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Ambiguous political power and contest efforts

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Abstract When there is incomplete information on the source of power in a contest, the contestants may divide their lobbying efforts between the potential centers of power, only one of which determines the contests' winning probabilities. Our analysis focuses on the effect of ambiguity regarding the source of power on the contestants' aggregate effort in a symmetric, simple lottery contest with two potential centers of power. Specifically, we examine the effects of varying the informativeness of the contestants' private signals (i.e., the probability that a signal is correct) and the degree of correlation between them. Our benchmark case is the standard Tullock's model, in which the source of power is known, i.e., the contestants' signals are perfectly informative. We show that the level of aggregate effort in this case is reached also when the signals are perfectly uninformative. However, in any intermediate case the contestants' aggregate effort is lower, provided that the signals are not perfectly correlated. In other words, there is a U-shape relationship between the informativeness of the signals and the aggregate effort in the contest. The lowest level of effort is spent when the signals are independent and the probability that a signal is true is about 0.85. In this case, efforts are reduced by about one-fifth in comparison with the benchmark case: from a rent dissipation of 50% to slightly over 40%.

Keywords Contests · Incomplete information · Lobbying · Transparency

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1 Introduction

Contestants are often observed to direct resources to the source of power in the contest, in order to increase their winning probability. However, in many real-world cases, players have only incomplete information regarding the power distribution within a set of potential sources of power. They may have a list of potential decisive agents but no firm information regarding the identity of the decisive agent – the true target of their efforts (e.g., influence activities, persuasion attempts, lobbying efforts or rent-seeking expenditures). From the contestants' point of view, resources directed to the wrong agent are a pure waste. However, under incomplete information, it makes sense to direct resources to several potential "power centers" and the question the contestants face is: Given the prior information about the distribution of power and the signals they receive, how much effort to make and how to allocate it among the potential power centers?

In this study we consider a standard, symmetric, two-player lottery contest where the lobbyists do not recognize with certainty the "center of power" within the system.[1] In certain types of bureaucratic organizations, the identification of the center of power within the system is not an easy task, especially for an outsider, because the distribution of power is not fully correlated with the hierarchic ranks within the organization. Moreover, in some bureaucratic organizations, not only is the distribution of power among the acting figures within the system unknown, but even the set of potential decision makers may only be partially known. In particular, the contestants may not know the identity of the "wire-puller" who controls the decision-making system, possibly from behind the scenes. Lobbyists acting in partially transparent political or bureaucratic organizations without full information about the location of the power center therefore have to make decisions about the optimal allocation of their lobbying budget between several potentially decisive persons. The probability of winning the lobbying contest thus depends on the relative size of the investments in lobbying directed to the participating politicians and the probability that each of them is the actual decision maker.[2] We model the contestants' information about the latter as private signals. Restricting ourselves to the case of two contestants and two potential power centers who are equally likely to be the actual one, our main objective is to study the effect of ambiguous political

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[1] The role of transparency in bureaucratic systems has been studied in numerous papers. In a comprehensive survey about transparency of a monetary policy, Geraats (2002) defines transparency as the absence of asymmetric information between decision makers and economic agents, which, he suggests, "reduces uncertainty and this is often believed to be beneficial (although it need not be)". Geraats also mentions that transparency affects incentives and reputation building. In his survey, Geraats mentions five aspects of transparency that correspond to different stages of the policy-making process: political, economic, procedural, policy and operational transparency. It seems that transparency regarding power distribution has not been studied in the literature. And in the context of lobbying contests, the issue of such transparency, that involves both incomplete information and asymmetry, was not raised.

