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If You Can't Beat them – Join Them. A Cooperative Game Theoretical approach to Rent-Seeking Contest

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Abstract

This article integrates cooperative game theory with non-cooperative rent-seeking and captured regulator theories, to explain cartel stability and provides conditions for monopolization or cartelization of an industry. It is shown that the difference between the Shapley value of the and the rent-dissipation in a stochastic rent-seeking contest depends on the "return to scale" measure, α. Protected and comprehensive cartels (as defined in the article) are the only stable cartels in a rent-seeking environment. Surprisingly, stable cartels can sustain if and only if the firms' gain from cooperation is non-positive.

Keywords: Rent-seeking, Regulation, Cartel, Shapley value

JEL classification: D4, H8, K2, L5

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1. Introduction

In a seminal article, Stigler (1964) conjectured that although collusion may maximize aggregate profits of all firms in an oligopolistic industry, loyalty to a cartel policy is not a Nash equilibrium strategy because each member firm is incentivized to cheat. This logic might raise some questions about the necessity for anti-trust legislation and expensive regulatory agencies, but Levenstein and Suslow's (2006) survey of the empirical literature has abundant examples of successful cartels that remained stable for significant periods. The authors also indicate that: (a) Cheating is absolutely not the main cause of the collapse of cartels but what they call "bargaining problems", and (b) Cartels do not rely on punishment mechanisms (like "price wars"), but on administrative organs like sales agencies or even an external regulatory or inspective agency. These empirical facts are incompatible with classical cartel theory.

Clearly, with seriously enforced anti-trust legislation, sales agencies or other cartel managing organizations cannot operate in the open. But the naïve Pigouvian view of a benevolent regulator has been challenged by economists who recognized that politicians and bureaucrats have their own interests and preferences that do not always coincide with social welfare maximization.

This article integrates cooperative game theory with non-cooperative rent-seeking and captured regulator theories to explain cartel stability, and provides conditions for monopolization or cartelization of an industry. I show that only *protected comprehensive* cartels (as defined below), can be stable. Secondly, I show that the difference between the Shapley value of a cartel member and his expected payoff in a conventional rent-seeking contest is positively related to the return-to-scale measure of the rent-seeking technology, but the stability of a cartel is inversely related to it. Moreover, I show that a cartel is stable if and only if the gain of a risk-neutral member firm from cooperation is non-positive. I also show that if the firm's gain from cooperation is positive, a rent-seeking contest evolves and the market will be monopolized by a winner firm.

The reminder of this article is organized as follows: The next section surveys related literature. Section 3 presents the benchmark Tullock (1980) rent-seeking model and a cooperative game theoretic analysis of rent-seeking cartel game. Section 4 summarizes and contains a brief discussion of the results. All proofs are relegated to the appendix.

2. Related Literature

Stigler (1971) postulated that regulation is merely a device used by the regulated industry to control entry and maintain its cartelistic arrangements. Moreover, Stigler claimed that in many cases oligopolistic industries seek a regulatory umbrella and "capture" the regulator to obtain exemptions from legal restrictions[1].

Incumbent firms are naturally not the only agents that expend resources to capture the regulator. Entrants (and consumer organizations) do the same. Several schools offered different methods to model this kind of competition.

The Political Influence Competition School (cf. Becker, 1983) described political influence competition as a zero sum game between two groups expending resources to produce "pressure" on the political system to adopt their favored policy. The equilibrium investment of each competitor in producing political pressure on the regulator and the public policy adopted is the consequence of a political parallelogram of forces. Noting that the core of simple cooperative voting games is usually empty, Becker explicitly refrains from using the cooperative approach to explain the formation of coalitions and collusions within his pressure groups, and his analysis is non-cooperative in nature.[2] However, Becker is well aware of the free riding problem, which is equivalent to Stigler's cheating problem; both undermine the stability of the cartel, and his analysis provides no explanation for the cartel stability puzzle.

Contrary to the Chicago School of Becker and Stigler, the Virginia School of Buchanan and Tullock (1962) suggested modeling the political influence competition as a probabilistic contest like a lottery where "the winner takes all," later termed by

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1 In addition to Stigler's interesting examples, taken from the US economy, the Israeli economy provides no less convincing and outrageous examples of full cooperation of the regulator with the regulated industry, which goes even beyond the standard "capture of the regulator" theory. The most interesting examples relate to the Association of Banks. The Association represents the banks and negotiates on their behalf with the government, the Knesset and the central bank even over banking commissions, although one might expect commissions to be determined competitively. There is no real competition in the Israeli banking system, and it is also worth noting that in the last 30 years, no new license has been issued in Israel to a new entrant bank. Another interesting example of protected cartels is the fuel and gas cartels. These cartels do not have any legal right either to exist or to punish a deviator, because synchronization of a business strategy is criminalized by law. Nevertheless, these cartels are very powerful and implement their collusive policy efficiently, effectively and above all, publically. The common denominator of these cartels (and others, like the fuel and gas cartels) is an open secret: governmental backing and cooperation.

