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Subgame-Perfect Compromise in the Shadow of Conflict

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Abstract

In this paper I analyze the subgame-perfect equilibrium of *n*-player biform contests in which the second stage is a bargaining game. It is shown that in subgame-perfect equilibrium, agents' expenditures depend on social norms while the implemented bargaining solution characteristics depend on regulatory policies.

Keywords: biform game, biform contest, rent-seeking.
JEL classification: C7, D7

1. Introduction

Contestants in rent-seeking contests may find it beneficial to cooperate and share the rent according to a certain sharing rule. In Schwarz (2011b) I suggested a taxonomy of cooperation in competitive environment model in 3 main categories: *wolf-pack* vs. *lion-pack* games on the one hand, and *compromise in the shadow of conflict* on the other hand. Usually, these models are based on a two-stage game framework. In a wolf-pack game, contestants cooperate in a group rent-seeking contest against rival groups in the first stage, but since there is no pre-known specific sharing rule in the second stage among the winning group, its members are re-engaged in a rent-seeking contest among themselves. The main result of *wolf-pack game* models is that a collective action problem within groups usually reduces total expenditures[1]. But this result holds in stochastic contests in which the winning probability is a

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1 Baik (1993), Katz and Tokatlidu (1996), Wärneryd (1998), Inderst, Müller & Wärneryd (2007).

continuous function of contestants' outlays. In deterministic contests, in which the contestant who exerts the highest effort wins with certainty, this result may be reversed[2].

In a lion-pack game, the prize is distributed in the second stage among the winning group members according to a pre-known specific sharing rule. This category also includes models in which the prize is a club good for the winning group[3]. Models of compromise in the shadow of conflict studied the endogenous choice of the sharing rule[4]. In these games, agents bargain over the allocation of the rent in the first stage, and only if bargaining fails are they engaged in a rent-seeking conflict in the second stage[5]. An evolving subclass of this category includes models in which the sharing rules are determined endogenously and specifically for each group in the first stage, and in the second stage, all individuals cooperate within the group rent-seeking contest and determine their contribution level[6].

These categories of two-stage games for modeling cooperation in contests are compatible, each with certain modifications, with Brandenburger and Stuart's (2007) *biform game* framework. A *biform (or hybrid)* n*-player game* is a two-stage game. The first stage is non-cooperative and is designed to describe the strategic moves of the players, but the consequences of these moves are not payoffs. Instead, each profile of strategic choices in the first stage leads to a second stage cooperative game. Similarly, I define a *biform contest* as a two-stage game. In the first stage, agents exert resources to build their position towards the second stage. The second stage is either a bargaining game or a cooperative game. If the bargain fails or if no cooperative solution is achieved, the second stage turns to a rent-seeking contest. The resources exerted in the first stage are aimed at being in optimal position in the second stage under every contingency.

As I indicated in Schwarz (2011b), although biform contests as a stylized model of cooperation in contests are related to numerous real world situations (like plea-bargaining,

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2 Konrad (2004)). For an analysis of a deterministic contest, see Ellingsen (1991)

3 The pre-known sharing rule in the *lion-pack* game neutralizes agents' risk-aversion (Nitzan (1991), Wärneryd (1998) and Skaperdas & Gan (1995)).

4 For studies of Compromise in the shadow of conflict, see, for example, McDonald & Solow (1981), Svejnar (1986), Alexander (1992), Skaperdas (1992) and Skaperdas & Gan (1995), Esteban & Sákovics (2002), Bayindir-Upmann & Gerber (2003) and Skaperdas (2006).

5 The studies mentioned in previous footnotes assumed arbitrary coalitional structures. For analyses of endogenous formation of alliances, see, for example, Hart (1974), Aumann & Meyerson (1988), Katz (1988), Linster (1994), Huck, Konrad & Müller (2002), Lee & Cheong (2005), Bloch, Sánchez-Pagés & Soubeyran (2006), Sánchez-Pagés (2007) and Glazer (2008).

6 See, for example, Nitzan & Ueda (2011) and references there.

arms races and peace negotiations, strikes and wage negotiations, allocation of cartel profits and more), this approach was rarely applied for analyzing these situations, probably because compromise in the shadow of conflict is known as a very complicated situation for modeling, and comparative statics was considered as impossible[7]. In addition to the well-known difficulties mentioned in the literature, the definition of the second stage as a cooperative game raises another difficulty, due to externalities which characterize the rent-seeking competitive environment and undermine the traditional definition of the coalitional worth function and the application of any cooperative solution concept. The externalities problem was addressed in Schwarz (2011b). In this current article, I analyze the biform contest with a bargaining game, for which the externalities problem is irrelevant.

Moulin (1984) suggested a protocol which non-cooperatively implements the Kalai-Smorodinsky (1975) bargaining solution in subgame-perfect equilibrium. Based on Moulin's basic idea, other protocols were suggested for subgame-perfect implementation of various bargaining and cooperative game solutions[8]. All these models are based on multi-stage game forms in which agents bid in a lottery for the right to submit the first proposal. However, the interpretation of this lottery bid and its application to real-world situations is vague. In Schwarz (2011a), I analyzed a 2-player biform judicial contest assuming that its second stage is a plea-bargaining game, and showed that as a stylized model of plea-bargaining practice, it can be viewed as a real-world application of Moulin's protocol. The purpose of this article is to generalize Schwarz' (2011a) model to n-player biform contests.

The remainder of this paper is organized as follows: Section 2 presents the benchmark one-shot rent-seeking contest. Section 3 sets up the biform contest model and the bargaining protocol. Section 4 analyzes the biform contest equilibrium and Section 5 summarizes.

2. The Benchmark One-Shot Rent-Seeking Contest

Consider a set N of n contestants who are engaged in a Tullock (1980) type stochastic rent-seeking contest. The prize value for agent $i \in N$ is z_i. Denote contestant i's expenditures by

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7 See McDonald & Solow (1981), Alexander (1992), Anbarci, Skaperdas & Syropoulos (2002), Bayindir-Upmann & Gerber (2003) and Skaperdas (2006).