[2] In our setting, the contestants face asymmetric information, that is, different signals about the identity of the actual decision maker. The effect of asymmetry and uncertainty on the outcome of rent-seeking or lobbying contests has been extensively studied in the political economy literature. The studies concerned with the effect of asymmetry have focused on prize, value or stake asymmetry (Epstein & Nitzan, 2004, 2003, 2002; Hillman & Riley, 1989; Hurley, 1998; Konrad & Schlesinger, 1997; Nti, 1999, 1997; Stein, 2002), asymmetric lobbying capabilities (Baik, 1999, 1994; Gradstein, 1995; Keem, 1998; Kohli and Singh, 1999; Leininger, 1993; Singh & Wittman, 2001; Stein, 2002), asymmetric information (Hurley, 1998; Hurley & Shogren, 1998a,b; Wärneryd, 2003), asymmetric constrained budgets (Che & Gale, 1998, 1997) and asymmetric sharing rules within groups competing on a collective rent (Davis & Reilly, 1999; Nitzan, 1991). The studies concerned with the effect of uncertainty have focused on uncertainty regarding the awarded prize (Chung, 1999), uncertainty regarding the existence of opposition (Cairns and Long, 1991; Ellingsen, 1991; Epstein & Nitzan, 2003) and uncertainty regarding the mere award of the contestable rent (Kahana and Nitzan, 1999).

power on the contestants' efforts. In particular, we examine how the efforts are affected by the informativeness of the contestants' signals (i.e., the probability p that each signal is true) and the degree of correlation between them ρ, and establish under what combination of p and ρ total efforts are minimal.

2 The symmetric lottery contest with incomplete information on political power

There are two bureaucrats (or politicians): a decisive bureaucrat, who determines the contest outcome, and a dummy one, who pretends to be the real decision maker but is actually powerless. There are two identical, risk-neutral contestants (or lobbyists), who are affected by the decisive bureaucrat's actions and are engaged in a standard probabilistic lobbying contest, as in Tullock (1980; see also Nitzan, 1994). However, the contestants are uncertain as to the identity of the real decision maker, which we denote by d. We assume that they share the common prior that d is equally likely to be one bureaucrat or the other. Each contestant receives a signal s regarding d's identity, which is correct (i.e., $s = d$) with probability $0 < p < 1$ and is incorrect ($s \neq d$) with probability $q = 1 - p$. Because of the half–half prior, p is also the *posterior* probability, given s, that the real decision maker's identity is indeed s. We do not specify the nature of the signals, which may be private, public, or a mixture of both. In particular, the two contestants' signals may or may not be conditionally independent, given d. We only assume that the conditional coefficient of correlation between the signals ρ is the same regardless of whether one bureaucrat or the other is the real decision maker, and that they are not *negatively* correlated, i.e., $0 \leq \rho \leq 1$.

The parameters p and ρ uniquely determine the probability α that both contestants' signals are true and the probability β that they are both false. These probabilities are easily seen to satisfy

$$\alpha - \beta = p - q. \tag{1}$$

They also determine the probability γ that any particular contestant receives a true signal while the other receives a false signal, which is given by

$$\gamma = (1 - \rho)pq. \tag{2}$$

If this happens, the contestants' posterior beliefs regarding the identity of the real decision maker are (generally) not the same, even though they share the same prior. This does not contradict Aumann's (1976) agreeing-to-disagree theorem, since the beliefs are not commonly, or even mutually, known. Since, clearly,

$$\alpha + \beta + 2\gamma = 1, \tag{3}$$

any two of the parameters α, β and γ uniquely determine the conditional joint distribution of the contestants' signals, given d.

A contestant's probability of winning the contest is determined by the lobbying efforts directed by him and by his rival to the decisive bureaucrat. This probability is given by the simple and often used version of Tullock's (1980) contest success function, namely, the contestant's lobbying effort divided by the sum of the two contestants' efforts directed to this bureaucrat. Without loss of generality, the contest prize is normalized to 1. To determine the symmetric Bayesian equilibrium in our game, let x and y denote the lobbying effort a

contestant directs to the bureaucrat his signal indicates is the decisive one and to the other bureaucrat, respectively. In equilibrium, (x, y) maximizes the contestant's expected payoff, taking into account the four possible combinations of signals he and his rival receive. The equilibrium condition is that a contestant cannot increase his expected payoff by changing x and y to some alternative X and Y. If a contestant unilaterally deviates from the equilibrium efforts by making these changes, his expected payoff becomes:

$$\frac{\alpha X}{X + x} + \frac{\gamma X}{X + y} + \frac{\gamma Y}{Y + x} + \frac{\beta Y}{Y + y} - X - Y.$$

