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Minmax Shapley Value and Biform Contests Subgame Perfect Nash Equilibrium

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Mordechai E. Schwarz[1]

Abstract

A biform contest is a two-stage game. In the first stage, agents exert resources to build up their position towards the second stage. The second stage is either a bargaining game or a cooperative game, but if no cooperative solution is achieved it turns to a rent-seeking contest. It is shown that with a passive regulator, the second stage is a cooperative game and if all contestants are risk-averse, the biform contest non-cooperatively implements the *minmax Shapley value*, as defined in the paper, in subgame perfect equilibrium.

Keywords: biform game, biform contest, rent-seeking, externalities, minmax approach, minmax coalitional worth, minmax Shapley value.

JEL classification: C7, D7

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1. Introduction

Cooperation among rent-seekers is usually modeled as a two-stage game. There are at least three main categories of these models. One category which I call *wolf-pack games* studied a two-stage group rent-seeking without a specific sharing rule of the prize among the winning group members. Contestants cooperate within group rent-seeking against rival groups in the first stage, and then the individual members of the winning group engage in another rent-seeking contest for the prize in the second stage[2]. A second category, which I call *a lion-pack game,* assumes that the prize is distributed in the second stage among the winning group members according to a pre-known specific sharing rule. (In this category, I also include models in which the prize is a club good for the winning group)[3]. The third category contains models of *compromise in the shadow of conflict*, which study the endogenous choice of the sharing rule[4]. In these games, agents bargain on the allocation of the rent in the first stage, and only if bargaining fails are they engaged in a rent-seeking conflict in the second stage[5]. An evolving subclass of this category includes models in which the sharing rules are determined endogenously and specifically for each group in the first stage, and in the second stage all individuals cooperate within the group rent-seeking contest and determine their contribution level[6].

These three categories of two-stage games of cooperation in contests are compatible, each with certain modifications, with Brandenburger and Stuart's (2007) *biform game* framework. A *biform (or hybrid)* n*-player game* is a two-stage game. The first stage is non-cooperative and is designed to describe the strategic moves of the players, but the consequences of these moves are not payoffs. Instead, each profile of strategic choices at the first stage leads to a second stage cooperative game. Similarly, I define a *biform contest* as a two-stage game. In

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[2] The main result of *wolf-pack game* models is that the collective action problem within groups usually reduces total contest expenditures (Baik (1993), Katz and Tokatlidu (1996), Wärneryd (1998), Inderst, Müller and Wärneryd (2007)). But this result holds in stochastic contests in which the winning probability is a continuous function of contestants' outlays. In deterministic contests, in which the contestant who exerts the highest effort wins with certainty, this result may be reversed (Konrad (2004)). For an analysis of a deterministic contest, see Ellingsen (1991).

[3] The main result of *lion-pack game* models is that the pre-known sharing rule neutralizes agents' risk-aversion. (Nitzan (1991), Wärneryd (1998) and Skaperdas and Gan (1995)).

[4] For studies of Compromise in the shadow of conflict, see, for example, McDonald and Solow (1981), Svejnar (1986), Alexander (1992), Skaperdas (1992) and Skaperdas and Gan (1995), Esteban and Sákovics (2002), Bayindir-Upmann and Gerber (2003) and Skaperdas (2006).

[5] The studies mentioned in previous footnotes assumed arbitrary partition of the contestants' set into groups. For analyses of endogenous formation of alliances, see, for example, Hart (1974), Aumann and Meyerson (1988), Katz (1987) and Linster (1994), Huck and Konrad (2002), Lee and Cheong (2005), Bloch, Sánchez-Pagés and Soubeyran (2006), Sánchez-Pagés (2007) and Glazer (2008).

[6] See, for example, Nitzan and Ueda (2011) and references there.

the first stage, agents exert resources to build their position towards the second stage. The second stage is either a bargaining game or a cooperative game. If the bargain fails or if no cooperative solution is achieved, the second stage turns to a rent-seeking contest. The resources exerted in the first stage are aimed at being in an optimal position in the second stage under every contingency.

Although biform contests as a stylized model of cooperation are related to numerous real world situations, (like plea-bargaining, arms races and peace negotiations, strikes and wage negotiations, allocation of cartel profits and more), this approach was rarely applied for analyzing these situations, probably because compromise in the shadow of conflict is known as a very complicated situation for modeling, and comparative statics were considered impossible[7].

In Schwarz (2011a) and (2011b), I analyzed the biform contest model with a bargaining game in its second stage. A biform contest in which the second stage is a cooperative game deserves a separate analysis since, in addition to the well-known difficulties mentioned in the literature, the definition of the second stage as a cooperative game raises another difficulty, due to externalities which characterize the rent-seeking competitive environment. As explained below, these externalities undermine the traditional definition of the coalitional worth function and the application of any cooperative solution concept.

