

Might Makes (Contingent) Right: Equilibrium in the Stochastic Jungle

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Abstract

In the stochastic jungle, the outcome of a confrontation between two agents is contingent, and depends on their relative effort. It is shown that the stochastic jungle subgame perfect equilibrium allocation coincides with the nucleolus of the corresponding bankruptcy game. This allocation is usually Pareto-inefficient, but can be viewed as the exchange economy initial endowment. The normative implications of this result are also briefly discussed.

Keywords: Jungle economy, stochastic jungle, nucleolus, cooperative game, contest

JEL classification: C71, C72, D5.

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1. Introduction

Piccione and Rubinstein (2007) (henceforth PR 2007) introduced an elementary model of anarchy which they called *a jungle*, where economic transactions are governed by coercion. The jungle consists of a set of individuals, each characterized by a preference relation over a bounded set of consumption bundles (implying that their capacity to consume is finite), and strength. Individuals are ranked according to their strength, and this ranking is unambiguous and common knowledge. In Piccione and Rubinstein's jungle, power has a simple and strict meaning: "a stronger agent is able to take resources from a weaker agent." Piccione and Rubinstein's jungle is deterministic, and characterized with perfect foresight regarding the outcome of any confrontation between two agents. Jungle equilibrium is a feasible allocation of goods such that no agent would like to change it or can change it. In Piccione and Rubinstein's jungle, power distribution is exogenous, implying that as the incentives to produce or collect the goods are ignored in an exchange economy, so are the incentives to build strength in the jungle. Thus, Piccione and Rubinstein's jungle is deterministic, and its equilibrium reflects the exogenous distribution of power.

As Piccione and Rubinstein indicated, their model of involuntary exchange resembles the Walrasian model and yields comparable properties. The distribution of power in the jungle is the counterpart of the distribution of endowments in the exchange economy. The authors' objective is twofold: "...On the one hand, we have a genuine interest in investigating the effects of power on the distribution of resources. On the other hand, we wish to uncover some of the rhetoric hidden in standard economic theory." Regarding their first objective, Piccione and Rubinstein show that jungle equilibrium exists, and under certain assumptions is also unique and efficient. But they differ about the applicability of their model regarding their second objective, thus they have written two separate conclusion sections.

My objectives in this article are the same, but my approach is different. Regarding the first objective, I present an alternative model of anarchistic economy, *a stochastic jungle* in which there is no unambiguous ranking of agents according to an exogenous distribution of power. Thus the outcome of a confrontation between two agents is not certain but contingent. The winning probability of each agent is endogenously determined by their relative efforts according to a Contest Success Function (CSF).

The main result of the analysis is that the stochastic jungle implements the nucleolus allocation in subgame perfect equilibrium.

Regarding the second objective, it is reasonable to expect that Piccione and Rubinstein's controversy about the interpretation of their results also applies here. Unfortunately, single authors do not enjoy the privilege of writing two alternative conclusion sections (unless they prove to be schizophrenic). Therefore, the final section of this article presents my personal view only. It goes without saying that every reader has the full right to suggest different interpretations and reach alternative conclusions.

The jungle shares certain characteristics with a bankruptcy problem. A bankruptcy problem consists of a set of creditors, each characterized by a claim over a bundle of available assets which is insufficient to satisfy all creditors' claims. A bankruptcy problem solution is a vector that efficiently allocates the available assets among creditors. Similarly, the jungle consists of agents where each is characterized by his capacity to consume (the agent's "claim"), and a bundle of commodities, smaller than the aggregate jungle population consumption capacity. Thus, efficient jungle equilibrium is apparently a bankruptcy problem solution. However, there are two features which distinguish a jungle from a bankruptcy problem. First, a regular bankruptcy solution is (explicitly or implicitly) based on the benevolent central planner assumption while the jungle equilibrium is a decentralized solution of the bankruptcy problem. Second, a bankruptcy solution is usually based on a normative axiomatic approach, while the jungle as a lawless anarchy governed by the rule of force implies that agent's capacity to consume cannot be a moral basis for his claim over the jungle commodities, unless one accepts the jungle sole moral imperative "*might makes right.*" In the final section of this article, I argue that these differences lead not only to different equilibria, but to different normative evaluations of the deterministic and stochastic jungle economies equilibria.

Classical economic analyses ignored the role of force in the formation of property rights and equilibrium allocation of resources, and early analyses of crime economics, e.g. Becker (1968), studied abnormal illegal behavior within a normative society. Bush and Mayer (1974) introduced a model of anarchistic society in which the move from the initial to the final allocation is affected by theft, and show that for this kind of anarchistic economy a natural equilibrium exists. Umbeck (1981) described a

deterministic anarchy in which agents' force is determined by their common knowledge choice of effort where agreements are allowed, and show that a reliable threat to use force is the dominant factor in determining the equilibrium allocation. Umbeck refers to competition among the agents and also to coalitions but does not develop a theory of contest or coalition formation in anarchy. Skaperdas (1992) studied contest equilibrium in the absence of property rights and shows that in certain circumstances *full cooperation* equilibrium, namely an equilibrium where no agent invests in power, exists. However, his analysis is restricted to two-player games and explicitly excludes coalition formation stating that "... this a priori agnosticism reflects the difficulties not just in solving but in simply formulating the long-standing problems of coalition formation." McGuire and Olson (1996) and Olson (2000) analyzed the strongest agent's incentives in appropriating resources and supplying public goods. More recently, Acemoglu and Robinson (2008) demonstrated that even in democracies characterized by the rule of law, de-facto power is no less important than de-jure power in determining allocations, prices or public goods production, but in their analysis the player-set partition is exogenous.

