be that positive consumption yields negative utility (if $0 < c < 1$). In other words, some people prefer death over old-age poverty and starving.

31 Keep in mind that people in this model do not derive utility from “life”, but from consumption that life enables.
case, they must be purely altruistically-motivated.\textsuperscript{32} Thus, if \( \hat{p}'(V^{'}) = 0 \) then the opposite is true and intergenerational transfers are mainly compensation to the next generation for life expectancy extending services. This means that parents manage to “extract” maximum life expectancy from their children.

If the second possibility of case 1 holds, namely if \( p = 1 \), then one may conclude that intergenerational transfers are purely altruistic, since in this case strategic transfers are not effective. The following proposition negates this possibility.

**Proposition 2:**

Assuming \( p = 1 \) is inconsistent with an interior solution of a stationary equilibrium.\textsuperscript{33}

\textbf{Proof:} See appendix.

Note that there is no reason to negate the possibility that all first order conditions would hold as equalities with \( p = 0 \). So in an interior solution, when intergenerational transfers take place life expectancy may be three periods.

In case 2, both sides of (8’) and (9’) are negative. The right side of (8’) and (9’) can be negative only if \( \hat{p}'(V^{'}) < 0 \). The left side of (8’) can be negative (when \( p < 1 \)) only if \( \delta < \frac{1}{\beta R} \). It follows from (8’) that this condition is fulfilled if:

\textsuperscript{32} In the case of logarithmic utility function, as mentioned in the previous footnote, the utility may even be negative if the consumption amounts to less than one unit. In this case the incentive is not to extend the life expectancy.

\textsuperscript{33} Or alternately stated, the critical value of \( \delta \) ensuring no intergenerational transfers (under the assumption that \( p = 1 \)) is negative, which means that people in this society are motivated by jealousy. However, negative \( \delta \) contradicts the model’s fundamentals.

\textsuperscript{34} As mentioned already in previous footnotes, there are utility functions which satisfy Inada conditions but give negative utility to certain amounts of positive consumption. But, if \( u(C_{3,t}) < 0 \) then since \( \hat{p}'(V^{'}) \leq 0 \) the right side of (8’) must be non-negative. In this case the left side of (8’) will, of course, also be non-negative and this can happen only if \( \delta \beta R \leq 1 \). However such parameters are unlikely and not consistent with the existence of intergenerational transfers. Therefore we shall ignore this possibility.
\[
\frac{1}{\beta} \left( \frac{\hat{p}'(V^t)}{(1 - \hat{p}(V^t))} \cdot \frac{u(C_{3,t})}{u'(C_{3,t})} \cdot \frac{1}{R} + 1 \right) < \frac{1}{\beta R} \quad (13)
\]

Rearranging this expression gives:

\[
\psi > \frac{C_{3,t}}{\eta_{u,c}} \cdot \frac{1}{r} \quad (14)
\]

implying the floor limit for strategic motivation for holding (8’) as equality, namely for intergenerational transfers during the second period with a given altruistic coefficient. Note that there is no reason to consider \( \delta = 0 \) as inconsistent with this contingency.

Using the same technique we can show that the left side of (9) is negative if:

\[
\psi > \frac{C_{3,t}}{\eta_{u,c}} \cdot \frac{1 + \hat{p}(V^t)}{\hat{p}'(V^t)} \cdot \frac{1}{\delta R} \quad (15)
\]

And that is the lower bound for holding (9’) as equality. Note that there is no a-priori way to tell which lower bound is higher. So there is no way to tell what lower bound for strategic motive ensures a full interior solution, given a certain level of \( \delta \).

Recall that earlier we found the lower limit of \( \psi \) for no intergenerational transfers at all (equation (11)). Here we found that there is a floor value for \( \psi \) that ensures that a full interior solution takes place. What happens if the actual value of \( \psi \) is in between is unknown.

**Altruism, Level of Income and Longevity**

A widely accepted empirical fact is that life expectancy is positively correlated to capital stock (Wilkinson, 1992, Ichiro, Kennedy and Wilkinson, 1999). The common explanation is that in rich economy people eat more nutritional food and enjoy high levels of sanitation and medical services that extend their life expectancy. In other
words, higher capital stock causes longevity. Alternative explanation turns the arrow of causality to the opposite direction, asserting that a postponement in transferring bequests forces the younger people to invest in accumulating human capital, therefore a rise in longevity causes an increase in capital stock (see Stark, 1995).