This is a concave function of X and Y, with a critical point where

$$\frac{\alpha x}{(X + x)^2} + \frac{\gamma y}{(X + y)^2} = 1$$

and

$$\frac{\gamma x}{(Y + x)^2} + \frac{\beta y}{(Y + y)^2} = 1.$$

Therefore, the conditions that characterize an interior equilibrium are

$$\frac{\alpha}{4x} + \frac{\gamma y}{(x + y)^2} = 1$$

and

$$\frac{\beta}{4y} + \frac{\gamma x}{(x + y)^2} = 1,$$

or

$$x (x + y)^2 = \frac{1}{4}\alpha (x + y)^2 + \gamma xy$$

and

$$y (x + y)^2 = \frac{1}{4}\beta (x + y)^2 + \gamma xy.$$

Subtracting and adding these equations, respectively, gives

$$x - y = \frac{1}{4}(\alpha - \beta)$$

and

$$(x + y)^3 = \frac{1}{4}(\alpha + \beta)(x + y)^2 + 2\gamma xy.$$

By (1), (2) and (3), this yields

$$x - y = \frac{1}{4}(p - q) \tag{4}$$

and

$$\frac{1}{4}(x+y)^2 - (x+y)^3 = (1-\rho)\,pq\left(\frac{1}{2}(x+y)^2 - 2xy\right) = \frac{1}{8}(1-\rho)(1-\delta^2)(x-y)^2,$$

where

$$\delta = p - q. \tag{5}$$

It follows that, at a symmetric equilibrium, each contestant's total effort,

$$z = x + y, \tag{6}$$

satisfies the cubic equation

$$\frac{1}{4}z^2 - z^3 = \frac{1}{128}\Delta, \tag{7}$$

where

$$\Delta = (1-\rho)\,\delta^2\left(1-\delta^2\right) \tag{8}$$

$(= 4\gamma(\alpha - \beta)^2)$. By (4), (5) and (6), the lobbying efforts directed to the two bureaucrats are given by

$$x = \frac{1}{2}z + \frac{1}{8}\delta$$

and

$$y = \frac{1}{2}z - \frac{1}{8}\delta.$$

These lobbying efforts are both positive if and only if

$$z > \frac{|\delta|}{4}.$$

The cubic polynomial $R(z) = (1/4)z^2 - z^3$ is nonnegative only for $z \le 1/4$. Between zero and 1/4, it is unimodal, peaking at $z = 1/6$ (see Figure 1). Since the assumption $0 < p < 1$ implies that $-1 < \delta < 1$, for $\delta \neq 0$

$$R\left(\frac{|\delta|}{4}\right) = \frac{\delta^2}{64}(1 - |\delta|) = \frac{\delta^2}{64} \cdot \frac{1-\delta^2}{1+|\delta|} > \frac{\Delta}{128}.$$

