

On the efficiency of producing gamma-ray bursts from isolated Population III stars

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ABSTRACT

The rate of long-duration gamma-ray bursts (GRBs) from isolated Pop III stars is not well known, as it depends on our poor understanding of their initial mass function (IMF), rotation rates, stellar evolution, and mass loss. Some massive ($M_{\text{ZAMS}} \gtrsim 20 M_{\odot}$) Pop III stars are expected to suffer core-collapse and launch a relativistic jet that would power a GRB. In the collapsar scenario, a key requirement is that the pre-supernova star imparts sufficient angular momentum to the remnant black hole to form an accretion disc and launch a relativistic jet, which demands rapid initial rotation of the progenitor star and suppression of line-driven mass-loss during its chemically homogeneous evolution. Here, we explore a grid of stellar evolution models of Pop III stars with masses $20 \leq M_{\text{ZAMS}}/M_{\odot} \leq 100$, which are initially rotating with surface angular velocities $0.6 \leq \Omega_0/\Omega_{\text{crit}} \leq 0.9$, where centrifugally driven mass-loss ensues for $\Omega > \Omega_{\text{crit}}$. Realistic accretion and jet propagation models are used to derive the initial black hole masses and spins, and jet breakout times for these stars. The GRB production efficiency is obtained over a phase space comprising progenitor initial mass, rotation, and wind efficiency. For modest wind efficiency of $\eta_{\text{wind}} = 0.45$ – 0.35 , the Pop III GRB production efficiency is $\eta_{\text{GRB}} \sim 10^{-5} - 3 \times 10^{-4} M_{\odot}^{-1}$, respectively, for a top-heavy IMF. This yields an observable all-sky equivalent rate of ~ 2 – 40 yr^{-1} by *Swift*, with 75 per cent of the GRBs located at $z \lesssim 8$. If the actual observed rate is much lower, then this would imply $\eta_{\text{wind}} > 0.45$, which leads to significant loss of mass and angular momentum that renders isolated Pop III stars incapable of producing GRBs and favours a binary scenario instead.

Key words: stars: evolution – gamma-ray burst: general – stars: Population III.

1 INTRODUCTION

The first stars in the Universe, often called Population III (Pop III) stars, emerged from metal-free primordial gas (i.e. comprising only H and He) and ended the so-called dark-ages (see e.g. V. Bromm & R. B. Larson 2004; R. S. Klessen & S. C. O. Glover 2023, for reviews). The copious amounts of UV photons produced by these stars re-ionized the Universe (e.g. N. Y. Gnedin & J. P. Ostriker 1997; J. Tumlinson & J. M. Shull 2000), and the supernovae that ended the lives of massive Pop III stars enriched the intergalactic medium with heavy elements (e.g. A. Heger & S. E. Woosley 2002; J. P. Ostriker & N. Y. Gnedin 1996; N. Yoshida, V. Bromm & L. Hernquist 2004) that affected the next generation of stars. In the lambda cold dark matter (Λ CDM) model of cosmology, Pop III stars are predicted to form at redshift $z \gtrsim 30$ and dominate the star-formation rate at $z \sim 15$ – 20 , after which epoch slightly metal-enriched Pop II stars begin to emerge (e.g. T. Hartwig et al. 2022). Significant uncertainty exists in predicting the formation rate and scenarios of the first stars (R. Barkana & A. Loeb 2001) as exploration is only limited to advanced numerical

simulations (e.g. T. Abel, G. L. Bryan & M. L. Norman 2002; V. Bromm, P. S. Coppi & R. B. Larson 1999, 2002), that in many cases are low resolution or lack relevant physical processes, with no near-term possibility of direct observations (A. T. P. Schauer, N. Drory & V. Bromm 2020). Alternative strategies involving surveys of local metal poor stellar populations to constrain the effect of the first stars on the nucleosynthetic makeup of Pop II stars (J. Tumlinson 2006) will have to wait future large, high-resolution spectroscopic surveys (M. Jeon et al. 2021). Therefore, at present, it is very challenging to probe the initial mass function and star formation rate at high redshifts.

Most simulations find an approximately top-heavy initial mass function of Pop III stars (K. M. J. Wollenberg et al. 2020) in comparison to the present day one (G. Chabrier 2003). Stars at the higher end of the mass distribution ($M \gtrsim 8 M_{\odot}$) are expected to explode as core-collapse supernovae, pair-instability supernovae (PISNe; A. Heger & S. E. Woosley 2002), or collapse directly to a black hole with no associated supernova (A. Heger et al. 2003; S.-C. Yoon, A. Dierks & N. Langer 2012). Out of these, some are expected to launch powerful relativistic jets if the collapsed core retains sufficient angular momentum to form an accretion disc, which would also power a more energetic supernova – a hypernova

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with kinetic energy $E_{\text{kin}} \gtrsim 10^{52}$ erg (K. Iwamoto et al. 1998; A. M. Khokhlov et al. 1999; K. Nomoto et al. 2003). If the jet manages to break out of the star then internal dissipation can produce a bright GRB (see e.g. P. Kumar & B. Zhang 2015 for a review). The radio afterglows of hypernovae can be detected up to $z \sim 20$ (K. Ioka & P. Mészáros 2005), the prompt GRB to $z \sim 100$ (D. Q. Lamb & D. E. Reichart 2000) and their afterglows to $z \sim 30$ in infrared, X-rays (B. Ciardi & A. Loeb 2000; L. J. Gou et al. 2004) and radio (S. Inoue 2004; K. Ioka 2003). The current record holders include GRB 090423 at $z = 8.2$ (N. R. Tanvir et al. 2009) and GRB 090429B at $z = 9.4$ (A. Cucchiara et al. 2011). Therefore, by obtaining deeper and more sensitive observations, it is possible to probe the initial mass function of Pop III stars using GRBs and their afterglows (see, e.g. K. Toma, S.-C. Yoon & V. Bromm 2016, for a review).

This prospect has attracted proposals of several new missions with greater sensitivity towards detecting high-redshift transients. The current front-runner in probing the distant Universe is the *James Webb Space Telescope* (JWST) that will not detect the Pop III stars themselves but will be sensitive to intrinsically bright transients, such as GRBs and PISNe (D. J. Whalen et al. 2013), instead. Another upcoming observatory that has the potential for detecting supernovae from Pop III stars for $z \gtrsim 6$ is the *Roman Space Telescope* (D. Spergel et al. 2015; D. J. Whalen et al. 2013). A few other notable missions that have been proposed to probe the highest redshift include the High- z Gamma-ray bursts for Unraveling the Dark Ages Mission (HiZ-GUNDAM; D. Yonetoku et al. 2014; K. Yoshida et al. 2016), the Gamow Explorer (N. E. White et al. 2021), and the Transient High-Energy Sky and Early Universe Surveyor (THESEUS; L. Amati et al. 2018).

In anticipation of these missions, several works have calculated the intrinsic and observable rate of GRBs at high redshifts (V. Bromm & A. Loeb 2006; M. A. Campisi et al. 2011; C. L. Fryer et al. 2022; G. Ghirlanda & R. Salvaterra 2022; T. Kinugawa, Y. Harikane & K. Asano 2019; S. Naoz & O. Bromberg 2007; R. S. Souza, N. Yoshida & K. Ioka 2011). Such a calculation requires knowledge of the star-formation rate (SFR) at high redshifts, the efficiency of producing GRBs per unit stellar mass (e.g. N. M. Lloyd-Ronning, J. L. Johnson & A. Aykutalp 2020), and the luminosity function. While the luminosity function can be constructed from existing observations of GRBs with redshifts measured using their afterglows (e.g. N. M. Lloyd-Ronning, C. L. Fryer & E. Ramirez-Ruiz 2002; D. Wanderman & T. Piran 2010), the SFR of Pop III stars and GRB production efficiency (η_{GRB} = number of GRBs per formed stellar mass) are not so well constrained. In particular, any inference of η_{GRB} is further complicated by our poor understanding of stellar evolution of metal-free stars which is significantly impacted by rotation and mass loss (see below for a discussion of mass-loss in Pop III stars). The not so well understood physics of core-collapse, properties of the remnant, and jet launching mechanisms further add to the challenges in ascertaining η_{GRB} .

Therefore, many works fix this parameter by comparing the theoretically expected rate of GRBs for a given instrument with the observed one. For example, V. Bromm & A. Loeb (2006) constrain $\eta_{\text{GRB}} \simeq 2 \times 10^{-9} M_{\odot}^{-1}$ for GRBs produced by Pop I/II progenitors, and assume that the same applies to Pop III stars, by comparing with the rate of *Swift* GRBs. Although consistent with upper limits in later works, it is several orders of magnitude lower than expected. For example, the local observed all-sky rate of GRBs with luminosity $L_{\gamma, \text{iso}} \geq 10^{50} \text{ erg s}^{-1}$ is $\simeq 1.3 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (D. Wanderman & T. Piran 2010). Correcting this for beaming, with $\eta_{\text{beam}} \simeq 1/500$ (D. A. Frail et al. 2001), since not all GRBs are beamed towards us, we obtain the intrinsic rate of $\sim 6.5 \times 10^{-7} \text{ Mpc}^{-3} \text{ yr}^{-1}$. When taking

the density of Milky way (MW) like galaxies in the local Universe to be $\sim 10^{-2} \text{ Mpc}^{-3}$, this rate is equivalent to $\sim 6.5 \times 10^{-5} \text{ yr}^{-1}$ per MW-galaxy. The SFR of MW is $0.68\text{--}1.5 M_{\odot} \text{ yr}^{-1}$ (T. P. Robitaille & B. A. Whitney 2010), which serves as a good proxy of the local SFR, then the local GRB production efficiency is $\eta_{\text{GRB, local}} \sim 4.3 \times 10^{-5}\text{--}10^{-4} M_{\odot}^{-1}$. In later works, S. Naoz & O. Bromberg (2007) obtain an upper limit on the efficiency, with $\eta_{\text{GRB}} < 3.2 \times 10^{-4} M_{\odot}^{-1}$, by comparing with the observed GRB rate of 15 yr^{-1} , out of the 90 yr^{-1} , detected by *Swift* with no optical counterpart and $T_{90} > 50 \text{ s}$. The assumption of optically ‘dark’ GRBs, and therefore no redshift information, is valid for very high- z GRBs as their optical afterglow photons observed at $z = 0$, but emitted with energies $E_z = (1 + z)E_{\text{obs}}$ and thus with wavelengths smaller than the Lyman limit, would be absorbed en route by photoionizing H/He atoms (D. Q. Lamb & D. E. Reichart 2000). M. A. Campisi et al. (2011) calculate $\eta_{\text{GRB}} = f_{\text{GRB}} \times \eta_{\text{BH}} \lesssim 7 \times 10^{-5} M_{\odot}^{-1}$, where η_{BH} is the efficiency per unit solar mass for producing a BH and f_{GRB} is the fraction that produces a GRB. They arrive at this limit by assuming a Salpeter initial mass function for Pop III stars in the mass range of $100 < M/M_{\odot} < 140$ and $260 < M/M_{\odot} < 500$, which yields $\eta_{\text{BH}} \sim 3.2 \times 10^{-3} M_{\odot}^{-1}$, and that $f_{\text{GRB}} \sim 2.2 \times 10^{-2}$ as inferred from *Swift* observations. Stellar evolution models show that very massive Pop III stars with $M > 260 M_{\odot}$ end their lives as red-supergiants that have extended stellar envelopes which makes it difficult for the jet to penetrate out over the engine lifetime (S.-C. Yoon et al. 2012; S.-C. Yoon, J. Kang & A. Kozyreva 2015).