2. The Stochastic Jungle

Consider an economy with a set of commodities $K = \{1, \dots, k\}$ and a set of agents, $N = \{1, \dots, n\}$. An aggregate bundle $w = (w_1, \dots, w_k) \gg 0$ is available for distribution among the agents. Each agent i is characterized by a preference relation \succsim^i on \mathbb{R}_+^K and a bounded consumption set $X_i \subseteq \mathbb{R}_+^K$. The preferences of each agent satisfy the standard assumptions of strong monotonicity and continuity, and can be represented by von-Neumann-Morgenstern utility functions $u_i(w_i)$ satisfying $u_i' > 0$, $u_i'' < 0$. The bounded set X_i is interpreted as agent i 's consumption capacity. Assume that X_i is compact, convex and satisfies free disposal¹. The satiation frontier of X_i is denoted by $B(X_i)$.

¹ Namely, that $x_i \in X_i$, $y \in \mathbb{R}_+^K$ and $x_i \leq y$ implies that $y \in X_i$.

In the deterministic jungle, unbounded individual consumption set yields uninteresting equilibrium, as the strongest agent takes all the resources. But Piccione and Rubinstein (2007) argue that a bounded consumption set is not a less genuine or a less plausible assumption than the unbounded set, due to physical limits of what people can consume. Moreover, even if modeling the desire for forever increasing wealth should differ from modeling the desire to satisfy basic needs, the difference reflects the fact that desire for wealth stems from the seeking of power and influence, which is absent in the deterministic jungle. In addition, they argue that the bounded consumption set can be interpreted as reflecting limitations of an agent's ability to protect his possessions.

In the stochastic jungle, the satiation assumption is not so desperately required in order to yield "interesting" equilibria, but rather to enrich the applicability of the model. As indicated above, the jungle economy corresponds to a bankruptcy problem. Under the non-satiation assumption, the jungle corresponds to a bankruptcy problem where every creditor claims all available assets, while a jungle under the satiation assumption corresponds to a bankruptcy problem in which some creditors claim less than all available assets. My interpretation for the satiation assumption is also slightly different. Satiation reflects asymptotic utility function. Namely, that agent's utility function satisfies $\partial u_i(x_i)/\partial x_{i,k}|_{x_i \in B(X_i)} \leq \varepsilon, \forall k \in K$, where $x_{i,k}$ denotes the amount of commodity k held by agent $i \in N$ and $\varepsilon \geq 0$ is an arbitrary constant. The satiation frontier of X^i is defined as $B(X_i) = \{x_i \in X_i | \partial u_i / \partial x_{i,k} \leq \varepsilon\}$. In other words, satiation does not necessarily mean a physical limitation on consumption ability, but rather the diminishing of marginal utility implying that provided that the agent already possesses bundle $x_i \in B(X_i)$, the additional utility induced by an additional amount of commodity k does not exceed ε and thus can be neglected. By this interpretation, the satiation frontier reflects the agent's frugality while consumption capacity can be used as a measure of greed. I shall return to this point in the final section.

The basic law of any jungle is "might makes right." However, the stochastic jungle allocation is not determined according to exogenous distribution of power among agents, because the outcome of a confrontation between two agents is

contingent. The winning probability of an agent is given by the Tullock's (1980) Contest Success Function (CSF)²,

$$(1) \quad p_i(\mathbf{e}) = \frac{e_i^\alpha}{\sum_{j \in N} e_j^\alpha}, \quad \forall i, j \in N, \quad \mathbf{e} = \{e_i\}_{i=1}^n$$

Where e_i denotes the effort exerted by agent i in the confrontation. The parameter α measures the return to scale of the rent-seeking efforts. If $\alpha \rightarrow 0$, $p_i = 1/n$, $\forall i \in N$. On the other hand, if $\alpha \rightarrow \infty$, the rent-seeking contest is reduced to an all-pay-auction under which the prize is awarded to the contestant who exerts the greatest effort³. I limit the analysis to games with symmetric pure-strategy Nash equilibria, and in order to assure a unique interior equilibrium I assume $0 < \alpha < n/n - 1$ ⁴.

An *exchange economy* is defined as a tuple $\langle \{\tilde{\omega}^i\}_{i \in I}, w, \{w_i\}_{i \in I} \rangle$, where w_i is the initial endowment of agent i and $\sum_{i=1}^n w_i = w$. A *deterministic jungle* is defined as a tuple $\langle \{\tilde{\omega}^i\}_{i \in N}, \{X_i\}_{i \in N}, w, \Psi \rangle$, where Ψ is the jungle strength relation. A *stochastic jungle* is defined as a tuple $\langle \{\tilde{\omega}^i\}_{i \in N}, \{X_i\}_{i \in N}, w, p \rangle$, where p is the CSF defined in (1). The exchange economy differs from the jungle as it is characterized by initial endowments, while the deterministic jungle is characterized by a specification of power relation and the stochastic jungle studied in this article is characterized by a contest success function. The replacement of power relation by a contest success function implies the following differences between the deterministic jungle and the stochastic jungle:

- (i) The source of power is stochastic and endogenous.
- (ii) The exercise of power is costly.
- (iii) Coalition formation is allowed.

² For axiomatization of this function, see Skaperdas (1996). See also Fullerton and McAfee (1999).

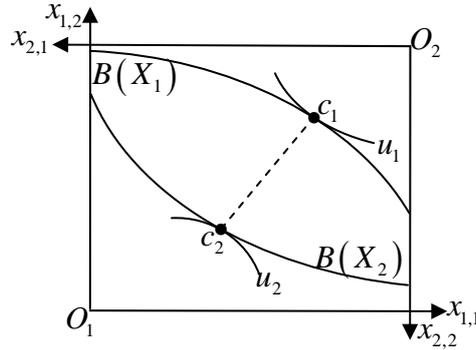
³ Baye, Kovenock and de Vries (1993).