This implies that Equation (7) has a unique solution z yielding positive x and y, which lies on the downward-sloping part of the graph of R, where $z \ge 1/6$. Therefore, the equilibrium
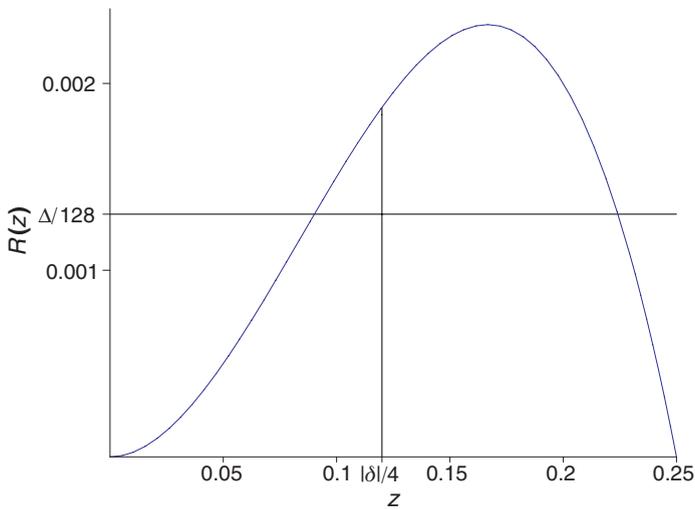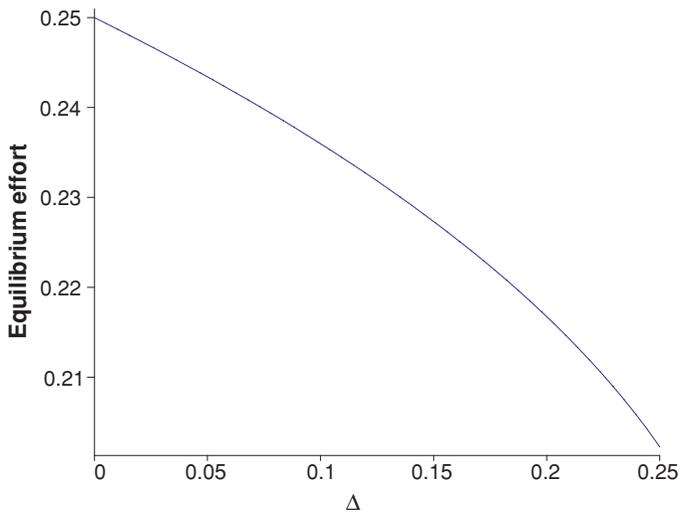
Fig. 1 A graphical representation of Equation (7)

Fig. 2 The relationship between the equilibrium total lobbying effort z of each contestant and the parameter Δ

effort z is determined by Δ as a strictly decreasing function (see Figure 2). Specifically,[3]

$$z = \frac{1}{12} + \frac{1}{6} \cos \frac{\theta}{3},$$

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[3] This can be checked by substitution into $R(z)$, which gives $\frac{1}{864} + \frac{1}{288} \cos \frac{\theta}{3} - \frac{1}{216} \cos^3 \frac{\theta}{3}$, and using the identity $\cos^3 \frac{\theta}{3} = \frac{3}{4} \cos \frac{\theta}{3} + \frac{1}{4} \cos \theta$.

where

$$\theta = \arccos\left(1 - \frac{27\Delta}{4}\right).$$

This gives, in particular, that the minimum value of z, which is attained when Δ attains its maximum value of $1/4$, is about 0.202.

3 Comparative statics

Our comparative statics analysis is based on the dependence of the equilibrium total effort on Δ, which is established above, and on the latter's dependence on the parameters ρ and p, which is spelled out in Equation (8). Our first result follows immediately from the fact that for $\Delta = 0$, Equation (6) gives $z = 0.25$.

Proposition 1. *Each contestant's total lobbying effort z is maximal when the contestants' signals are perfectly uninformative ($p = 0.5$) or are perfectly correlated ($\rho = 1$). In these cases, the aggregate effort $2z$ is equal to 50% of the prize. This level of aggregate effort, or rent dissipation, equals that in Tullock's benchmark case, in which the source of power is known, and is approached also in the limit when the signals become perfectly informative, i.e., $p \to 1$ or $p \to 0$.*

The second comparative statics observation that we make is that z is positively related to ρ, holding p constant. Specifically, for fixed $\delta \neq 0$, Δ decreases as ρ increases from 0 to 1. Consequently, z increases monotonically, reaching its maximum of 0.25 at $\rho = 1$.

Proposition 2. *If the signals are not perfectly uninformative, then the aggregate effort increases with increasing conditional correlation between the signals ρ.*

For fixed $0 \leq \rho < 1$, Δ is determined by δ as a unimodal function that peaks where $\delta^2 = 1/2$. Since $p = (1 + \delta)/2$, this gives the following result.

