Quasi-Hyperbolic Discounting and Social Security Systems

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Abstract

Hyperbolic discounting has become a common assumption for modeling bounded rationality with respect to individual savings decisions. We examine the effects of hyperbolic discounting on the comparison of alternative social security systems. We show that this form of bounded rationality breaks the equivalence between *funded* and *pay-as-you-go* systems established in Sheshinski and Weiss (1981). Intergenerational transfers within a PAYG economy are usually secured by the social security system and independent of longevity, whereas this is not the case for the funded economy. The savings level under hyperbolic discounting is lower than under exponential discounting (Laibson et al. 1996), but the ratio between the savings level under hyperbolic discounting within a funded economy and a PAYG economy depends on the effectiveness of the commitment devices. It is shown that if individuals are hyperbolic discounters, then in a PAYG economy any change in the mandated level of intergenerational transfers is neutralized by individuals' voluntary bequests. This does not apply to a funded system.

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1. Introduction

Time inconsistency of individual preferences and the notion of dynamic preferences or with evolving "selves" is a well-known and studied phenomenon (Pollak 1968, Phelps and Pollak 1968, Yaari and Peleg 1972, Hammond 1976, Thaler and Shefrin 1981, Schelling 1984). The prevalent tool for modeling time inconsistency of individual preferences in the last decades of the twentieth century was the hyperbolic discounting function (Ainslie 1992, Lowenstein and Prelec 1992, Laibson et al. 1996, Laibson 2001), and such modeling can be traced back to the mid-twentieth century (Strotz 1956, Chung and Herrnstein 1961).

The hyperbolic discounting function was applied to the study of undersaving and the sharp reduction in consumption of old people (Laibson 1996), to the early retirement pattern of workers (Laibson et al. 1998, Diamond and Kőszegi 2002), and to job search behavior (Paserman and Della-Vigna 2000).

The hyperbolic or quasi-hyperbolic discounting hypothesis (henceforth, HDH[1]), as an empirical finding, was based on experiments performed on humans and animals that the researchers interpreted as supporting this hypothesis (for a survey, see Laibson *et al.* 1998). However, as an empirical fact, HDH is controversial (Rubinstein 2001, 2002, Read 2001, Besharov and Coffey 2003). From a theoretical point of view, hyperbolic discounting raises several problems. For instance, time inconsistency reflects irrationality, or at least bounded rationality, especially in its naïve version. However, in certain circumstances of uncertainty, hyperbolic discounting can be reconciled with rationality and does not necessarily generate time inconsistency and reversal of preferences (Weitzman 1998, Azfar 1999, 2002, Dasgupta and Maskin 2002).

Our purpose in this paper is neither to decide on the empirical controversy nor to find any further theoretical justifications for assuming hyperbolic discounting by rational individuals. Rather, taking HDH for granted and leaving aside the empirical controversy, we explore its implications on optimal social security systems. The main result of this paper is that the equivalence between an optimal *pay-as-you-go* intergenerational transfers system (henceforth a PAYG system) and an optimal *funded*

[1] We shall use the terms "hyperbolic discounting" and "quasi-hyperbolic discounting" interchangeably.

pension system, established in Sheshinski and Weiss (1981, henceforth SW), does not hold in general, under hyperbolic discounting. The model we analyze in this paper is an extension of the SW model. In order to enable analysis of the hyperbolic discounting effect, we have added a third period to the standard two periods' model.

We show that the equivalence between funded and PAYG pension systems, established in SW, holds only under exponential discounting, (which is a special case of the hyperbolic discounting). A social security system provides perfect insurance under PAYG structure, whereas in a funded economy bequests are random and depend on realized longevity. The correlation between bequests and longevity in a funded economy depends on the elasticity of the parents' marginal utility from bequests. A similar result is obtained for savings. The savings level is identical in the two economies only if effective commitment devices exist in the PAYG economy. We also analyze the effect of government intervention in private decisions (i.e., introducing a compulsory pension scheme) and show that under hyperbolic discounting, such intervention has different consequences on the two types of economies.

The theoretical equivalence of PAYG and funded social security systems in various theoretical models was generally ignored in pension reforms carried out in western countries. The common direction of all these reforms is a shift from a PAYG system to a funded system. However, the logic of this shift and its costs are controversial. It has been argued that one social security method has no advantages over the other (Orszag and Stiglitz, 1999). Our results imply that the ability to rank these systems holds only under the limiting assumption of exponential discounting. If people are hyperbolic discounters then each pension method has its own unique advantages and disadvantages.

In the next section we briefly describe the hyperbolic discounting function and the time inconsistency that it causes in individual decisions. Next, we present the familiar SW model modified for the hyperbolic discounting analysis using a three-period version. The following sections include a number of propositions and a discussion about their implications for optimally-designed pension systems. The final section is a discussion and conclusions.

2. Quasi-Hyperbolic Discounting and Time Inconsistency

Assume an individual who lives T periods $(T \geq 3)$ and derives utility from consumption at each period. The utility function of the individual in period t is $u(c_t)$ and we make the standard assumptions regarding this function, namely $u'(c_t) > 0, \quad u''(c_t) < 0$. Suppose that life-cycle utility is a Von-Neuman Morgenstern, additively separable utility function. Namely, it is the sum of all *discounted* one-period utilities. The form of the discounting function and its implications are the core of our analysis. Assuming hyperbolic discounting implies that the individual life-cycle utility is,

(1) $$V_0\left(\{c_t\}_{t=0}^{T}\right) = u(c_0) + \beta \sum_{t=1}^{T} \delta^t u(c_t)$$

Where $0 < \delta < 1$ and $0 < \beta \leq 1$.