2 As Becker himself indicates, the emptiness of the core led Aumann and Kurz to employ the Shapley Value and limit the majority power to extract resources from the minority (cf. Aumann and Kurz, 1977a, 1977b).

Krueger (1974) as "rent-seeking" contest. In Tollison's (1998) words: "The interest-group theory of government is about lobbying, and the theory of rent-seeking is about the costs of lobbying."

Collusion among firms may take several forms. One prevailing form appears to be that firms collude in lobbying against consumer groups; for example, to buy less regulation, but they still compete in the product market. Thus, they are cartels in the market for less regulation but are competitors in the product market. There are also different types of regulation: there is regulation against mergers and getting monopolies, regulation against pricing policies, financial disclosure of a firm's operations, the quality of the product and more. The rent-seeking model encompasses all types of regulation and cartel formation. Rent-seeking competition may be conducted through cash transfers, but also through various methods lobbyists use in campaigning for their agenda.

The Industrial Organization branch of the literature combined the capture of the regulator approach with rent-seeking and asymmetric information of the principal-agent model. For instance, Laffont and Tirole (1991, 1993) assumed a regulatory agency supervised by a political principal (Congress). The regulated firm possesses private information about its costs, and the regulator can discover this by expending resources (financed by the political principal). The firm can collude with the regulator to hide this information from Congress. The collusion between the regulated firms and the regulatory agency is aimed mainly at extracting resources from the political principal.[3]

The economic schools mentioned above produced vast literature, theoretical and empirical, applying a variety of modeling methods and solution concepts. The extraordinary richness of the literature precludes any honest survey within the framework of this article.[4] Nevertheless, it is worth mentioning a few articles that tried to integrate some cooperative approaches with rent seeking contests. In a pioneering article Appelbaum and Katz (1987) considered the case when firms act "sophisticated," namely, form coalitions. As they indicate, *"it is clearly preferable for sophisticated firms not to engage in individual rent seeking and furthermore, form a*

3 Another branch of the Industrial Organization literature is based on "life cycles" models of the regulatory agencies (cf. Martimort 1999).

4 For a partial survey of the political influence competition literature, see Mitchel and Munger, (1991). For a rent-seeking literature survey, see Nitzan (1994), Baye, Kovenock and deVries (1998), and also Epstein and Nitzan (2007). For a detailed discussion of Industrial Organization approach, see for instance, Tirole (1995) and Pepall, Richards and Norman (2008).

coalition to face the regulator and try to affect his policies. The formation of such a coalition is of course subject to the usual problems facing coalitions, namely enforcement, monitoring, free riders etc." However, their analysis ignores these problems and they address neither the question of rent-seeking coalition stability, nor the question of equilibrium allocation of the prize among the winning group members.

Linster (1994) analyzed a simple version of the Tullock (1980) model, and, using the Nash Bargaining Solution concept, shows that repetition of the game can serve as an enforcement mechanism for stable cooperation. Huck and Konrad (2002) analyzed the merger-collusion dilemma of competing firms and show that the preferred option depends on the convexity properties of the contest success function.

Lee and Cheong (2005) studied a cooperative rent-seeking contest where n incumbent firms compete against m potential entrants and show that collusion of the incumbent firm on the one hand and the entrants on the other hand usually increases the aggregate expenditures, or the rate of rent-dissipation. In their analysis, collusion takes place within each group, while the regulator is inactive.

Sánchez-Pagés (2007) analyzed a model of a two-stage contest. In the first stage, agents form groups and then compete for prizes in the second stage by investing resources. Rivalry persists within coalitions once victory is attained, implying that prizes are assumed to decrease with the size of the group. He shows that bigger groups tend to drop out of the contest and that coalition formation generates positive spillovers on non-members. When coalitions form simultaneously, the contest among individual agents is the only stable structure where deviations are assumed to leave the rest of the structure intact. When, on the contrary, coalitions break apart completely after a member withdraws, more concentrated coalition structures, including the grand coalition, can be stable, provided that intra-group rivalry is not too strong. The grand coalition is likely to form in certain circumstances in a sequential game.