Naturally, analysis this kind of problems requires a general equilibrium model. But even without extending our model we can see, intuitively, that there are two effects here, that act in opposite directions. One effect is accumulating capital to finance living while waiting for the bequest (as in Stark), and also to finance inter-vivos transfers. The other effect is consuming part of the capital by the old people. Therefore it is impossible to conclude what the overall net effect is, based on a-priori theoretical considerations. The same is true for the correlation between life expectancy and altruism, as stated in the following,

**Proposition 3:**

> In full interior solution equilibrium, the endogenous life expectancy depends only on the subjective rate of time preference, and is independent of the rate of altruism. The difference between the two types of interior solutions is in the effect of the interest rate on life expectancy.

**Proof:** See appendix.

Apparently, the explanation for this result is that the strategic motive is connected to the time preferences of the individuals, but not to the altruistic motive. A higher time preference coefficient \( \beta \) means that individuals assign a relatively higher weight to future consumption so they have a stronger incentive to make third period consumption possible. On the other hand, if \( \beta \) is relatively low, the weight of third period consumption in the individual utility function is even lower \( \beta^2 \) and the “profitability” of giving up current consumption in order to buy longer life expectancy

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35 In Stark’s model, the expected bequest is land, which does not decay while in the possession of the old people. Therefore, in his model, extending life expectancy increases the capital stock because the young people must increase their investments. In this model there are two opposite sign effects and the net effect is unclear.
is diminished. Altruistic transfers, on the other hand, are not at all connected to the desire to extend life expectancy and are therefore not supposed to have any influence on it.\footnote{A similar conclusions from empirical data see Deaton (2001)}

The conclusion from this discussion is that when an economy is in a stationary equilibrium, it may be possible to construct an empirical test to enable a distinction between altruistic and strategic transfers, assuming that in such equilibrium the interest rate represents the time preferences of individuals. The higher the time preference coefficient accompanied by a higher volume of intergenerational transfers, the more apparent it is that these transfers are mostly strategic. If, on the other hand, we observe a high volume of transfers and a relatively low time preference coefficient, the more reasonable it is to assume that these transfers are mostly altruistic. However, the empirical test is valid only in steady states, implying that much of the empirical research hitherto performed to explore the motives for intergenerational transfers is doubtful, since most economies are not in steady state.

Proposition 4:

\textit{In an interior solution (namely, when bequests exist), the individual maximizes his efforts to extend his parents’ life expectancy, given that the interest rate is positive.}

\textbf{Proof:} See appendix.

Note that if for some reason the timing of receiving the bequest is unimportant for the individual, he has no incentive to invest \(l\) to extend his parents’ life expectancy. Since the individual will not be damaged by postponement of the transfer of the bequest, he needs no compensation for it. Therefore it is unlikely to assume that in this case \[
\frac{\partial V^m}{\partial l} = 0 \quad (\text{see (7)}).
\]

Such an individual will invest \(l\) from altruistic motives only.

Of course, this result is by no means novel. It is clear that children’s investment in their parents comes either from altruistic or strategic motives. But this
result shows why an empirical distinction between these motives is not easy. The difference between strategic and altruistic individuals is not behavioral but cognitive, namely intangible. Revealed preferences seem useless here.

**Proposition 5:**

In an interior solution and $\hat{p} < 1$, parents maximize their life expectancy by using inter-vivos transfers of income to their children, given that the interest rate is positive.

**Proof:** See appendix.

Interest rate, subjective time preference rate and the altruism coefficient of the individual determine the timing of the transfers. It may be possible to create an empirical test for the existence of strategic motives, since if we observe only bequests and no inter-vivos transfers in an economy with liquidity constraints, that indicates that the strategic motive is relatively weak. However, as mentioned above, most intergenerational transfers are inter-vivos transfers.

**Perfect Altruism and Perfect Egoism**

We now turn to analyzing the behavior of extreme individuals that were so extensively studied in the prevailing literature, namely pure altruists and pure egoists, but this time from the integrative point of view. We define a pure egoist as a person with $\delta = 0$. In contrast to the pure egoist, defining a pure altruist involves philosophical difficulties and the definition is actually based on the moral tenets of the definer. Basically, it is possible to assume that there are righteous people for whom even $\delta > 1$ holds (I can testify that I have had the privilege of knowing such people). Some philosophers even try to define a pure altruist as a person who assigns no weight to his own utility in his utility function, but only to the utility of his fellow

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37 An additional conclusion is that it is reasonable to assume that (usually) people know how to distinguish between altruistic and strategic gifts, where their donors expect $l$ in return.
man. But such people can hardly be considered to be representative agents. An alternative definition for a pure altruist may be a person who assigns equal weights to his and to his fellow man’s utility. Without going deeply into this philosophical issue, let us simply define a pure altruist as a person with $\delta = 1$.