A glance at (1) reveals that standard exponential discounting is actually a particular case of hyperbolic discounting, with $\beta = 1$. The discount factor is δ. When $\beta < 1$, the individual assigns a relatively higher weight to current period's consumption than to that of future periods, as can readily be verified by comparing the ratio of the utilities between second and first period utility, $\beta\delta$, to the ratio between any following periods' utility for $t \geq 1$, which is δ.

Equation (1) presents the optimization problem that an individual faces at $t = 0$. The solution to this problem being a sequence, $\left\{c_t^*\right\}_{t=0}^{T}$, which corresponds to his view at this point in time. However, at $t = 1$, for instance, his view of the relative weight of utility of different periods changes. In period 1, the life-cycle utility function describing the individual's preferences is

(2) $$V_1\left(\{c_t\}_{t=1}^{T}\right) = u(c_1) + \beta \sum_{t=2}^{T} \delta^t u(c_t)$$

and the optimal consumption sequence from the first period's point of view is therefore $\left\{\hat{c}_t\right\}_{t=1}^{T}$. Time inconsistency arises from the fact that first period's solution implies $\frac{u'\left(c_1^*\right)}{u'\left(c_0^*\right)}=\beta\delta$ and $\frac{u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)}=\delta,\ \forall t\geq 1$, while the solution from the second period's point of view implies $\frac{u'\left(\hat{c}_2\right)}{u'\left(\hat{c}_1\right)}=\beta\delta$ and $\frac{u'\left(\hat{c}_{t+1}\right)}{u'\left(\hat{c}_t\right)}=\delta\ \ \forall t\geq 2$. Thus, $c_t^*\neq\hat{c}_t$. In other words, in each period the individual changes his mind regarding his optimal life-cycle path of consumption. The critical issue in this framework is the reversibility of decisions made in the past and the individual's level of sophistication. If decisions were irreversible, a sophisticated consumer would consider this when deciding on his optimal level of consumption at each decision node during the life cycle.

Irreversibility of past decisions seems to be a very important feature for social security systems, especially if individuals are time-inconsistent. In a PAYG system, for example, if representatives of the old generations control the intergenerational transfers, insufficient savings while young might result in smaller net bequests rather than smaller consumption. Namely, if past decisions are reversible, future generations will have to pay the bill for insufficient old generation savings, while old generations will continue to smooth their life-cycle consumption. On the other hand, in a funded economy, past decisions seem less reversible.

3. The model

In order to demonstrate the time inconsistency, analysis of a hyperbolic discounter's optimization problem requires at least a three-period model. For the sake of convenience, we analyze the consumer's optimization problem under a funded system and under a PAYG system separately. In the next section, we compare the results of the analysis of the standard model (with exponential discounting) to the more general case of hyperbolic discounting.

Denote by c_0^i and c_1^i the consumption of generation i individual during the first and second periods, respectively. Assume a three-period overlapping-generations model. In this economy, all individuals of the same generation have identical instantaneous preferences represented by a utility function $u\left(c\right)$, and future utilities are discounted using hyperbolic discounting. An individual begins life with an initial wealth B_0 inherited from his parents, and derives utility from bequeathing to G children.[2] Denote by θ the fraction of potential retirement period that is actually realized, $(0 \le \theta \le 1)$. We assume that the distribution of θ is the same for all generations.

Funded system

In a funded economy the individual has to decide during the first period how much to consume, and how much to contribute to a funded pension scheme and how much to save. So the optimization problem of a representative hyperbolic discounter within a funded economy in his first period is,

$$\begin{aligned} & V^i = \max_{s,a} \left\{ u\left(c_0^i\right) + \beta\delta \mathop{E}_{\theta}\left[v\left(c_1^i, \theta\right) + \delta G h\left(B^i\right)\right]\right\} \\ (3) \qquad & s.t. \\ & c_0^i = w + B^{i-1} - s^i - a^i \end{aligned}$$

Where $0 \le \beta, \delta \le 1$ are discount factors, $v\left(c_1^i, \theta\right)$ is second period utility of the flow of consumption c_1^i given θ, $(v_1 > 0,\ v_{11} < 0,\ v_2 > 0)$, $h\left(B^i\right)$ is the individual's utility from bequests B^i $(h' > 0,\ h'' < 0)$ which occurs in the subsequent period (three) and hence discounted by δ, a and s are pension contributions and savings, respectively and w is wages (obtained only in the first-period and the same for all generations) and B^{i-1} is initial endowment (bequests from the previous generation).

However, due to the hyperbolic discounting function, V^i is not a contraction mapping and therefore has no fixed point (Laibson et al. 1996). Solving this problem is possible by applying a backwards induction technique.