⁴ Pérez-Castrillo and Verdier (1992).

2.1. The Jungle as a Decentralized Bankruptcy Problem

A bankruptcy problem is a pair, (w, \mathbf{c}) , where $\mathbf{c} = \{c_i\}_{i=1}^n$ is the claims' vector of n creditors, $0 < c_1 \leq \dots \leq c_n$ and $0 < w < \sum_{i=1}^n c_i$. An agent's claim in the jungle, c_i , is defined as $c_i = \arg \max_{x_i \in B(X_i)} u_i(x)$. Namely, agent i 's claim, c_i , is actually the solution of his utility maximization problem, subject to his consumption capacity constraint. A bankruptcy problem solution is a vector $\{y_i\}_{i=1}^n$ satisfying $\sum_{i=1}^n y_i = w$. Figure 1 describes the bankruptcy problem corresponding to a jungle with two agents and two commodities using the familiar Edgeworth box diagram. Agent i 's claim, c_i , is the optimal bundle in his consumption set. Namely, the tangent point between the agent's $B(X_i)$ and indifference curves. A solution is a point on the $c_1 c_2$ line.

Figure 1



Recall the two main differences between a regular bankruptcy problem and the jungle. First, a regular bankruptcy solution is based on a normative axiomatic approach and imposed by a benevolent central planner, while the jungle equilibrium is a self-imposed decentralized solution free of any norms or imperatives, except the basic law of the jungle: “might makes right.” Second, in a regular bankruptcy problem, the judicial validity of the claims is assumed undoubted⁵, while due to the absence of law, no agent in the jungle has a legal claim on a certain share of w . Nevertheless, it is worth

⁵ See for example O’Neill (1982) (1982), Aumann and Maschler (1985).

noting that the famous Contested Garment rule for bankruptcy problems is stated in the Mishna⁶ explicitly in the case where the claims validity is doubtful:

Two persons, who hold a garment, and each of them claims that he has found it, or that the whole belongs to him, (in such a case) each of them shall take an oath that no less than a half belongs to him, and then its value shall be divided. If, however, one claims the whole and the other half of it, then the oath for the first must be for no less than three quarters, and for the second no less than a quarter, and it is to be divided accordingly.

The two claimants should take an oath because their claims are not based on valid evidence. The solution principle of the Mishna (henceforth – the CG-principle) is obvious. First, each claimant takes what the other claimant concedes to him. Then, the disputed part of the contested garment is divided equally between the two claimants⁷.

Mathematically, the Mishna describes a two-creditor bankruptcy problem (w, \mathbf{c}) where $\mathbf{c} = \{c_1, c_2\}$ and $c_1 + c_2 > w$. By claiming c_i , claimant i concedes $\max(w - c_i, 0)$ for claimant j . Hence, the disputed part of the garment is,

$$(2) \quad w - \max(w - c_1, 0) - \max(w - c_2, 0)$$

The Mishna rules that each claimant receives the amount conceded to him by the other claimant, and half of the disputed amount. Namely,

$$(3) \quad y_i = \max(w - c_j, 0) + \frac{1}{2} [w - \max(w - c_i, 0) - \max(w - c_j, 0)], \quad i, j = 1, 2$$

Alternatively, using the identity $\frac{1}{2} [w - \max(w - c_i, 0)] = \frac{1}{2} \min(w, c_i)$, (3) can be rewritten as,

⁶ Baba-Metzia Ch. 1 §1.

⁷ The Babylonian Talmud explains that although only Sumchus holds that “money which is doubtful is to be divided without an oath” and the majority of the Sages hold that המוציא מחברו עליו הראיה (Lit. “It is always incumbent on the plaintiff to bring evidence,” Namely, possession is nine-tenths of the law), this Mishna can be reconciled with the ruling of Sumchus and the Sages. (see the Babylonian Talmud, tractate Baba-Metzia 2b)

$$(4) \quad y_i = \frac{1}{2} \left[\min(w, c_i) + \max(w - c_j, 0) \right], \quad i, j = 1, 2.$$

A n -creditor bankruptcy solution is *CG-consistent* if for all $i \neq j$ the division prescribed by the CG-principle for claims c_i and c_j is (y_i, y_j) . Namely, a solution is CG-consistent if any couple of creditors uses the contested garment rule to divide $y_i + y_j$ assigned to them by the solution.

Define a *rule* as a function that assigns a solution to each bankruptcy problem. A rule f is *self-consistent* if $f(w, \mathbf{c}) = y$ implies $f(y_S, \mathbf{c}_S) = y_S$ for each set $S \subseteq N$ of creditors, where \mathbf{c}_S is the claims' vector of S members and $y_S = \sum_{i \in S} y_i$. Namely, a rule is consistent if *any* subset $S \subseteq N$ of creditors apply f to divide y_S assigned to them when f is applied to the original bankruptcy problem.

3. The Stochastic Jungle Game

A bankruptcy problem is not a game, because it lacks a definition of agents' strategy spaces and payoffs. Associating the jungle decentralized equilibrium with a bankruptcy solution requires a definition of a corresponding jungle game. Actually, the jungle bankruptcy problem can be associated with either a non-cooperative or a cooperative game, as shown in this section. Firstly, I define the stochastic jungle corresponding to a 2-player contest. Secondly, I show that this contest equilibrium is also a subgame perfect equilibrium in a 2-stage jungle game. Lastly, in order to generalize the results to n -agent jungle, I define the corresponding jungle cooperative game and associate the subgame perfect jungle allocation with a cooperative solution concept.