**Proposition 6:**

*Perfect egoism is inconsistent with a full interior solution of a stationary equilibrium.*

**Proof:** See appendix.

**Proposition 7:**

*Perfect altruism is inconsistent with steady state equilibrium. In a utopian economy, the system does not converge to a stationary equilibrium.*

**Proof:** See appendix.

No matter what is the intergenerational share in accumulated wealth, this share is significant and consist from both types of inter-vivos and posthumous transfers. Namely, real world data comply with interior solution. Furthermore, we started our analysis with the assumption that people *might* consider both altruistic and strategic considerations when they have to decide whether, when and how much to transfer to their kids. The last two propositions imply that in an interior solution of a steady state, it must be that people have both kinds of motives on the same time, otherwise only one type of transfers would take place.

Nevertheless, keep in mind that this is true only in steady states. So the last two propositions imply that economies with extreme individuals are not expected to converge to steady state equilibrium. Thus an econometric test of these results

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38 A possible difficulty in creating such an empirical test might be the need to construct a reliable index of $l$, which also contains an intangible component.
depends on the general empirical debate about convergence. It seems that additional research is required on this point.

**Government Intervention**

After three decades of stormy polemic discussions on Ricardian Equivalence, it seems out-dated, because since the publication of Barro’s (1974) intriguing paper, so many ways were found to debunk the Equivalence, that finding another one, per-se, does not seem a great achievement. However, although numerous empirical studies were conducted to test the Ricardian Equivalence, the empirical literature has not been able to decide this old question. This fact justifies, in my opinion, continuing the search for the theoretical reasons, not for the failure of the Ricardian Equivalence, but for the failure of the empirical branch of the economic science to decide on it.

To enable the examination of the influence of government intervention we shall now assume that there are two types of savings: mandatory and voluntary. Contributions to mandatory saving are denoted as before by $a_{i,j}$ and voluntary contributions are denoted by $(i=1,2) s_{i,j}$. A simple arbitrage argument implies that in a competitive market with full information, all savings tracks in an economy should bear the same rate of return. But this is not the case when one of the tracks is compulsory. The compulsory saving rate of return are denoted by $R_p$ and the voluntary saving contributions are denoted by $R_g$. The analysis below is based on the assumption of a full interior solution.

When two tracks of savings – pension schemes and provident funds – exist, it seems that the purpose of pension saving is insurance against longevity risk. That is the risk that an individual faces if he lives into the third period and does not have sufficient resources to finance his living. Saving in a provident fund seems naturally to aim at accumulating wealth, for instance, for bequests or for shock buffering (Sheshinsky and Weiss, 1981).

There are two main ways that governments can encourage savings for old age: mandatory pension schemes and tax incentives. If contributions to mandatory saving are denoted by $a_{i,j}$ and contribution to voluntary saving by $s_{i,j}$ and assuming that tax
incentives are given when the contribution to the provident fund is made, the net contribution is $(1 - \mu)s_{it}$. Naturally, the fundamental situation the individual faces at the beginning of his life has not changed. Namely, there are two contingent states of nature. Therefore the constraints in state of nature $I$ are:

\[
C_{1i} = w_i - a_{it} - (1 - \mu)s_{it} + V_{it}^m + p(l)B^{t-1} \quad (1')
\]
\[
C_{2i} = w_2 - a_{2i} - (1 - \mu)s_{2i} - V_{2i}^x + B^{t-1} \quad (2I')
\]
\[
C_{3i} = R_p^2 a_{it} + R_p a_{2i} + R_p^2 s_{it} + R_g s_{2i} - V_{3i}^x - B' \quad (3I')
\]

And in state of nature $II$ the constraints are:

\[
C_{1i} = w_i - a_{it} - (1 - \mu)s_{it} + V_{it}^m + p(l)B^{t-1} \quad (1I')
\]
\[
C_{2i} = w_2 - a_{2i} - (1 - \mu)s_{2i} - V_{2i}^x \quad (2II')
\]
\[
C_{3i} = R_p^2 a_{it} + R_p a_{2i} + R_p^2 s_{it} + R_g s_{2i} - V_{3i}^x - B' + B^{t-1} \quad (2II')
\]

The first order conditions of the individual problem under the new set of constraints are basically the same as in the original problem. The only additional condition is $R_g = (1 - \mu)R_p$. This means that in the consumer’s equilibrium, the difference between the two tracks of saving equals the tax subsidy on the subsidized track\(^{39}\).