In this work, we calculate the efficiency of producing a GRB from a distribution of isolated (i.e. single and not in a binary) Pop III stars based on stellar evolution numerical calculations done using the public code MESA. We leave the investigation of Pop III stars in a binary to a future work. After a brief review in Section 1.1 of the stellar evolution expected, and as shown by earlier works using different stellar evolution codes, for massive and rapidly rotating Pop III stars, we describe our simulation set-up in Section 2.1. The remaining Section 2 describes the results from our model grid spanning an initial mass range of $20 \leq M_{\text{ZAMS}}/M_{\odot} \leq 100$ as well as initial rotation of $0.6 \leq \tilde{\Omega}_0 \equiv \Omega_0/\Omega_{\text{crit}} \leq 0.9$ and shows the radial profiles of density, angular velocity and specific angular momentum. We evolve most of the stars in our grid to advanced nuclear burning stages approaching imminent core-collapse, as shown in the HR diagram, which has not been achieved in many earlier works on Pop III stars. These stellar profiles are then used to analytically calculate the properties of core-collapse and accretion in Section 3.1, where we give the mass and spin of the newly formed BH when the accretion disc forms. Based on the rate of accretion, we calculate the jet power in Section 3.2 and its subsequent evolution through the stellar interior after launch using an analytic formalism in Section 3.3. Using our jet breakout criteria, here we also present the efficiency of GRB production as a function of initial progenitor mass, rotation, and stellar wind efficiency. We use this phase space producing successful GRBs to calculate the rate of GRBs in Section 4 as a function of redshift. In Section 5, we conclude that if the all-sky detection rate of Pop III GRBs by any instrument with *Swift*/BAT sensitivity is significantly less than $\sim 2 \text{ yr}^{-1}$, then the stellar wind efficiency is constrained from below, which severely limits the prospects of producing GRBs from isolated Pop III stars.

1.1 Stellar evolution of rapidly rotating massive Pop III stars

Stellar rotation has been shown to alter the evolution of massive stars (A. Heger, N. Langer & S. E. Woosley 2000; N. Langer 2012; A. Maeder & G. Meynet 2000), and this effect is even more pronounced in lower metallicity stars (G. Meynet & A. Maeder 2002; S.-C. Yoon

et al. 2012). In comparison to their non-rotating counterparts, rapid rotation tends to produce significantly larger (by ~ 25 per cent in mass) He cores due to rotationally induced chemical mixing that supplies fresh fuel to the core from the outer stellar layers. Since the He-core mass is a proxy for the remnant mass, rotation reduces the limit on the zero-age-main-sequence (ZAMS) mass that can produce core-collapse supernovae. Rotationally induced chemical mixing also leads to chemically homogeneous evolution (CHE; A. Maeder 1987), that produces dramatic changes in the evolutionary tracks on the HR diagram. As a result, massive metal-poor stars show a blueward evolution if they are rotating sufficiently fast as compared to a redward one for stars with lower rotation (I. Brott et al. 2011; N. Langer 1992; A. Maeder 1987; S.-C. Yoon et al. 2012).

As demonstrated in I. Brott et al. (2011), CHE is sensitive to the initial stellar mass, rotation, and metallicity. In general, rotation tends to lower the local effective gravity due to the centrifugal acceleration, which affects the radiative energy flux (H. Zeipel 1924) at the stellar surface. This leads to cooler and less luminous stars, a latitude dependence of the surface temperature and luminosity, as well as polar and equatorial outflows (A. Maeder 1999). However, these effects are canceled in rapidly rotating massive stars (e.g. $\gtrsim 15 M_\odot$) due to the development of relatively more massive cores that yield higher luminosities when compared to non-rotating stars of the same mass.

Rotational mixing becomes more efficient at lower metallicity (I. Brott et al. 2011; S. C. Yoon, N. Langer & C. Norman 2006). Stars with higher metallicity are prone to stronger line-driven winds that drive mass-loss and along with it the loss of angular momentum (N. Langer 1998). This slows down the star and reduces the efficiency of rotational mixing. In slow rotators, chemical mixing is inhibited due to the emergence of a chemical gradient between the convective core and radiative envelope (A. Maeder 1987), when the mixing time-scale becomes longer than the thermonuclear time. In contrast, very metal-poor rapid rotators, and in particular Pop III stars, do not lose angular momentum to line-driven winds and therefore become fully chemically mixed and show a blueward rise in luminosity in the HR diagram.

Rotation also alters the limits on the ZAMS mass of metal-free stars that are capable of producing core-collapse and pair-instability supernovae. In non-rotating stars, a type IIP supernova is expected where a star with mass $8 \lesssim M_{\text{ZAMS}}/M_\odot \lesssim 25 M_\odot$ undergoes core-collapse and forms a neutron star (A. Heger et al. 2003). Above this limit and in the initial mass range of $25 \lesssim M_{\text{ZAMS}}/M_\odot \lesssim 140$ the remnant collapses to a BH, either by fallback accretion ($M \lesssim 40 M_\odot$) or directly (C. L. Fryer 1999). When the core collapses directly to a BH, there is no accompanying type II supernova. In the mass range of $140 \lesssim M_{\text{ZAMS}}/M_\odot \lesssim 260$, the core suffers from pulsational pair-instability and is disrupted entirely, leaving no remnant (A. Heger & S. E. Woosley 2002). Stars more massive than $\sim 260 M_\odot$ again form BHs directly as their cores implode due to photodisintegration (C. L. Fryer, S. E. Woosley & A. Heger 2001; A. Heger & S. E. Woosley 2002). S.-C. Yoon et al. (2012) carried out a comprehensive study of rotating massive Pop III stars and showed that the above limits are modified in rapid rotators. Stars born with initial surface angular velocity $\Omega \gtrsim 0.5\Omega_k$, where $\Omega_k = \sqrt{GM/R_\star^3}$ is the Keplerian angular velocity at the stellar surface, do not produce type II supernovae in the mass range $13 \lesssim M_{\text{ZAMS}}/M_\odot \lesssim 190$. Instead, they experience CHE and end their lives in type Ib/c supernovae, with some in the mass range $13 \lesssim M_{\text{ZAMS}}/M_\odot \lesssim 84$ producing GRBs and hypernovae.

2 SIMULATIONS WITH MESA

2.1 Model set-up

We present a grid of simulations covering the stellar evolution of massive zero-metallicity ($Z = 0$) stars in the mass range of $20 \leq M_{\text{ZAMS}}/M_\odot \leq 100$. These were performed using the public stellar evolution code MESA (version 22.05.1), which is a one-dimensional code that includes the effects of convection, rotation, and mass loss (B. Paxton et al. 2011, 2013, 2015, 2018, 2019). Our models follow the stellar evolution of rapidly rotating stars from the pre-main sequence (pre-ZAMS) to the advanced burning stages (e.g. core silicon burning) just before core-collapse.

All stars are initialized at the ZAMS with solid-body rotation at an angular velocity Ω which is some fraction of the critical angular velocity $\Omega_{\text{crit}} = \left(1 - \frac{L}{L_{\text{Edd}}}\right)^{1/2} \Omega_k$, where the ratio $L/L_{\text{Edd}} = \kappa L/4\pi cGM$ is the Eddington factor, κ is the opacity to electron scattering, L and M are the stellar luminosity and mass, and G is the gravitational constant. The critical angular velocity, evaluated at the stellar equator, represents the break-up limit and includes the effect of radiative acceleration via the Eddington factor. Here we consider a range of initial rotation rates, with $0.6 \leq \hat{\Omega}_0 \equiv \Omega_0/\Omega_{\text{crit}} \leq 0.9$, that are subcritical. However, as the star evolves, the ratio $\Omega/\Omega_{\text{crit}}$ at the surface approaches unity which is then followed by substantial rotationally driven mass loss until the surface angular velocity becomes subcritical.

Line-driven mass loss is initially unimportant for Pop III stars due to the complete absence of metals. However, rapid rotation drives the stars towards CHE that mixes the metals produced during core He burning and at later stages into the stellar envelope. At that point, stellar winds due to non-zero surface metallicity become important. For the treatment of stellar winds, we adopt the *Dutch* wind scheme as described in E. Glebbeek et al. (2009) for both cool and hot phases, setting temperature thresholds of 0.8×10^4 K and 1.2×10^4 K, respectively, to switch between mass-loss prescriptions. Specifically, we employ the wind prescription by C. Jager, H. Nieuwenhuijzen & K. A. van der Hucht (1988) in the low-temperature regime, which is well suited for cool, extended stellar envelopes. On the other hand, for the high-temperature regime and given the high-helium-mass fraction at the surface, the mass-loss rate transitions to the T. Nugis & H. J. G. L. M. Lamers (2000) Wolf–Rayet prescription. In both regimes, the wind mass-loss rate, $\dot{M}_{\text{wind}} = \eta_{\text{wind}} \dot{M}_{\text{wind}}^{\text{model}}$, is obtained by scaling the model mass-loss rate by a dimensionless parameter $0 < \eta_{\text{wind}} \leq 1$. Different wind schemes and their efficiencies are calibrated by comparing theoretical expectations with observations of stars in our Galaxy and those in the small and large Magellanic clouds. Since there are no direct observations of Pop III stars, it is difficult to prescribe a certain wind efficiency η_{wind} in removing stellar mass. Earlier works discussing stellar evolution of GRB progenitors used $\eta_{\text{wind}} \sim 0.1$ – 0.3 (e.g. S. E. Woosley & A. Heger 2006; S.-C. Yoon et al. 2012) to retain sufficient angular momentum to launch relativistic jets. We employ the wind schemes in a parametrized way and consider a range of efficiencies with $0.2 \leq \eta_{\text{wind}} \leq 1.0$. These choices allow us to model a range of mass-loss regimes, from fully efficient winds to scenarios with significantly reduced mass-loss rates, thereby capturing the potential diversity in the evolution of metal-free massive stars.

The effects of rotation are incorporated with the explicit inclusion of magnetic fields following the Spruit–Taylor dynamo (H. C. Spruit 2002) and fully accounting for the Solberg–Høiland instability, the secular shear instability, Eddington–Sweet circulation, and the