3.1. The Jungle Contest

Any dispute in the jungle is settled either by confrontation or by agreement. Let us first analyze the confrontation equilibrium. Consider a set N of n contestants who are engaged in a Tullock (1980) type stochastic rent-seeking contest over a prize, for

which its value for agent $i \in N$ is denoted by c_i . Denote contestant i 's effort by e_i and assume that agent i 's winning probability is given by (1).

From agent i 's point of view, any confrontation (contest) has two contingent results: winning (state I) or losing (state II). If the contestant wins, his net payoff is $w_i^I = c_i - e_i$. If the contestant loses, his net payoff is $w_i^{II} = -e_i$. Each agent $i \in N$ seeks to maximize the following expected utility function,

$$(5) \quad Eu_i(w_i) = p_i(\mathbf{e})u_i(w_i^I) + (1 - p_i(\mathbf{e}))u_i(w_i^{II}).$$

Solving the contestants' optimization problem simultaneously yields the following first order conditions for an interior solution,

$$(6) \quad p_i' \Delta u_i - Eu_i' = 0, \quad \forall i \in N$$

where $p_i' = \partial p_i / \partial e_i$, $\Delta u_i = u_i(w_i^I) - u_i(w_i^{II})$ and $Eu_i' = p_i u_i'(w_i^I) + (1 - p_i) u_i'(w_i^{II})$.

With risk-neutral contestants, (6) is reduced to $p_i' c_i - 1 = 0$, $\forall i \in N$. Denote the equilibrium effort profile and equilibrium winning probabilities with risk-averse and risk-neutral contestants by \mathbf{e}^* , $\hat{\mathbf{e}}$, p_i^* and \hat{p}_i , respectively, and it follows that with risk-neutral agents, the equilibrium winning probability of agent $i \in N$ is

$\hat{p}_i = 1 - \sum_{i=1}^n \hat{e}_i / \alpha c_i$, implying that agent $i \in N$ is an active contestant if and only if

$c_i \geq \frac{1}{\alpha} \sum_{i=1}^n \hat{e}_i$. If the prize value is equal for all agents ($c_i = c$, $\forall i \in N$), then it can

easily be verified that $\hat{e}_i = (n - 1/n^2) \alpha c$ and $\hat{p}_i = 1/n$ $\forall i \in N$.

Assuming risk-aversion complicates the analysis significantly⁸. Generally, the effect of risk-aversion on expenditures is ambiguous although some progress has been

⁸ Probably, this is the main explanation for the prevalence of the apparently unrealistic risk-neutrality assumption in the rent-seeking literature. See Millner and Pratt (1991).

achieved⁹. Konrad and Schlesinger (1997) show that $e_i^* \gtrless \hat{e}_i \Leftrightarrow Eu_i'(\hat{\mathbf{e}}) \lesseqgtr \Delta u_i(\hat{\mathbf{e}})/c$.

Applying a second order Taylor expansion on this condition yields a simplified version of Cornes and Hartley's (2010) result: $e_i^* \gtrless \hat{e}_i \Leftrightarrow p_i^* \gtrless \frac{1}{2}, \forall i \in N$ ¹⁰. Denote by

$R_i = -u_i''/u_i'$ the Pratt (1964) absolute risk-aversion index, and it follows that if $\forall i \in N c_i = c$ and $R_i = R$ then $\mathbf{e}^* = \hat{\mathbf{e}}$ and $\mathbf{p}^* = \hat{\mathbf{p}} = \left\{\frac{1}{n}\right\}_{i=1}^n$.

Proposition 1: *In a 2-agent stochastic jungle contest, if both agents have an identical attitude towards risk ($R_i = R, i = 1, 2$), $p_i^* = \frac{1}{2}, i = 1, 2$ and the expected allocation of w is CG-consistent.*

Proof: Define $z_i = \min(w, c_i) - \max(w - c_j, 0), i = 1, 2$, as the difference between contestant i 's truncated claim and what is conceded for him by contestant j . Clearly, z_i is the real prize for winning the contest from contestant i 's point of view. There are four contingencies that should be checked separately¹¹, and it can be verified that always $z_i = z_j$. Hence, by the simplified version of version of Cornes and Hartley's (2010) result, $e_1^* = e_2^*$ implying that $p_1^* = p_2^* = \frac{1}{2}$. Thus the expected share of agent $i = 1, 2$ is

$$(7) \quad E(y_i) = \frac{1}{2} \left[\min(w, c_i) + \max(w - c_j, 0) \right], i, j = 1, 2$$

Comparing (7) with (4) completes the proof. ■

It should be emphasized that Proposition 1 relates to agents' *expected payoffs* only, not to agents' *expenditures*. The effect of risk-aversion on the rent-dissipation

⁹ Hillman and Katz (1984) show that for "small" prizes, risk-aversion reduces rent-dissipation. (See also Hillman and Samet (1987) (1987) and Nitzan (1994)). But as Konrad and Schlesinger (1997) indicated, generalizing this result is non-trivial. (See also Cornes and Hartley (2003)).

¹⁰ This is a "simplified" form of Cornes and Hartley's (2010) result because it is based on the Taylor expansion technique, and hence valid only for relatively "small" prize contests. Cornes and Hartley (2010) proved that the prudence condition (namely $u_i''' > 0 \forall i \in N$) is sufficient for this result to hold generally.