Now suppose that the individual is required (by law) to increase his contributions to a mandatory pension scheme beyond his voluntary saving rate. Most authors predict that savers will offset, in full or in part, this compulsory increase in pension saving, by a decrease in contributions to voluntary saving. However, all the models with which I am familiar (for example, Spivak, 1994) take into account only the “substitution effect” between the various saving tracks. But in our model there is also an “income effect” which reflects the change in the saving rate as a consequence of a change in the individual’s income caused by changes in intergenerational transfers. In other words, we also have to take into account the possibility that if people are forced to increase their contributions to pension schemes beyond their

\(^{39}\) Recall that the uncertainty in this model refers only to life expectancy and not to the rate of return.
voluntary rate, they may partially offset the public social security policy effect. This may be done either by transferring part of the “over income” of the third period back to their children as inter-vivos transfers, or by increasing planned bequests. In a particular case, these intergenerational transfers may fully offset the intergenerational policy of the government. This phenomenon might be more important when capital markets are imperfect.

Formally, we denote the total influence of a change in compulsory saving on voluntary saving by \( \frac{\partial S}{\partial a} \). This is actually the sum of all “income” and “substitution” effects, both on saving during first period by young people and on second period saving by middle age people. Assume for convenience that the population growth rate \( n \) is zero,\(^{40}\) implying that the proportion of young people (who are in their first or second periods of life) in the total population is \( \frac{1}{3-p} \), and the proportion of old people in the population is \( \frac{1-p}{3-p} \). Thus, the total effect of a change in compulsory saving rate on total saving rate is given by:

\[
\frac{\partial S}{\partial a} = \frac{1}{3-p} \sum_{i} \sum_{j} \left( \frac{\partial S_{i,j}}{\partial a_{j,i}} + \frac{\partial S_{i,j}}{\partial V^{x,i}} \cdot \frac{\partial V^{x,i}}{\partial a_{j,i}} \right) + \frac{1}{3-p} \sum_{i} \sum_{j} \left( \frac{\partial S_{i,j-1}}{\partial a_{j,i}} + \frac{\partial S_{i,j-1}}{\partial V^{x,i}} \cdot \frac{\partial V^{x,i}}{\partial a_{j,i}} \right) + \frac{1-p}{3-p} \sum_{i} \left( \frac{\partial B}{\partial a_{i,i}} + \frac{\partial B}{\partial V^{x,i}} \cdot \frac{\partial V^{x,i}}{\partial a_{i,i}} \right) \tag{16}
\]

The first term within each of the first two brackets of (16) is the direct substitution effects of increasing mandatory contributions on voluntary contributions. The second

\(^{40}\) As already explained in a previous footnote, this assumption should actually be proved mathematically, since in this model \( p \) is endogenous. Therefore it is required to prove that in steady state, \( p \) is determined so that the population is stable (namely that \( n = 0 \)). This is impossible to prove without making further assumptions about birth rates and the factors that influence them. However, this is not a critical assumption for the qualitative results of the model and it is made for the sake of convenience only.
expression is the income effect, reflecting the change in voluntary saving caused by changes in net intergenerational transfers. Discussion of this influence was hitherto lacking in all studies on offset effects. The last brackets contain both income and substitution effects of increasing compulsory saving on bequests.

It is very reasonable to assume that some of the derivatives in (16) are zeroed.\(^{41}\) Therefore, the total effect under this kind of assumptions is:

\[
\frac{\partial S}{\partial a} = \frac{\partial s_{1,t}}{\partial a_{1,t}} + \frac{\partial s_{2,t}}{\partial a_{1,t}} + \frac{\partial s_{1,t}}{\partial a_{2,t}} + \frac{\partial s_{2,t-1}}{\partial a_{2,t}} + \frac{\partial s_{1,t}}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{1,t}} + \frac{\partial s_{2,t}}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{2,t}} + \frac{\partial s_{1,t}}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{1,t}} + \frac{\partial s_{2,t-1}}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{2,t-1}}
\]

\[+ \frac{\partial B}{\partial a_{1,t}} + \frac{\partial B}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{1,t}} + \frac{\partial B}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{2,t}} + \frac{\partial B}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{2,t}}\]