More recently, Glazer (2008) suggested a two-stage bargaining rent-seeking model and shows that if firms collude in the first stage they can maximize joint profits by investing on a non-infinitesimal level, and restrict investment even if the cost of the rent-seeking effort is zero.

However, none of these studies addressed the stability problem of collusion, explicitly specified how the rent from cooperation is allocated among coalition members, or characterized the interrelations between the sharing formula and the collusion arrangement's stability.[5]

3. The Model

Firms in oligopolistic industrial branch have to choose one of the three possible types of strategies, two of them non-cooperative and one cooperative. Namely, the firms may conduct an oligopolistic competition, engage in a rent-seeking contest for acquiring a monopoly in the market, or agree on a cartelistic collusion arrangement of production and pricing, and of course on allocation of the monopolistic rent among members of the cartel. The rent-seeking literature concentrates mainly on modeling the non-cooperative type of rent-seeking contest. In this section I compare the well known results of the non-cooperative rent-seeking game with the cooperative rent-seeking game where the Shapley value is applied as the solution concept. As a benchmark, let us start with the simplest version of the conventional well-known non-cooperative rent-seeking contest.

3.1. The Stochastic Rent Seeking Non-Cooperative Contest

Consider n risk-neutral firms engaging in a rent-seeking contest for acquiring monopolistic status by expending resources to the regulator. Denote the expenditure of a contestant $i \in N$ by x_i, the firms' set by N and assume that the winning probability in the contest is given by a Tullock (1980) type contest success function[6],

(1) $$\Pr_i\left(\mathbf{x}\right) = \frac{x_i^{\alpha}}{\sum_{j \in N} x_j^{\alpha}}, \quad \forall i, j \in N, \quad \mathbf{x} = \left\{x_i\right\}_{i=1}^{n}$$

5 For discussions of the endogenous formation of coalitions, see, for instance, Aumann and Meyerson (1988) and Hart (1974). For studies that applied a specific cooperative solution in biform games, see, for instance, Grossman and Hart (1986), who applied the Shapley Value; Hart and Moore (1990), who applied the Nash Bargaining Solution; and Branderburger and Stuart (2007), who applied the Core. For detailed discussion of biform games, see Muto, Nakayama, Potters and Tijs (1988) and Tijs (1990).

6 For axiomatization of this function, see Skaperdas (1996). See also Fullerton and McAfee (1999).

The parameter α measures the return to scale of the rent-seeking efforts. If $\alpha \to 0$, the winning probability is the same for all contestants. On the other hand, if $\alpha \to \infty$, the rent-seeking contest becomes an all-pay-auction under which the prize is awarded to the contestant who makes the highest effort. (Baye, Kovenock and de Vries, (1993). I limit the analysis to games with symmetric pure-strategy Nash equilibria, therefore I assume $0 \le \alpha < \frac{n}{n-1}$, (Pérez-Castrillo and Verdier, 1992).

For the sake of simplicity, the prize value of the contest, denoted by $\overline{v}$, is assumed to be identical for all contestants. Denote the contestant's net payoff by w_i. The expected net payoff of a risk-neutral contestant firm $i \in N$ is,

(2) $$E(w_i) = \Pr_i(\mathbf{x})\overline{v} - x_i$$

The optimization problem of the firm is to choose x_i^* which maximizes (2). Solving this optimization problem yields the well-known results,

(3)
$$x_i^* = \left(\frac{n-1}{n^2}\right)\alpha\overline{v}, \qquad \forall i \in N$$
$$E(w_i) = \frac{\left[\alpha + n(1-\alpha)\right]}{n^2}\overline{v}, \quad \forall i \in N$$
$$\sum_{i=1}^{N} x_i^* = X^* = \left(\frac{n-1}{n}\right)\alpha\overline{v}$$

It follows from (3) that rent-dissipation is proportional to the number of contestants, and $\lim_{n\to\infty} X^* = \alpha\overline{v}$, which are well-known results as well.

3.2. Cooperative Rent-Seeking Competition

Define a *coalition* as a subset $S \subseteq N$ of firms colluding as a cartel against the set F of non-colluding firms $(S \cup F = N, \quad S \cap F = \varnothing)$. The main goal of coalition S (the cartel) is to drive the non-colluding firms F (henceforth the "fringe" firms) out of the market. In order to achieve this goal, the cartel seeks the regulator's assistance.