¹¹ (a) $c_1 > w$ and $c_2 > w$, (b) $c_1 > w$ and $c_2 < w$, (c) $c_1 < w$ and $c_2 > w$ and (d) $c_1 < w$ and $c_2 < w$.

rate in jungle rent-seeking contests is still unknown. Nevertheless, Proposition 1 states that even if risk-aversion affects expenditures, it has no effect on expected payoffs. The intuitive explanation of Proposition 1 is as follows. Suppose first that $c_i > w$, $i=1,2$. In this case, the ratio c_1/c_2 is irrelevant as both claimants compete over the same prize, w . Now, without loss of generality, assume $c_i < w$ and $c_j \geq w$. In this case, since claimant i concedes $w-c_i$ to claimant j , the real dispute is over c_i only, implying that, in this case too, the contestants compete over the same (truncated) rent. Therefore, although the effect of risk-aversion on expenditures is ambiguous, the agents' equilibrium winning probabilities still equal $\frac{1}{2}$, implying that each claimant is expected to receive half of the disputed part of w , which is exactly his share according to the CG-principle.

3.2. The Jungle Subgame-Perfect Equilibrium

Even in anarchistic environments, disputes can be settled by compromises and agreements. For example, when no party has adequate power for victory or if the costs associated with building superior power exceed the expected prize. Of course, with the absence of an enforcement authority, agreements in the jungle should be self-imposed.

Consider the following 2-stage 2-player jungle game. In the first stage, agents bargain over w , where each agent claims c_i . If the agents fail to reach an allocation agreement, they are engaged in a rent-seeking confrontation in the second stage.

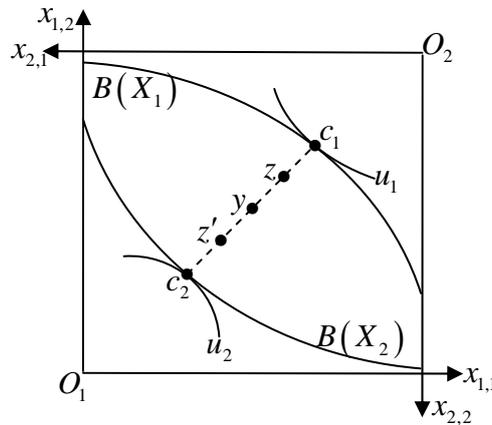
Proposition 2: *In a 2-agent stochastic jungle game, if both agents have an identical attitude towards risk, an allocation agreement is subgame perfect equilibrium if and only if it is CG-consistent.*

Proof: The first direction is straightforward. By Proposition 1 the expected share of each agent in a rent-seeking confrontation coincides with his share according to the

CG-principle. Thus, if w is divided according to the CG-principle by agreement, no agent is incentivized to deviate¹².

To see that a subgame perfect equilibrium allocation must be CG-consistent, let y be the CG-consistent allocation, and suppose that z is a non-CG-consistent subgame perfect equilibrium allocation. Namely, z represents an allocation agreement which is not CG-consistent. Suppose, without loss of generality that $z_1 > y_1$, implying by efficiency that $z_2 < y_2$ (see Figure 2). Clearly, agent 1 has no reason to deviate, contrary to agent 2, who is incentivized to take z_2 according to the agreement, and then deviate and confront agent 1 over $c_2 - z_2$ in the second stage. The expected payoff for each agent in this confrontation is $\frac{1}{2}(c_2 - z_2)$, represented by point z' in Figure 2, contrary to the assumption that z is a non-CG-consistent but subgame perfect equilibrium allocation¹³. ■

Figure 2



The crucial point in the proof of Proposition 2 is that in a jungle, there is no government which enforces agreements. In a civilized economy governed by the rule of law, agents can not deviate. However, recall that in a civilized economy, agents are not expected to sign “discriminating” agreements and bankruptcy problems are settled by an arbitrator who is expected to impose a “fair” sharing rule.

¹² Notice that this is true even for risk-neutral agents, since the agreement saves their confrontation efforts \hat{e} , and thus increases their welfare.

¹³ Notice that z' also cannot be a subgame perfect equilibrium allocation, since now agent 1 is incentivized to deviate and confront agent 2 over $c_1 - z'_1$. It follows that for any allocation $z \neq y$ one of the two agents is incentivized to deviate.

3.3. The Jungle Cooperative Game

A *transferable utility cooperative game* is a pair (N, v) where N is a finite set of agents and $v: 2^N \rightarrow \mathbb{R}$ is a function that associates a real number $v(S)$ with each subset (or *coalition*) $S \subseteq N$, where $v(\emptyset) = 0$. The function v is called the *coalitional, characteristic* or simply the *value* function of coalition S , and represents the payoff that coalition S can obtain collectively by itself, independent of actions taken by players outside S . A *solution concept* for a cooperative game is a mapping $\varphi: (N, v) \rightarrow \mathbb{R}^N$ that associates a set of payoff vectors with each cooperative game. The payoff of agent $i \in S$ according to solution concept φ is denoted by $\varphi_i(S)$. The set φ can contain several payoff vectors, be empty or contain a unique solution¹⁴. Usually, solution concepts are based on an axiomatic approach, implying that each solution concept satisfies a different set of normative axioms.

Aumann and Maschler (1985) already indicated that a bankruptcy problem is not a cooperative game, because coalition formation does not appear explicitly in its formulation and it lacks an explicit definition of a coalitional function. Generalizing Proposition 2 to n -agent jungle requires allowing coalition formation and a definition of the jungle coalitional function.