To overcome the externalities problem, I follow Tauman and Watanabe's (2007) *minmax approach* and define the *minmax value* of a coalition, and the *minmax Shapley value*, as explained below. The main result of the analysis is that this type of biform contest non-cooperatively implements the minmax Shapley value in subgame perfect equilibrium, provided that all contestants are risk-averse.

This introduction is followed by four sections. Section 2 presents the benchmark one-shot rent-seeking contest. Section 3 sets up the biform contest model, discusses the externalities problem of competitive environments, presents the *minmax Shapley value* approach, defines the biform contest cooperative solution concept and briefly refers to the role of social norms and regulatory policies. Section 0 presents the main results of this article. Section 0

[7] See McDonald and Solow (1981), Alexander (1992), Anbarci, Skaperdas and Syropoulos (2002), Bayindir-Upmann and Gerber (2003) and Skaperdas (2006).

summarizes, discusses the implication of the results of the analysis and suggests some directions for further research.

2. The Benchmark One-Shot Rent-Seeking Contest

Consider a set N of n contestants who are engaged in a Tullock (1980) type stochastic rent-seeking contest over a prize commonly valued by z. Denote contestant i's expenditures by x_i and assume that agent i's winning probability is given by the Tullock's (1980) Contest Success Function[8],

(1) $$p_i(\mathbf{x}) = \frac{x_i^{\alpha}}{\sum_{j\in N} x_j^{\alpha}}, \quad \forall i, j \in N, \quad \mathbf{x} = \{x_i\}_{i=1}^{n}$$

The parameter α measures the return to scale of the rent-seeking efforts. If $\alpha \to 0$, $p_i = 1/n, \ \forall i \in N$. On the other hand, if $\alpha \to \infty$, the rent-seeking contest becomes an all-pay-auction under which the prize is awarded to the contestant who exerts the greatest effort[9]. I limit the analysis to games with symmetric pure-strategy Nash equilibria, and in order to assure a unique interior equilibrium I assume $0 < \alpha < n/n-1$[10].

The preferences of agent $i \in N$ are represented by von-Neumann-Morgenstern utility functions $u_i(w_i)$ satisfying $u_i' > 0, \ u_i'' < 0$. From contestant i's point of view, the contest has two contingent results: winning (state *I*) or losing (state *II*). Denote contestant i's initial wealth by A_i and his post-contest wealth by w_i. If the contestant wins, $w_i^I = A_i + z - x_i$. If the contestant loses, $w_i^{II} = A_i - x_i$. Each agent $i \in N$ seeks to maximize the following expected utility function,

(2) $$Eu_i(w_i) = p_i(\mathbf{x})u_i(w_i^I) + (1 - p_i(\mathbf{x}))u_i(w_i^{II}).$$

Solving the contestants' optimization problem simultaneously yields the following first order conditions for an interior solution,

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8 For axiomatization of this function, see Skaperdas (1996). See also Fullerton and McAfee (1999).
9 Baye, Kovenock and de Vries, (1993).
10 Pérez-Castrillo and Verdier, (1992).

(3) $$p_i' \Delta u_i - Eu_i' = 0, \ \ \forall i \in N .$$

Where $p_i' = \partial p_i / \partial x_i$, $\Delta u_i = u_i\left(w_i^I\right) - u_i\left(w_i^{II}\right)$ and $Eu_i' = p_i u_i'\left(w_i^I\right) + \left(1 - p_i\right) u_i'\left(w_i^{II}\right)$. With risk-neutral contestants, (3) is reduced to $p_i' z - 1 = 0, \ \ \forall i \in N$. Denote equilibrium expenditures profile with risk-averse and risk-neutral contestants by $\mathbf{x}^*$ and $\hat{\mathbf{x}}$, respectively. It can easily be shown that,