8 See, for example, Bergantiños & Méndez-Naya (2000) and Miyagawa (2002).

x_i and assume that agent i's winning probability is given by the Tullock (1980) Contest Success Function[9]

(1) $$p_i(\mathbf{x}) = \frac{x_i^\alpha}{\sum_{j \in N} x_j^\alpha}, \quad \forall i, j \in N, \quad \mathbf{x} = \{x_i\}_{i=1}^n$$

The parameter α measures the return to scale of the rent-seeking efforts. If $\alpha \to 0$, $p_i = 1/n, \ \forall i \in N$. On the other hand, if $\alpha \to \infty$, the rent-seeking contest becomes an all-pay-auction under which the prize is awarded to the contestant who makes the highest effort[10].
The preferences of agent $i \in N$ are represented by von-Neumann-Morgenstern utility functions $u_i(w_i)$ satisfying $u_i' > 0, \ u_i'' < 0$. From contestant i's point of view, the contest has two contingent results: winning (state *I*) or losing (state *II*). Denote contestant i's initial wealth by A_i and his post-contest wealth by $w_i^I = A_i + z_i - x_i$ and $w_i^{II} = A_i - x_i$, respectively. Agent $i \in N$ seeks to maximize the following expected utility function,

(2) $$Eu_i(w_i) = p_i(\mathbf{x}) u_i(w_i^I) + (1 - p_i(\mathbf{x})) u_i(w_i^{II}).$$

The solution for the contestants' optimization problem is an expenditures vector, $\mathbf{x}^*$, which simultaneously solves the following set of first order conditions,

(3) $$p_i' \Delta u_i - Eu_i' = 0, \quad \forall i \in N$$

where $p_i' = \partial p_i / \partial x_i$, $\Delta u_i = u_i(w_i^I) - u_i(w_i^{II})$ and $Eu_i' = p_i u_i'(w_i^I) + (1 - p_i) u_i'(w_i^{II})$. I limit the analysis to games with symmetric pure-strategy Nash equilibria, and in order to assure a unique interior equilibrium, I assume $0 < \alpha < n/n-1$[11].

Assuming risk-aversion complicates the analysis significantly. Nevertheless, the effect of risk-aversion on contest equilibrium is ambiguous[12]. Probably, this is the main reason for the

[9] For axiomatization of this function, see Skaperdas (1996). See also Fullerton & McAfee (1999).
[10] Baye, Kovenock & de Vries (1993)
[11] Pérez-Castrillo & Verdier, (1992)

prevalence of the risk-neutrality assumption in the rent-seeking literature, although this assumption is seldom real[13]. Denote equilibrium expenditures profile with risk-averse and risk-neutral contestants by $\mathbf{x}^*$ and $\hat{\mathbf{x}}$, respectively. Konrad and Schlesinger (1997) show that $x_i^* \gtreqless \hat{x}_i \Leftrightarrow Eu_i'(\hat{\mathbf{x}}) \lesseqgtr \Delta u_i(\hat{\mathbf{x}})/z_i$. Taking the second order Taylor expansion of the right-hand side of this expression yields a simplified version of Cornes and Hartley's (2010) result: $x_i^* \gtreqless \hat{x}_i \Leftrightarrow p_i^* \gtreqless \frac{1}{2},\ \forall i \in N$ [14]. Denote by $R_i = -u_i''/u_i'$ the Pratt (1964) absolute risk-aversion index, and it follows that if $\forall i \in N\ z_i = z$ and $R_i = R$ then $\mathbf{x}^* = \hat{\mathbf{x}}$ and $\mathbf{p}^* = \hat{\mathbf{p}} = \left\{\frac{1}{n}\right\}_{i=1}^{n}$.

3. The Biform Contest

Brandenburger and Stuart (2007) described a *biform n-player game* (or *hybrid* game) as a two-stage game. The first stage is non-cooperative and is designed to describe the strategic moves of the players. However, the consequences of these moves are not payoffs. Instead, each profile of strategic choices in the first stage leads to a second stage cooperative game. This gives the competitive environment created by the choices that the players made in the first stage. Formally, a biform game is a collection (Σ, v, C), where $\Sigma = \sigma^1 \times \sigma^2 \times \ldots \times \sigma^n$ is the strategies profile, $v : \Sigma \times 2^n \to \mathbb{R}$ is a cooperative game coalitional function and $\mathbf{c} = \{c_i\}_{i=1}^{n} \in C$ is a vector of "confidence indices" for each player $(c_i \in [0,1])$ [15].

Similarly, a *biform n-player contest* is a two-stage game. In the first stage, agents non-cooperatively and irreversibly chose their efforts, but this stage does not end with a winner who takes all the rent. Each expenditures' profile implies a corresponding winning probabilities vector and induces a second stage bargaining game. If bargaining fails, agents are engaged in a rent-seeking contest, thus the bargaining game disagreement point is the contestants' expected payoffs vector. Agents' expenditures in the first stage and their induced

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[12] Hillman & Katz (1984) show that for "small" prizes, risk-aversion reduces rent-dissipation. (See also Hillman & Samet (1987) and Nitzan (1994)). But as Konrad and Schlesinger (1997) indicated, generalizing this result is non-trivial. See also Cornes & Hartley, (2003).

[13] See, for example, Millner &Pratt (1991).

[14] This is a "simplified" form of Cornes and Hartley"s (2010) result because it is based on the Taylor expansion technique, and hence valid only for relatively "small" prize contests. Cornes and Hartley (2010) proved that the prudence condition (namely $u_i''' > 0\ \forall i \in N$) is sufficient for this result to hold generally.