(17)

In (17), \(\frac{\partial s_{1,t}}{\partial a_{1,t}}\) is the simple offset coefficient, discussed in most empirical studies since Feldstein (1974) and \(\frac{\partial s_{2,t}}{\partial a_{1,t}}\) is the influence of mandatory saving of the second period on voluntary saving of the first period, assuming that a rational person takes into account that he is obliged to make compulsory savings in his second period as well. \(\frac{\partial s_{1,t}}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{1,t}}\) and \(\frac{\partial B}{\partial V^x} \cdot \frac{\partial V^x}{\partial a_{1,t}}\) is the income effect, hitherto missing from all papers about the offset question. In general, papers like Feldstein (1974) concentrated mostly on substitution effect while the polemic around Barro’s (1974) Ricardian Equivalence Theorem concentrated mainly on income effects. All authors discussed direct and partial effects only, ignoring cross derivatives.

The components of (17) can be derived from first order conditions, using the implicit function rule. Since there are two contingent states of nature in this model, we can talk about the expectancy of (17) only, which naturally depends on the

\[^{41}\text{For instance, it is very likely that } \frac{\partial s_{2,t-1}}{\partial a_{1,t}} = 0.\]
expectancies of its components. A distinction should be made between all contingent combinations of intergenerational transfers: inter-vivos transfers without bequests, bequests with no inter-vivos transfers, bequests and inter-vivos and a situation of no intergenerational transfers at all. Namely, three dozens of derivatives. But actually the number of derivatives is smaller, since under the model assumptions, some of them are zeroed. We are exempt from justifying these assumptions because the sign of (16) is anyway indeterminable since some of its components are positive, some negative and the sign of the remainder is unclear. For example:

\[
E\left(\frac{\partial s_t}{\partial a_{t,i}}\right) = E\left[ -\frac{u^*(C_{1,i}) + \beta^2(1-p)R_p^2u^*(C_{3,i})}{(1-\mu)[u^*(C_{1,i}) + (1-\mu)\beta^4(1-p)R_p^4u^*(C_{3,i})]} \right] < 0 \tag{18}
\]

\[
E\left(\frac{\partial s_t}{\partial V^x}\right) = E\left[ \frac{u^*(C_{1,i})}{-(1-\mu)[u^*(C_{1,i}) + (1-\mu)\beta^2(1-p)R_p^2u^*(C_{3,i})]} \right] > 0 \tag{19}
\]

Also, (9) yields:

\[
E\left(\frac{\partial V^x}{\partial a_{t,i}}\right) = E\left[ -\frac{-\delta u^*(C_{1,i+1}) - \beta p'(V^x)u'(C_{3,i})R_p^2}{\delta u^*(C_{1,i+1}) - \beta p^x(V^x)u'(C_{3,i})} \right] > 0 \tag{20}
\]

and (10) yields:

\[
E\left(\frac{\partial V^x}{\partial B}\right) = E\left[ -\frac{\delta u^*(C_{2,i+1}) - \beta(1-p(V^x))u'^*(C_{3,i})}{\delta u^*(C_{2,i+1}) - \beta[(\hat{p}(V^x) - 1)u'^*(C_{3,i}) - p'(V^x)u'(C_{3,i})]} \right]. \tag{21}
\]

But unlike (20) there is no way to determine the sign of (21), so the sign of \(E\left(\frac{\partial B}{\partial V^x}, \frac{\partial V^x}{\partial a_{t,i}}\right)\) is also unknown.

Also note that
\[
E\left(\frac{\partial B}{\partial a_{t,j}}\right) = E\left[ -\frac{\beta(1 - p(V^x))u^*(C_{3,j})}{\partial u^*(C_{1,i+1})} - \beta p^*(V^x)u(C_{3,j}) \right],
\]

and again, there is no way to know the sign of this derivative.

Using the same techniques, it can be shown that \( E\left(\frac{\partial S_{2,i}}{\partial a_{t,j}}\right) < 0 \) and there are other derivatives (like \( \frac{\partial S_{1,i}}{\partial a_{2,j}} \)), whose expectancy sign is unknown. Going through the whole list of ingredients of (16) (or (17)) is cumbersome and the above examples are sufficient to show that the overall sign of (16) is indeterminate on a theoretical basis. Indeed, the vast amount of empirical research about offset and Ricardian Equivalence have not yielded an unambiguous result because, as far as I know, none of them dealt with all the direct, indirect and cross effects expressed in (16).\(^{42}\)

Although there is as yet no empirical proof, I believe that it is reasonable to assume that intergenerational transfers dampen the offset effect. It also seems reasonable to assume that usually the substitution effect is dominant and therefore offset for some extent is expected. However, further research is required.