(4) $$\hat{x}_i = \left(\frac{n-1}{n^2}\right)\alpha z, \ \forall i \in N .$$

Unfortunately, the effect of risk-aversion on expenditures is generally ambiguous, although some progress has been achieved[11]. On the other hand, assuming risk-aversion complicates that analysis significantly[12]. Konrad and Schlesinger (1997) show that $x_i^* \gtreqless \hat{x}_i \Leftrightarrow Eu_i'\left(\hat{\mathbf{x}}\right) \lesseqgtr \Delta u_i\left(\hat{\mathbf{x}}\right)/z$, and applying a second order Taylor expansion on this condition yields a simplified version of Cornes and Hartley's (2010) result: $x_i^* \gtreqless \hat{x}_i \Leftrightarrow p_i^* \gtreqless \tfrac{1}{2}, \ \forall i \in N$ [13]. Denote by $R_i = -u_i'' / u_i'$ the Pratt (1964) absolute risk-aversion index, and it follows that if $\forall i \in N \ z_i = z$ and $R_i = R$ then $\mathbf{x}^* = \hat{\mathbf{x}}$ and $\mathbf{p}^* = \hat{\mathbf{p}} = \left\{\tfrac{1}{n}\right\}_{i=1}^n$. In Figure 1, Konrad and Schlesinger's condition states that $x_i^* \gtreqless \hat{x}_i \Leftrightarrow \tan\left(a_1\right) \lesseqgtr \tan\left(a_2\right)$, and by the simplified version of Cornes and Hartley's result this condition is equivalent to $x_i^* \gtreqless \hat{x}_i \Leftrightarrow \hat{\hat{w}}_i \lesseqgtr \tfrac{1}{2}\left(\hat{w}_i^I - \hat{w}_i^{II}\right) \Leftrightarrow p_i^* \gtreqless \tfrac{1}{2}$.

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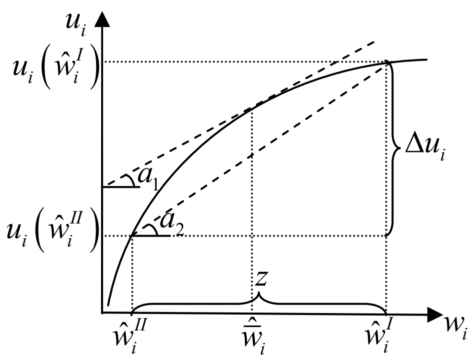
11 Hillman and Katz (1984) show that for "small" prizes, risk-aversion reduces rent-dissipation. (See also Hillman and Samet (1987) and Nitzan (1994)). But as Konrad and Schlesinger (1997) indicated, generalizing this result is non-trivial. (See also Cornes and Hartley, (2003)).

12 Probably, this is the main explanation for the prevalence of the apparently unrealistic risk-neutrality assumption in the rent-seeking literature. See Millner and Pratt (1991).

13 This is a "simplified" form of Cornes and Hartley's (2010) result because it is based on the Taylor expansion technique, and hence valid only for relatively "small" prize contests. Cornes and Hartley (2010) proved that the prudence condition (namely $u_i''' > 0 \ \forall i \in N$) is sufficient for this result to hold generally.

Figure 1



3. The Biform Contest

Brandenburger and Stuart (2007) described a *biform* n*-player game* (or *hybrid* game) as a two-stage game. The first stage is non-cooperative and is designed to describe the strategic moves of the players. However, the consequences of these moves are not payoffs. Instead, each profile of strategic choices at the first stage leads to a second stage cooperative game. This lends the competitive environment created by the choices that the players made in the first stage. Formally, a biform game is a collection (Σ, V, C), where $\Sigma = \sigma^1 \times \ldots \times \sigma^n$ is a profile of strategies, $V : C \times 2^n \rightarrow \mathbb{R}$ is a cooperative game coalitional function and $\mathbf{c} = \{c_i\}_{i=1}^n \in C$ is a vector of "confidence indices" for each player $(c_i \in [0,1])$[14]. Roughly speaking, the confidence indices evaluate every contingent outcome in the core according to player i's preferences[15].

Similarly, I define a *biform contest* as a two-stage game. In the first stage, agents exert resources to optimize their position towards the second stage, namely, to be well-equipped and prepared in case the bargaining fails, or in case where no cooperative solution is achieved and the rent-seeking contest is to be played. Formally, a biform contest is a collection $(\mathbf{x}, p, \mathbf{u}, v)$ where $\mathbf{x} = \{x_i\}_{i=1}^n$ is the contestants rent-seeking expenditures vector, p is the contest success function, $\mathbf{u}$ is a vector of contestants' utilities and v is a TU cooperative game characteristic function.

[14] For detailed discussion of biform games, see also Muto, Nakayama, Potters and Tijs (1998) and Tijs (1990). For a more accurate definition and explanation of confidence index, see Brandenburger and Stuart (2007) or Stuart (2005).

[15] For studies that applied a specific cooperative solution in biform games see, for instance, Grossman and Hart (1986), who applied the Shapley value; Hart and Moore (1990), who applied the Nash Bargaining Solution and Brandenburger and Stuart (2007), who applied the core.