The set of players in the cooperative game is extended therefore to $\{N, r\}$, were N is the set of firms and r denotes the regulator, $s = \#S$ is the number of elements in S, (so $1 \leq s \leq n$ where $n = \#N$ is the number of firms in the industry). Any coalition is allowed. The aggregate value of a coalition $S \subseteq N$ is represented by a *coalitional function* $v(S)$ satisfying $v(\varnothing) = 0$. Denote the payoff of a firm $i \in S$ under a cartel arrangement by $v_i(S)$ and the firm's payoff implied by the Shapley (1953) formula as $\varphi v(i)$.

Stability, Cheating and Betraying

For the purpose of our analysis, it is crucial to distinguish between ***cheating*** and ***betraying. Cheating*** relates to underground deviation from prices or quota agreements. A member firm ***betrays*** the cartel by attempting to bribe the regulator to obtain a monopolistic status in the market, at the expense of the other member firms. In other words, cheating is with consumers, whereas betraying is with the regulator.

Definitions

A ***comprehensive cartel*** is a coalition S which includes the entire industry $(S = N)$.

A ***partial cartel*** is a coalition of a subset of the firms $(S \subset N)$.

A ***protected cartel*** is a coalition that includes the regulator as an active member. A particular case of a protected cartel is the legal monopoly of firm $i \in N$, which forms a coalition $\{i\} \cup r$ with the regulator.

An ***effective coalition*** has the power to appropriate the entire monopolistic rent in a certain market. Namely, if S is an effective coalition, $v(S) = \overline{v}$.

A cooperative rent-seeking cartel is ***stable****, or* ***betrayal robust****, if* $v_i(S) \geq v_i(T) \quad \forall i \in S, \quad \forall S, T \subset N, \quad S \cap T = \varnothing$. Namely, coalition S is stable if no member firm has any incentive to betray it and join a different coalition T.

Assumption 1: *Comprehensive and protected cartels are effective coalitions, implying* $v\left(N\right) = v\left(S \cup \left\{r\right\}\right) = \overline{v}, \quad \forall S \subseteq N$[7].

Proposition 1:

Any unprotected cartel is unstable.

Proof: see appendix.

Proposition 1 simply states that being loyal to any unprotected cartel is not a Nash equilibrium strategy.

Corollary 1:

Any partial cartel, either protected or unprotected, is unstable.

The intuition underlying this corollary is obvious. As long as fringe firms are operative, the rent-seeking contest is not over, and every firm (either a member of the cartel or a fringe firm), is incentivized to offer additional ε to the regulator. On the other hand, as explained in the appendix, the regulator has no incentive to cooperate with a partial cartel. Hence, only a *comprehensive* cartel can be formed, but only a *protected comprehensive* cartel can be stable.

3.3. The Shapley Value of the Cooperative Rent-Seeking Game

In this subsection I analyze the rent-seeking industry's behavior using the cooperative game theory toolkit, with the Shapley value as solution concept. Calculating the Shapley value requires the additional two assumptions:

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7 Notice carefully that by assumption 1 this game is not a "Big-Boss" game. See Tijs (1990) and Muto, Nakayama, Potters and Tijs (1998).

Assumption 2: *(Ineffective coalition value),* $r \notin S \Rightarrow v(S) = 0, \quad \forall S \subset N$[8].

Assumption 3: $v(\{r\}) = X^* = \left(\frac{n-1}{n}\right)\alpha\overline{v}$ *. Namely, if no coalition is formed, the conventional rent-seeking contest is played, and the regulator receives* X^*.

Assumptions 1-3 imply the following definition of the coalitional function for the cooperative rent-seeking game,

(4)
$$v(S) = \begin{cases} \overline{v} & S = N \\ \overline{v} & S \subset P \\ \left(\frac{n-1}{n}\right)\alpha\overline{v} & S = \{r\} \\ 0 & \text{otherwise} \end{cases}$$

Define $\Delta(r) = \varphi v(r) - X^*$ and $\Delta(i) = \varphi v(i) - E(w_i)$. $\Delta(r)$ is the difference between the Shapley value of the regulator and his expected payoff in a rent-seeking contest (the rent-dissipation rate), and measures the regulator's gain from cooperation with the cartel. Similarly, $\Delta(i)$ is the difference between the Shapley value of firm i and its expected payoff in rent-seeking contest.

Proposition 2

(5)
$$\Delta(r) \gtreqless 0 \Leftrightarrow \alpha \lesseqgtr 1,$$
$$\Delta(i) \gtreqless 0 \Leftrightarrow \alpha \gtreqless 1, \quad \forall i \in S$$

Proof: see appendix.