During the third period, the individual has no decision to make. So we begin our analysis in the second period in which the consumer's optimization problem is:

[2] The timing of the children's birth on the individual life-cycle is unimportant in this model.

$$\max_{c_1^i} \underset{\theta}{E}\left\{v\left(c_1^i,\theta\right)+\beta\delta G h\left(B^i\right)\right\}$$
(4) $$s.t.$$
$$GB^i = Rs^i + \left(\frac{Ra^i}{\bar{\theta}} - c_1^i\right)\theta$$

The first order condition (F.O.C) for an interior solution is:

(5) $$\underset{\theta}{E}\left[v_1^i\left(\hat{c}_1^i,\theta\right)-\beta\delta h'\left(\hat{B}^i\right)\theta\right]=0$$

With solutions $\hat{c}_1^i = \hat{c}_1^i\left(s^i,a^i\right)$ and $\hat{B}^i\left(s^i,a^i,\theta\right)$. Since the objective function of the second period is strictly concave, the solutions are unique, which is also the case for all solutions in this paper.[3]

Inserting $\hat{c}_1^i$ and $\hat{B}^i$ into (3) yields the hyperbolic consumer first-period optimization problem:

$$V^i = \max_{s^i,a^i}\left\{u(c_0^i)+\beta\delta \underset{\theta}{E}\left[v\left(\hat{c}_1^i,\theta\right)+\delta G h\left(\hat{B}^i\right)\right]\right\}$$
(6) $$s.t.$$
$$c_0^i = w + B^{i-1} - s^i - a^i$$

The F.O.C are:

(7) $$\frac{dV^i}{ds} = -u'(\hat{c}_0^i) + \beta\delta \underset{\theta}{E}\left[\left(v_1^i(\hat{c}_1^i,\theta)-\delta h'\left(\hat{B}^i\right)\theta\right)\frac{d\hat{c}_1^i}{ds}+\delta h'\left(\hat{B}^i\right)R\right]=0$$

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[3] In view of our assumptions about the concavity of $u\left(c_0^i\right)$, $v\left(c_1^i,\theta\right)$ with respect to c_0^i and c_1^i, and also of $h\left(B^i\right)$, the maximization of (4) has a unique solution.

$$\begin{aligned}\frac{dV^i}{da} &= -u'(\hat{c}_0^i)\\ &+\beta\delta \underset{\theta}{E}\left[\left(v_1^i(\hat{c}_1^i,\theta)-\delta h'\left(\hat{B}^i\right)\theta\right)\frac{d\hat{c}_1^i}{da}+\delta h'\left(\hat{B}^i\right)\frac{R\theta}{\bar{\theta}}\right]=0\end{aligned} \tag{8}$$

With solutions $\hat{s}^i$, $\hat{a}^i$ and the corresponding $\hat{c}_0^i$.

Pay as You Go System

The first period optimization problem of a hyperbolic discounter in a PAYG economy is:

$$\begin{aligned}&V^i = \max_s \left\{u\left(c_0^i\right)+\beta\delta \underset{\theta}{E}\left[v\left(c_1^i,\theta\right)+\delta Gh\left(B^i-a^i\right)\right]\right\}\\ &s.t.\\ &c_0^i = w+B^{i-1}-s^i-a^{i-1}\end{aligned} \tag{9}$$

Note that the contribution of the individual to the PAYG system, a^{i-1} is given while the contribution of the next generation, a^i, is endogenous. This reflects the assumption that the "older" population determines the taxes paid by workers. Again, this problem can be solved by using backwards induction. In his second period, the consumer's problem is:

$$\begin{aligned}&\max_{c_1^i,a^i} \underset{\theta}{E}\left\{v\left(c_1^i,\theta\right)+\beta\delta Gh\left(B^i-a^i\right)\right\}\\ &s.t.\\ &GB^i = Rs^i+\left(\frac{Ga^i}{\bar{\theta}}-c_1^i\right)\theta\end{aligned} \tag{10}$$

which yields the following F.O.C:

$$\underset{\theta}{E}\left[v_1^i\left(c_1^{i*},\theta\right)-\beta\delta h'\left(B^{i*}-a^{i*}\right)\theta\right]=0 \tag{11}$$

(12) $$\beta\delta \underset{\theta}{E}\left[h'\left(B^{i*}-a^{i*}\right)\left(\frac{\theta}{\overline{\theta}}-1\right)\right]=0$$

Denote the solutions by $c_1^{i*}\left(s^i\right)$, $a^{i*}\left(s^i\right)$ and $B^{i*}\left(s^i\right)$.

Now going back to the first period, inserting $c_1^{i*}\left(s^i\right)$, $a^{i*}\left(s^i\right)$ and $B^{i*}\left(s^i\right)$ into (9) yields:

(13) $$\begin{aligned}&V^i=\max_{s^i}\left\{u\left(c_0^i\right)+\beta\delta \underset{\theta}{E}\left[v\left(c_1^{i*},\theta\right)+\delta Gh\left(B^{i*}-a^{i*}\right)\right]\right\}\\&s.t.\\&c_0^i=w+B^{i-1}-s^i-a^{i-1}\end{aligned}$$

Which has the first order condition:

(14) $$\frac{\partial V^i}{\partial s}=-u'(c_0^{i*})+\beta\delta \underset{\theta}{E}\left[\begin{array}{r}\left(v_1^i\left(c_1^{i*},\theta\right)-\delta h'\left(B^{i*}-a^{i*}\right)\theta\right)\dfrac{dc_1^{i*}}{ds}\\+\delta h'\left(B^{i*}-a^{i*}\right)\left(R+G\left(\dfrac{\theta-\overline{\theta}}{\overline{\theta}}\right)\dfrac{da^{i*}}{ds}\right)\end{array}\right]=0\,.$$

With solution s^{i*} and the corresponding c_0^{i*}.