[15] Roughly speaking, the confidence indices evaluate every contingent outcome in the core according to player i 's preferences. For a more accurate definition and explanation of confidence index, see Brandenburger & Stuart (2007) or Stuart (2005).

winning probabilities determine their bargaining power in the second stage, and since these expenditures are irreversible, the disagreement point is well-defined and unique. Formally, a biform contest is a collection $(\mathbf{x}, p, \mathbf{u}, B, \mathbf{d})$ where $\mathbf{x} = \{x_i\}_{i=1}^{n}$ is the expenditures vector, p is the contest success function, $\mathbf{u}$ is a vector of contestants' utilities, B is the bargaining set and $\mathbf{d}$ is the disagreement point[16].

3.1 Bargaining

A bargaining problem is a pair $(B, \mathbf{d})$ where $\mathbf{d}$ is the disagreement point and B is the bargaining set, namely the set of feasible allocations. By definition $\mathbf{d} \in B$, hence $B \neq \varnothing$, but an interesting bargaining problem has a non-degenerate bargaining set, $B \setminus \mathbf{d} \neq \varnothing$. The bargaining set frontier, $P(B)$, is the set of all Pareto efficient allocations. A bargaining problem solution is a function $f:(B,\mathbf{d}) \to P(B)$ that picks a point on the bargaining set frontier. To ensure existence and uniqueness of a bargaining solution, we assume that B is a bounded compact and convex set.

Nash (1950) suggested a bargain solution, $\mathbf{b}^{Nash} = \arg\max_{\mathbf{b} \in P(B)} \prod_{i \in N} u_i(\mathbf{b})$, and proved that this solution uniquely satisfies four basic axioms: Invariance to affine transformation (IAT), Efficiency or Pareto Optimality (PO), Symmetry (S) and Independence of Irrelevant Alternatives (IIA). But the IIA axiom was criticized by Luce and Raiffa (1957), and replaced with the monotony axiom (M) by Kalai and Smorodinsky (1975), who suggested an alternative unique bargain solution that satisfies IAT, PO, S and M. Define $\mathbf{b}_i^M = \arg\max_{\mathbf{b} \in P(B)} u_i(\boldsymbol{\beta})$ as the most preferred feasible solution from agent i's point of view. $\mathbf{u}(\mathbf{b}^M)$ is known as the *utopia point*. The Kalai-Smorodinsky bargaining solution is the $\mathbf{b}^*$ which solves

(4)
$$\frac{u_i(\mathbf{b}^*) - u_i(\mathbf{d})}{u_i(\mathbf{b}_i^M) - u_i(\mathbf{d})} = \lambda_B, \quad \forall i \in N .$$

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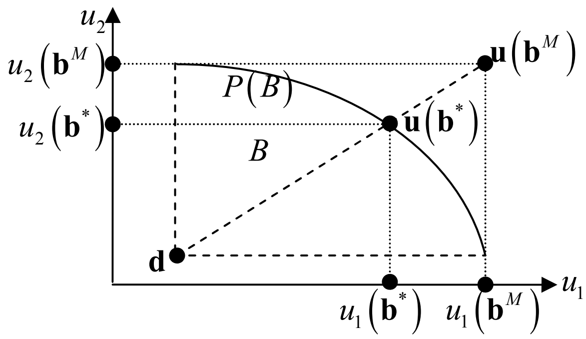
16 For early analyses of biform contest situations, see McDonald & Solow (1981), Svejnar (1986), Alexander (1992) and Skaperdas & Gan (1995). More recent studies include, for example, Eesteban & Sákovics (2002), Bayindir-Upmann & Gerber (2003) and Skaperdas (2006).

Kalai-Smorodinsky's solution for a 2-player bargaining game is depicted in Figure 1. The solution $\mathbf{u}\left(\mathbf{b}^{*}\right)$ is the intersection point between the $P(B)$ curve and the $\mathbf{d}-\mathbf{u}\left(\mathbf{b}^{M}\right)$ line.

Figure 1



A cooperative game solution concept that is related to the Kalai-Smorodinslky bargaining solution is Tijs' (1981) compromise τ-value. Let (v,N) be a TU cooperative game where $v:2^n \to \mathbb{R}$ is the characteristic function which assigns a *value*, namely a real number $v(S)$ for every coalition $S \subseteq N$, satisfying $v(\varnothing)=0$. Henceforth, we follow the convention and refer to the cooperative game simply as v instead of (v,N).

The upper bound of an agent's value in a TU game v, $M_i(v)$, which is the agent's *maximal claim*, is defined by

(5) $$M_i(v)=v(N)-v(N\setminus\{i\}).$$

Namely, an agent's maximal claim equals his marginal contribution to the grand coalition's value. On the other hand, the lower bound of an agent's value, or the agents' *minimal claim*, $m_i(v)$ is defined by

(6) $$m_i(v)=\max_{S\subset N, i\in S}\left\{v(S)-\sum_{j\in S\setminus\{i\}} M_j(v)\right\}.$$

Namely, an agent's minimal claim is the maximal remainder of a coalitional value after all other coalition members received their maximal claim. A cooperative game v is *quasi-balanced* if $M_i(v) \geq m_i(v)$, $\forall i \in N$ and $\sum_{i\in N} m_i(v) \leq v(N) \leq \sum_{i\in N} M_i(v)$ [17]. Denote the upper and lower bounds vectors by $M(v) = \{M_i(v)\}_{i=1}^{n}$ and $m(v) = \{m_i(v)\}_{i=1}^{n}$, respectively. The τ-value is defined as

$$\tau(v) = \lambda_v M(v) + (1-\lambda_v) m(v) \tag{7}$$

where $\lambda_v \in [0,1]$ and $\sum_{i\in N} \tau_i(v) = v(N)$. Namely, the τ-value is a weighted average of $M(v)$ and $m(v)$. The τ-value is well defined only for quasi-balanced games and is the only efficient point between the upper and lower bounds[18].