8 Notice that this assumption actually stems from corollary 1. In particular, notice that since $v(i)=0, \forall i \in N$, it follows that $\varphi v(i) = \varphi v(j), \quad \forall i, j \in N$.

Proposition 2 states that the difference between the Shapley value of each player and his expected payoff in a rent-seeking competition depends on the "return to scale" parameter , α .

Proposition 3

a. *The protected comprehensive cartel* $N \cup \{r\}$ *is stable if and only if* $\alpha \leq 1$.

b. *If* $\alpha > 1$ *a protected cartel is not sustainable, and the market will be monopolized by a rent-seeker contestant firm.*

Proof: see appendix.

In other words, proposition 3 states that when $\alpha \leq 1$, betraying the comprehensive protected cartel is not a Nash equilibrium strategy. On the other hand, if $\alpha > 1$ no cartel is sustainable implying that the industry will be monopolized by the firm that wins the rent-seeking contest[9].

Corollary 2:

The stability of a protected comprehensive cartel is inversely related to the return to scale parameter, α .

9 Hirshleifer (1995) distinguished between chaos and anarchy, which he termed "spontaneous order" and defined as "a system in which participants can seize and defend resources without regulation from above". Hirshleifer show that "an anarchic system, to be sustained, must be *dynamically stable* and *viable*". Dynamic stability holds when the "decisiveness of conflict" (measured by α in his model) is sufficiently low. Namely, when $0 < \alpha < 1$. (This result was generalized in Hausken (2006)). Hirshleifer emphasized that he has not attempt to model the problems of group formation and collective action, or "how to get agreement on a social contract and, even more important, how to enforce it?", but more importantly, in Hirshleifer's anarchy there is no regulator and no cartels are formed. When α is sufficiently high "the most militarily effective contender would become a hegemon". Piccione and Rubinstein (2007) show a similar result using different modeling of anarchical equilibrium. (See also Bush and Mayer (1974), McGuire and Olson , (1996) and Acemuglu and Robinson (2008)). In our model there is a regulator, and our results imply that if $0 < \alpha < 1$, cartels can be formed and sustain in a rent-seeking environment if the Shapley formula is applied to allocate the rent between the cartel and the regulator, and among the cartel members.

Proposition 3 seems counter intuitive, as it follows that cartel stability requires that $\alpha \le 1$ while by Proposition 2 the firms' gain from cooperation with the cartel is non-negative if and only if $\alpha \ge 1$. In other words, even risk-neutral firms are expected to cooperate with the protected cartel although their gain from cooperation is non-positive. The intuitive explanation to this surprising result, as explained in the appendix, is that when $\alpha < 1$, although risk-neutral firms prefer to engage in a rent-seeking contest, the regulator has a clear incentive to enforce cartelization and will not accept any offer to betray (a betrayer firm may be driven out of the market by the regulator). When $\alpha < 1$ the firms prefer to form a protected cartel with the regulator, but in this case the regulator is clearly incentivized to enforce a rent-seeking competition[10].

4. Summary and Discussion

In the previous section I compared the non-cooperative and the cooperative rent-seeking games applying the Nash equilibrium and the Shapley value as solution concepts, respectively. It was shown that assuming risk-neutrality, the difference between a player's expected payoff in a rent-seeking contest and his Shapley value depends on the rent-seeking technology return-to-scale measure, α. It was also shown that a regulator can be captured by a cartel if $\alpha \le 1$, while if $\alpha > 1$ the regulator will be captured by a winner firm in a rent-seeking contest, and the industry will be monopolized. Surprisingly, a protected cartel can be formed and sustain if and only if the firms' gain from cooperation is non-positive.

The interrelations between cooperative and non-cooperative game solution concepts are well known since the "equivalence theorems" were proven in the pioneering articles of Debreu and Scarf (1963) and Aumann (1964), who formulated the Core Equivalence Theorem which states that in perfectly competitive economies, the core coincides with the set of competitive allocations. Later works[11] produced the *Value Principle: "In perfectly competitive economies, every Shapley value allocation*

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[10] It should be emphasized that the "loss" of the risk-neutral firms from the enforced cartelization is only in comparison with their expected gain from rent-seeking contest. They are still better off in comparison to their expected payoff from playing a Cournot game.

[11] Shapley (1964), Shapley and Shubik (1969), Aumann and Shapley (1974), Hart (1979) to mention only few.

is competitive, and the converse holds in the differentiable case" (Hart 2001, see also Anderson 1992).