3. Comparing the Two Systems under HDH

In this section we show that assuming hyperbolic discounting breaks the equivalence between funded and PAYG systems, established in Sheshinski and Weiss (1981). Thus, the equivalence holds only in the special case $\beta=1$, i.e. under exponential discounting.

Proposition 1

Bequests under a PAYG system are independent of θ, while under a funded system bequests are random and dependent on θ.

Proof: see appendix

Proposition 1 states that the equivalence of PAYG and funded systems, established by SW (1981), is violated if individuals are hyperbolic discounters, PAYG system isolates both bequests and second period consumptions from fluctuations of longevity while funded systems fails to do so. The intuition of this result stems from the well known fact that hyperbolic discounting creates time inconsistency whenever there is a time gap between taking a decision and implementing it and there are no commitment devices. In a funded economy, all decisions are made in the beginning of the first period, but the execution of second period consumption and bequest is delayed one period. Since preferences are not time consistent, this delay opens the gate for reversals and corrections of past taken decisions. In a PAYG system, the decisions about second period consumptions and bequests are all made simultaneously during the second period leaving no room for time inconsistency caused by hyperbolic discounting. When individuals discount future utilities using the conventional exponential discounting function, no time inconsistency arises and the two alternative systems are equivalent.

Since by proposition 1 bequests under a funded system with hyperbolic discounters are random and depend on θ, risk averse individuals (regarding the well being of their descendants) would try to lower the hazard rate over their planed bequests by contributing "enough" to their pension fund, ensuring that they will not have to reduce bequest because insufficient pension benefits. Following Kimball (1990), we define the "relative prudence coefficient" regarding welfare of future generations as $\eta\left(\hat{B}^i\right) \equiv -\frac{\hat{B}^i h'''\left(\hat{B}^i\right)}{h''\left(\hat{B}^i\right)}$. We also define $\varepsilon\left(\hat{B}^i\right) \equiv \frac{\theta}{\hat{B}^i}\eta\left(\hat{B}^i\right)$, the "modified coefficient of relative prudence", which is Kimball's coefficient multiplied by the ratio between the hazard rate and the sum under risk.

Proposition 2:

In a funded economy with hyperbolic discounters,

$$(15)\qquad \begin{aligned} &(a) \qquad \varepsilon\left(\hat{B}\right) \mathop{\gtrless} 1 \Rightarrow \frac{R\hat{a}}{\bar{\theta}} - \hat{c}_1^i \mathop{\gtrless} 0 \\ &(b) \qquad \varepsilon\left(\hat{B}^i\right) \neq 1 \end{aligned}$$

Proof: *See appendix.*

Proposition 2 states that as individuals are more "prudent" regarding the welfare of their descendants, they will insure their bequests by contributing more to pension fund, lowering the probability that they will have to reduce planed bequest to finance consumption due to insufficient pension benefits.

Proposition 3 provides a necessary and sufficient condition for a higher saving rate under a funded system.

Proposition 3

Assuming hyperbolic discounting

$$(16)\qquad \operatorname{sgn}\left(\hat{c}_0^i - c_0^{i*}\right) = \operatorname{sgn}\left(\frac{dc_1^{i*}}{ds^i} - \frac{d\hat{c}_1^i}{ds^i}\right) = -\operatorname{sgn}\left(\frac{da^{i*}}{ds^i}\right)$$

Proof: *See appendix.*

Proposition 3 states that first period savings of a hyperbolic individual in a funded economy is higher (lower) than first period savings of the same individual in a PAYG economy if $\frac{da^{i*}}{ds^i}$ is negative (positive). This derivative reflects the exogenous institutional arrangements of commitment devices within an economy. In the funded economy, the consumer decides about both $\hat{s}$ and $\hat{a}$ at the same time (his first period), and there is no withdrawal from that decision. In a PAYG economy, on the other hand, the consumer decides about s^{i*} during his first period, while postponing the decision about $\left(B^{i*} - a^{i*}\right)$ to the second period. Therefore, the institutional arrangements of commitment devices are critical in this economy, and for this very reason no wonder why we can not infer $\text{sgn}\left(\frac{da^{i*}}{ds^i}\right)$ analytically, as explained in the proof of this proposition, (see appendix). Put differently, a funded economy is equivalent to an economy with effective commitment devices, since the consumer cannot reverse his past decisions about savings during his second period. Things are quite different in the PAYG system; the reversibility of past decisions depends on the political system, and especially on the age distribution of the ruling politicians. If the political system is ruled by old people, they may try to increase the PAYG pension benefits at the expense of young generations. Sophisticated young individuals should take this into account. Namely, they should take into account that their first period plan regarding $\left(B^{i*} - a^{i*}\right)$ will change when they reach their second period. If the institutional arrangements within the economy establish effective commitment devices, then $\frac{da^{i*}}{ds^i} = 0$ and savings levels within the two economies will coincide.

As explained in the proof for proposition 3 (see appendix), although $\frac{da^{i*}}{ds^i} < 0$ is a fairly plausible assumption, we have no way of proving it. However, under this assumption it follows from (16) that a hyperbolic consumer in funded economy saves less than in a PAYG economy, although the funded system with hyperbolic consumers provides no insurance for intergenerational transfers. This means that if $\frac{da^{i*}}{ds^i} < 0$, the "hyperbolic effect" or the "myopia" is dominant.