Intuitively, $M(v)$ and $m(v)$ correspond to $\mathbf{u}(\boldsymbol{\beta}^M)$ and $\mathbf{d}$, respectively. Notice also that (4) can be written slightly different as $\mathbf{u}(\boldsymbol{\beta}^*) = \lambda_B \mathbf{u}(\boldsymbol{\beta}_i^M) + (1-\lambda_B)\mathbf{u}(\mathbf{d})$. Since both the τ-value and the Kalai-Smorodinsky solution are unique and efficient, $\lambda_B = \lambda_v$ implying that these two solutions should coincide. This intuition is true, as proved by Dagan and Volij (1993)[19].

3.2 The Bargaining Set of a Biform Contest

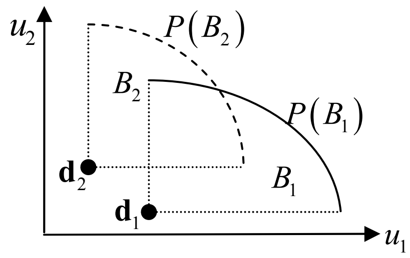
Apparently, a bargaining solution of a biform contest is a sharing rule $\boldsymbol{\beta} = \{\beta_i\}_{i=1}^{n}$, $\beta_i \in [0,1]$ that assigns each contestant $i \in N$ a share $\beta_i z_i$ of the rent and a utility level of $u_i(A_i + \beta_i z_i - x_i^*)$ where $\mathbf{u}(\boldsymbol{\beta}) \in P(B)$. The set of all efficient sharing vectors is denoted by $F = \{\boldsymbol{\beta} | \mathbf{u}(\boldsymbol{\beta}) \in P(B)\}$.

However, in a biform contest, disagreement in the second stage means a confrontation in a rent-seeking contest. Namely, $\mathbf{d} = \{Eu_i\}_{i=1}^{n}$, implying that the whole bargaining set is endogenous. For instance, an increase in agent 1 first-stage efforts shifts the disagreement

[17] If the inequalities are strict, the game is *strict* quasi-balanced.
[18] See Tijs (1981).
[19] See also Tijs &Otten (1993) and Bergantiños & Méndez-Naya (2000).

point from $\mathbf{d}_1$ to $\mathbf{d}_2$ and, consequently, the bargain set from B_1 to B_2 (see Figure 2). This has led many authors to conclude that deriving equilibrium of compromise in the shadow of conflict is complicated, and comparative statics is even impossible[20].

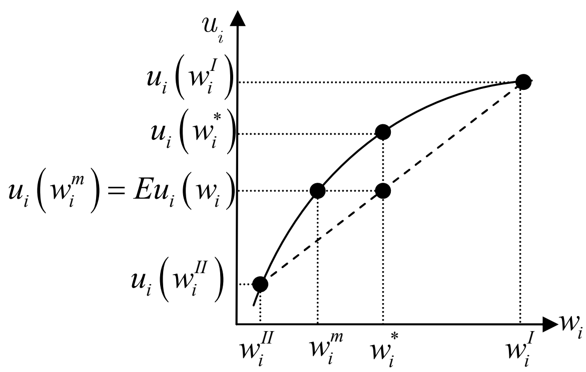
Figure 2



But this conclusion is too pessimistic. T**he irreversibility of first stage expenditures, $\mathbf{x}^*$, implies that in the second stage they are sunk-costs, thus in the second stage, the bargaining set is fixed. But it is still left for us to show that the biform contest bargaining set is non-degenerate, compact and convex.**

In the benchmark one-shot contest, the expected equilibrium net wealth of contestant $i \in N$ is $w_i^*(\mathbf{x}^*) = A_i + p_i(\mathbf{x}^*) z_i - x_i^*$. Assuming risk-aversion and applying Jensen's inequality implies that $u_i(w_i^*) \geq Eu_i(w_i)$, and also that there is a *certainty equivalent share*, β_i^m, which yields a *certainty equivalent wealth,* $w_i^m = A_i + \beta_i^m z_i - x_i^* < w_i^*$, satisfying $u_i(w_i^m) = Eu_i(w_i)$, (see Figure 3).

Figure 3



20 See references in footnote 16.

A sharing vector $\boldsymbol{\beta}$ is *privately rational* if $u_i\left(\beta_i, x_i^*\right) \geq Eu_i\left(w_i, \mathbf{x}^*\right)$, and *rational* if this inequality is satisfied for all $i \in N$. The *bargaining set*, B, is the set of all rational sharing vectors. For the sake of convenience define an agent's *gain function* by

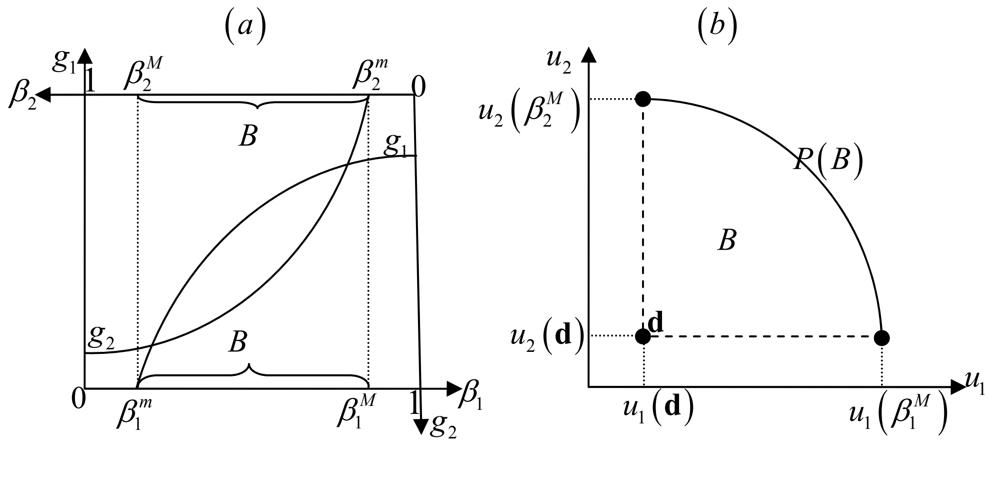
(8) $$g_i\left(\boldsymbol{\beta}, \mathbf{x}^*\right) = u_i\left(\beta_i, x_i^*\right) - Eu_i\left(\mathbf{x}^*\right).$$