However, there are two important features that differentiate our results from the above-mentioned theorems. First, Equivalence Theorems relate to purely competitive environments, whereas our results relate to an oligopolistic environment.[12] Secondly, in our setting, the "competitive" equilibrium is not characterized by a price vector or a budget set. Nevertheless, these theorems underlie the intuition behind our results, pointing out that the interrelations between competitive and uncompetitive solution concepts also hold for rent-seeking contests and cartelistic collusions in oligopolistic markets. Using much simpler mathematics, I show the equivalence between the Shapley value and the *expected* Nash equilibrium allocation in a rent-seeking lottery contest.

Our results imply that structural reforms, where new entrants are licensed to challenge a legal monopoly in an oligopolistic competition, do not insure an increase in consumers' surplus. The crucial point is whether the market structure changed from a monopoly to an oligopolistic competition or to a protected cartel. There is no reason to expect that a market shift from legal monopoly (a rent-seeking contest winner) to a protected cooperative rent-seeking cartel bear any promise for improvement in consumers' welfare. On the contrary, in case of $\alpha < 1$ the excess rent-dissipation from cooperation, $\Delta(r)$, is positive, and if consumers' demand is not infinitely elastic, their welfare will decrease.

Our results are compatible with Stigler (1971), who pointed out that "the political decisions take account also of the political strength of the various firms, so small firms have a larger influence than they would posses in an unregulated industry. Thus, when quotas are given to firms, the small firms will almost always receive larger quotas than cost minimizing practices would allow."[13] This pattern of reducing the share of large companies and augmenting the

[12] In fact, the original theorems of Aumann (1964) relate to "non-atomic" economies. For a discussion of the Core Equivalence Theorem within an "atomic environment" context see Anderson (1992).

[13] Stigler illustrates this assertion with data on oil import quotas, saying that "The smallest refiners were given a quota of 11.4 percent of their daily consumption of oil, and the percentage dropped as refinery size rose". In an interesting footnote, Stigler added "Largest refineries were restricted to 75.7 percent of their historical quota under the earlier voluntary import quotas."

share of small companies is compatible with allocation according to the Shapley formula.

Our results can be applied to many other social situations. Here is one example: Consider a newly elected parliament with n parties; none of them holds a majority of the seats. There are two options: A "narrow" coalition S, which will have to conduct a costly rent-seeking political contest to survive, or a stable "national unity" coalition where the ruling power is shared with a key opposition party r. Our results imply that if the ruling power is allocated among the coalition members according to their Shapley Value, risk-averse parties will prefer the national unity coalition option. As a matter of fact, this is what happened in Israel after the 1984 and 2009 general elections.[14] Nevertheless, further research is required to determine whether in this case a national unity coalition will be formed even when $\alpha > 1$.

The above analysis was conducted under the risk-neutrality assumption. The effect of risk-aversion on contestants' behavior, winning probabilities, expected utilities and rent-dissipation rate is generally unknown, or requires further assumptions[15]. Nevertheless, the more relevant question in this article context is the effect of risk-aversion on the stability of coalitions. As mentioned above, by proposition 3 a protected cartel is stable only if the gain of the firms is non-positive. Further research is required to check the validity of this counter intuitive result under risk-aversion assumption[16]. In particular, notice that by (A5) (see the appendix), under risk-neutrality $\frac{\Delta(r)}{n} = -\Delta(i), \ \forall i \in N$, hence $\Delta(r)$, can be interpreted as the aggregate risk premium paid by the firms in order to refrain from competing in a rent-seeking contest. Further research is required to check if this interpretation is valid also for the risk-aversion case, and to determine the effect of risk-aversion on both $\Delta(i)$ and $\Delta(r)$.

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14 For an assessment of Shapley-Shubik index in the Knesset, see Owen (1971), and Rapoport and Golan (1985), who applied six power indices (including the Banzhaf and the Shapley-Shubik indices).

15 Hilmann and Katz (1984), Skaperdas and Gan (1995), Konrad and Schlesinger (1997).

16 Intuitively, risk-aversion may have contradicting effects. On the one hand, it may happen that when $\alpha > 1$, the gain of a risk-averse firm from cooperation with the protected cartel is still non-negative. On the other hand, if the risk-aversion effect on $\Delta(r)$ is negative, the regulator may not be willing to cooperate with the protected cartel.