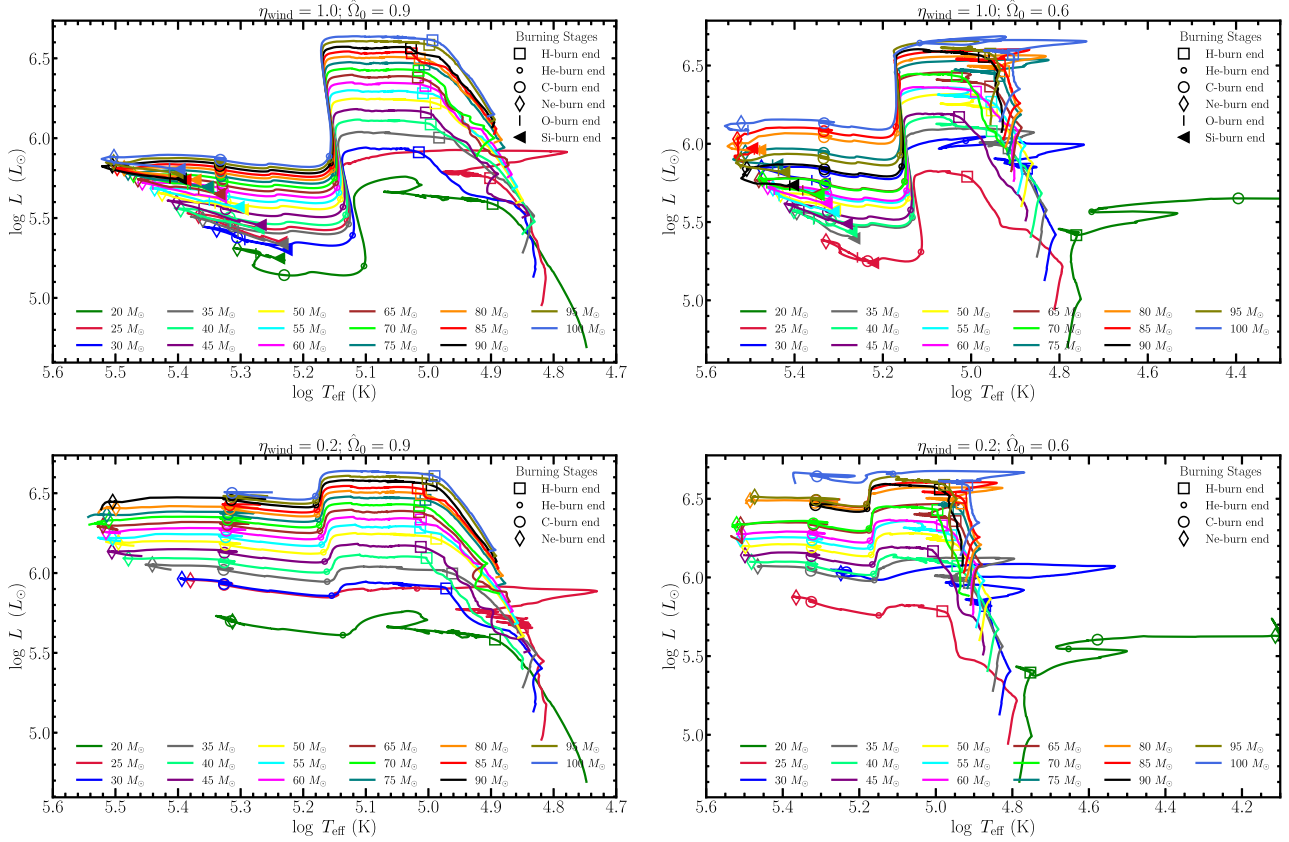


Figure 1. Evolutionary tracks in the HR diagram for stellar models with initial masses ranging from 20 to 100 M_{\odot} , assuming an initial rotation fraction of $\hat{\Omega}_0 \equiv \Omega_0/\Omega_{\text{crit}} = 0.9$ (left) and $\hat{\Omega}_0 = 0.6$ (right). Each track shows luminosity evolution as a function of the effective temperature from the ZAMS to advanced core nuclear burning stages before core-collapse. These different nuclear burning stages are highlighted with different symbols. (*Top*) Stellar evolution with Dutch wind scaling factor of $\eta_{\text{wind}} = 1.0$, with evolutionary tracks shown up to a central temperature of $\log T_{\text{core}} < 9.6$. (*Bottom*) The panel presents the scenario for $\eta_{\text{wind}} = 0.2$, where tracks are presented up to $\log T_{\text{core}} < 9.4$. These temperature thresholds are imposed to prevent numerical instabilities that arise in the late evolutionary stages.

Goldreich–Schubert–Fricke instability (B. Paxton et al. 2013). Convective and mixing processes are treated via mixing-length theory (MLT), where we set the mixing-length parameter to $\alpha_{\text{MLT}} = 1.5$ and adopt the Henyey option, which yields stable convergence in convective regions. The convective boundaries are determined using the Ledoux criterion, ensuring that the chemical composition gradients are taken into account. Semiconvection is incorporated using the N. Langer, M. F. El Eid & K. J. Fricke (1985) mixing scheme with an efficiency parameter of $\alpha_{\text{sc}} = 0.02$, allowing moderate mixing in semiconvective zones. It is worth noting that in the late stages of evolution, the time scales associated with semiconvective mixing become longer than those for nuclear reaction processes. As a result, semiconvection is unlikely to have a considerable impact on evolution. Its induction causes numerical instabilities due to rapid changes in the stellar structure, and therefore we set $\alpha_{\text{sc}} = 0$ during advanced burning before core-collapse. A step overshoot scheme during core H burning is used with overshoot parameter $f = 0.345$, while the scale length at the convective boundary is set to $f_0 = 0.01$.

In most of our models, stellar evolution is followed up to late stages before core-collapse when the core Silicon (^{28}Si) fraction is $X_{\text{Si}28} < 10^{-3}$. In some models, the evolution is terminated earlier than this limit due to onset of instabilities. However, in all cases, the stars are evolved up to a point when mass-loss has terminated and the stellar structure, including the radial density and angular momentum

profiles, remain unchanged in the subsequent stellar evolution before the instabilities set in. This ensures that the total angular momentum remains unchanged and profiles obtained at this stage are very similar to what would have been obtained just before core-collapse.

2.2 Stellar evolution models

Here, we present the results of these simulations, focusing primarily on their evolutionary tracks, and an exhaustive compilation of their physical properties throughout their evolution, but particularly during the final stage of stellar evolution that precedes core-collapse and in which, in most cases, the iron core has already formed.

Fig. 1 shows the evolutionary track in the HR diagram for all masses in our grid from the ZAMS to advanced stages prior to core-collapse. The models are initialized with a rigidly rotating star with angular velocity $\hat{\Omega}_0 = \{0.6, 0.9\}$. These diagrams are shown up to central temperatures of $\log T_c < \{9.6, 9.4\}$ for $\eta_{\text{wind}} = \{1.0, 0.2\}$ to avoid numerical difficulties in resolving the outer stellar layers that cause erratic behaviour in the HR diagram. In contrast, the core remains stable and the models are in fact evolved to higher temperatures. Key nuclear burning stages are marked with distinct symbols, as indicated in the legend, corresponding to the end of hydrogen, helium, carbon, neon, oxygen, and silicon burning, when their respective central mass fractions are $X < 0.01$.

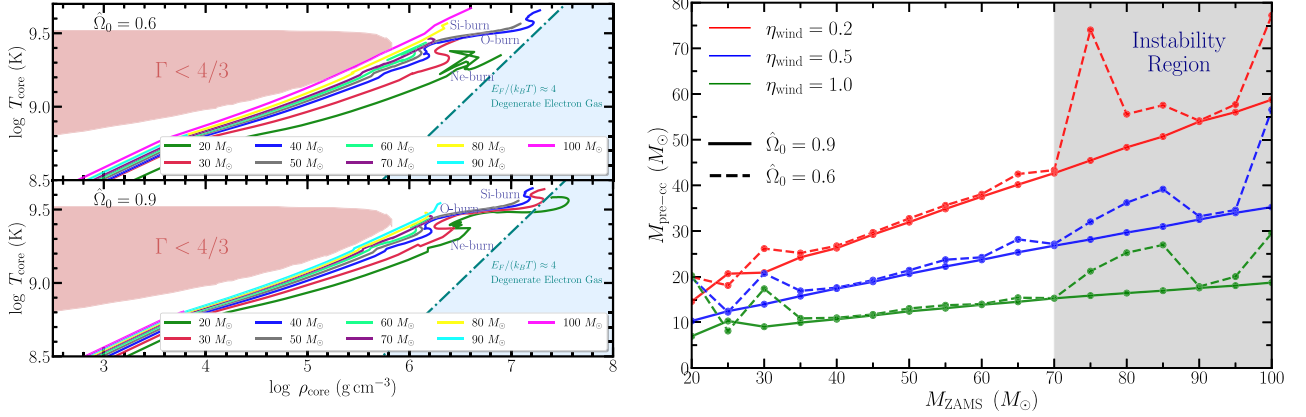


Figure 2. (Left) Evolution of the core temperature as a function of core density for stellar models with masses ranging from $20 M_{\odot}$ to $100 M_{\odot}$ for $\eta_{\text{wind}} = 0.2$ and for an initial rotation of $\hat{\Omega}_0 = 0.6$ (top) and $\hat{\Omega}_0 = 0.9$ (bottom). The tracks illustrate the progression through different nuclear burning stages (C, Ne, O, Si) as the core evolves. The red shaded area indicates the $\Gamma < 4/3$ instability criterion, valid particularly for non-rotating stars, where the stellar core becomes dynamically unstable due to the creation of electron-positron pairs that reduce the adiabatic index below the critical threshold, while the blue region corresponds to the criteria for a degenerate electron gas. (Right) Evolution of the pre-core-collapse mass as a function of the initial mass (M_{ZAMS}), for different values of η_{wind} and initial rotation of $\hat{\Omega}_0 = 0.9$ (solid) and $\hat{\Omega}_0 = 0.6$ (dashed). Slower rotating massive stars ($M_{\text{ZAMS}} > 70 M_{\odot}$) enter the (pulsational) pair-instability region and their evolution was not followed all the way to core-collapse. Hence, the non-monotonic behavior in the pre-core-collapse mass.

When $\hat{\Omega}_0 = 0.9$, all stellar models undergo CHE due to rapid rotation (I. Brott et al. 2011; S.-C. Yoon et al. 2012), which causes the stars to evolve bluewards where they become hotter and more luminous due to an increase in the helium fraction at the surface, which in turn causes the surface to contract and heat up rather than expand towards cooler temperatures. This trend is different from that obtained in the non-rotating counterparts (P. Marigo et al. 2001), where the evolution is redwards towards cooler effective temperatures.

Left panels in Fig. 2 show the evolutionary tracks in the $\log T_{\text{core}} - \log \rho_{\text{core}}$ plane for the cores of stars in our grid. The different rows show the evolution for different wind mass-loss efficiencies. These evolutionary tracks reflect the typical behaviour of the stars in our model, beginning with low core densities and temperatures (due to hydrogen burning) and progressing to higher densities and temperatures as the core contracts and heavier isotopes burn. The high-initial rotation imposed in our models also favours rotational mixing, incorporating fresh hydrogen into the core during the main sequence and increasing the convective core mass (S. Ekström et al. 2008). The right panel of this figure shows the evolution of the pre-core-collapse mass ($M_{\text{pre-cc}}$) as a function of the initial mass (M_{ZAMS}), considering different stellar wind efficiencies ($\eta_{\text{wind}} = 0.2, 0.5, 1.0$) and initial rotation rates ($\hat{\Omega}_0 = 0.9$ for solid lines and $\hat{\Omega}_0 = 0.6$ for dashed lines). Here, we can notice the effect of wind efficiency, with a low η_{wind} value resulting in minimal mass-loss, thus allowing the stars to retain a more significant fraction of their initial mass. The relation between the two masses shows a linear trend, where lower wind efficiencies allow for greater mass retention. In the mass range of $35 \leq M_{\text{ZAMS}}/M_{\odot} \leq 70$, the initial rate of rotation produces minimal changes in the outcome. However, results diverge at both the low and high ends. At the high-mass end, in particular, the slower rotators become susceptible to the pulsational pair instability (PPI), which causes thermonuclear pulsations that eject part of the envelope without destroying the star (S. E. Woosley 2017; S.-C. Yoon et al. 2012). This effect is illustrated in the top-left panel of Fig. 2, corresponding to models with $\hat{\Omega}_0 = 0.6$. As rotation decreases, stellar evolution paths approach the instability

region where the adiabatic index Γ drops below $4/3$. In this regime, the formation of e^{\pm} pairs softens the equation of state, reducing Γ and weakening the pressure support against gravity. This loss of support leads to rapid core contraction.

2.3 Rotation and mass-loss

The evolution of rotational and mass-loss profiles in our Pop III star models is strongly influenced by the choice of initial parameters, such as initial mass M_{ZAMS} , initial rotation rate $\hat{\Omega}_0$, and wind efficiency factor η_{wind} . We examine these effects to understand the behaviour of each model. In general, we find that stars with high initial rotation and low η_{wind} (i.e. weak winds) retain more angular momentum in the core, which would eventually favour producing a successful GRB. In contrast, models with higher initial mass and high η_{wind} (i.e. strong winds) and low initial rotation experience significant loss of mass and angular momentum. Such stars will ultimately lack the requisite angular momentum to launch relativistic jets and will fail to produce GRBs.

Fig. 3 shows the evolution of the dimensionless surface angular velocity ($\hat{\Omega} \equiv \Omega/\Omega_{\text{crit}}$) as a function of stellar age for 20 and $100 M_{\odot}$ models, with varying η_{wind} factors across panels. As noted in prior studies (E. Chatzopoulos & J. C. Wheeler 2012; S. Ekström et al. 2008; L. J. Murphy et al. 2021; S.-C. Yoon et al. 2012), $\hat{\Omega}$ approaches critical rotation ($\hat{\Omega} \rightarrow 1$) over time, regardless of the initial rotation rate, due to the growth in stellar radius, which reduces Ω_k and consequently Ω_{crit} . For both cases with $\hat{\Omega}_0 = 0.9$ (purple curve), critical rotation is reached early, and then it is progressively delayed for slower rotators. When a star reaches critical rotation, it is accompanied by a significant increase in the rate of mass loss, and angular momentum, that allows the star to remain at $\hat{\Omega} = 1$ and not exceed it. This is the scenario that is characterized as the *forbidden region* in fig. 12 of S.-C. Yoon et al. (2012). Different wind efficiencies only have a mild effect on the time when the star reaches critical rotation.