4. Compulsory Pension Schemes

Hyperbolic discounting, sometimes referred to as "myopia", is a common argument put forward by proponents of compulsory pension schemes in many economic and political debates. Compulsory savings means government intervention in private decisions, which is in essence a violation of individual freedom. In democracies, such intervention in private decisions is justified if and only if negative externalities are proved. In the social security context, myopic low savings rates lead to old age poverty which has negative externalities on society as a whole. Compulsory pension proponents claim that mandatory savings is the perfect correcting tax on myopic or hyperbolic consumers to prevent this negative externality.

In this section we will not go into the political, philosophical or moral aspects of bureaucratic intervention in private decisions "for the sake of the citizens". Our main interest here is to determine the economic consequences of such intervention on the pension market, either funded or PAYG.

Government Intervention in the Funded Economy

Suppose that the government sets a mandatory contribution for a pension fund of $\tilde{a} > \hat{a}$.

Second period optimization of the individual becomes:

(17)
$$\begin{aligned} &\max_{c_1^i} \underset{\theta}{E}\left\{v\left(c_1^i,\theta\right)+\beta\delta Gh\left(B^i\right)\right\} \\ &s.t. \\ &GB^i = Rs^i + \left(\frac{R\tilde{a}^i}{\bar{\theta}} - c_1^i\right)\theta \end{aligned}$$

The first order condition is:

(18)
$$\underset{\theta}{E}\left[v_1^i\left(\tilde{c}_1^i,\theta\right)-\beta\delta h'\left(\tilde{B}^i\right)\theta\right]=0$$

Implying $\tilde{c}_1^i = \tilde{c}_1^i\left(s^i,\tilde{a}^i\right)$, $\tilde{B}^i\left(s^i,\tilde{a}^i,\theta\right)$.

Proposition 4

$$\text{(A19)} \qquad \begin{aligned} &(a) \quad \tilde{c}_1^i\left(s^i,\tilde{a}^i\right) > \hat{c}_1^i\left(s^i,\hat{a}^i\right), \;\; \tilde{c}_0^i < \hat{c}_0^i \\ &(b) \quad \tilde{B}^i\left(s^i,\tilde{a}^i,\theta\right) > \hat{B}^i\left(s^i,\tilde{a}^i,\theta\right) \end{aligned}$$

Proposition 4 states that compulsory additional savings for pensions in the funded economy will result in higher second period consumption, higher rates of savings and larger bequests. The entire burden of the additional savings is carried by first period consumption. However, the randomness of bequests remains unchanged after government intervention. In other words, public policy will not insure intergenerational transfers or second period welfare by compulsory savings to funded schemes, if people discount future utilities using hyperbolic discounting function.

Proof: *See Appendix.*

Government Intervention in a PAYG system

Governmental intervention in a PAYG system means setting the rate of intergenerational transfers a^i from young to old generations at each period. Without loss of generality, suppose that the government sets a compulsory intergenerational transfer of $\widehat{a}^i > a^{i*}$.

Proposition 5

Compulsory intergenerational transfers in a PAYG economy cause bequests to be random, depending on longevity.

Proof: *See Appendix.*

Corollary

Governmental intervention in a PAYG system reduces it to an effectively funded system, with the same equilibrium of consumption and (random) net intergenerational transfers.

Proof: *See Appendix.*

The intuition here is quite clear. As mentioned above, the difference between these two systems under hyperbolic discounting stems from the fact that in a funded economy decisions regarding pension premiums are taken during the first period while decision about pension benefits are taken during the second, and this time gap is crucial when time preferences are inconsistent. In a PAYG economy, on the other hand, both decisions are taken during the same period (the second). If the government sets a^i arbitrarily, it is no more a decision variable of the individuals and they have no decision to make about it in their first period. They take both premiums and benefits of the pension system as given for the second period, implying that their optimization problem during all periods is the same as in the funded economy.

5. Discussion and Concluding Remarks

The global trend during the last decades of the twentieth century was a shift from PAYG DB pension systems to funded DC systems. The rationale for this shift is controversial (see Orszag and Stiglitz 1999). However, as Sheshinski and Weiss (1981) show, assuming standard exponential discounting provides the positive question of how this shift affects the real variables of the economy with a strict and clear answer. The two types of social security are equivalent and therefore, as long as the pension systems are optimally designed, no real change is expected.

In this paper we show that if people assign a higher weight to current period consumption relative to future periods as well as future generations consumption, this answer is no longer valid. While a PAYG system operates within the economy very similarly no matter what discounting function the individual actually uses, provided that there are effective commitment devices, the effect of the funded system and its equivalence to the PAYG system depends on the discounting function and on $\varepsilon\left(\hat{B}^i\right)$.

In an economy with hyperbolic discounters and effective commitment devices, a PAYG system might be preferable to a funded system, because it neutralizes bequests from the randomness of longevity. However, this neutrality is conditional on the refraining of the government from any attempt of intervention in the pension market. Effective governmental intervention in a PAYG economy with hyperbolic discounters reduces it to the funded equilibrium, but with random bequests and without the advantages of the funded system.

As we emphasized above, we have not attempted to solve the controversy surrounding hyperbolic discounting, especially not the empirical aspect. However, since it is unreasonable to assume that the features of a utility function change by the shift from a PAYG system to a funded system; if future evidence shows that savings and bequeathing patterns have changed because of the shift from one type of pension system to another, this may also serve as an empirical piece of evidence in favor of hyperbolic discounting.