Apparently, the conventional definition of $v(S)$ is usually inappropriate for the competitive rent-seeking environment. To see this, consider the expected value of coalition $S \subset N$ in a competitive rent-seeking environment, $E[v(S)] = p_S c_S - e_S$ where $e_S = \sum_{i \in S} e_i^*$. For convenience, assume $c_i = c \forall i \in N$. In this case, $e_i^* = e^*$, $\forall i \in N$, implying that $E[v(S)] = \frac{1}{r} c_S - s e_i^*$, $\forall S \subseteq N$, where r denotes the total number of coalitions. Namely, the expected worth of a coalition $S \subset N$ depends on the coalitional size s but also on the total number of coalitions, r . In other words, the competitive environment creates externalities on the worth of coalitions, implying that the value of a coalition depends also on the coalitional structure and particularly

¹⁴ Solution concepts which associate a unique payoff vector with every cooperative game are the *value*, introduced by Shapley (1953), and the *nucleolus*, introduced by Schmeidler (1969).

on the partition of the complementary set, $N \setminus S$. These externalities undermine the traditional definition of coalitional function¹⁵.

Nevertheless, the externalities problem is irrelevant to the jungle environment. Aumann and Maschler (1985) suggested that each bankruptcy problem can be associated with a cooperative game by taking the value of a coalition S to be what it can obtain without confrontation; i.e. by accepting either nothing or what is left of w after each member i of the complementary set $N \setminus S$ receives his complete claim c_i . Namely, the *jungle cooperative game* corresponding to the (w, \mathbf{c}) bankruptcy problem, is defined by the following coalitional function,

$$(8) \quad v_{(w, \mathbf{c})}(S) = \max \left(w - \sum_{i \in N \setminus S} c_i, 0 \right).$$

Notice that the definition of (8) as the jungle coalitional function neutralizes the externalities problem¹⁶.

Proposition 3: *If all agents have an identical attitude towards risk, the subgame perfect allocation of the stochastic jungle is the nucleolus of the corresponding jungle cooperative game.*

Proof: By Theorem D in Aumann and Maschler (1985), the unique CG-consistent solution of a n -person bankruptcy problem is the nucleolus of the corresponding cooperative game. It is left to show that the nucleolus is the unique subgame perfect allocation of a n -agent stochastic jungle, but this is straightforward. Suppose that y is the nucleolus allocation, but is not a subgame perfect equilibrium. Thus, there is at least one agent $i \in N$ who is incentivized to deviate and confront either a single agent $j \in N$ or a coalition $S \subseteq N \setminus \{i\}$. By CG-consistency, $y_i + y_j$ (or $y_i + y_S$) is allocated between i and j (or between i and S) according to (4), implying by Proposition 2 that neither i nor j (or S) are incentivized to deviate. ■

¹⁵ For further discussions on cooperative games with externalities see Macho-Stadler, Pérez-Castrillo and Wettstein (2006), Tauman and Watanabe (2007), Chander and Tulkens (2006), Chander (2007), Chander and Wooders (2010), Chander (2010) and Schwarz (2011).

¹⁶ It can also be verified that (8) defines the jungle cooperative game as super-additive and 0-monotonic.

4. Public Policy and the Stochastic Jungle

“Public policy” and “jungle” together sound like an oxymoron. Nevertheless, public policy is a relevant variable within the stochastic jungle context. Viewing the stochastic jungle as a competitive environment in which the outcome of a confrontation between two agents is determined endogenously according to their relative efforts, implies that the stochastic jungle model is applicable to many political economy situations. Actually, rent-seeking contests, lobbying, sports tournaments, master artists competitions and certain governmental auctions are all examples of designed civilized stochastic jungles. An interesting case of a designed stochastic jungle contest appears in a very famous Talmudic topic which discusses a case when the court could not reach an evidence-based decision. In tractate Baba-Batra (34b) we read:

[If there are two claimants to a property¹⁷ and] one says, ‘It belonged to my father,’ while the other says, ‘To my father’ [without either of them bringing any evidence], R. Nahman says: *kol dealim gvar*. [Lit. might makes right].

According to R. Nahman’s ruling, when evidence is insufficient for any decision, the decision is determined by confrontation between the litigants. However, the Talmud emphasizes that a confrontation is a legal judicial decision device if and only if the following conditions are fulfilled:

- (a) No claimant can prove his claim.
- (b) The object is not possessed now by any of the claimants.
- (c) The object cannot belong to both of them.

The legal interpretation of *might makes right* is controversial among the medieval commentators of the Talmud. Some commentators hold that this is a metaphor, meaning that the claimant with the relatively stronger evidence wins, although his evidence is not strictly convincing¹⁸. However, most classical commentators and Halachic¹⁹ authorities interpreted R. Nahman’s ruling literally. Namely, when no

¹⁷ Whether landed property or other.

¹⁸ Rashbam on Baba-Batra (34a starting at Hatam). R. Moshe Sofer, *Innovations on Baba-Batra* (ibid.), Raban (ibid), Tosfot Rid (Baba-Metzia 2a), Ramban (ibid), Aliot D’Rabeinu Yona (Baba-Batra 35b), Rashba, responsa (atr.) ch. 138 and more.