The bargaining set is defined by

(9) $$B\left(\boldsymbol{\beta}, \mathbf{x}^*\right) = \left\{\boldsymbol{\beta} \middle| g_i\left(\boldsymbol{\beta}, \mathbf{x}^*\right) \geq 0,\ \forall i \in N\right\}.$$

It can easily be verified that $\partial g_i / \partial \beta_i > 0$ and $\partial^2 g_i / \partial \beta_i^2 < 0$, implying that the g_i curve is upward sloping and concave. For $n = 2$ we can use the familiar Edgeworth box, depicted in panel (a) of Figure 4. β_i^m is the minimal value of β_i satisfying $g_i \geq 0$. For each β_i^m there is a corresponding β_i^M, which is the maximal β_j implied by β_i^m, as demonstrated in Figure 4. The bargaining set is represented by the closed segment $\left[\beta_i^m, \beta_i^M\right]$. Panel (b) of Figure 4 presents the biform contest bargaining set, B, drawn on the utilities plain.

Figure 4



It can immediately be verified that since $g_i\left(\boldsymbol{\beta}, \mathbf{x}^*\right)$ is bounded and concave, B is compact and convex. The following proposition provides a sufficient condition for a non-degenerate B.

Proposition 1:

(10) $$B \setminus \mathbf{d} \neq \varnothing \Leftrightarrow \exists \boldsymbol{\beta} \in F \text{ such that } \beta_i^* > \frac{1-\sqrt{1-p_i^* R_i z_i \left(2-R_i z_i\right)}}{R_i z_i}, \quad \forall i \in N .$$

Proof of Proposition 1:

Taking the Taylor expansion of g_i^* (with a remainder) implies that $g_i^* > 0$ if,

(11) $$u_i'\left(A_i - x_i^*\right) z_i \left(\beta_i^* - p_i^*\right) + \frac{1}{2} u_i''\left(A_i - x_i^*\right) z_i^2 \left(\beta_i^{*2} - p_i^*\right) + \mathcal{R} > 0$$

where $\mathcal{R}$ denotes the remainder of the Taylor series (assumed negligible). (10) is derived by dividing (11) by $u_i'\left(A_i - x_i^*\right)$, solving for β_i and taking the relevant (lower) solution.

Notice that by (10) $\operatorname{sgn}\left(\partial \beta_i^* / \partial R_i\right)$ depends on $\operatorname{sgn}\left(\partial p_i^* / \partial R_i\right)$ which is unknown, implying that the effect of risk-aversion on the bargaining set boundaries (namely, on agreement availability) is indeterminate.

3.3 Social Norms and Regulatory Policies

Social norms relate to "fair" bargaining solutions and sharing proposals. For example, a *socialist* norm postulates that effort should be compensated and compensation should take into account the individual's effort and also his relative position in total societal effort. Thus, a socialist sharing rule determines the agent's share according to a monotonically increasing function $\beta_i = \beta_i\left(\mathbf{x}\right)$. On the other hand, a *competitive* norm postulates that every sharing rule and proposal, $\boldsymbol{\beta}$, is legitimate, conditional on being freely and unanimously approved in a proper and fair bargaining process.

From a positivist economic point of view, the difference between socialist and competitive sharing rules relates to their effect on uncertainty. Socialist sharing rules reduces uncertainty, contrary to competitive sharing rules, which do not require any a-priori functional relation between expenditures and share, and consequently enhance uncertainty. As demonstrated below, although risk-aversion effect on contestants' expenditures is ambiguous, increased uncertainty unambiguously enhances expenditures in biform contests.

Regulatory policies express the regulator's social philosophy. A *passive* regulator believes in "laissez-faire" and does not intervene at all in the second stage. Consequently, there is no imposed bargaining protocol in the second stage. Wolf-pack, lion-pack and compromise in the shadow of conflict are all particular cases of biform contests with a passive regulator, as well as the biform contest in which the second stage is a cooperative game, which was analyzed in Schwarz (2011b).

An *active regulator* is concerned about "procedural justice," but indifferent to "distributive justice." Thus, an active regulator determines the rules of the game (the bargaining protocol), without any attempt to implement a specific sharing rule. Under an active regulator, any feasible sharing rule $\boldsymbol{\beta} \in F$ is allowed. On the other hand, an *active benevolent regulator* also takes "fair division" or "distributive justice" considerations into account and seeks to implement a sharing rule which maximizes a social welfare function $W(\mathbf{u}(\boldsymbol{\beta}))$ where $W : \mathbb{R}^n \to \mathbb{R}$ is continuous, monotone and quasi-concave. An active benevolent regulator seeks to implement

$$\text{(12)} \qquad \boldsymbol{\beta}^W = \arg\max_{\boldsymbol{\beta} \in F} W(\mathbf{u}(\boldsymbol{\beta})).$$

For example, a Benthamite regulator seeks to implement $\boldsymbol{\beta}^W = \arg\max_{\boldsymbol{\beta} \in F} \sum_{i \in N} u_i(\boldsymbol{\beta})$, and a Rawlsian regulator seeks to implement $\boldsymbol{\beta}^W = \arg\max_{\boldsymbol{\beta} \in F} [\min \mathbf{u}(\boldsymbol{\beta})]$[21]. With an active benevolent regulator, a feasible counter proposal $\boldsymbol{\beta}^i \in F$ is allowed if and only if $W(\mathbf{u}(\boldsymbol{\beta}^i)) \geq W(\mathbf{u}(\boldsymbol{\beta}^1))$. Namely, given a first feasible proposal $\boldsymbol{\beta}^1$ which induces $W(\mathbf{u}(\boldsymbol{\beta}^1))$, no counter proposal is allowed if it induces lower social welfare.