The above discussion implies that we can divide all economies into two main groups: “pure” economies and “mixed” economies. In a “pure” Ricardian economy, \( \frac{\partial S}{\partial a} = 1 \); in a “pure” Keynesian economy, \( \frac{\partial S}{\partial a} = 0 \) and “mixed” economies lie on the continuum between these two kinds of “pure” economies. The “mixed” economies exhibit partial equivalence, namely \( 0 < \frac{\partial S}{\partial a} < 1 \), and they differ from each other by the interior allocation of the components of (16), as explained above\(^{43}\). Namely, in this model the (partial or full) equivalence comes from two main sources: the one is offset by contributions to voluntary saving as a response to a compulsory government increase in pension contribution. The other comes from intergenerational transfers.

\(^{42}\) Apparently, empirical studies that examine the correlation between consumption and fiscal deficits (or between saving and deficits) bypass this problem. In Schwarz (2000), I discuss Kotlikoff’s (1993) Generational Accounting approach and show that this assertion is inaccurate.

\(^{43}\) In fact, it is absolutely uncertain that in any case \( 0 \leq \frac{\partial S}{\partial a} \leq 1 \), implying that overshooting of individuals’ reaction for governmental policy is certainly possible. Deep analysis of necessary and sufficient conditions for overshooting, not to mention explaining it, is beyond the scope of this paper.
from old people, receiving a higher replacement ratio than they planned, to young people who are forced to increase their contribution to pension schemes. The plausible assumption that most economies have \( 0 < \frac{\partial S}{\partial a} < 1 \) explains why the empirical literature could not decide on the question of Ricardian Equivalence. An additional conclusion from the above is that the radical conclusions by Bernheim & Bagwell (1988) depend on the critical assumption that \( \frac{\partial S}{\partial a} = 1 \). Therefore, I think that Seater (1993) who claims that full Ricardian Equivalence is a special extreme case is more reasonable. After all, full Ricardian Equivalence is actually based on unreal assumptions. Nevertheless, Seater writes that Ricardian Equivalence is a very good approximation of reality; namely, economies are usually “mixed” but their \( \frac{\partial S}{\partial a} \) is close to unity.

5. Conclusion

In this paper I analyzed individuals’ behavior within a partial equilibrium microeconomic framework. The core of the paper is an integrative model, combining both altruistic and strategic considerations as arguments in the individual’s objective function. It was shown that this model is a multi-equilibria model.

The analysis shows that the larger the altruism coefficient (for a given level of strategic motive), the greater the tendency of individuals to anticipate intergenerational transfers from the third to the second period of life, and maybe even to expand them. This result is invariant to the income profile of the individuals in the economy.

Interior solutions require that both motives for intergenerational transfers are operative. With a positive interest rate, interior solutions and a large enough time preference coefficient, the individuals can maximize the level of services they extract from their children, as well as their life expectancy, through intergenerational transfers. However, the effect of a rise in life expectancy on the amount of capital in the economy is ambiguous, since it is the sum of two opposite sign effects. In interior solution of a steady state (other things being equal), life expectancy depends on the
subjective time preference rate of the individuals and on the interest rate, and is independent of the altruism coefficient.

The ability to characterize economies with radical objective functions (pure egoists or altruists) is limited since the absence of altruistic motive does not ensure corner solution. However, it seems that pure altruism cannot be reconciled with stationary equilibrium.

We have shown that the existence of Ricardian Equivalence hinges on many contrasting effects. Empirical studies have so far concentrated on part of these effects only. The important conclusion that emerges from this study is that there is an intrinsic difficulty in establishing the existence of Ricardian Equivalence.