In Fig. 4, we present the temporal evolution of the mass-loss rate from our grid of simulations, considering various initial masses

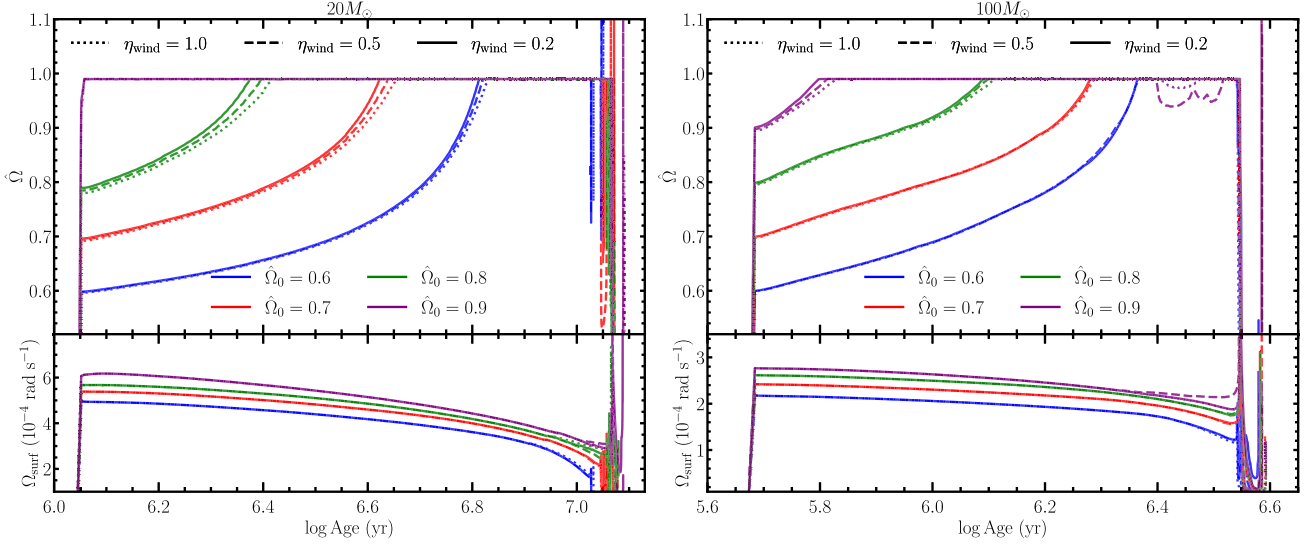


Figure 3. Temporal evolution of the stellar surface rotation for $20 M_{\odot}$ (left) and $100 M_{\odot}$ (right) stars. (Top) Normalized surface angular velocity $\hat{\Omega} \equiv \Omega / \Omega_{\text{crit}}$, as a function of stellar age, for different wind scaling factors: $\eta_{\text{wind}} = 1.0$ (dotted lines), $\eta_{\text{wind}} = 0.5$ (dashed lines), $\eta_{\text{wind}} = 0.2$ (solid lines), and for different initial rotation rates: $\hat{\Omega}_0 = 0.6$ (blue), $\hat{\Omega}_0 = 0.7$ (red), $\hat{\Omega}_0 = 0.8$ (green), and $\hat{\Omega}_0 = 0.9$ (purple). (Bottom) Corresponding evolution of the surface angular velocity.

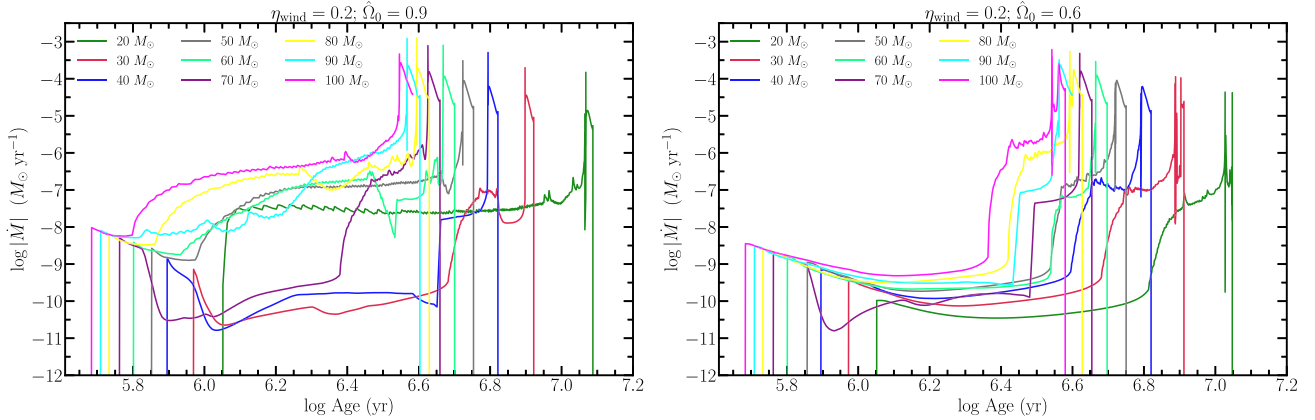


Figure 4. Mass-loss rates as a function of stellar age for the entire mass grid with wind efficiency factor of $\eta_{\text{wind}} = 0.2$, shown for two different initial rotation rates: $\hat{\Omega}_0 = 0.9$ (left), and $\hat{\Omega}_0 = 0.6$ (right). Higher η_{wind} yields similar mass loss rates. Models with higher initial $\hat{\Omega}_0$ reach critical rotation earlier (see Fig. 3) and therefore show a sudden increase in mass loss caused by centrifugal effects.

(M_{ZAMS}), initial rotation rates ($\hat{\Omega}_0$), and wind efficiencies (η_{wind}). In our simulations, mass-loss is driven by both rapid rotation and mixing of nuclear products from the core into the outer stellar layers due to CHE, which again is facilitated by rapid rotation. When comparing the two panels, showing the rate of mass-loss for two different initial rotation rates, the onset of significant mass loss at early times can be seen for $\hat{\Omega}_0 = 0.9$. For example, in the $M_{\text{ZAMS}} = 100 M_{\odot}$ case, the onset of rapid mass-loss is triggered by the star achieving super-critical rotation, i.e. when $\hat{\Omega} > 1$, as discussed above and shown in the right panel of Fig. 3. The same temporal coincidence can be seen for all of the stars in our grid. The rate of mass loss becomes even more extreme near the terminal stages of the star, when due to the CHE, the metal abundance in the stellar envelope rises greatly. This results in the removal of the outer stellar layers due to powerful line-driven winds. Mass-loss is explicitly disabled when the central temperature reaches $T_{\text{core}} = 1.1 \times 10^9$ K, during the terminal stage of the star, to maintain numerical stability. This is the reason for the very sharp drop in the rate of mass loss at late times.

2.4 Stellar structure at core-collapse

In this section, we examine the stellar structure of our Pop III star models at the onset of core collapse, first focusing on their density profiles, as shown in Fig. 5. These profiles are extracted from the final snapshot of MESA simulations and they capture the state of the star immediately before collapse. The density distribution reveals two distinct regions: (a) the stellar core characterized by high and nearly constant density and (b) the outer layers that display a near power-law decline in density, with $d \ln \rho / d \ln r \sim -3.1$. Near the stellar surface the density declines very rapidly. We model the density profile as a piecewise function,

$$\rho(r) = \begin{cases} \rho_{\text{core}} & \text{if } r \leq R_{\text{core}}, \\ \rho_{\text{core}} \left(\frac{r}{R_{\text{core}}} \right)^{-m} \left(1 - \frac{r}{R_*} \right)^3 & \text{if } R_{\text{core}} < r \leq R_*, \\ 0 & \text{if } r > R_*, \end{cases} \quad (1)$$

where the core density varies over a large range for the different masses in our model, with $10^4 \lesssim \rho_{\text{core}} / (\text{g cm}^{-3}) \lesssim 10^8$. Likewise,

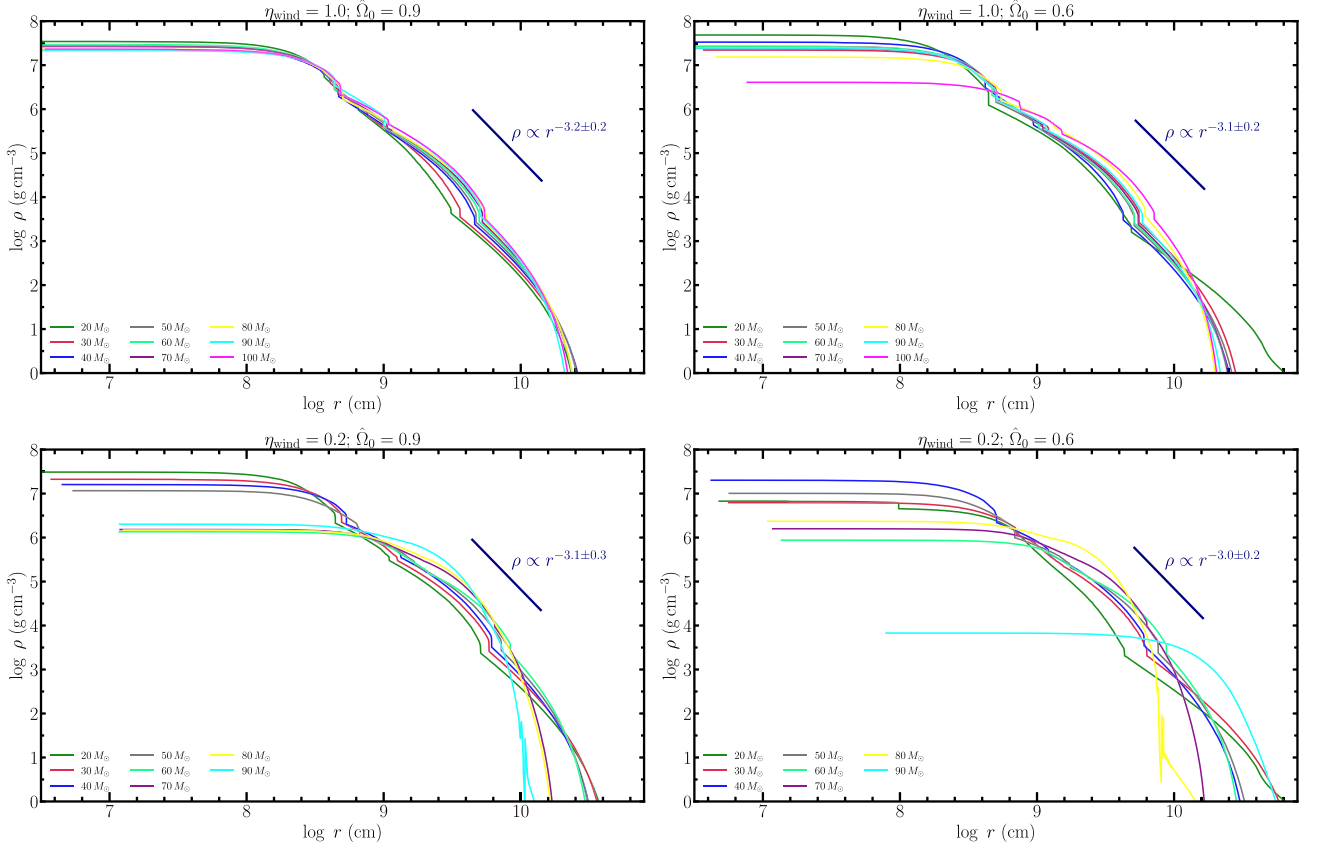


Figure 5. Pre-core-collapse density profiles as a function of radius for stellar models with initial masses ranging from 20 to $100 M_{\odot}$, shown for initial $\hat{\Omega}_0 = 0.9$ (left) and $\hat{\Omega}_0 = 0.6$ (right) and wind scaling factor of $\eta_{\text{wind}} = 1.0$ (top) and $\eta_{\text{wind}} = 0.2$ (bottom).

the core radius varies in the range $1 \lesssim R_{\text{core},8} \lesssim 200$ and the stellar radius in the range $1 \lesssim R_{*,10} \lesssim 6$. The density power-law index appears to show sensitivity to $\hat{\Omega}$, η_{wind} , and M_{ZAMS} , however, in many cases the power-law decline does not have enough dynamic range over which the asymptotic value of m can be determined. When this is not the case, $m \sim 3.1$.