6. Appendix

Proof of Proposition 1

It follows from (12) that

(A1) $$\underset{\theta}{E}\left[h'\left(B^{i*}-a^{i*}\right)\left(\theta-\bar{\theta}\right)\right]=\operatorname{cov}\left[h'\left(B^{i*}-a^{i*}\right),\theta\right]=0,$$

Since the optimal net bequest, $B^{i*}-a^{i*}$, is monotone in θ and $h''<0$, it follows that $\left(B^{i*}-a^{i*}\right)$ must be *independent* of θ. This is a well-known result, since Yaari (1965) has shown that annuities, when chosen optimally, provide the consumer with insurance that isolates his economic well-being from random shocks.

The striking fact here is that we do not have a condition like (12) under the funded system, indicating that the funded system cannot insure the well being of the consumer, provided that the individual is a hyperbolic discounter. Formally, by subtracting (8) from (7) we obtain:

(A2) $$\underset{\theta}{E}\left[v_1(\hat{c}_1^i,\theta)-\delta h'\left(\hat{B}^i\right)\theta\right]\left(\frac{d\hat{c}_1^i}{ds^i}-\frac{d\hat{c}_1^i}{da^i}\right)=\frac{\delta R}{\bar{\theta}}\underset{\theta}{E}\left[h'\left(\hat{B}^i\right)(\theta-\bar{\theta})\right].$$

Totally differentiating (5) with respect to s and a gives:

(A3)
$$\underset{\theta}{E}\left[v_{11}^{i}(\hat{c}_1^i,\theta)+\frac{\beta\delta}{G}h''\left(\hat{B}^i\right)\theta^2 \right]d\hat{c}_1^i-\frac{\beta\delta R}{G}E\left[h''\left(\hat{B}^i\right)\theta \right]ds=0$$
$$\underset{\theta}{E}\left[v_{11}^{i}(\hat{c}_1^i,\theta)+\frac{\beta\delta}{G}h''\left(\hat{B}^i\right)\theta^2 \right]d\hat{c}_1^i-\frac{\beta\delta R}{G\bar{\theta}}E\left[h''\left(\hat{B}^i\right)\theta^2 \right]da=0$$

Thus,

(A4)
$$\frac{d\hat{c}_1^i}{ds^i}=\frac{\beta\delta R}{\Delta G}\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta \right]>0$$
$$\frac{d\hat{c}_1^i}{da^i}=\frac{\beta\delta R}{\Delta G\bar{\theta}}\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta^2 \right]>0$$

Where $\Delta\equiv\underset{\theta}{E}\left[v_{11}^{i}\left(\hat{c}_1^i,\theta\right)+\frac{\beta\delta}{G}h''\left(\hat{B}^i\right)\theta^2 \right]<0$.

It follows from (A4) that:

(A5)
$$\frac{d\hat{c}_1^i}{ds^i}-\frac{d\hat{c}_1^i}{da^i}=-\frac{\beta\delta R}{\Delta G\bar{\theta}}\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right) \right]$$

From equation (5) we have:

(A6)
$$\begin{aligned}
&\underset{\theta}{E}\left[v_1^i\left(\hat{c}_1^i,\theta\right)-\delta h'\left(\hat{B}^i\right)\theta \right] \\
&=\underset{\theta}{E}\left[v_1^i\left(\hat{c}_1^i,\theta\right)-\beta\delta h'\left(\hat{B}^i\right)\theta+(\beta-1)\delta h'\left(\hat{B}^i\right)\theta \right] \\
&=(\beta-1)\delta E\left[h'\left(\hat{B}^i\right)\theta \right]
\end{aligned}$$

Inserting (A5) and (A6) into (A2) we get:

(A7) $$(1-\beta)\delta \underset{\theta}{E}\left[h'\left(\hat{B}^i\right)\theta\right]\frac{\beta}{\Delta G}\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right]=\underset{\theta}{E}\left[h'\left(\hat{B}^i\right)\left(\theta-\bar{\theta}\right)\right]$$

The right-hand side of (A7) is $\operatorname{cov}\left[h'\left(\hat{B}^i\right),\theta\right]$. Thus, setting $\beta=1$ on the left-hand side of (A7) yields $\operatorname{cov}\left[h'\left(\hat{B}^i\right),\theta\right]=0$. In other words, $\beta=1$ reduces our model to the familiar exponential discounting model of SW (1981) and the equivalence between the two types of social security systems is restored.

If $\beta<1$, than according to (A7) the other case in which $\operatorname{cov}\left[h'\left(\hat{B}^i\right),\theta\right]=0$ is when $\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right]=0$, implying (by (A5)) that $\frac{d\hat{c}_1^i}{ds^i}=\frac{d\hat{c}_1^i}{da^i}$, or $\frac{d\hat{a}}{d\hat{s}}=1$. Namely, a full offset of savings in one track as a result of increased savings in the other track. However, $\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right]=0$ if and only if $\varepsilon\left(\hat{B}^i\right)=1$, but this possibility is excluded by proposition 2. ***QED***.