Proposition 3. *If the signals are not perfectly correlated (i.e., if $0 \leq \rho < 1$), each contestant's total effort z is determined by the informativeness of the contestants' signals p as a U-shape function, with a minimum at*

$$p = \frac{1}{2} + \frac{1}{4}\sqrt{2} \approx 0.854. \tag{9}$$

Combining the above results with the numerical one at the end of Section 2 gives the following result, which is illustrated by Figure 3.

Proposition 4. *If the signals are not perfectly correlated and are not perfectly informative or uninformative, the aggregate equilibrium lobbying effort is lower than 50% of the prize. The minimum aggregate lobbying effort, which is about 40% of the prize, is attained when the contestants' signals are conditionally uncorrelated ($\rho = 0$) and their informativeness is given by (9).*
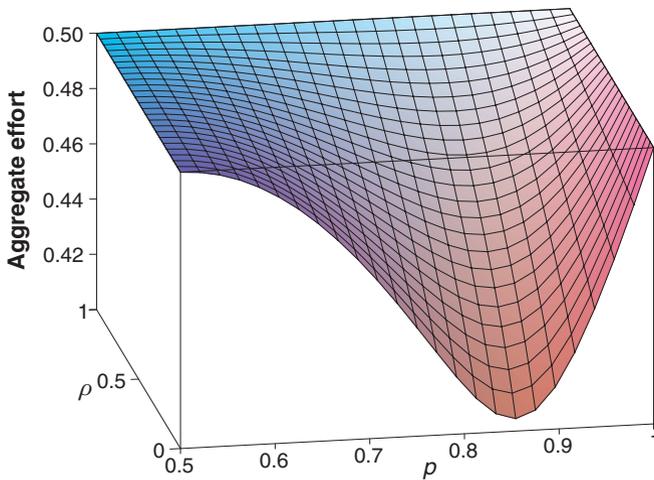
Fig. 3 The aggregate equilibrium lobbying effort as a function of the informativeness of the contestants' signals p and the conditional coefficient of correlation between them ρ

Our results strongly suggest that the contestants' equilibrium total efforts reflect their signals' informational content: both about the decision maker's identity d (e.g., whether the signals are informative) and about each other (e.g., whether they are conditionally correlated, given d). In particular, the efforts are maximal in the cases of perfect uninformativeness or conditional correlation. These two extreme cases share a common feature. Namely, given one contestant's signal s, knowing the other's signal s' would not add any information about d. In the case of uninformative signals, this is simply because both signals are valueless. In the case of perfectly correlated signals, no information is added since always $s = s'$.

This observation raises the question of whether, more generally, the contestants' equilibrium efforts are a function of the difference between the information about d conveyed by the pair of signals s and s' and the information conveyed by the contestant's own signal s alone. To give 'information' an exact, quantitative meaning we may resort to information theory (Cover & Thomas, 1991). This task is left for future research.

4 Discussion

Recall that in Tullock's two-player, symmetric, simple lottery contest without ambiguity regarding the source of power, the rent dissipation is equal to 50% of the contested prize. In other words, the aggregate effort is equal to 0.5. Proposition 1 establishes that the extent of rent dissipation is invariant to ambiguity regarding the source of power if $p = 1/2$, p is close to 1, or $\rho = 1$. Thus, under such extreme circumstances, incomplete information is inconsequential in terms of the total lobbying outlays spent by the contestants in attempting to win the contested prize. The explanation for this seemingly surprising result is simple. Under the assumed Tullock's contest success function, which is homogeneous of degree zero in the contestants' lobbying efforts, the contestants' winning probabilities depend on the relative size of the lobbying expenditures (See Epstein & Nitzan, 2006). When the signals are symmetric in one of the above three senses, the contestants allocate their lobbying budgets between the two bureaucrats according to their "symmetric" signals and this does not affect

the relative investment in the decisive bureaucrat in comparison to the situation where the source of power is known. Note that a situation of perfectly uninformative, useless signals means symmetry because the signals do not matter and, therefore, they have no effect on the contestants' incentives to make efforts. A situation of perfectly informative signals implies that the two contestants receive the same correct signal, that is, the power structure is fully transparent. This means that the situation is essentially identical to the one where the source of power is known. Finally, when the signals are perfectly correlated, they need not be correct. However, the contestants are equal in terms of the information they get, they do allocate their efforts between the two potential decision makers, but these efforts are unaltered relative to the situation where the identity of the source of power is known.