We use these density profiles later to calculate the rate of accretion and jet propagation along the rotation axis of the star. In this way, the density of material in the path of the jet is over-estimated. Since the stellar material near the rotational poles of the star has very little angular momentum, it readily plunges towards the center upon losing pressure support, thus creating a lower density funnel. To what extent the density is diluted along the pole when the jet is launched can be understood with core-collapse numerical simulations that allow proper accounting for free-fall and hydrodynamic effects. This was done in G. Halevi et al. (2023), where they found that pre-core-collapse density profiles, with $d \ln \rho / d \ln r \sim -2.5$, flatten during the collapse to $d \ln \rho / d \ln r \sim -1.5$ before BH formation.

Fig. 6 illustrates the radial variation of angular velocity in the star prior to core collapse, shown for $\hat{\Omega}_0 = \{0.6, 0.9\}$ and $\eta_{\text{wind}} = \{0.2, 1\}$. In most 1D stellar evolution codes, like MESA, rotational effects are included using the *shellular approximation* (e.g. G. Meynet & A. Maeder 1997) that makes the assumption of a constant $\Omega(r)$ over isobars (constant pressure) which is valid in the presence of strong anisotropic turbulence acting along isobars (B. Paxton et al. 2013). This is the reason behind the discontinuous behavior seen in the $\Omega(r)$ profile. It is evident that in the absence

of strong winds ($\eta_{\text{wind}} = 0.2$), the models with higher rotation ($\hat{\Omega}_0 = 0.9$) show greater angular velocities. Some models at the high mass end, particularly with $M_{\text{ZAMS}} > 70 M_{\odot}$, show a smooth $\Omega(r)$ profile when $\eta_{\text{wind}} = 0.2$. These stars suffer from the PPI, as discussed above, making it numerically challenging to evolve them to advanced nuclear burning stages close to core-collapse. Since the evolution was stopped at an earlier time in their evolutions, the angular velocity profiles do not yet show the shellular structure which is apparent in other models that were evolved to near core-collapse.

Fig. 7 shows the distribution of the specific angular momentum as a function of the enclosed mass (M_r) for our stellar models. Like the angular velocity profiles, the angular momentum profiles also show the discontinuous behavior due to the shellular approximation. The star is able to retain a larger angular momentum when the wind efficiency is lowered. For example, when comparing the $\eta_{\text{wind}} = 1.0$ with $\eta_{\text{wind}} = 0.2$ case for the $20 M_{\odot}$ model with $\hat{\Omega}_0 = 0.9$, we find that the specific angular momentum is around a factor of 3 higher for a $3 M_{\odot}$ core (see Tables A1 and A3) when winds are suppressed. Furthermore, the total angular momentum of the entire star is an order of magnitude larger when $\eta_{\text{wind}} = 0.2$ over that when $\eta_{\text{wind}} = 1.0$. Comparison of these profiles with previous results (S.-C. Yoon et al. 2012) reveals consistency in the observed trends. In particular, our models with reduced values of η_{wind} resemble the more favourable models for GRBs previously reported by S. E. Woosley & A. Heger (2006). For a thin accretion disc, the requisite specific angular momentum at the inner-most stable circular orbit (ISCO) is given by (J. M. Bardeen, W. H. Press &

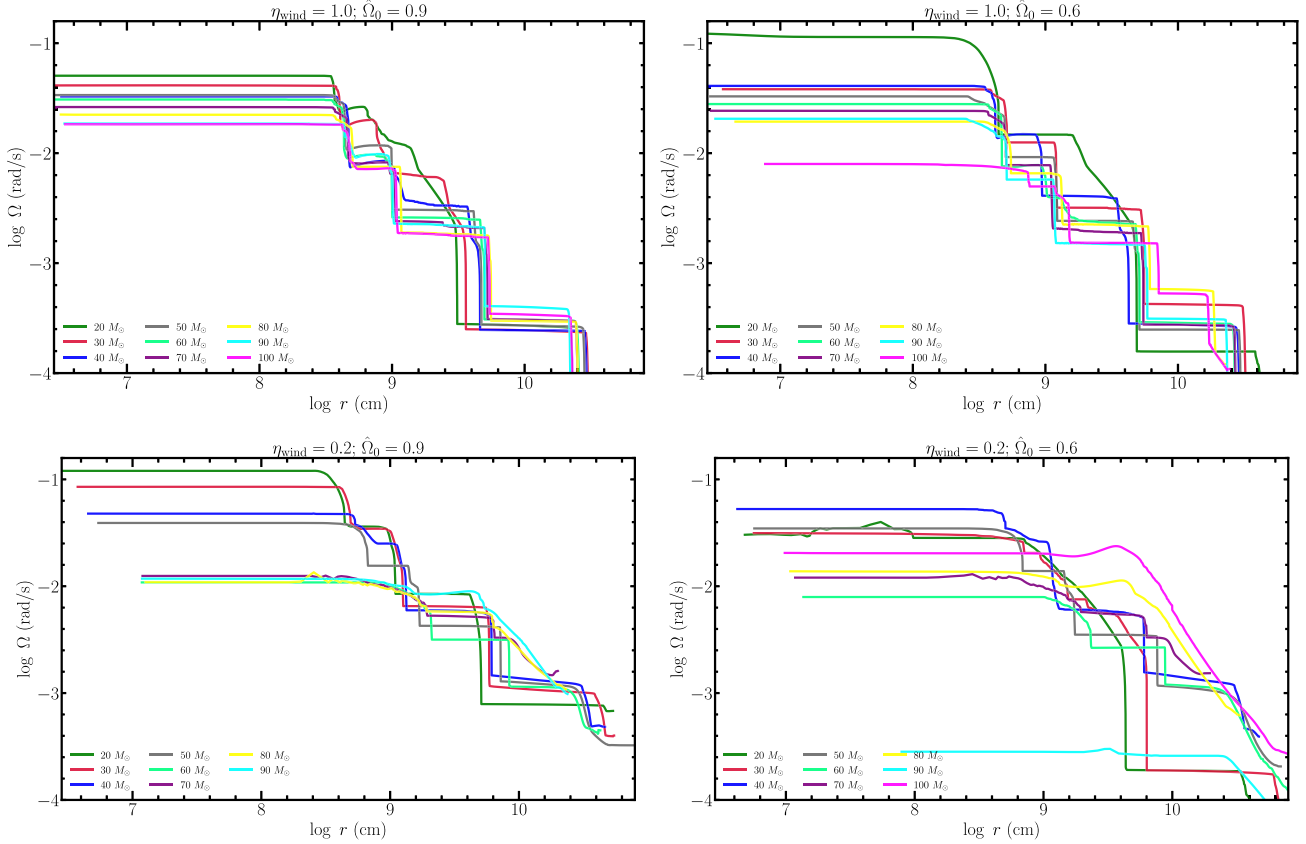


Figure 6. Pre-core-collapse angular velocity profiles as a function of radius for stellar models with initial masses ranging from 20 to 100 M_{\odot} , shown for initial $\Omega_0 = 0.9$ (left) and $\Omega_0 = 0.6$ (right) and with $\eta_{\text{wind}} = 1.0$ (top) and $\eta_{\text{wind}} = 0.2$ (bottom). The staircase pattern arises due to the shellular assumption in MESA that evolves the radial profile over several isobaric shells of stellar material.

S. A. Teukolsky 1972)

$$j_{\text{ISCO}} = \frac{2GM_{\text{BH}}}{3^{3/2}c} [1 + 2(3\hat{R}_{\text{ISCO}} - 2)^{1/2}], \quad (2)$$

where $\hat{R}_{\text{ISCO}} = R_{\text{ISCO}}/R_g$ and $R_g = GM_{\text{BH}}/c^2$ is the gravitational radius of a BH with mass M_{BH} . To launch a successful jet, the basic requirement of $\hat{R}_{\text{ISCO}} > 1$ must be met. When considering, e.g. a BH with mass $M_{\text{BH}} = 5 M_{\odot}$, this condition is met for $j_{\text{ISCO}} \gtrsim 3 \times 10^{16} \text{ cm}^2 \text{ s}^{-1}$ (e.g. A. I. MacFadyen & S. E. Woosley 1999). When taking this to be the fiducial scenario, we find that stars with $\Omega_0 = 0.9$ and higher wind efficiencies will not have enough angular momentum to form an accretion disc. Only in the case with $\eta_{\text{wind}} = 0.2$ there is enough material with angular momentum larger than the minimum needed to have a successful accretion disc and even launch a relativistic jet.

For low η_{wind} values, such as $\eta_{\text{wind}} = 0.2$, reaching core collapse in MESA becomes particularly challenging due to numerical instabilities, which leads to the difficulty of obtaining specific angular momentum profiles for high-mass, low-rotation models ($\Omega_0 = 0.6$). Consequently, profiles are presented only for the mass range of 20–70 M_{\odot} for this case.

2.5 Remnant properties and associated supernovae

Here, we analyse pre-core-collapse (pre-cc) stellar parameters such as the final pre-cc mass ($M_{\text{pre-cc}}$), the specific angular momentum of the central $3 M_{\odot}$ core ($\log j_{3M_{\odot}}$), the compactness parameter $\xi_{2.5}$, and

the carbon-oxygen core mass (M_{CO}). These parameters are presented in detail in Tables (A1, A2, and A3). We use these data to infer the types of supernovae (SNe) and resulting remnants. The compactness parameter, valid particularly for non-rotating stars, is defined as (E. O'Connor & C. D. Ott 2011)

$$\xi_{2.5} \equiv \frac{M_{2.5}/M_{\odot}}{R(M_{2.5})/10^3 \text{ km}}, \quad (3)$$

where $M_{2.5} \equiv 2.5 M_{\odot}$ and $R(M_{2.5})$ is the radial coordinate that includes that mass, indicates whether the core is compact enough to collapse to a BH. When $\xi_{2.5} \gtrsim 0.45$, it becomes very difficult to explode the star, as in a supernova, and BH formation via collapse is the most likely option. Here we use the pre-collapse properties of the progenitor to calculate the compactness, but its actual value should be strictly determined at the time of core bounce. The latter can only be obtained using an advanced numerical code, which is outside the scope of this work. Consequently, the values that we obtain are approximative. Our simulation grid finds $\xi_{2.5} \sim 0.3$, a value typically associated with neutron star formation in non-rotating models for some equations of states. When factoring in the uncertainty in reliably calculating the compactness parameter from stellar evolution codes alone, we make the assumption in later sections that all of our models do not explode and collapse directly to a BH remnant.