Proof of Proposition 2:

Suppose that $\frac{Ra}{\bar{\theta}}-\hat{c}_1^i<0$. In this case, as can be seen from (4), $\hat{B}^i$ depends negatively on θ. Assume that $\varepsilon\left(\hat{B}^i\right)>1$, or $\frac{d}{d\theta}\left(\theta h''\left(\hat{B}^i\right)\right)=\theta h'''\left(\hat{B}^i\right)+h''\left(\hat{B}^i\right)>0$. Then,

(A8) $$\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right]>h''\left(\hat{B}^i(\bar{\theta})\right)\bar{\theta}\,\underset{\theta}{E}\left[\theta-\bar{\theta}\right]=0,\ \ \forall\theta,$$

Therefore:

(A9) $$\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right]>h''\left(\hat{B}^i(\bar{\theta})\right)\bar{\theta}\,\underset{\theta}{E}\left[\left(\theta-\bar{\theta}\right)\right]=0$$

If, on the other hand, $\frac{Ra^i}{\bar{\theta}}-\hat{c}_1^i>0$, then B_1^i increases in θ. Assume $\varepsilon\left(\hat{B}^i\right)<1$, namely $\frac{d}{d\theta}\left(\theta h''\left(\hat{B}^i\right)\right)=\theta h'''\left(\hat{B}^i\right)+h''\left(\hat{B}^i\right)<0$, thus, (A10) $\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right] < h''\left(\hat{B}^i(\bar{\theta})\right)\bar{\theta}\,\underset{\theta}{E}\left[\left(\theta-\bar{\theta}\right)\right]=0$. ***QED (a).***

It follows from (4) that the correlation between B^i and θ depends on the sign of $\frac{Ra^i}{\bar{\theta}}-\hat{c}_1^i$. Therefore, assuming that $h'''\left(\hat{B}^i\right)$ is monotonic, B^i either increases or decreases in θ. If $h'''\left(\hat{B}^i\right)$ is constant in θ then $h''\left(\hat{B}^i\right)\theta$ decreases in θ, therefore $\underset{\theta}{E}\left[h''\left(\hat{B}^i\right)\theta\left(\theta-\bar{\theta}\right)\right]$ is never equal to zero, implying that if $\beta<1$, the right side of (A7) is also never equal to zero; this means that $\frac{Ra^i}{\bar{\theta}}-\hat{c}_1^i \neq 0$, implying that $\varepsilon\left(\hat{B}^i\right)\neq 1$. ***QED (b).***

Proof of Proposition 3:

First period savings in the funded economy is determined by (7) and (8), and first period savings in the PAYG economy is determined by (14), and a quick glance at these two equations systems reveals that the only functional difference between them is $G\left(\frac{\theta-\bar{\theta}}{\bar{\theta}}\right)\frac{da^{i*}}{ds^i}$ but $\underset{\theta}{E}\left[G\left(\frac{\theta-\bar{\theta}}{\bar{\theta}}\right)\frac{da^{i*}}{ds^i}\right]=0$; therefore, first period savings in the funded economy differs from first period savings in the PAYG economy iff $\frac{d\hat{c}_1^i}{ds^i}\neq\frac{dc_1^{i*}}{ds^i}$.

By totally differentiating (A1) we obtain,

$$\underset{\theta}{E}\left[v_{11}\left(c_1^{i*},\theta\right)+\beta\delta h''\left(B^{i*}-a^{i*}\right)\frac{\theta^2}{G}\right]dc_1^* - \beta\delta\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\theta\left(\frac{\theta}{\overline{\theta}}-1\right)\right]da^i$$

$$=\frac{\beta\delta R}{G}\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\theta\right]ds^i$$

(A11)

$$-\frac{1}{G}\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\theta-\overline{\theta}\right)\theta\right]dc_1^* + \underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\frac{\theta}{\overline{\theta}}-1\right)^2\right]da^i$$

$$=-\frac{R}{G}\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\theta-\overline{\theta}\right)\right]ds^i$$

As we already know (by proposition 1), $\left(B^{i*}-a^{i*}\right)$ is independent of θ, therefore, $h'\left(B^{i*}-a^{i*}\right)$ as well as $h''\left(B^{i*}-a^{i*}\right)$ are also independent of θ, and this fact allows us to set $\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\theta-\overline{\theta}\right)\right]=0$. Thus, by the second equation of (A11),

(A12)
$$\frac{dc_1^{i*}}{da^i}=\frac{G\,\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\frac{\theta}{\overline{\theta}}-1\right)^2\right]}{\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\theta-\overline{\theta}\right)\theta\right]}$$

It also follows from (A11) that

(A13)
$$\frac{dc_1^*}{ds^i}=\frac{\beta\delta}{\tilde{\Delta}}\underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\theta\left(\left(\frac{\theta}{\overline{\theta}}-1\right)\frac{da^{i*}}{ds^i}+\frac{R}{G}\right)\right]$$

Where $\tilde{\Delta}=\underset{\theta}{E}\left[v_{11}\left(c_1^{i*},\theta\right)+\beta\delta h''\left(B^{i*}-a^{i*}\right)\frac{\theta^2}{G}\right]$.