¹⁹ Halacha is the collective corpus of Talmudic law. The term is derived from the Hebrew הליכה (Lit. a walk, or going, namely “the path to go”). Jewish law does not distinguish between religious and

claimant presents adequate evidence, *might makes right* and the winner takes possession²⁰. The dispute relates also to the Halachic status of might in cases of litigations without evidence. Some commentators hold that R. Nahman did not mean to rule that the stronger gets formal legal ownership of the object, but rather that the court disengages from the process, leaving the two rivals in their “natural” state. However, other important authorities²¹ hold that R. Nahman’s ruling means that if the above-mentioned three conditions are fulfilled, the stronger indeed obtains formal legal ownership of the object. The controversy also relates to the possibility of repeated confrontations. The authorities who hold that a confrontation does not award legal ownership usually permit repeated confrontations, while the authorities who hold that a confrontation awards the winner with a legal ownership emphasize that confrontation is allowed *only once*, although its outcome is conditional²². Namely, if the loser manages to collect evidence in the future supporting his claim, the file will be reopened by the court. The rationale of R. Nahman’s ruling is explained by R. Asher (“the Rosh”, 1250-1327)²³:

This ruling is a verdict. Namely, the one who wins this time gets the object until his rival brings evidence. But as long as the loser fails to present evidence the court will prevent him from retrying to acquire the object forcefully, because it is unlikely that the Sages meant that these two persons will spend their lives in quarrels and confrontations, this person wins today and his rival tomorrow. The Sages ruled *might one time makes right*, assuming that the true owner is more likely to obtain evidence and is also more likely to defend his property than his rival is willing to fight for a stolen object. In addition, the false claimant probably says, what is the use of fighting if later, today or tomorrow, he might present evidence and take it anyway.

In modern terminology, R. Asher assumes that the value of an object is higher to its true owner due to the *endowment effect*²⁴, and also because the false claimant bears a

non-religious spheres of life. Hence, Halachic corpora contain guidelines referring to religious practice as well as beliefs, national issues, commercial law, personal status, daily life and more.

²⁰ See Rosh on Baba-Batra Ch. 3 §22 and his response Ch. 87 §1. See also Rosh Baba-Metzia Ch. 1 §1. Maimonides, *Laws Pertaining to Disputes between Plaintiffs and Defendants*, Ch. 15 §4, Tur and Shulchan Aruch Hoshen Mishpat, Ch. 139 §1 (but see the Shach’s gloss there), and more.

²¹ i.e., Maimonides, R. Asher, the Tur and the Shulchan Aruch (ibid).

²² For a survey on repeated contests see Konrad (2010).

²³ Rosh on Baba-Batra, Ch. 3 §22. See also Rosh, Responsa, Ch. 77 §1.

²⁴ See, for instance, Thaler (1980), Kahneman, Knetsch and Thaler (1990) and Knetsch (1989).

risk that his fallacy may be discovered, implying that the return for his effort is uncertain. Thus, R. Asher assumes that in confrontation the true owner will exert stronger efforts, implying that his winning probability is higher. In other words, according to R. Asher, *might makes stochastic right*, since the outcome of a confrontation is uncertain, as winning probability is endogenously determined by relative efforts exerted by the parties.

The above analysis results imply that this rationale is, in fact, an application of the CG-principle for cases where decisions cannot be founded on evidence. The parties may choose a confrontation, or alternatively divide the contested object between them according to a certain sharing rule. Our results imply that the subgame perfect allocation of the contested object when *might makes stochastic right* coincides with the CG-principle. In case of n -person dispute, *might makes right* implies a subgame perfect allocation which coincides with the nucleolus.

It follows that the crucial point, according to Talmudic law, is not the *absolute* validity of a claim, but its *relative* validity. The classical study of Aumann and Maschler (1985) refers to the famous case of bankruptcy from the Mishna (Kethubot 10 §4) where the validity of all claims is beyond any doubt. On the other hand, as mentioned above, the CG-principle is stated in the Mishna in case where claims validity is equally doubtful and R. Nahman's ruling *might makes right* refers to a case where all claims are equally baseless. In the first two cases, the Mishna imposes either the CG-principle or the nucleolus, while in the third case, where no claimant has any legal basis for his claim, R. Nahman ruled *might makes right* which, according to the above analysis, implies that the unique subgame perfect equilibrium in this case is a CG-consistent compromise allocation²⁵.

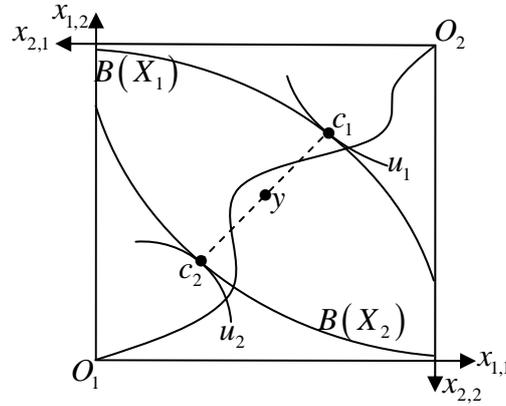
5. Welfare Economics of the Stochastic Jungle

Apparently, the nucleolus is an efficient solution as it allocates all available assets. Nevertheless, nothing in the above analysis indicates that the nucleolus is also Pareto-

²⁵ The Talmud (Baba-Metzia 3a) explains that in the case of the contested garment, condition (c) is not satisfied. Namely, it may be that both claimants found the garment simultaneously, implying that it really belongs to both of them. R. Nahman's rule is applied only in cases where obviously at least one claim is false, but there is no way to distinguish between a true claim and a false one.

efficient. For example, in Figure 3 although y is the CG-consistent allocation, it is not necessarily located on the contract curve O_1O_2 .

Figure 3



Nevertheless, the conclusion that a stochastic jungle is Pareto-inefficient is too hasty. A plausible interpretation of y is that it represents the exchange economy's initial endowment. From now on, agents can exchange commodities according to an agreed price ratio and reach a point on the contract curve. These transactions are protected by the fact that they stem from a subgame perfect initial allocation, thus no agent is incentivized to deviate from exchange to confrontation.