The chemical composition of the stellar envelope at the pre-core-collapse stage, as illustrated in Fig. 8, is crucial for classifying the engine-powered supernova types. These figures present mass fraction profiles of key elements (^4He , ^{12}C , ^{16}O) for initial rotation rates of

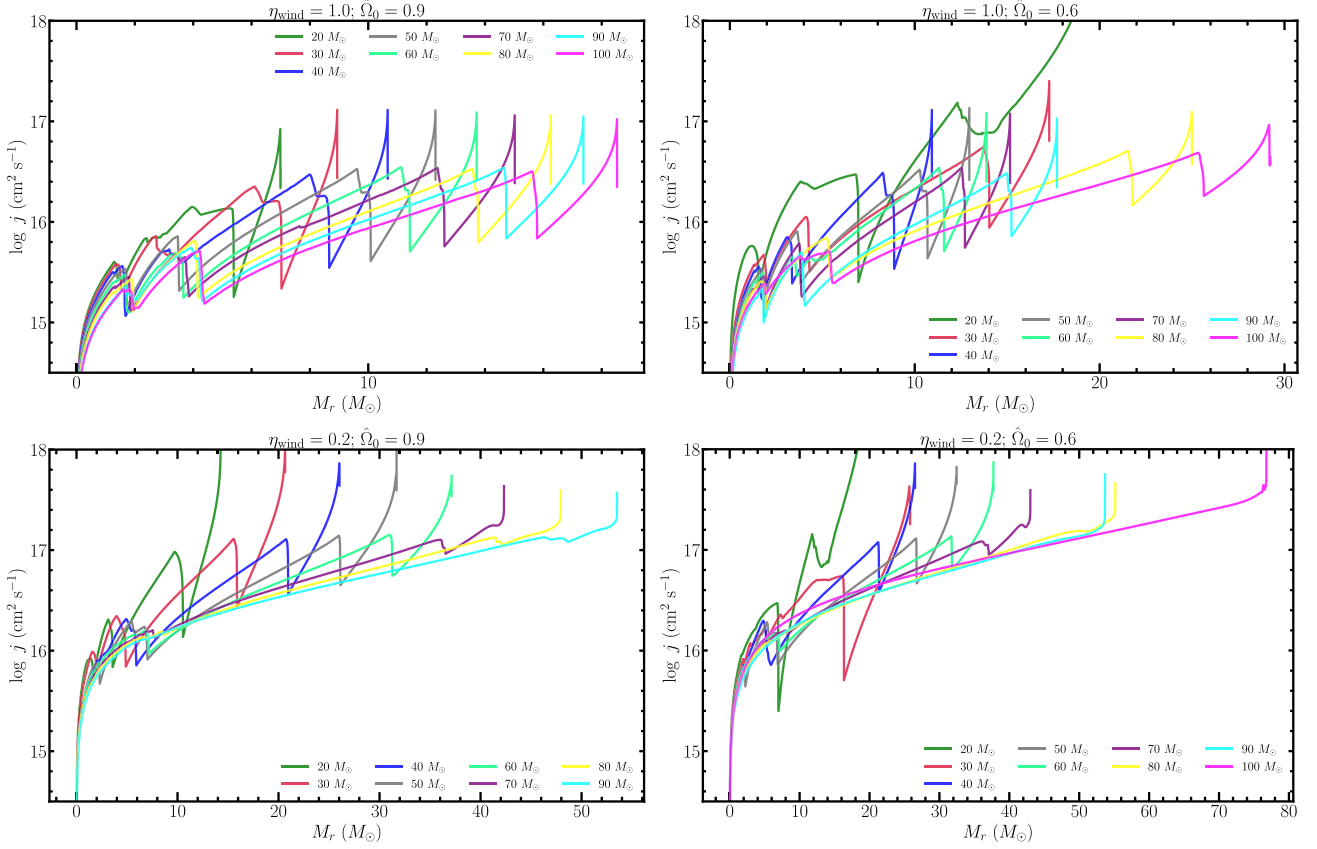


Figure 7. Pre-core-collapse specific angular momentum profiles as a function of enclosed mass for stellar models with initial masses ranging from 20 to $100 M_{\odot}$, with initial rotation $\hat{\Omega}_0 = 0.9$ (left) and $\hat{\Omega}_0 = 0.6$ (right) and $\eta_{\text{wind}} = 1.0$ (top) and $\eta_{\text{wind}} = 0.2$ (bottom). The sawtooth profile corresponds to the staircase profile seen in Fig. 6 that results from the shellular approximation in MESA.

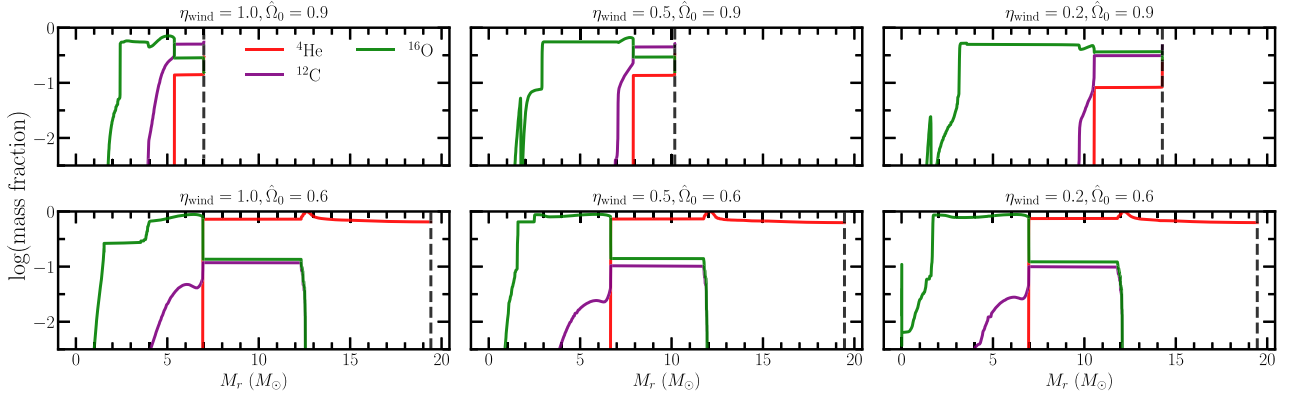


Figure 8. Mass fraction of key elements (${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$) as a function of the mass coordinate (M_r) for a model with an initial mass of $M_{\text{ZAMS}} = 20 M_{\odot}$, shown for initial $\hat{\Omega}_0 = 0.6$ (top) and $\hat{\Omega}_0 = 0.9$ (bottom), and $\eta_{\text{wind}} = \{1.0, 0.5, 0.2\}$ (left to right). The vertical dashed line in each subplot indicates the pre-core-collapse mass at the final stage prior to core-collapse. The bottom row represents a special case, as the evolution of this model at $\hat{\Omega}_0 = 0.6$ is the only one lacking C and O in the envelope, likely resulting in a type Ib SN, while all other models (showing fractions similar to the top row of our grid) exhibit C and O in the envelope, leading to a type Ic SN.

$\hat{\Omega}_0 = 0.6$ and 0.9 , with different wind efficiencies. The complete absence of hydrogen in all models, resulting from rotation-induced mass-loss, prevents the formation of Type II supernovae (SN II), which require a significant hydrogen envelope (A. Heger et al. 2003). Initially, the surface metal abundance decreases gradually due to dilution from rotational mixing that transports hydrogen from the

surface inward, but once critical rotation is reached it increases abruptly as the hydrogen-rich envelope is ejected, exposing the elements resulting from nuclear reactions in the core. Consequently, the outer envelopes become dominated by helium, while the cores exhibit substantial fractions of ${}^{12}\text{C}$ and ${}^{16}\text{O}$, influencing the final remnant and favouring Type Ib or Ic supernovae over SN II. In fact,

several SNe associated with observed population of long-duration GRBs, that are produced by more metal-rich progenitors, are of type Ic (Z. Cano et al. 2017; S. E. Woosley & J. S. Bloom 2006). This provides additional motivation to relate GRBs with rapidly rotating massive stars that may have undergone CHE.

The SN outcome is sensitive to initial mass and rotation. For example, the model with $M_{\text{ZAMS}} = 20 M_{\odot}$ and $\hat{\Omega}_0 = 0.6$ shows no ^{12}C or ^{16}O on the surface, as can be seen in the bottom row of Fig. 8. Instead, the surface is dominated by ^4He . Unlike all other models undergoing CHE, this star evolves toward the red giant phase in the HR diagram (Fig. 1), indicating non-CHE. The combination of low rotation and minimal mass-loss, driven by the low η_{wind} , favours the formation of a Type Ib supernova, characterised by a helium-rich envelope devoid of hydrogen. In contrast, all other models with higher rotation ($\hat{\Omega}_0 = 0.9$) or initial masses ($M_{\text{ZAMS}} \geq 25 M_{\odot}$) develop massive CO cores with significant surface fractions of ^{12}C and ^{16}O , leading to a direct core-collapse.

3 GRB PRODUCTION EFFICIENCY

3.1 Core collapse and accretion

In contrast to the type II core-collapse that delivers a successful supernova explosion in massive and non-rotating stars, rapidly rotating such stars may form an accretion disc after the iron-core collapses into a black hole (A. I. MacFadyen & S. E. Woosley 1999). In the collapsar model of long-GRBs, infalling stellar material with specific angular momentum in the range $3 \lesssim j/(10^{16} \text{ cm}^2 \text{ s}^{-1}) \lesssim 20$ will form a compact disc at the optimal radius away from the Kerr BH to power relativistic outflows that produce GRBs. To calculate the rate of accretion in this scenario, we use the model of P. Kumar, R. Narayan & J. Johnson (2008) that considers the infall of stellar material from an axisymmetric, rotating star onto an accretion disc after the star has lost pressure support. In our formulation below, many quantities that depend on the polar angle θ have been averaged since MESA, being a 1D code, only provides stellar profiles as a function of radius.

After a temporal delay given by the sound crossing time, which can be approximated using the free-fall time, $t_s(r) \sim t_{\text{ff}}(r) = \sqrt{3\pi/32G\bar{\rho}(r)}$, where $\bar{\rho}(r) = 3M_r/4\pi r^3$ is the mean density of material within radius r , the material at a spherical coordinate (r, θ, φ) starts to free-fall. Here, M_r is the mass enclosed within radius r ,

$$M_r = \int_0^r 4\pi r'^2 \rho(r') dr'. \quad (4)$$

and $\rho(r)$ is the local density. Given its angular velocity $\Omega(r, \theta)$ the material follows an elliptical trajectory, with eccentricity $e = 1 - \Omega^2(r, \theta) \sin^2 \theta / \Omega_k^2$, that intersects the equatorial plane after duration

$$t_{\text{eq}}(r, \theta) = \frac{1}{\Omega_k} \left(\arccos(-e) + e\sqrt{1-e^2} \right) (1+e)^{-3/2} + t_s(r), \quad (5)$$

and then circularizes at the fallback radius in the equatorial plane at

$$R_{\text{fb}} \approx r \frac{\Omega^2(r)}{\Omega_k^2(r)}. \quad (6)$$

This radius is approximated using the polar-angle averaged angular velocity of the infalling material from a given radius r and where $\Omega_k = \sqrt{GM_r/r^3}$ is the Keplerian angular velocity of that material. Mass falls at the equatorial plane at the rate

$$\dot{M}_{\text{fb}} \equiv \frac{dM_{\text{fb}}(r)}{dt} = \frac{dM(r)}{dr} \left[\frac{d\langle t_{\text{eq}}(r) \rangle}{dr} \right]^{-1}, \quad (7)$$

where $dM/dr = 4\pi\rho(r)r^2$ and

$$\langle t_{\text{eq}}(r) \rangle = \frac{1}{2} \int t_{\text{eq}}(r, \theta) \sin \theta d\theta \quad (8)$$

is the angle averaged equatorial fallback time. It is possible that regions close to the rotational axis of the star might experience an outflow instead, in which case the above estimate needs to be appropriately modified considering the solid angle of the material that falls back instead of the 4π factor. Here, we do not consider such possibility due to its inherent uncertainty and instead consider material from all polar angles to contribute either to the BH mass or the accretion disc. In what follows, we also do not consider general relativistic effects near the newly formed BH and our treatment remains non-relativistic (see, however, S. Ghodla & J. J. Eldridge 2024, for a formalism including a dynamical space-time).