A *coalition* $S \subseteq N$ is a subset of agents colluding in rent-seeking against the complementary set $F = N \setminus S$. Denote by $\mathcal{P}(N)$ the collection of all subsets of N and by $s = \# S$ the number of elements in S, (so $1 \le s \le n$ where $n = \# N$ is the total number of contestants). Let $v : \mathcal{P}(N) \to \mathbb{R}$ be a TU cooperative game characteristic function which assigns a *value*, namely a real number $v(S)$ for every coalition $S \subseteq N$, satisfying $v(\varnothing) = 0$. The payoff of agent $i \in S$ is denoted by $v_i(S)$. A solution concept is an imputation $\varphi(v) \in \mathbb{R}^N$ satisfying certain normative axioms like efficiency and individual rationality.

Shapley (1953) suggested five normative axioms for a cooperative game solution: fairness, efficiency, symmetry, additivity and the null player axiom[16], and proved that the unique solution which satisfies these axioms assigns each agent $i \in N$ a *value* according to the following formula,

(5) $$sh_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \left[v(S \cup \{i\}) - v(S) \right].$$

$v(S) - v(S \setminus \{i\})$ is the marginal contribution of agent i to the value of coalition S. So the Shapley value of agent $i \in N$ is his average marginal contributions to each coalition, assuming that every order of agents is equally likely.

Externalities

The conventional definition of $v(S)$ implicitly assumes that the coalitional worth is independent of what the players outside of S do. This assumption, however, does not hold for a competitive environment, implying that the conventional definition of $v(S)$ is inappropriate for biform contests. To see this, note that the expected worth of coalition $S \subset N$ is

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16 Formally:
Fairness, $\varphi_i(v) \ge v(\{i\})$, $\forall i \in N$;
Efficiency, $\sum_{i \in N} \varphi_i(v) = v(N)$;
Symmetry, $v(S \cup \{i\}) = v(S \cup \{j\}) \Rightarrow \varphi_i(v) = \varphi_j(v)$, $\forall i, j \notin S$;
Additivity, $\varphi_i(v+w) = \varphi_i(v) + \varphi_i(w)$, $\forall i \in N$;
The null player axiom, $v(S \cup \{i\}) = v(S) \Rightarrow \varphi_i(v) = 0$, $\forall i \in S$.

(6) $$E\left[v(S)\right]=p_S z-x_S .$$

Apparently, (6) characterizes a risk-neutral contestant. Suppose that the coalitional effort, x_S, is the sum of voluntary contributions of the coalition's members, $x_S=\sum_{i\in S}x_i^*$, where x_i^* is the first stage individual equilibrium expenditure. Plugging x_S into (6) yields

(7) $$E\left[v(S)\right]=\frac{\left(\sum_{i\in S}x_i^*\right)^{\alpha}}{\sum_{S\subseteq N}\left(\sum_{i\in S}x_i^*\right)^{\alpha}}z-\sum_{i\in S}x_i^*,\ \forall S\subseteq N .$$

In symmetric equilibrium, $x_i^*=x^*$, $\forall i\in N$, implying that

(8) $$E\left[v(S)\right]=\frac{1}{k}z-sx_i^*,\ \forall S\subseteq N ,$$

where k denotes the total number of coalitions. Namely, the expected worth of a coalition $S\subset N$ depends on the total number of coalitions, k, and on the coalition's size, s. In other words, the competitive environment creates externalities on the worth of coalitions as the expected worth of a coalition $S\subset\mathcal{P}(N)$ depends on the partition of the complementary set, $\mathcal{P}(F)$. Therefore, the worth function in a competitive environment is actually $v(S,\mathcal{P}):\mathcal{P}(N)\times\mathcal{P}(F)\to\mathbb{R}$. Apparently, these externalities undermine the traditional definition of coalitional worth and complicate the application of any cooperative solution concept.

To overcome this difficulty, I apply Tauman and Watanabe's (2007) *minmax approach*[17]. The coalitional *minmax worth*, denoted by $mv(S)$, is defined as the coalitional expected worth under the worse scenario from the coalition's point of view, which occurs when F contains

[17] An alternative *average value* approach was suggested by Macho-Stadler, Pérez-Castrillo and Wettstein (2006). For further discussions of externalities in coalition formation, see, for example, Chander and Tulkens (2006), Chander (2007), Chander and Wooders (2010) and Chander (2010).

$n-s$ singleton coalitions and the total number of contestants is $k=n-s+1$. It follows from (8) that

$$\text{(9)} \qquad mv(S)=\min_{k}\max_{x(S,k)}\left\{\frac{1}{k}z-sx_i^*\right\}.$$

The application of Shapley value to our competitive environment requires some modification to overcome the externalities problem. Firstly, define the cooperative game mv as the *minmax corresponding game* of v. The *minmax Shapley value*, denoted by $sh(mv)$ is the Shapley value of minmax corresponding game mv. Agent i's minmax Shapley value is denoted by $sh_i(mv)$.