[21] According to Miyagawa (2002), a Rawlsian regulator seeks to implement the Kalai-Smorodinsky (1975) bargaining solution. This claim is inaccurate. Rawlsian social welfare maximization is equivalent to implementation of the Kalai-Smorodinsky bargaining solution if and only if $\forall i \in N \; u_i = u$ and $z_i = z$.

3.4 The Biform Contest Bargaining Game Form

The biform bargaining contest is a two-stage game with a non-passive regulator who imposes a modified version of Moulin's (1984) bargaining protocol in the second stage. The game form is:

Stage 1: Every contestant $i \in N$ exerts x_i^* in order to establish a bargaining position. Each agent is characterized by his equilibrium winning probability $p^*\left(\mathbf{x}^*\right)$, and contestants are ordered according to their winning probabilities in decreasing order, namely $i \geq j \Leftrightarrow p_i^* \geq p_j^*$. (If case of $p_i^* = p_j^*$, i and j are ordered randomly).

Stage 2: The second stage bargaining protocol has *n-1* potential rounds.

Round 1: Contestant 1 submits a sharing rule $\boldsymbol{\beta}^1 = \left\{\beta_j^1\right\}_{j=1}^n$. If $\boldsymbol{\beta}^1$ is unanimously approved, the game is over. Otherwise, proceed to round 2.

Round 2: Agent 2 submits his proposal $\boldsymbol{\beta}^2$ for unanimous approval. If this proposal is unanimously accepted, the regulator imposes a lottery $p_1^*\boldsymbol{\beta}^2 + \left(1 - p_1^*\right)\mathbf{d}$. Else, proceed to round 3, and so on.

…

Round n-1: Agent n submits his proposal $\boldsymbol{\beta}^n$. If this proposal is unanimously accepted, the regulator imposes a lottery $p_1^*\boldsymbol{\beta}^n + \left(1 - p_1^*\right)\mathbf{d}$. Else, the game ends with $\mathbf{d}$.

4. Equilibrium

A subgame-perfect equilibrium of a biform contest is a pair $\left(\mathbf{x}^*, \boldsymbol{\beta}^*\right)$ of expenditures' vector and a bargaining solution. By the following propositions, social norms affect expenditures, while regulatory policies affect the equilibrium bargaining sharing rule.

Proposition 2:

A socialist sharing rule neutralizes contestants' risk-aversion. Namely, in a socialist society, contestants' subgame perfect expenditures' profile is proportional to a risk-neutral expenditures profile in the benchmark one-shot contest.

Proof of Proposition 2[22]:

With a socialist norm, a second stage sharing rule is common knowledge, implying that no real bargaining is going to take place in this stage. Agents know in advance that their second stage payoff is $\beta_i = \beta_i(\mathbf{x})$, implying that the first stage target function of agent $i \in N$ is

(13) $$\max_{x_i} u_i\left(A_i + \beta_i(\mathbf{x}) z_i - x_i\right).$$

First order conditions for interior solutions are

(14) $$u_i'(\bullet)\left[\beta_i'(\mathbf{x}) z_i - 1\right] = 0,\ \forall i \in N.$$

With solutions $x_i^* = \beta_i'(\mathbf{x})\hat{x}_i,\ \forall i \in N$. Namely, equilibrium expenditures' profile is proportional to risk-neutral agents' expenditures' profile.

The intuition of Proposition 2 is that a socialist norm removes uncertainty, as agents know in advance the payoff function and their efforts' effect on their expected share. Hence, their expenditures are proportional to risk-neutral agents' expenditures profile and the proportion coefficient of agent $i \in N$ is his marginal share $\beta_i'(\mathbf{x})$.

Proposition 3:

A competitive sharing rule enhances subgame-perfect expenditures.

Proof of Proposition 3:

In a competitive society, there is no a-priori relation between the sharing rule, $\boldsymbol{\beta}$, and agents' expenditures. Thus, what agents know in advance is that a real bargaining game is going to be played in the second stage. However, its consequences, particularly whether the second stage is going to end with an agreement or a confrontation, are unknown yet. In a competitive society, agents do not exert efforts to be compensated according to a socialist pre-known formula, but in order to optimize their bargaining position. The first stage target function of agent $i \in N$ is

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22 This proof follows Skaperdas & Gan (1995).

(15) $$\max_{x_i}\left\{u_i\left(A_i+\beta_i z_i-x_i\right)-\left[p_i\left(\mathbf{x}\right)u_i\left(A_i+z_i-x_i\right)+\left(1-p_i\left(\mathbf{x}\right)\right)u_i\left(A_i-x_i\right)\right]\right\}.$$

And the first order conditions for interior solution are

(16) $$p_i'\Delta u_i - Eu_i' = -u_i'\left(\beta_i\right),\ \ \forall i\in N.$$

Denote the solution of (16) by $\tilde{\mathbf{x}}$. A comparison of (16) with (3) immediately reveals that $\tilde{\mathbf{x}} > \mathbf{x}^*$.

Intuitively, the left-hand side of (16) is the marginal utility of expenditure in case the second stage bargaining fails and agents are engaged in a rent-seeking contest. The right-hand side of (16) is the marginal utility of expenditure in case of agreement. Since agents in a competitive society do not know in advance what contingency going to be realized, they equalize the two marginal utilities. It follows that in biform contests, increased uncertainty enhances expenditures.

Proposition 4:

With an active regulator, the biform game non-cooperatively implements Tijs' (1981) compromise τ-value in subgame-perfect equilibrium.