Initially, material in the central regions falls freely onto the proto-neutron-star and turns it into a BH when the mass of the compact remnant grows beyond the threshold mass of $M_{\text{th}} \simeq 1.2 M_{\text{TOV}} \simeq 2.4 M_{\odot}$ (e.g. C. Breu & L. Rezzolla 2016), for a maximum mass of a non-rotating NS $M_{\text{TOV}} \simeq 2 M_{\odot}$ (e.g. B. Margalit & B. D. Metzger 2017; L. Rezzolla, E. R. Most & L. R. Weih 2018; M. Shibata et al. 2017). Subsequently, more mass with specific angular momentum less than that required to circularise continues to fall, and the mass and spin of the BH continues to grow, with

$$M_{\text{BH}} = \int_0^{t_0} \dot{M}_{\text{fb}}(t_{\text{eq}}) dt_{\text{eq}}, \quad a_* = \frac{cJ_r}{GM_{\text{BH}}^2} \quad (9)$$

where J_r is the angular momentum of the mass M_r that fell into the BH until some critical time $t \leq t_0$. At this critical time, a disc begins to form of material that has enough specific angular momentum to launch it into orbit around the BH at radii beyond R_{ISCO} , the innermost stable circular orbit (J. M. Bardeen et al. 1972),

$$R_{\text{ISCO}}(M_{\text{BH}}, a_*) = \frac{GM_{\text{BH}}}{c^2} \left\{ 3 + z_2 - \sqrt{(3 - z_1)(3 + z_1 + 2z_2)} \right\}, \quad (10)$$

with

$$z_1 = 1 + (1 - a_*^2)^{1/3} [(1 + a_*)^{1/3} + (1 - a_*)^{1/3}], \quad (11)$$

$$z_2 = \sqrt{3a_*^2 + z_1^2}. \quad (12)$$

The critical time to disc formation is determined from the condition, $R_{\text{ISCO}}(M_r, a_*) = R_{\text{fb}}(r)$, when the fallback radius in the equatorial plane becomes comparable to R_{ISCO} . Material with $R_{\text{fb}} < R_{\text{ISCO}}$ falls directly into the nascent BH and that with $R_{\text{fb}} > R_{\text{ISCO}}$ has sufficient angular momentum to circularize and form an accretion disc. We first obtain the critical radius $R_{\text{fb,crit}}$ from the stellar profiles and then calculate the corresponding t_0 as well as the BH mass and spin at t_0 . This is demonstrated in Fig. 9, using a $20 M_{\odot}$ model (referred to as 20R9W2 henceforth) with $\hat{\Omega}_0 = 0.9$ and $\eta_{\text{wind}} = 0.2$, that illustrates the intersection between the R_{fb} and the R_{ISCO} as a function of the mass coordinate M_r . The red point denotes the critical intersection at $M_r = M_{\text{BH},0} = 7.5 M_{\odot}$, with spin $a_{*0} = 0.72$ at the critical time $t_0 = 9.4$ s after core-collapse. For the innermost part of the stellar core, the R_{ISCO} curve rises at a later mass coordinate as it is not well defined since a_* exceeds unity there. As a result, we only show the range in mass coordinate where R_{ISCO} is well defined, according to equation (10). The critical solution depends on the rotational profile of the star at core-collapse and it is not obtained in all models, in which we find $R_{\text{fb}} < R_{\text{ISCO}}$ for the majority of the stellar material lacking of centrifugal support and leaving very little for accretion. Such models are not capable of launching

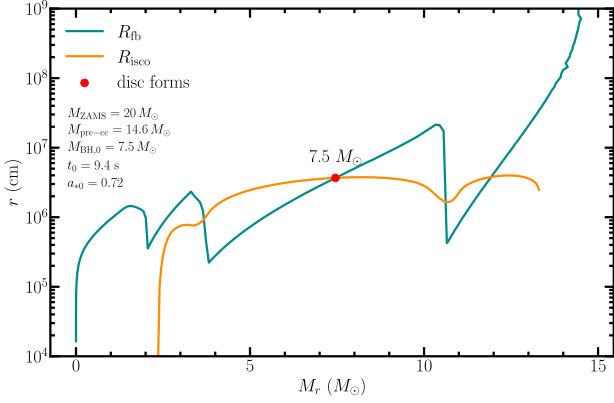


Figure 9. Intersection of the fallback radius profile, R_{fb} , with R_{ISCO} , shown for a $M_{ZAMS} = 20 M_{\odot}$ star post core-collapse as a function of the mass coordinate M_r . This model assumes a wind scaling factor of $\eta_{wind} = 0.2$ and an initial rotation rate of $\hat{\Omega}_0 = 0.9$. All mass below the intersection point (red dot) falls directly into the BH, while that above it goes into an accretion disc. The mass and spin of the BH at the critical time ($t_0 = 9.4$ s) when the disc forms are $M_{BH,0} = 7.5 M_{\odot}$ and $a_{*0} = 0.72$.

relativistic jets and do not produce GRBs. We have verified the results of our model using the pre-collapse stellar profiles for the 16TI model from S. E. Woosley & A. Heger (2006), for which we find $M_{BH,0} \simeq 3 M_{\odot}$ and $a_{*0} \simeq 0.7$. These are consistent with results from advanced numerical simulations of core-collapse for the same stellar model (e.g. C. Dean & R. Fernández 2024; O. Just et al. 2022).

We consider the formation of a thick accretion disc that accretes matter onto the BH over the viscous time-scale (R. Narayan, T. Piran & P. Kumar 2001). Accretion in such a disc may proceed in three different regimes, namely (i) NDAF (neutrino-dominated accretion flow; R. Popham, S. E. Woosley & C. Fryer 1999), (ii) ADAF (accretion-dominated accretion flow; R. Narayan & I. Yi 1994), and (iii) a mix of both (i) and (ii). The mass accretion rates, \dot{M}_{BH} , onto the black hole in all three regimes are given in equations (22)–(24) of P. Kumar et al. (2008); the interested reader should look there for more details. When accretion occurs via an ADAF, a significant amount of disc mass is lost due to outflows, in which case the accretion rate at any given radius, $\dot{M}(r)$, is suppressed as a power-law,

$$\dot{M}(r) = \dot{M}_{acc}(R_d) \left(\frac{r}{R_d} \right)^s \quad \text{for} \quad 0 \leq s \leq 1. \quad (13)$$

The exact value of the power-law index s is unclear and numerical simulations in different works find different values (J. C. McKinney, A. Tchekhovskoy & R. D. Blandford 2012; R. Narayan et al. 2012). We keep it as a free parameter in what follows. Here, $\dot{M}_{acc} = \dot{M}_d/t_{acc}$, is the mass accretion rate out of the disc of mass M_d . The accretion proceeds over the standard characteristic viscous time-scale $t_{acc} \sim 2/\alpha\Omega_k$, where $\alpha \sim 10^{-2}$ – 10^{-1} is the dimensionless viscosity parameter. The mass of the disc changes due to fallback and accretion, with $\dot{M}_d = \dot{M}_{fb} - \dot{M}_{acc}$ and some fraction of the disc mass is then accreted onto the BH.

The mass and spin of the BH grows over time as mass accretes from the disc. To calculate the temporal evolution of disc mass, and the mass and spin of the BH, we numerically solve the coupled equations (25)–(32) from P. Kumar et al. (2008). One caveat is that this formalism does not include the effect of outflows at higher latitudes (i.e. near the rotational pole) as well as accretion of low-angular momentum stellar material from the same polar regions. The latter may influence the spin evolution of the BH, and that effect is

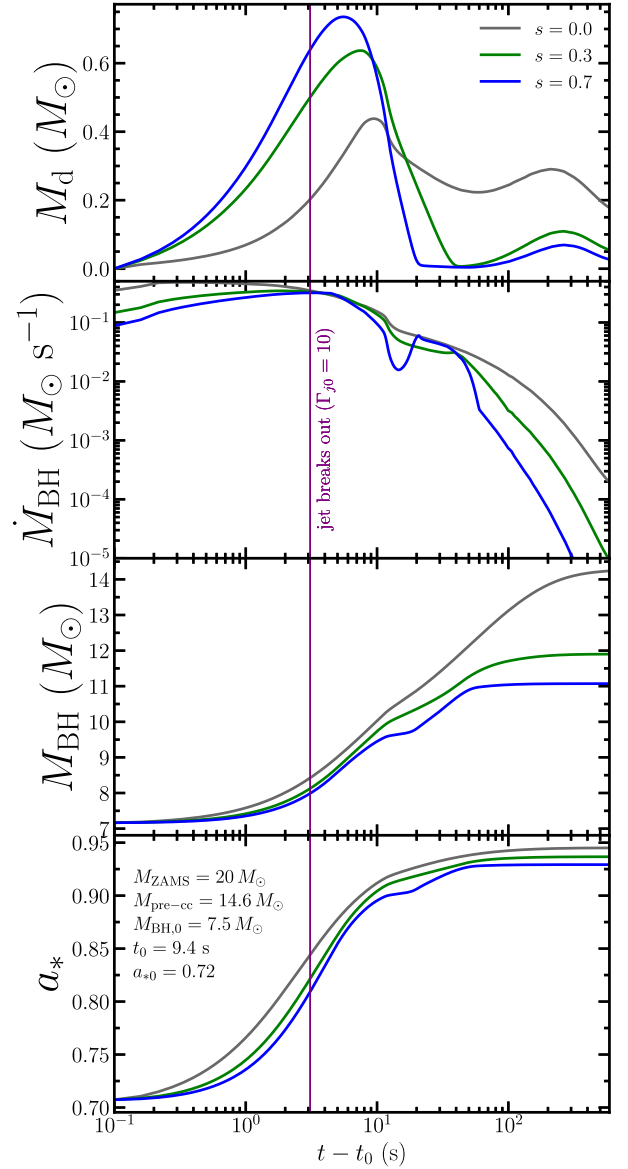


Figure 10. Temporal evolution of key quantities describing the accretion disc and central black hole properties in the 20R9W2 model. From top to bottom, the panels show the temporal evolution of i) the disc mass, ii) the accretion rate \dot{M}_{BH} , iii) the black hole mass, and iv) dimensionless spin parameter a_* for different values of the parameter $s = \{0.0, 0.3, 0.7\}$. The vertical dashed line represent the moment when the jet breaks out of the surface of the progenitor star considering an initial Lorentz factor of $\Gamma_{j0} = 10$ with jet opening angle $\theta_0 = 0.1$. See Fig. 9 for further details.

not included here due to its complexity which requires numerical simulations.

Fig. 10 shows the temporal evolution of accretion and the properties of the BH for the 20R9W2 model. From top to bottom, the panels show (i) the temporal evolution of the disc mass (M_d), that initially grows, reaches a peak, and then declines. The disc mass is controlled by the rate of mass fallback to the equatorial plane and that of accretion, where the former is governed by the angular momentum profile at core-collapse. The disc mass initial grows due to the infalling material having sufficient angular momentum to join the accretion disc, until it reaches the point (shown by the sharp drop in the green curve in Fig. 9) where some material from larger radii

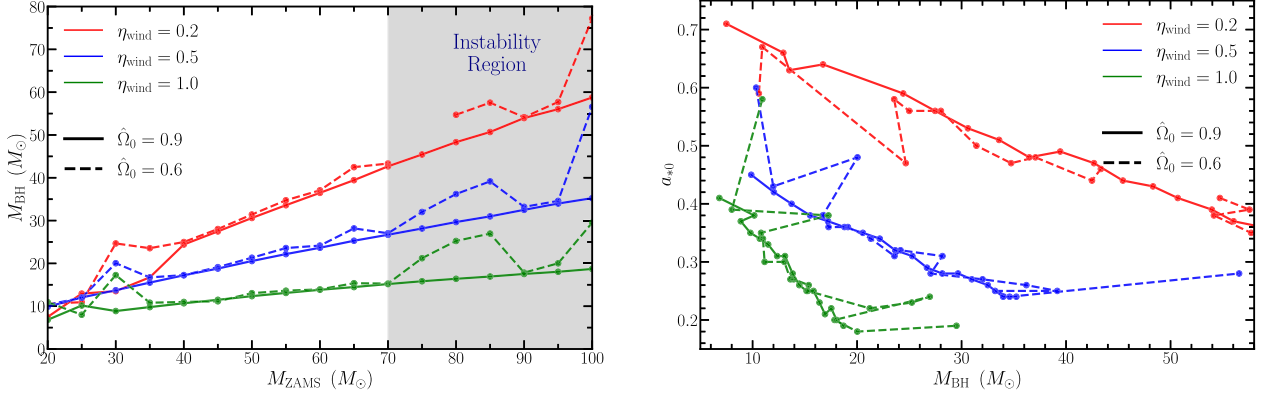


Figure 11. (Left) Evolution of the initial black hole mass ($M_{\text{BH},0}$) as a function of the initial stellar mass (M_{ZAMS}). Results are shown for different values of η_{wind} . Solid lines correspond to models with an initial rotation rate of $\hat{\Omega}_0 = 0.9$, while dashed lines represent those with $\hat{\Omega}_0 = 0.6$. (Right) Evolution of the dimensionless spin parameter (a_{*0} when disc forms) as a function of M_{ZAMS} .