The Biform Contest Cooperative Solution Concept

A *sharing rule* $\boldsymbol{\beta}=\{\beta_i\}_{i=1}^n$, $\beta_i\in[0,1]$ assigns each contestant $i\in N$ a share $\beta_i z$ of the contest prize. If $\boldsymbol{\beta}$ is applied in the second stage, agent i's utility is $u_i\left(A_i+\beta_i z-x_i^*\right)$. If no sharing rule is applied in the second stage, or when no cooperative solution is achieved, all contestants are engaged in a rent-seeking contest and the expected utility of agent $i\in N$ is $Eu_i(w_i)=p_i^*(\mathbf{x}^*)u_i(w_i^I)+(1-p_i^*(\mathbf{x}^*))u_i(w_i^{II})$. Define agent i's *gain function* as

$$\text{(10)} \qquad g_i(\boldsymbol{\beta},\mathbf{x}^*)=u_i\left(A_i+\beta_i z-x_i^*\right)-Eu_i(w_i).$$

The gain function measures agent i's gain from applying the sharing rule $\boldsymbol{\beta}$, over competing in the rent-seeking contest. A sharing rule $\boldsymbol{\beta}$ is *rational* if $g_i(\boldsymbol{\beta},\mathbf{x}^*)\geq 0,\ \forall i\in N$. The set of all rational sharing rules is defined as $\mathrm{B}=\left\{\boldsymbol{\beta}\middle| g_i(\boldsymbol{\beta},\mathbf{x}^*)\geq 0,\ \forall i\in N\right\}$. It can be verified that $g_i(\boldsymbol{\beta},\mathbf{x}^*)$ is bounded and concave, implying that B is bounded, compact and convex[18]. Define the set of all feasible utility vectors $U(\boldsymbol{\beta})=\left\{\mathbf{u}(\boldsymbol{\beta})\middle|\sum_{i=1}^n\beta_i\leq 1\right\}$, and by the same logic, it can be also shown that $U(\boldsymbol{\beta})$ is also bounded, compact and convex, implying that its

[18] See Schwarz (2011b).

efficient frontier, $P(U)=\left\{\mathbf{u}(\boldsymbol{\beta})\middle|\boldsymbol{\beta}\in \mathrm{B},\ \sum_{i=1}^{n}\beta_i=1\right\}$, which is the set of all utility vectors induced by rational sharing rules, is concave. A biform contest cooperative solution concept is a feasible and rational sharing rule $\mathbf{u}(\boldsymbol{\beta})\in P(U)$.

Social Norms and Regulatory Policies

Social norms relate to "fair" solutions and sharing rules. For example, a *socialist* norm postulates that effort should be compensated and that compensation should take into account the individual's effort but also his relative position in the total societal effort. Thus, a socialist sharing rule determines the agent's share according to a monotonically increasing function $\beta_i=\beta_i(\mathbf{x})$. On the other hand, a *competitive* norm postulates that every sharing rule, $\boldsymbol{\beta}$, is legitimate conditional on being freely and unanimously approved in a proper and fair process[19].

Regulatory policies express the regulator's social philosophy. A *passive* regulator believes in "laissez-faire" and does not intervene at all in the second stage. An *active regulator* is concerned about "procedural justice," but indifferent to "distributive justice." Thus, an active regulator determines the rules of the game (the bargaining protocol), without any attempt to implement a specific sharing rule. Under an active regulator, any feasible sharing rule $\boldsymbol{\beta}\in P(U)$ is allowed. On the other hand, an *active benevolent regulator* also takes "fair division" or "distributive justice" considerations into account and seeks to implement a sharing rule which maximizes a certain social welfare function $W(\mathbf{u}(\boldsymbol{\beta}))$ where $W:\mathbb{R}^n\to\mathbb{R}$ is continuous, monotone and quasi-concave. In other words, an active benevolent regulator seeks to implement $\boldsymbol{\beta}^W=\arg\max_{\boldsymbol{\beta}\in P(U)} W(\mathbf{u}(\boldsymbol{\beta}))$[20].

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[19] In Schwarz (2011b) , I showed that socialist norms neutralize contestants' risk-aversion, implying that in a socialist society, contestants' expenditures in subgame perfect equilibrium are proportional to risk-neutral agents' expenditures in the benchmark one-shot contest. On the other hand, in a competitive society, contestants' subgame perfect expenditures are unambiguously higher than in the benchmark case. The intuition of this result is that socialist norms reduce uncertainty, contrary to competitive norms which do not require any a-priori functional relation between expenditures and share, and consequently enhance uncertainty.