Throughout the analysis, the rent-seeking technology, and especially the return-to-scale measure, α, were assumed exogenous. If firm can optimally chose α_i, a Prisoner-Dilemma may evolve. On the one hand, by proposition 3, as α increases, the regulator tends to impose a rent-seeking contest, hence high value of α is an advantage. On the other hand, if all firms collectively chose a relatively low level of α, the industry is more likely to be cartelized. Nevertheless, even in case where risk-averse firms strictly prefer cartelization over engaging in a rent-seeking contest, the Nash equilibrium strategy may be inefficient, implying relatively high values of α, and rent-dissipation rate, and monopolization of the industry. Of course, further research is required to rigorously check this intuition.

Appendix

Proof of Proposition 1:[17] The proof of the comprehensive cartel case is straightforward. By Assumption 1, the payoff of a member firm $i \in N$ in a comprehensive cartel is $\overline{v}/n$, and the regulator's payoff is 0. Consider an alternative coalition $T = \{i\} \cup r$. By assumption 1, $v(T) = \overline{v}$. Therefore, each firm $i \in N$ is incentivized to betray and offer the regulator a bribe $\varepsilon > 0$ in order to acquire monopolistic status and drive all the $N \setminus \{i\}$ firms out of the market. It follows that $v_i(T) = \overline{v} - \varepsilon > v_i(S)$. Since this is true for every firm $i \in N$, it follows that $0 < \varepsilon \leq x_i^*$ (where x_i^* is the optimal rent-seeking investment as calculated in (3)), and a comprehensive unprotected cartel is unstable.

The argument for the partial cartel case is slightly different. Suppose that a subset $S \subseteq N$ of firms merges into a cartel that competes against the "fringe" set F. In one extreme case, the set F contains $n-1$ singleton coalitions. In another extreme case, the fringe firms also form a coalition F.

Consider the first extreme case. In this case, the number of contestants is reduced from n to $n-s+1$. Apparently, by (3), the regulator's share in this reduced rent-seeking contest, X_R^*, is,

[17] If fact, this result stems directly from the well-known result that an equilibrium in which all players in the Tullock rent-seeking game bid zero is not a Nash equilibrium (Ellingsen 1991). Nevertheless, I present this proof for what follows.

(A1) $$X_R^* = \left(\frac{n-s}{n-s+1}\right)\alpha\overline{v}$$

Comparing (A1) to (3) reveals that under the reduced rent-seeking contest every firm is better off and the regulator is worse off. Nevertheless, (A1) does not describe a Nash equilibrium of the reduced contest, because every firm $i \in N$ has a clear incentive to offer the regulator an additional bribe, ε $\left(0 < \varepsilon \leq x_i^*\right)$, in order to become a monopoly. Hence, loyalty to the partial coalition S in the reduced rent-seeking contest is ***not*** a Nash equilibrium strategy.

Moreover, being a partial cartel requires cooperation by the regulator in order to drive the fringe firms out of the market. But since $X_R^* \leq X^*$, $\frac{\partial X^*}{\partial n} > 0$ and $\frac{\partial X_R^*}{\partial s} < 0$, the regulator surely has no incentive to reduce the number of contestants. As a matter of fact, the best case for the regulator is when $n \to \infty$, while the worst case, from the regulator's point of view, occurs on the other extreme, namely, when the fringe firms also form a coalition F. In this case, there are only two contestants, coalition S and coalition F, and the regulator's share is therefore $\overline{v}/2$. Needless to say, the same argument holds in this case as well. Namely, each firm $i \in S \cup F$ has the incentive to offer the regulator an additional bribe ε, $\left(0 < \varepsilon \leq x_i^*\right)$ in order to become a monopoly. The generalization of this argumentation for any partition k of N is straightforward. Hence, every unprotected cartel is unstable. QED.

Proof of Proposition 2: The Shapley value of a player $\varphi v(i)$ in a cooperative game is defined by,

(A2) $$\varphi v(i) = \sum_{S \subset N} \frac{(n-s)!(s-1)!}{n!}\left[v(S) - v\left(S \setminus \{i\}\right)\right]$$

The conventional interpretation of (A2) is probabilistic. $v(S)-v(S\setminus\{i\})$ is the marginal contribution of player i to the value of coalition S. $(n-s)!(s-1)!/n!$ is the probability that player i joins a coalition with $s-1$ players chosen randomly. In other words, the Shapley value of a player is the expectancy of his marginal contribution.