Proof of Proposition 4:

A subgame-perfect equilibrium is computed by backwards induction. Suppose that in round n-1 contestant n-1 suggests $\boldsymbol{\beta}^{n-1}$. By rejecting $\boldsymbol{\beta}^{n-1}$ and making a counter proposal of $\boldsymbol{\beta}^n=\left\{\beta_1^m,\beta_2^m,...,\beta_n^M\right\}$, contestant n ensures an expected utility of

(17) $$Eu_n\left(\boldsymbol{\beta}^n\right)=p_1^*u_n\left(\boldsymbol{\beta}^n\right)+\left(1-p_1^*\right)u_n\left(\mathbf{d}\right)$$

But agent n's proposal, $\boldsymbol{\beta}^n$, will be approved if and only if it satisfies

(18) $$u_i\left(\boldsymbol{\beta}^n\right)=p_1^*u_i\left(\beta_i^m\right)+\left(1-p_1^*\right)u_i\left(\mathbf{d}\right)=0,\ \forall i\in N,\ i\neq n.$$

Hence, agent $n-1$'s proposal, $\boldsymbol{\beta}^{n-1}$, must satisfy

(19)
$$u_n\left(\boldsymbol{\beta}^{n-1}\right) \geq p_1^* u_n\left(\beta_n^M\right) + \left(1-p_1^*\right) u_n\left(\mathbf{d}\right)$$
and
$$u_i\left(\boldsymbol{\beta}^{n-1}\right) \geq p_1^* u_i\left(\beta_i^m\right) + \left(1-p_1^*\right) u_i\left(\mathbf{d}\right) = 0, \ \forall i \in N, \ i \neq n$$

Using the same argumentation, it can be easily verified that agent $n-2$'s best response strategy is to submit a proposal, $\boldsymbol{\beta}^{n-2}$, satisfying

(20)
$$u_n\left(\boldsymbol{\beta}^{n-2}\right) \geq p_1^* u_n\left(\beta_n^M\right) + \left(1-p_1^*\right) u_n\left(\mathbf{d}\right)$$
$$u_{n-1}\left(\boldsymbol{\beta}^{n-2}\right) \geq p_1^* u_{n-1}\left(\beta_{n-1}^M\right) + \left(1-p_1^*\right) u_{n-1}\left(\mathbf{d}\right)$$
and
$$u_i\left(\boldsymbol{\beta}^{n-2}\right) \geq p_1^* u_i\left(\beta_i^m\right) + \left(1-p_1^*\right) u_i\left(\mathbf{d}\right) = 0, \ \forall i \in N, \ i \neq n, n-1$$

Going back to round 1, the best response of agent 1 is to propose $\boldsymbol{\beta}^*$ satisfying

(21)
$$\mathbf{u}\left(\boldsymbol{\beta}^*\right) = p_1^* \mathbf{u}\left(\boldsymbol{\beta}^M\right) + \left(1-p_1^*\right) \mathbf{u}\left(\mathbf{d}\right).$$

Recall that since first stage expenditures $\mathbf{x}$ are irreversible, in the second stage $p_1^* = \lambda_B$ is fixed and constant, thus $\mathbf{u}\left(\boldsymbol{\beta}^*\right)$ is a weighted average of the upper and lower bound vectors $\mathbf{u}\left(\boldsymbol{\beta}^M\right)$ and $\mathbf{u}\left(\mathbf{d}\right)$, implying that $\boldsymbol{\beta}^*$ satisfies (21) and induces the τ-value compromise solution.

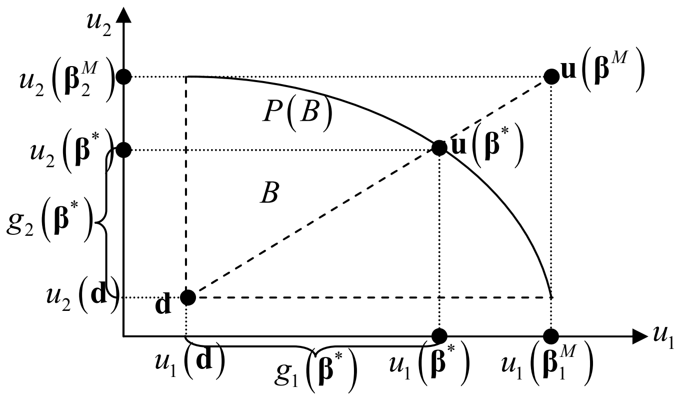
To understand the intuition behind Proposition 4: suppose that $n=2$ and refer to Figure 5. Suppose that $p_1^* > p_2^*$, and agent 1 suggests $\boldsymbol{\beta}_1^M = (1,0)$. By rejecting this proposal and suggesting $\boldsymbol{\beta}_2^M = (0,1)$, agent 2 can assure himself an expected utility of

$p_1^* u_2(\boldsymbol{\beta}_2^M) + (1 - p_1^*) u_2(\mathbf{d})$. Hence, agent 1's best response is to suggest $\boldsymbol{\beta}^*$ satisfying $u_2(\boldsymbol{\beta}^*) \geq p_1^* u_2(\boldsymbol{\beta}_2^M) + (1 - p_1^*) u_2(\mathbf{d})$. Now suppose that $p_2^* > p_1^*$ and agent 2 makes the first suggestion. By the same argument, agent 2's best response is to suggest $\boldsymbol{\beta}^*$ satisfying $u_1(\boldsymbol{\beta}^*) \geq p_1^* u_1(\boldsymbol{\beta}_1^M) + (1 - p_1^*) u_1(\mathbf{d})$. Combining these two conditions yields

(22) $$u_i(\boldsymbol{\beta}^*) = u_i(\mathbf{d}) + p_1^* \left[u_i(\boldsymbol{\beta}_i^M) - u_i(\mathbf{d}) \right],\ i = 1,2$$

And the $\boldsymbol{\beta}^*$ which solves (22) actually applies the compromise τ-value, which coincides with the Kalai-Smorodinsky solution, as demonstrated in Figure 5.