[20] For instance, a Benthamite regulator seeks to implement $\boldsymbol{\beta}^W=\arg\max_{\boldsymbol{\beta}\in P(U)}\sum_{i\in N}u_i(\boldsymbol{\beta})$. Miyagawa (2002) claimed that a Rawlsian regulator seeks to implement the Kalai-Smorodinsky (1975) bargaining solution. This claim is inaccurate. A Rawlsian regulator seeks to implement

Equilibrium

A biform contest subgame perfect Nash equilibrium is a pair $\left(\boldsymbol{\beta}^*, \mathbf{x}^*\right)$ where $\mathbf{u}\left(\boldsymbol{\beta}^*\right) \in P(U)$. Namely, $\left(\boldsymbol{\beta}^*, \mathbf{x}^*\right)$ is a biform contest subgame perfect Nash equilibrium, if applying $\boldsymbol{\beta}^*$ as the second stage sharing rule results in $\mathbf{x}^*$ as equilibrium expenditure profile, and if $\mathbf{x}^*$ is the first stage expenditures profile and $\boldsymbol{\beta}^*$ is applied in the second stage, no contestant can benefit from deviation.

The results in Schwarz (2011a) and Schwarz (2011b) were based on the assumption of an active regulator who imposes in the second stage a modified version of Moulin's (1984) bargaining protocol. In this article, I assume a passive regulator, implying that no bargaining protocol is imposed in the second stage. Thus, in the second stage, agents can either form coalitions and play a cooperative game, or compete in a rent-seeking contest. The following proposition provides a necessary and sufficient condition for a subgame perfect cooperative solution.

Proposition: *A biform contest with a passive regulator non-cooperatively implements the Minmax Shapley value in subgame perfect equilibrium, if and only if* $R_i \geq 0, \ \forall i \in N$.

Proof: Denote first stage expenditures profile by $\mathbf{x}^*$. In the second stage, agents' expenditures are sunk-costs and each contestant $i \in N$ is characterized by his expenditure level, x_i^*, and his corresponding winning probability, p_i^*. Hence,

$$
\begin{aligned}
mv\left(\{i\}\right) &= \min_k \max_{x(S,k)} \left\{ p_i^*\left(\mathbf{x}^*\right) z - x_i^* \right\} \\
&= \min_k \max_{x(S,k)} \left\{ \frac{x_i^{*\alpha}}{\sum_{S \subseteq N} \sum_{i \in S} x_i^{*\alpha}} z - x_i^* \right\} = \frac{1}{n} z - x_i^*, \ \forall i \in N.
\end{aligned} \tag{11}
$$

$\boldsymbol{\beta}^W = \arg\max_{\boldsymbol{\beta} \in P(U)} \left[\min \mathbf{u}\left(\boldsymbol{\beta}\right) \right]$, which is equivalent to implementation of the Kalai-Smorodinsky bargaining solution if and only if $\forall i \in N \ u_i = u$ and $z_i = z$.

Similarly plugging $x_S = \sum_{i \in S} x_i^*$ into (9) yields,

$$mv(S) = \min_k \max_{x(S,k)} \left\{ \frac{1}{k} z - s x_i^* \right\}, \ \forall S \subseteq N . \tag{12}$$

With k coalitions, the average size of a coalition is $\overline{s} = n/k$. Inserting $\overline{s}$ into (12) yields the *average minmax value* (amv) of coalition $S \subseteq N$,

$$amv(S,k) = \frac{1}{k}\left(z - n x_i^*\right). \tag{13}$$

Now if agent $\{i\} \in F$ $(F \cap S = \varnothing)$ joins S, the total number of coalitions is reduced to $k-1$, implying that the *average marginal contribution* of agent $\{i\} \in F$ to the average minmax value of S, denoted by $amc_i(S,k)$, is

$$amc_i(S,k) = av(S,k-1) - av(S,k) = \frac{1}{k(k-1)}\left(z - n x_i^*\right). \tag{14}$$

On the other hand, if agent $i \in T$ leaves coalition T and join coalition S, the total number of coalitions is unchanged, implying that agent i's average marginal contribution to coalition S is non-zero if and only if by joining (leaving) S agent i decreases (increases) the total number of coalitions. Since by definition, the minmax Shapley value of agent $i \in N$ is his average marginal contribution to the minmax value of all coalitions, it follows that

$$\begin{aligned} sh_i(mv) &= \frac{1}{n-1}\sum_{k=2}^{n} amc_i(S,k) \\ &= \frac{1}{n-1}\sum_{k=2}^{n} \frac{1}{k(k-1)}\left(z - n x_i^*\right) = \frac{1}{n} z - x_i^* = mv(\{i\}), \ \forall i \in N \end{aligned} \tag{15}$$

Inserting (15) into (10) yields

$$g_i\left(sh(mv),\mathbf{x}^*\right)=u_i\left(A_i+\frac{1}{n}z-x_i^*\right)-\left[\frac{1}{n}u_i\left(A_i+z-x_i^*\right)+\left(\frac{n-1}{n}\right)u_i\left(A_i-x_i^*\right)\right] \tag{16}$$

Taking the second order Taylor expansion of (16) and rearranging yields

$$g_i\left(sh(mv),\mathbf{x}_i^*\right)=\frac{1}{2}\frac{R_i(n-1)z^2}{n^2}+\mathcal{R} \tag{17}$$

where $\mathcal{R}$ denotes the remainder of the Taylor series (assumed negligible). It follows immediately from (17) that $g_i\left(sh(mv),\mathbf{x}^*\right)\gtreqless 0 \Leftrightarrow R_i \gtreqless 0,\ \forall i\in N$.