Let us first compute the Shapley value of the regulator. By assumption 3, the "marginal" contribution of the regulator to the value of a partial protected coalition is $\overline{v}$. (The regulator is a pivot player in a partial protected coalition). There are $n!(n-1)$ ordered partial protected coalitions. In addition, there are $n!$ ordered comprehensive coalitions, in which the marginal contribution of the regulator is zero, and also $n!$ ordered sets for the n firms when no coalitions are formed at all (in these cases, by assumption 3, the regulator receives X^*). Hence,

(A3) $$\varphi v(r)=\frac{n!}{(n+1)!}\left[0+(n-1)+\left(\frac{n-1}{n}\right)\alpha\right]\overline{v}=\frac{(n-1)(n+\alpha)\overline{v}}{(n+1)n}$$

Then, as $\varphi v(S)=\overline{v}-\varphi v(r)\quad \forall S\subseteq N$, calculating the Shapley value for every member firm is straightforward. Namely $\varphi v(i)=\dfrac{\varphi v(S)}{n}\quad \forall i\in S$, implying,

(A4) $$\varphi v(i)=\frac{\overline{v}-\varphi v(r)}{n}=\frac{\left[\alpha+(2-\alpha)n\right]\overline{v}}{(n+1)n^2}=E(w_i),\ \ \forall i\in N\,.$$

From (A3), (A4) and (3) we obtain:

(A5)

$$\Delta(r) = \frac{(n-1)(n+\alpha)\overline{v}}{(n+1)n} - \left(\frac{n-1}{n}\right)\alpha\overline{v} = \frac{(n-1)(1-\alpha)\overline{v}}{n+1}$$

$$\Delta(i) = \frac{\left[\alpha+(2-\alpha)n\right]\overline{v}}{(n+1)n^2} - \frac{\left[\alpha+n(1-\alpha)\right]}{n^2}\overline{v} = \frac{(n-1)(\alpha-1)\overline{v}}{(n+1)n}$$

And it can easily be verified that that $\Delta(r) = \gtreqless 0 \Leftrightarrow \alpha \lesseqgtr 1$ and $\Delta(i) \gtreqless 0 \Leftrightarrow \alpha \gtreqless 1$. ***QED***.

Proof of Proposition 3: Suppose first that $\alpha = 1$. In this case, by proposition 2 $\Delta(r) = 0$ and $\Delta(i) = 0, \ \ \forall i \in N$. Clearly, the regulator will not cooperate with the cartel unless he is assured at least X^*, which, by, ***(3)*** is his alternative payoff from the non-cooperative rent-seeking contest, and, by ***(A5)***, is also his Shapley value. Hence, every cartel member must pay the regulator a "membership fee" of $f_i^* = \frac{X^*}{n}$. But, by the definition of X^* (see ***(3)***), it follows that $f_i^* = x_i^*, \ \ \forall i \in N$, which is the optimal investment of a firm in a regular non-cooperative rent-seeking contest. Thus, a member firm in a protected cartel is expected to gain nothing from betraying. Hence, the protected cartel is stable.

Now suppose that $\alpha < 1$. In this case, by proposition 2 $\Delta(r) > 0$, but $\Delta(i) < 0$. There is no doubt that the regulator prefers in this case to cooperate with the protected comprehensive cartel, and it is also clear that no risk-neutral firm is incentivized to engage in a protected comprehensive coalition. However, the firms have no choice, because the regulator will not accept any offer to betray. On the contrary, a betrayer firm may be driven out of the market by the regulator.

The opposite is true, of course, when $\alpha > 1$. In this case the firms would prefer to form a comprehensive protected cartel with the regulator, but the regulator has no reason to accept this offer. It follows that a stable comprehensive protected cartel can be formed if and only if $\alpha \leq 1$. **QED**[18]**.**

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18 Note carefully that this proposition is based on the implicit but standard assumption in rent-seeking literature that assigns all bargaining power to the firms (or the cartel), and none to the regulator. In other words, it is the firms that make a take-it-or-leave-it offer to the regulator. Alternately, one can assume that the regulator can give one of the firms monopoly power by demanding a bribe of $m = \varphi v\left(r\right) + \varepsilon$ from this firm and guarantee the firm the payoff of $\overline{v} - m \geq \varphi v\left(i\right)$. If $0 < \varepsilon \leq \left(n-1\right)\left[n\left(2-\alpha\right)+\alpha\right]\overline{v} \big/ n^2\left(n+1\right)$, both the regulator and this firm will be better off relative to their payoffs in the protected comprehensive cartel. However, note that no firm is incentivized to make this offer to the regulator because such an offer from the firms to the regulator is not a Nash equilibrium.

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