**Appendix**

**Proof of Proposition 1:** From (4) we conclude that in this sort of equilibrium $C_{3,t} = 0$, so the Inada conditions imply that $u'(C_{3,t}) = \infty$, which is an irrational solution, since if the marginal utility from consumption in the third period is infinite, it is advisable to reallocate the consumption and transfer some consumption from the previous two periods to the third by saving for the old age. It is possible to reconcile such a solution with rational choice, by assuming that the individual knows for sure that he is not going to live the third period, and he cannot change his fate by intergenerational transfers. That is $\hat{p}'(V') = 0, \ p = 1$. Such a solution implies that the strategic index of the individual, $\psi$, is zeroed. The consumption profile of an individual in this economy is given by:

$$C_{1,t} = w_1, \ C_{2,t} = w_2, \ C_{3,t} = 0. $$

However, even a very low absolute value of $\psi$ is not a sufficient condition to ensure that there will be no altruistically motivated intergenerational transfers. So the conclusion is that, when people consider a mixture of altruistic and strategic considerations, even a very low value of $\delta$ is insufficient to ensure no intergenerational transfers. Apparently intergenerational transfers may stem
from strategic motivations, since by (8) and (9), it can be seen that they can exist as equalities even if $\delta = 0$. Therefore, the additional condition for no intergenerational transfers is that the strategic motive, $\psi$, is also low enough, below a certain critical value.$^{44}$.

**Proof of Proposition 2:** Generally, when all first order conditions hold as equalities, we can isolate $\delta$ from (8') and have:

$$\delta = \frac{1}{\beta} \left( \frac{\hat{p}'(V^x)}{(1 - \hat{p}(V^x))} \cdot \frac{u(C_{3, t})}{u'(C_{3, t})} \cdot \frac{1}{R} + 1 \right).$$  \hfill (A1)

And by the same token, we can isolate $\delta$ also from (9) to get:

$$\delta = \frac{1}{R} \left( \frac{\hat{p}'(V^x)}{(1 - \hat{p}(V^x))} \cdot \frac{u(C_{3, t})}{u'(C_{3, t})} \cdot \frac{1 + p}{1 - p} \right).$$  \hfill (A2)

Now it is clear that the same $\delta$ that makes (8) equality, also makes (9') and (10') equalities. Therefore:

$$\frac{1}{\beta} \left( \frac{\hat{p}'(V^x)}{(1 - \hat{p}(V^x))} \cdot \frac{u(C_{3, t})}{u'(C_{3, t})} \cdot \frac{1}{R} + 1 \right) = \beta(1 - \hat{p}(V^x)).$$  \hfill (A3)

It can be easily verified that in such an interior solution, $p = 1$ is impossible, because as $p$ approaches 1, the expressions within the brackets approach infinity while $\beta(1 - \hat{p}(V^x))$ approaches zero. Therefore the inevitable

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$^{44}$ By inserting $\hat{p}'(V^x) = 0$, it follows that in case $\psi = \frac{1}{\hat{p}'}(V^x)$. 
conclusion is that \( p = 1 \) is inconsistent with an interior solution of stationary equilibrium.\(^{45}\).  

**Proof of Proposition 3:** It is not very complicated to calculate the endogenous life expectancy in such equilibrium. In an interior solution when the left side of (8’) is set to zero, \( \delta = \frac{1}{\beta R} \). From the left side of (9’) we see that in this case \( \delta = \frac{1 + p}{R(1 - p)} \). Since in an interior solution both conditions hold as equalities, the above two \( \delta \) must coincide. Thus:

\[
\frac{1}{\beta R} = \frac{1 + p}{R(1 - p)} \tag{A4}
\]

This enables us to figure out the (endogenous) probability of the individual living the third period, as shall be determined in the stationary equilibrium of an interior solution when both (8) and (9) are set to zero:

\[
(1 - \hat{p}) = \frac{2\beta}{1 + \beta} \tag{A5}
\]

Namely, in such equilibrium, life expectancy depends only on the subjective time preference of the individuals. The more patient the individuals, the more they are willing to sacrifice current consumption for extending (through intergenerational transfers) the probability of enjoying future consumption. Notice that if \( \beta = 1 \) then also \( (1 - \hat{p}) = 1 \) which means that if people assign the same weight to future consumption and to current consumption,\(^{46}\) they could maximize their life expectancy. The interesting point here is that the endogenous life expectancy depends on the subjective time preference coefficient, and is independent of the altruism coefficient \( \delta \).\(^{\square}\).

**Proof of Proposition 4:** Equation (7) is actually a sum of two non-negative expressions adding up to zero. This means that each of them must also be zero.

\(^{45}\) Or, that the critical value of \( \delta \) ensuring no intergenerational transfers is negative, which means that people in this society are motivated by jealousy.