Viewing the stochastic jungle as the starting point of the exchange economy implies that contrary to Rubinstein's (in PR 2007) conclusion, both the deterministic jungle and the stochastic jungle cannot be considered as merely semantic exercises. Although the deterministic jungle equilibrium allocation is Pareto-efficient, as Piccione (in PR 2007) indicated, in the deterministic jungle, there is no room for exchange since the efficient allocation is determined by an exogenous power distribution. On the other hand, the above analysis suggests that even in cases where the stochastic jungle subgame perfect equilibrium is Pareto-inefficient, it is only the starting point; the initial endowment of the exchange economy. Hence, in spite of some similarities in the semantics and the mathematical model, jungle and exchange economics lie in separate spheres of economic thought.

6. Discussion

As indicated by Aumann and Maschler (1985), the nucleolus is an order preserving solution concept. When $w \leq \frac{1}{2}nc_1$ it assigns egalitarian allocation, when $w \geq \sum_{i \in N} c_i - \frac{1}{2}nc_1$ it assigns equal “concessions” (measured by $c_i - y_i$) and when $\frac{1}{2}nc_1 \leq w \leq \sum_{i \in N} c_i - \frac{1}{2}nc_1$, it applies the CG-principle between any pair of agents (or coalitions). The allocation process can be described as follows. The claimants are ordered according to their claims. The resources are distributed equally until all the resources are exhausted or until every claimant receives $\frac{1}{2}c_1$, when claimant 1 leaves the game. The distribution process continues until all the resources are exhausted, or until each claimant (except claimant 1) have $\frac{1}{2}c_2$, when claimant 2 leaves the game, and so on. It follows that usually claimants receive half of their claim at most. Thus, the nucleolus as the stochastic jungle subgame perfect equilibrium allocation illustrates the saying of R. Yudan in the name of R. Eivu (4th century): “No one leaves the world with even a half of his desires fulfilled. If a person has 100 he must have 200, if he has 200 he must have 400”²⁶. Viewing the satiation frontier as a measure of frugality and the consumption capacity as a measure of greed, as I have suggested above, implies that R. Yudan’s saying reflects a gloomy view of human greed, leading most people to achieve during their lifetime no more than half of their desires.

Nevertheless, there is also an optimistic point in the stochastic jungle. In his concluding comments, Piccione (in PR 2007) argues that “Insofar as the jungle precludes ‘redistribution’ of power, it also precludes redistribution of resources.” Actually, this claim is inaccurate, because even in the deterministic jungle, the equilibrium allocation reflects not only power distribution, but also the distribution of consumption capacities, and its influence on equilibrium allocation is substantially aggravated in the stochastic jungle. If education can “redistribute” consumption capacities by enhancing frugality, for example, redistribution of resources is not precluded in the stochastic jungle.

The above analysis implies that as agents’ greed is higher, equilibrium allocation tends to be more egalitarian. This result is compatible with the observation that resources distribution in prosperous economies is less egalitarian than in poor

²⁶ Midrash Kohelet Rabbah, ch. 1.

economies, since as the economy is richer, satiation frontiers become more binding. Politically, egalitarian allocation of resources differs substantially from egalitarian allocation of “concessions.” Unequal allocation of resources is obvious, but if agents’ consumption capacity is private information, the egalitarian allocation of concessions is hidden. Therefore, it is expected that rich, greedy societies are characterized with more envy, jealousy and social tension. This prediction is compatible with the observation that radical and revolutionary egalitarian movements evolved in 19th century relatively rich Europe and not in relatively poor Africa or Asia although, as indicated by Adam Smith (1776), “the poverty of the lower ranks of people in China far surpasses that of the most beggarly nations of Europe.”²⁷

In his concluding remarks, Rubinstein (in PR 2007) wrote:

Overall, the relative comparison of the jungle and the market mechanisms depends on our assessment of the characteristics with which agents enter the model. If the distribution of the initial holdings in the market reflects social values which we wish to promote, we might regard the market outcome as acceptable. However, if the initial wealth is allocated unfairly, dishonestly or arbitrarily, then we may not favour the market system. Similarly, if power is desirable we might accept the jungle system but if the distribution of power reflects brutal force which threatens our lives we would clearly not be in favour.

In the deterministic jungle, might indeed makes absolute right, and this natural state of the world may be perceived as immoral. In the stochastic jungle, however, might is endogenously determined according to agents’ effort profile, implying that might is a sort of market good whose price is determined by the market demand and supply forces. In other words, as suggested by R. Asher, the equilibrium effort vector can be interpreted as revealed preferences. Following this path of thought suggests that the demand for equal distribution of initial endowments and the grievances against non-egalitarian distribution may stem from envy and greed, motivated by a large consumption capacity as indicated by R. Y. M. Kagan²⁸ (1838-1933), who argued that envy is actually the consequence of greed. As an agent’s greed is higher, he becomes more jealous realizing that other people’s consumption crowds out his

²⁷ Wealth of Nations, Book I, Chapter VIII, p.86.

²⁸ Known by his nickname Hafetz Haim, drawn from the title of his most famous book.

own consumption opportunities. It follows that as greed and envy are lower, society is expected to be more tolerant towards inequality and non-egalitarian distributions²⁹.

Finally, it should be emphasized that, in my opinion, frugality is not contradictory to seeking development and growth. Actually, although there is no satiation assumption in standard models of growth, all classical models³⁰ predict that the economy will converge into a steady state. More recent studies³¹ of growth allow for permanent growth. An interesting direction for further research may be the combined effect of personal frugality and social desire for growth and development as a device for relaxing the bankruptcy characteristics of the stochastic jungle, on convergence to a steady state.

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²⁹ This conclusion is related, of course, to the formal definition of the nucleolus as lexicographic minimization of coalitional excesses.

³⁰ i.e. Solow (1956).

³¹ For a survey see for example Grossman and Helpman (1991).

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