Imperfect correlation between the signals received by the contestants implies asymmetry between the contestants. By Proposition 2, such asymmetry tends to reduce the contestants' aggregate efforts. For a given intermediate level of correlation between the signals, aggregate efforts depend on the quality of the signals. By Proposition 3, the U-shape relationship between aggregate efforts and signal quality p is such that aggregate efforts decrease (increase) with signal quality when p is smaller (larger) than 0.854. By Proposition 4, ambiguity regarding the source of power in the contest usually reduces the extent of rent dissipation relative to the standard Tullock's benchmark case where the source of power is known. The minimal aggregate lobbying efforts of about 40% of the contested prize is obtained when the signals received by the contestants are not correlated at all and their quality is equal to about 0.854.

Although our preliminary analysis is based on a stylized contest setting, it illustrates the significance of incomplete information regarding the distribution of power in a bureaucratic or a political system. The propositions presented above demonstrate that the analysis of lobbying contests that involve bureaucratic or political aspirations in addition to economic interests may shed new light on the role of asymmetric information when the contestants face ambiguity regarding the identity of the real decision maker.

One possible generalization of our analysis of symmetric Bayesian Nash equilibrium that is worth pursuing is the introduction of asymmetry between the potential decision makers by assuming different priors that the bureaucrats are the real decision makers. Such a generalization adds a new parameter: the prior probability that one of the bureaucrats is the real decision maker. The question is, how does this asymmetry between the two bureaucrats, together with p and ρ, affects the lobbying efforts of the contestants. Another interesting issue is the clarification of the role of information in our setting as a factor determining the contestants' behavior. Both of these issues are left for future research.

Finally, while we have focused on the behavior of the contestants, given their signals regarding the identity of the actual decision maker, further research is required to examine the robustness of our results if one allows for the endogenous formation of signals that usually hinge on the behavior of bureaucrats or politicians. Clearly, there is a significant difference between the situations in a certain and an uncertain decision-making system because in the latter situation, some of the lobbying outlays go to irrelevant bureaucrats who therefore have a clear incentive to scatter more fog on the identity of the "power center" in the system. Obviously, the bureaucrats do not share the same view regarding the desired quality of signals. The higher this quality is, the higher the expected receipts of the decisive bureaucrat and the lower the receipts of the powerless one. As quality of beliefs declines, more non-decisive bureaucrats can pretend to be influential and share some of the contestants' outlays with the decisive bureaucrat. Bureaucrats therefore have a clear incentive to invest in propaganda campaigns to convince lobbyists that they control the power of decision-making. Apparently, the optimal investment level in propaganda for each bureaucrat depends on his ability to affect the contestants' signals and, in turn, their lobbying outlays directed to him.

These considerations are disregarded in the present study in which we have confined our attention to the simpler contest where the bureaucrats' propaganda investments and, in turn, the contestants' signals about the identity of the actual decision maker are exogenously given.

References

Aumann, R.J. (1976). Agreeing to disagree. *Annals of Statistics*, *4*, 1236–1239.

Baik, K.H. (1999). Rent-seeking firms, consumer groups, and the social costs of monopoly. *Economic Inquiry*, *37*, 542–554.

Baik, K.H. (1994). Effort levels in contests with two asymmetric players. *Southern Economic Journal*, *61*, 367–378.

Cairns, R.D., & Long, N.V. (1991). Rent seeking with uncertain opposition. *European Economic Review*, *35*, 1223–1235.

Che, Y.K., & Gale, I. (1998). Caps on political lobbying. *American Economic Review*, *88*, 643–651.