\(^{46}\) As Ramsey (1928), based on moral arguments, claimed should be done.
The first expression can be zero only if \( \frac{\partial V^m}{\partial l} = 0 \). To set the second expression to zero, at least one of the following contingencies must hold:

(a) \( p'(l) = 0 \)

(b) \( B'^{-1} = 0 \)

(c) \( u'(C^I_{2,l}) - u'(C^I_{2,l}) = -\beta(1 - \hat{p})[u'(C^I_{1,l}) - u'(C^I_{1,l})] \)

It is easily noted that contingency (a) means that the individual maximizes his effort to extend the life expectancy of his parents. If this is not the case, then we have contingency (b) or (c). In case (b), the individual does not expect any bequest. In case (c) the difference between marginal utilities from consumption in state of nature I, to that of state of nature II, equals the value of the discounted difference of marginal utilities from consumption in the third period at state of nature I to that of state of nature II. Hence, the individual is indifferent about receiving the bequest in his second period rather than in his third period. However, this is probably a relatively rare occurrence. In other words, if the case is not (b) or (c), the individual expects a positive bequest and the timing of receiving it is important to him.

**Proof of Proposition 5:** If our case is not (c) (see proof of proposition 4), which is probably the more frequent case, and if \( B'^{-1} > 0 \), then – as was proven above – \( p'(l) = 0 \). Let us write this equation in full:

\[
 p'(l(V^x)) = \frac{\partial p}{\partial l} \frac{\partial l}{\partial V^x} = p'(l) \cdot \frac{\partial l}{\partial V^x} = 0 \quad \text{(A6)}
\]

Since by the assumptions of the model \( p'(l) < 0, \ \forall l \), the only way that (12) can exist is when \( \frac{\partial l}{\partial V^x} = 0 \). Hence, if the interest rate is positive and there is a system of intergenerational transferring of bequests, there is also a system of inter-vivos transfers that aim to buy life expectancy and to compensate the young for postponing the transfer of the bequest.

**Proof of Proposition 6:** By setting \( \delta = 0 \) we can rewrite \((8')\) as
\[ \psi = \frac{C_{3,t}}{\eta_{u,e}} \cdot \frac{1}{\beta R} \]  

(A7)

But this equation is impossibility, since by definition \( \psi \) is strictly negative, while the expression on the right side of (A7) is positive. By the same token, we can substitute \( \delta = 0 \) into (9') to have:

\[ \psi = \frac{C_{3,t} + \eta_{u,e} (1 + \hat{p}(V^s))}{\delta R\eta_{u,e}} \]  

(A8)

Again, (A8) is an impossible equation, because it equates negative to positive. Thus, perfect egoism is inconsistent with a full interior solution of a stationary equilibrium \( \blacksquare \).

**Proof of Proposition 7**: Recall that under the pure altruism assumption, having (8) hold as inequality implies that \( R \) must satisfy \( R < \frac{1}{\beta} \), (which is a private case of \( R < \frac{1}{\delta \beta} \), the general result from above). At first glance, there is no reason to suppose that this case is impossible, therefore pure altruism is not a sufficient condition for inter-vivos transfers, at least for the second period of the donor. These transfers depend on the interest rate and the time preference coefficient. Similarly it can be shown that for the pure altruist (9) can hold as inequality if \( R < \frac{1 + p}{1 - p} \), which is also a special case of a result developed above. So the alleged conclusion is that a strong altruistic motive is not sufficient for inter-vivos transfers in the third period.

But the unlikeness of such a solution becomes clear after a close examination of (10) under the pure altruism assumption. In this case, we get \( \beta (1 - \hat{p}(V^s)) = 1 \). Obviously, this can happen only if \( p = 0 \) and \( \beta = 1 \), implying by (8) that \( R < 1 \) (or \( r < 0 \)). This is an extreme and unlikely case. Specifically, it seems to us unreasonable to assume a negative interest rate in a
stationary equilibrium.\textsuperscript{47} Thus, the conclusion is that pure altruism is inconsistent with a steady state and in such a utopian economy of pure altruism the system will not converge into a stable equilibrium.

\section*{References}


\textsuperscript{47} Usually it is assumed that if there is a retirement period and it is not possible to store inventories, there will be positive saving, even with a negative rate of return. The theoretical question as to whether there is likely to be a negative interest rate in equilibrium (especially stationary equilibrium), is controversial. In Samuelson (1958), the equilibrium interest rate is negative. However, Keynes and his followers, on the contrary, entirely reject this possibility. Eisner (1986, p. 170) even emphasizes that in monetary economy (with no inflation) people will never agree to invest their money and get a negative return as long as they can just put it under the floor of their homes and have a zero rate of return. In this


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paper we shall simply exclude a negative rate of return without discussing the theoretical question in depth.