Notice that by (17) $\partial g_i/\partial n \lesseqgtr 0 \Leftrightarrow R_i \gtreqless 0$. Inserting $\hat{x}_i$ from (4) into (17) yields $g_i\left(sh(mv),\mathbf{x}_i^*\right)=\mu_i\hat{x}_i$ where $\mu_i=R_iz/2\alpha$, implying that $\partial g_i/\partial\alpha \lesseqgtr 0 \Leftrightarrow R_i \gtreqless 0$. Namely, risk-averse (seeker) agent's gain (loss) from cooperation is proportional to risk-neutral agent's rent-seeking expenditure, and is negatively (positively) affected by the number of contestants and the return to scale rate to rent-seeking expenditures.

Summary and Discussion

A *biform n-player contest* is a two-stage game. In the first stage, agents exert resources to build up their position towards the second stage. The second stage is either a bargaining game or a cooperative game. If no cooperative solution is achieved in the second stage, contestants are engaged in a rent-seeking contest. The nature of the second stage depends on the regulator's type. An active regulator imposes a bargaining protocol, implying that the second stage is a bargaining game. This article was confined to the analysis of a biform contest with a passive regulator, namely a biform contest in which the second stage is a TU cooperative game. To overcome the externalities problem associated with the rent-seeking competitive environment, I applied Tauman and Watanabe (2007) *minmax approach* and defined the coalition's *minmax worth function* as its worth under the worse scenario from the coalition's point of view. The Minmax Corresponding Game is a cooperative game in which the coalitional worth function is replaced by the *minmax* coalitional worth function. An agent's *minmax Shapley value* is defined as his Shapley value in the Minmax Corresponding Game.

The main result of the above analysis is that if all contestants are risk-averse, the biform contest with passive regulator non-cooperatively implements the minmax Shapley value in subgame perfect equilibrium. It was also shown that the risk-averse agent's gain from cooperation is proportional to the risk-neutral agent's rent-seeking expenditure, and is negatively correlated with the number of contestants and the return to scale rate to rent-seeking expenditures.

The interrelations between cooperative and non-cooperative game solution concepts are well known since the "equivalence theorems" were proven in the pioneering articles of Debreu and Scarf (1963) and Aumann (1975), who formulated the Core Equivalence Theorem. This theorem states that in perfectly competitive economies, the core coincides with the set of competitive allocations. Later works[21] produced the *Value Principle: "In perfectly competitive economies, every Shapley value allocation is competitive, and the converse holds in the differentiable case."*[22] However, two important features differentiate our results from these theorems. First, equivalence theorems relate to "non-atomic" competitive environments, namely environments with infinite number of contestants[23], whereas our results relate "atomic" environments in which the contestants' number is finite[24]. Secondly and maybe more importantly, in our setting, the competitive equilibrium is not characterized by a price vector or a budget set. Nevertheless, these theorems underlie the intuition behind our results, pointing out that the interrelations between competitive and non-competitive solution concepts also hold for rent-seeking competitive environments, with or without externalities.

In a seminal article, Stigler (1964) conjectured that although collusion may maximize aggregate profits of all firms in an oligopolistic industry, cartels tend to be unstable because each member firm is incentivized to cheat. This logic might raise some questions about the necessity for anti-trust legislation and expensive regulatory agencies, but Levenstein and Suslow's (2006) survey of the empirical literature has abundant examples of successful cartels

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21 Shapley (1964), Shapley and Shubik (1969), Aumann and Shapley (1974), Hart (1979) to mention only a few.

22 See Hart (2001) and also Anderson (1992).

23 See also Aumann (1964).

24 For a discussion of the Core Equivalence Theorem within an "atomic" environment context, see Anderson (1992).

that remained stable for significant periods of time. The authors also indicate that: (a) Cheating is absolutely not the main cause of the collapse of cartels but what they call "bargaining problems," and (b) Cartels do not rely on punishment mechanisms (like "price wars"), but on administrative organs like sales agencies or even an external regulatory or inspective agency. These empirical facts are incompatible with classical cartel theory.