It follows from (A12) and (A13) that

$$\frac{dc_1^{i*}}{da^i} \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow \underset{\theta}{E}\left[h''\left(B^{i*}-a^{i*}\right)\left(\theta-\bar{\theta}\right)\theta\right] \begin{matrix} < \\ > \end{matrix} 0 \Leftrightarrow \varepsilon\left(B^{i*}-a^{i*}\right) \begin{matrix} < \\ > \end{matrix} 1$$

(A14) and

$$\frac{dc_1^{*}}{ds^i} > 0 \Leftrightarrow \frac{da^{i*}}{ds} < 0$$

Notice that by (A4) and (A13) if $\frac{da^{i*}}{ds^i} = 0$ then $\frac{dc_1^{*}}{ds^i} = \frac{d\hat{c}_1^i}{ds^i}$. Nevertheless, although $\frac{da^{i*}}{ds} < 0$ is a plausible assumption, we have no way to prove it. Therefore, the signs of $\frac{dc_1^{i*}}{da^i}$ and $\frac{dc_1^{*}}{ds^i}$ are unknown. Comparing (A13) to (A4), we obtain:

$$\text{(A15)} \qquad \operatorname{sgn}\left(\frac{dc_1^{*}}{ds^i} - \frac{d\hat{c}_1^i}{ds^i}\right) = -\operatorname{sgn}\left(\frac{da^{i*}}{ds^i}\right)$$

Returning to (7) and (14), it now follows that:

$$\text{(A16)} \qquad \operatorname{sgn}\left(\hat{c}_0^i - c_0^{i*}\right) = \operatorname{sgn}\left(\frac{dc_1^{*}}{ds^i} - \frac{d\hat{c}_1^i}{ds^i}\right) = -\operatorname{sgn}\left(\frac{da^{i*}}{ds}\right)$$

and this completes the proof of proposition 3. ***QED (a)***.

Proof of Proposition 4

By plugging $\tilde{c}_1^i = \tilde{c}_1^i\left(s^i, \tilde{a}^i\right)$ and $h\left(\tilde{B}^i\right)$ into (8) we get:

$$\text{(A17)} \qquad \begin{aligned} \frac{dV^i}{da^i} &= -u'(\tilde{c}_0^i) \\ &+ \beta\delta \underset{\theta}{E}\left[\left(v_1^i(\tilde{c}_1^i,\theta) - \delta h'\left(\tilde{B}^i\right)\theta\right)\left.\frac{d\tilde{c}_1^i}{da^i}\right|_{a=\tilde{a}} + \delta h'\left(\tilde{B}^i\right)\frac{R\theta}{\bar{\theta}}\right] = 0 \end{aligned}$$

Since $\left.\frac{d\tilde{c}_1}{da^i}\right|_{a^i=\tilde{a}^i} > 0$ (see (A4)), it follows that $v_1^i\left(\tilde{c}_1^i,\theta\right) < v_1^i\left(\hat{c}_1^i,\theta\right)$, therefore holding equation (18) implies $h'\left(\tilde{B}^i\right) < h'\left(\hat{B}^i\right)$, thus $\tilde{B}^i > \hat{B}^i$. The inevitable conclusion is that $\tilde{c}_0^i < \hat{c}_0^i$. Namely, first period consumption bears the entire burden of additional compulsory savings. Since $\varepsilon\left(\tilde{B}^i\right) \neq 1$ still hold, the randomness of bequests remains unchanged after government intervention. ***QED.***

Proof of Proposition 5:

Suppose that the government sets $a^i = \widehat{a}$ and this is no more a decision variable of individuals. In this case the second period optimization problem of the consumer is,

(A18)
$$\begin{aligned} &\max_{c_1^i} \underset{\theta}{E}\left\{v\left(c_1^i,\theta\right) + \beta\delta G h\left(B^i - \widehat{a}^i\right)\right\} \\ &s.t. \\ &GB^i = Rs^i + \left(\frac{G\widehat{a}^i}{\overline{\theta}} - c_1^i\right)\theta \end{aligned}$$

First order condition of this problem is,

(A19)
$$\underset{\theta}{E}\left[v_1^i\left(\widehat{c}_1^i,\theta\right) - \beta\delta h'\left(\widehat{B}^i - \widehat{a}^i\right)\theta\right] = 0$$

Implying $\widehat{c}_1^i\left(s^i\right)$ and $\widehat{B}^i\left(s^i\right)$. Notice that since a^i is no more a decision variable of the consumer, we do not have here a condition like (12), thus by (10) bequests in this PAYG economy will be random, depending negatively on θ. ***QED***

Proof of Corollary 5

Going back to the first period, inserting $\widehat{c}_1^i\left(s^i\right)$, $\widehat{a}^i$ and $\widehat{B}^i\left(s^i\right)$ into (9) yields:

$$V^i = \max_{s^i} \left\{ u\left(c_0^i\right) + \beta\delta \underset{\theta}{E} \left[v\left(\widehat{c}_1^i, \theta\right) + \delta G h\left(\widehat{B}^i - \widehat{a}^i\right) \right] \right\}$$

(20) $s.t.$

$$c_0^i = w + B^{i-1} - s^i - \widehat{a}^i$$

Which has the first order condition:

$$(21) \quad \frac{\partial V^i}{\partial s} = -u'(\widehat{c}_0^i) + \beta\delta \underset{\theta}{E} \left[\left(v_1^i\left(\widehat{c}_1^i, \theta\right) - \delta h'\left(\widehat{B}^i - \widehat{a}^i\right)\theta \right) \frac{d\widehat{c}_1^i}{ds} + \delta h'\left(B^{i*} - a^{i*}\right) R \right] = 0 .$$

With solution $\widehat{s}^i$ and the corresponding $\widehat{c}_0^i$.

Notice that equations (A19) and (21) actually identical to equations (5) and (7), respectively, implying that under governmental intervention both systems coincide into the same funded equilibrium with random bequests. ***QED.***

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