Figure 5



Proposition 5:

With an active benevolent regulator, the bargaining biform contest game form non-cooperatively implements $\boldsymbol{\beta}^W$ in subgame-perfect equilibrium.

Proof of Proposition 5:

Applying backwards induction, we begin at stage 2 round $n-1$. Recall that this stage is reached only if all previous proposals and counter-proposals have been rejected. Now it is agent n's turn to submit his counter-proposal. Suppose that agent n submits $\boldsymbol{\beta}^n$, subject to $W(\mathbf{u}(\boldsymbol{\beta}^n)) \geq W(\mathbf{u}(\boldsymbol{\beta}^1))$. Clearly, the best response strategy of agent $i \in N \setminus \{n\}$ to $\boldsymbol{\beta}^n$ is,

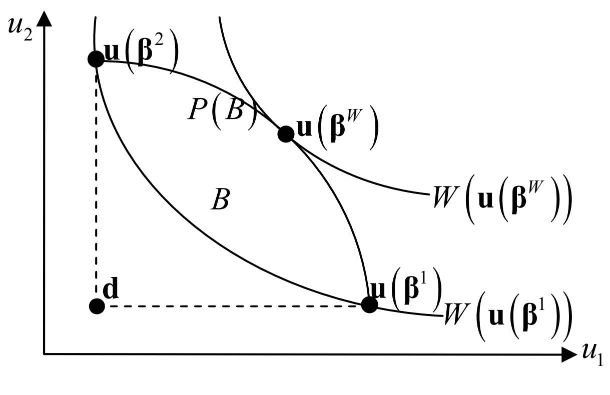
(23)
$$\begin{cases} accept \text{ if } p_1^* u_i(\boldsymbol{\beta}^n) + (1 - p_1^*) u_i(\mathbf{d}) \geq u_i(\mathbf{d}) \\ reject \quad \text{otherwise} \end{cases}$$

Implying *accept* if $u_i(\boldsymbol{\beta}^n) \geq u_i(\mathbf{d})$ and *reject* otherwise. Going back to round 1 reveals that agent 1's best response proposal to any expected counter-proposal, $\boldsymbol{\beta}^i$, is

(24)
$$\boldsymbol{\beta}^W = \arg\max W(\mathbf{u}(\boldsymbol{\beta})).$$

The intuition of Proposition 5 is illustrated in Figure 6, assuming again $n = 2$. Suppose that agent 1 suggests $\boldsymbol{\beta}_1^M$. If agent 2 rejects $\boldsymbol{\beta}_1^M$ he is allowed to submit a counter proposal $\boldsymbol{\beta}_2^M$, subject to $W(\mathbf{u}(\boldsymbol{\beta}_2^M)) \geq W(\mathbf{u}(\boldsymbol{\beta}_1^M))$ and in this case his expected utility is $p_1^* u_2(\boldsymbol{\beta}_2^M) + (1 - p_1^*) u_2(\mathbf{d})$, implying that agent 1's best response is $\boldsymbol{\beta}^W$ which induces $\mathbf{u}(\boldsymbol{\beta}^W)$, the tangent point of $W(\mathbf{u}(\boldsymbol{\beta}))$ with $P(B)$.

Figure 6



The above four propositions yield the following corollary.

Corollary:

Social norms' effect on subgame-perfect expenditures is independent of regulatory policies, and regulatory policies' effect on the bargaining solution is independent of social norms.

5. Summary

This article complements Schwarz (2011a) and Schwarz (2011b), where in the former, I studied a 2-player biform contest with a bargaining game in its second stage, and in the latter, I assumed that the second stage is a cooperative game. The purpose of this article was to generalize Schwarz' (2011a) results to an n-player biform contest.

The biform game form analyzed here is of a two-stage game, where in the first stage agents exert irreversible expenditures, and in the second stage, they bargain according to a modified version of Moulin's (1984) bargaining protocol, imposed by a non-passive regulator. If the bargaining fails, the rent-seeking contest is played.

The above analysis shows that in subgame-perfect equilibrium, contestants' expenditures depend on social norms while the implemented bargaining solution characteristics depend on the regulatory policy. It was shown that socialist norms neutralize contestants' risk-aversion, while competitive norms enhance agents' expenditures.

Wolf-pack, lion-pack, compromise in shadow of conflict and biform contests with cooperative games in their second stage are all particular cases of biform contests with a passive regulator. In Schwarz (2011b), I show that a biform contest with a cooperative game non-cooperatively implements the *minmax Shapley Value* as defined and explained there. In Schwarz (2011a), I show in a 2-player framework, that with an active regulator, the biform contest applies the Moulin (1984) protocol which non-cooperatively implements the Kalai-Smorodinsky (1975) bargaining solution in subgame-perfect equilibrium. The above analysis generalized this result to n-player biform contests, and showed that with an active regulator, the biform contest implements Tijs (1981) τ-value in subgame-perfect equilibrium, while with an active benevolent regulator, the biform contest non-cooperatively maximizes a social welfare function in a subgame-perfect equilibrium.

One may conjecture that rent-seeking contests among lobbyists and political pressure groups are characterized by passive regulators. On the other hand, as I explained in Schwarz (2011a), in countries where plea bargaining is regulated by law, the legal provisions primarily refer to the compatibility of the agreement with the public punitive policy and to the procedural justice ("fairness") of the bargaining process. Usually, "distributive justice" of the gain from a plea-deal between the prosecutor and the defendant is not considered as a social goal by legislators, implying that judicial contests with plea-bargaining may be modeled by biform contests with an active regulator (the court). Finally, biform contests with an active benevolent regulator may be viewed as a stylized model of intra-family competition between siblings over their altruistic parents' love (and bequest). All this indicates that the above analysis opens the gate for additional further interesting directions for research, theoretical, empirical and experimental.

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