Che, Y.K., & Gale, I. (1997). Rent dissipation when rent seekers are budget constrained. *Public Choice*, *92*, 109–126.

Chung, T.-Y. (1996). Rent-seeking contest when the prize increases with aggregate efforts. *Public Choice*, *87*, 55–66.

Cover, T.M., & Thomas, J.A. (1991). *Elements of information theory*. New York: Wiley-Interscience.

Davis, D.D., & Reilly, R.J. (1999). Rent-seeking with non-identical sharing rules: An equilibrium rescued. *Public Choice*, *100*, 31–38.

Epstein, G.S., & Nitzan, S. (2006). Effort and performance in public-policy contests. *Journal of Public Economic Theory*, *8*, 265-282 .

Epstein, G.S., & Nitzan, S. (2004). Strategic restraint in contests. *European Economic Review*, *48*, 201–210.

Epstein, G.S., & Nitzan, S. (2003). Political culture and monopoly price determination. *Social Choice and Welfare*, *21*, 1–19.

Epstein, G.S., & Nitzan, S. (2002). Endogenous public policy, politicization and welfare. *Journal of Public Economic Theory*, *4*, 661–677.

Ellingsen, T. (1991). Strategic buyers and the social cost of monopoly. *American Economic Review*, *81*, 648–657.

Geraats, P.M. (2002). Central bank transparency. *The Economic Journal*, *112*, F532–F565.

Gradstein, M. (1995). Intensity of competition, entry and entry deterrence in rent-seeking contests. *Economics and Politics*, *7*, 79–91.

Hillman, A.L., & Riley, J.C. (1989). Politically contestable rents and transfers. *Economics and Politics*, *1*, 17–39.

Hurley, T.M. (1998). Rent dissipation and efficiency in a contest with asymmetric valuations. *Public Choice*, *94*, 289–298.

Hurley, T.M., & Shogren, J.F. (1998a). Effort levels in Cournot-Nash contest with asymmetric information. *Journal of Public Economics*, *69*, 195–210.

Hurley, T.M., & Shogren, J.F. (1998b). Asymmetric information and contests. *European Journal of Political Economy*, *14*, 645–665.

Kahana, N., & Nitzan, S. (1999). Uncertain preassigned non-contestable and contestable rents. *European Economic Review*, *43*, 1705–1721.

Keem, J.H. (1998). The social cost of monopoly under asymmetric lobbying abilities. Mimeo.

Kohli, I., & Singh, N. (1999). Rent seeking and rent setting with asymmetric effectiveness of lobbying. *Public Choice*, *99*, 275–298.

Konrad, K.A., & Schlesinger, H. (1997). Risk aversion in rent seeking and rent augmenting games. *Economic Journal*, *107*, 1671–1683.

Leininger, W. (1993). More efficient rent seeking – a Munchhausen solution. *Public Choice*, *75*, 43–62.

Nitzan, S. (1994). Modelling rent-seeking contests. *European Journal of Political Economy*, *10*, 41–60.

Nitzan, S. (1991). Rent seeking with non-identical sharing rules. *Public Choice*, *71*, 43–50.

Nti, K.O. (1999). Rent seeking with asymmetric valuations. *Public Choice*, *98*, 415–430.

Nti, K.O. (1997). Comparative statics of contests and rent seeking games. *International Economic Review*, *38*, 43–59.

Stein, W.E. (2002). Asymmetric rent seeking with more than two contestants. *Public Choice*, *113*, 325–336.

Singh, N., & Wittman, D. (2001). Contests where there is variation in the marginal productivity of effort. *Economic Theory*, *18*, 711–744.

Tullock, G. (1980). Efficient rent seeking. In J.M. Buchanan, R.D. Tollison, & G. Tullock (Eds.) *Toward a theory of the rent seeking society* (pp. 97–112). College Station: Texas A&M University Press.

Wärneryd, K. (2003). Information in conflicts. *Journal of Economic Theory*, *110*, 121–136.