Clearly, with seriously enforced anti-trust legislation, sales agencies or other cartel managing organizations cannot operate in the open. But the naïve Pigouvian view of a benevolent regulator has been challenged by economists who recognized that politicians and bureaucrats have their own interests and preferences that do not always coincide with social welfare maximization. In a later article, Stigler (1971) himself postulated that regulation is merely a device used by the regulated industry to control entry and maintain its cartelistic arrangements. Moreover, Stigler claimed that, in many cases, oligopolistic industries seek a regulatory umbrella and "capture" the regulator to obtain exemptions from legal restrictions. Appelbaum and Katz (1987) also indicated that *"it is clearly preferable for sophisticated firms not to engage in individual rent-seeking and furthermore, form a coalition to face the regulator and try to affect his policies. The formation of such a coalition is of course subject to the usual problems facing coalitions, namely enforcement, monitoring, free riders etc."*

The biform contest approach shed some light on the puzzle of the grand coalition (cartel) stability. As mentioned above, with risk-averse agents, the grand coalition is stable and the cooperative solution coincides with the minmax Shapley value. This result is supported by **Stigler** (1971)**, who pointed out that "the political decisions take account also of the political strength of the various firms, so small firms have a larger influence than they would possess in an unregulated industry. Thus, when quotas are given to firms, the small firms will almost always receive larger quotas than cost minimizing practices would allow."** This pattern of reducing the share of large companies and augmenting the share of small companies is compatible with allocation according to the Shapley formula.

The deadweight loss of welfare caused by cartelization of an industry is a well- known fact, and appears in any microeconomics textbook. It is also well-known that rent-seeking contests to acquire a monopolistic position create additional welfare loss by rent-dissipation. However, as mentioned above and proved in Schwarz (2011b), rent-dissipation rate in biform contests depends on social norms. Competitive norms increase rent-dissipation unambiguously, but the *additional* effect of cartelization on welfare loss under socialist norms is ambiguous. Nevertheless, Cornes and Hartley's (2010) result implies that cartelization under socialist norms is more likely to increase rent-dissipation and welfare loss.

Our results also imply that structural reforms, where new entrants are licensed to challenge a legal monopoly in an oligopolistic competition, do not insure an improvement of social welfare. The crucial point is whether the market structure changed from a monopoly to an oligopolistic competition or to a protected cartel. There is no reason to expect that a market shift from a legal monopoly (a rent-seeking contest winner) to a legal cartel (a coalition of rent-seekers) bears any promise for improvement in social welfare. As mentioned above, a higher rate of rent-dissipation is a more reasonable prediction, and if consumers' demand is not infinitely elastic, their welfare is surely expected to decrease.

Finally, the simplified model of biform contest with a passive regulator analyzed in this article suggests some further research directions for better understanding of grand coalition stability in rent-seeking competitive environments. Here are 5 conjectures for coalitional instability factors; each deserves deep study.

a. **Attitude towards risk:** Our main result is conditional on risk-aversion of all contestants. Principal-agent problems may neutralize risk-aversion of organizations like firms or states, and undermine the stability of the cooperative solution.
b. **Social Norms and Stability:** As mentioned above, norms affect expenditures. Further research is required about norms' effect on stability. For example, with competitive norms, sharing rules do not directly relate effort to compensation. Thus, applying the minmax Shapley value may trigger deprivation feelings among certain members of the grand coalition.
c. **Heterogeneity:** The above analysis assumed homogenous evaluation of the prize by all contestants. Further research is required to check whether our results hold also with heterogeneous evaluation[25].
d. **Asymmetric Information:** The above analysis assumed common knowledge of the prize value and individuals' preferences. Further research is required to check whether our results still hold in case of private or asymmetric information.

[25] The results of Schwarz (2011a) and (2011b) regarding a biform contest with an active regulator are robust to heterogeneous evaluation.

e. **Repeated Games:** Our model is static. Further research is required to check whether our results still hold in a dynamic framework of repeated biform contests[26].

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[26] For example, consider a newly elected parliament with n parties; none of them holds a majority of the seats. There are two options: A "narrow" coalition, which will have to conduct a costly rent-seeking political contest to survive, or a "national unity" coalition where the ruling power is shared with a key opposition party. Apparently, applying a "reasonable" sharing rule would lead to a stable national unity coalition, as happened in Israel after the 1984 and 2009 general elections. However, national unity governments are not generally formed and my conjecture is that our simplified model ignores some important factors, as multidimensional ideological discrepancies, and even more importantly, the dynamic nature of the political process as a repeated game. By forming a national unity government, politicians convey their constituency the message that ideology is not that important. With ideological constituency, such messages are risky in terms of reelecting probability.

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