falls directly into the BH, having insufficient angular momentum to join the disc. This manifests as a sudden drop in the disc mass. The disc mass starts to grow at later times as material from the outer stellar layers continues to join the disc. (ii) The temporal evolution of the accretion rate, which for the first ten seconds remains above $0.1 M_{\odot} \text{ s}^{-1}$. (iii) and (iv) The temporal evolution of the BH mass and spin, where both are sensitive to the accretion rate radial profile power-law index s . The pre-core-collapse mass of the star in the 20R9W2 model is $14.6 M_{\odot}$ and not all of it makes it into the BH as some is lost to disc-driven winds and outflows during the accretion process. The BH spin increases from its initial value $a_{*0} = 0.72$ to $a_* > 0.9$ as more angular momentum is brought in by the accreting material. When the BH launches any outflows, some of the angular momentum will be lost to those, and therefore, its spin will be regulated by gain and loss of angular momentum (J. Jacquemin-Ide et al. 2024; B. Lowell et al. 2024; Z.-F. Wu et al. 2025). How much angular momentum is lost to outflows depends on the jet launching process, which is unclear.

Fig. 11 (left panel) shows the initial black hole mass ($M_{\text{BH},0}$) that forms promptly upon core-collapse as a function of the ZAMS mass for our grid. The results are presented considering different stellar wind efficiencies ($\eta_{\text{wind}} = 0.2, 0.5, 1.0$) and two initial rotation rates, $\hat{\Omega}_0 = 0.9$ (solid lines), and $\hat{\Omega}_0 = 0.6$ (dotted lines). The right panel shows the initial spin parameter a_{*0} as a function of initial BH mass, it decreases with increasing BH mass and correlates well with decreased wind efficiency that retains larger angular momentum in the star at core-collapse. Both figures highlight a region of instability for $M_{\text{ZAMS}} > 70 M_{\odot}$, previously discussed in Fig. 2 where significant effects due to pulsational pair-instability are observed.

3.2 Jet power

The composition of GRB jets, i.e. whether they are kinetic-energy-dominated or Poynting-flux-dominated, remains unclear, and so is the launching mechanism. Therefore, the main problem is how to convert, and with what efficiency, the power brought in by the accreting matter, which is $\dot{M}_{\text{BH}} c^2$. Two different jet launching mechanisms have been proposed thus far: neutrino annihilation (A. I. MacFadyen & S. E. Woosley 1999) and magnetohydrodynamic (MHD) processes in the vicinity of the rapidly rotating BH, e.g. the R. D. Blandford & R. L. Znajek (1977) (BZ) process. Several works have explored the efficiency of both mechanisms (W.-H. Lei et al.

2017; T. Liu et al. 2015; R. Popham et al. 1999), but it is still unclear which one dominates in GRBs.

In the neutrino-annihilation framework, the jet is powered by the energy deposition from neutrino and antineutrino annihilation above the accretion disc. Following M. Leng & D. Giannios (2014), the jet power for a black hole with spin $a_* = 0.95$ is approximated as

$$L_{\nu\bar{\nu}} \approx 1.3 \times 10^{52} \left(\frac{M_{\text{BH}}}{3 M_{\odot}} \right)^{-3/2} \times \left(\frac{\dot{M}_{\text{BH}}}{M_{\odot} \text{ s}^{-1}} \right)^{9/4} \text{ erg s}^{-1}, \quad (14)$$

with a more detailed discussion given in I. Zalamea & A. M. Beloborodov (2011). As shown in M. Leng & D. Giannios (2014), the jet power from neutrino annihilation is typically limited to a maximum of $\sim 5 \times 10^{51} \text{ erg s}^{-1}$ for an engine duration of $\sim 10 \text{ s}$, and it starts to fall for longer lasting bursts. This is problematic for ultra-long bursts, with duration of $\sim 10^3 \text{ s}$, where this scenario is unable to account for the radiated energy.

Alternatively, in the collapsar scenario, what is clear is that the jet is ultimately powered by accretion, and therefore its power can be parametrized using an efficiency parameter that is regulated by the spin of the BH, such that

$$L_j = \eta_j(a_*) \dot{M}_{\text{BH}} c^2 \quad \text{and} \quad \eta_j(a_*) \approx 0.07 \left[\frac{a_*}{1 + \sqrt{1 - a_*^2}} \right]^5, \quad (15)$$

where $\eta_j(a_*)$ is obtained from GRMHD simulations (J. C. McKinney 2005). In this scenario, the spin of the BH is only allowed to increase, from an initial value of $0 < a_{*0} < 1$ to the maximal value of $a_* = 1$, due to addition of angular momentum brought by the accreting matter.

However, torques due to large scale magnetic fields threading the accreting gas and the event horizon of the rapidly spinning BH, as posited in the BZ process, remove angular momentum from the BH and power a Poynting-flux-dominated outflow. As demonstrated by Z.-F. Wu et al. (2025) (also see J. Jacquemin-Ide et al. 2024; B. Lowell et al. 2024), the spin of the BH may thus be regulated to attain an equilibrium value of $a_{*\text{eq}} \sim 0.5$ when the accumulated magnetic flux at the event horizon (Φ_{BH}) in the BZ scenario is $\sim 0.4 \Phi_{\text{MAD}}$, where Φ_{MAD} is the magnetic flux when the accretion transitions to the *magnetically arrested disc* (MAD) state (R. Narayan, I. V. Igumenshchev & M. A. Abramowicz 2003; A. Tchekhovskoy, R. Narayan & J. C. McKinney 2011). In the MAD state, the accumulated magnetic flux is strong enough to disrupt the accretion of gas when the magnetic pressure exceeds the ram pressure of the accreting gas.

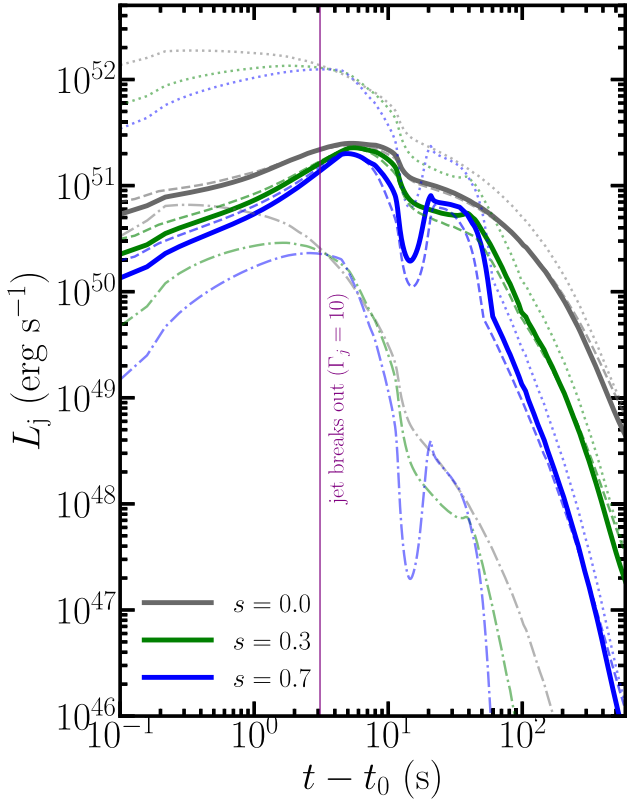


Figure 12. Temporal evolution of the true jet power L_j for our model, shown for $s = \{0.0, 0.3, 0.7\}$. The linestyles correspond to the different ways to calculate the jet power: (i) *Solid lines*: Standard jet power obtained with Eq. (15). (ii) *Dash-dotted lines*: Considering the energy deposition rate due to neutrino annihilation (equation 14). (iii) *Dotted lines*: Maximum jet power assuming that the system reaches equilibrium between the thin-disc accretion and MAD state. (iv) *Dashed lines*: Jet power for a new model 20R6W2 with higher metallicity ($M_{ZAMS} = 20 M_\odot$, $\tilde{\Omega}_0 = 0.6$, $\eta_{wind} = 0.2$, $Z = 0.01 Z_\odot$) and calculated with equation (15). The accretion disc forms at $t = t_0$ and it is assumed that a hydrodynamic jet is launched shortly thereafter at the injection radius of $z_{inj} = 10 r_g \approx 10^7$ cm. The initial bulk Lorentz factor is $\Gamma_{j0} = 10$ with jet opening angle $\theta_0 = 0.1$. For $s = 0.0$, the total injected energies after breakout for each case are: (i) $E_j \approx 1.00 \times 10^{53}$ erg, (ii) $E_j \approx 8.32 \times 10^{50}$ erg, (iii) $E_j \approx 2.33 \times 10^{53}$ erg, (iv) $E_j \approx 9.98 \times 10^{52}$ erg.

At the equilibrium spin value, the accretion efficiency is maximized in powering the jet, which reaches $L_{j,eq}/\dot{M}_{BH}c^2 \approx 0.022$ (Z.-F. Wu et al. 2025, see their fig. 2).

Fig. 12 shows the temporal evolution of the jet power obtained from the three different scenarios, as discussed above, for the 20R9W2 model and for different values of the parameter s . All three scenarios show different efficiencies, with the least efficient being the neutrino annihilation and the most efficient is in which the BH spin is regulated by the BZ process. The neutrino-annihilation-powered jets are up to an order of magnitude less luminous than those obtained from equation (15), especially at low accretion rates where neutrinos may not efficiently cool the disc.

The different values of the s parameter, that controls how much mass is lost to outflows when accretion occurs in the ADAF regime, has some effect on the jet power due to its effect on the accretion rate. What is particularly interesting is the dip in the jet power seen most prominently for $s = 0.7$. This can lead to the ejection of two different mass shells that can then power distinct prompt emission episodes followed by afterglow emission with the potential

for a re-brightening due to refreshed shocks (however, see e.g. B. D. Metzger, P. Beniamini & D. Giannios 2018 for an alternative scenario with fallback accretion onto a millisecond magnetar). The main parameters in the two-shell collision problem are the ratio of (kinetic) energies and that of the bulk Lorentz factors of the two shells. The former can be obtained from the jet power evolution, but the latter depends on the baryon loading of each shell, which is not so clear. This type of re-brightening has been seen in a small fraction of afterglows (see e.g. fig. 9 of M. Busmann et al. 2025) in long GRBs. Typically, both parameters are not known a priori and are instead constrained from fits to re-brightening episodes in the afterglow. Knowledge of the temporal evolution of jet power, and therefore the energy in the two shells reduces the unknown parameters by one.

We also compare the jet power (dashed lines), calculated using equation (15), for a metal rich ($Z = 0.01 Z_\odot$) progenitor of the same mass but with slower initial rotation ($\tilde{\Omega}_0 = 0.6$) in Fig. 12. This model is similar to a Pop II progenitor and its comparison with the Pop III model shows that stars in our model grid may ultimately produce GRBs with properties, e.g. energetics and durations, similar to that expected from Pop II stars. This adds to the difficulty of distinguishing between Pop III and more metal rich progenitors of high redshift GRBs.

In what follows, we use equation (15) to calculate the jet power, which is intermediate between the neutrino-annihilation and BZ powered jets.

3.3 Criteria for a successful GRB: jet launching & breakout

Given the expansive grid of simulations performed in this work, a variety of outcomes are possible, including successful and choked jets. In the former case, the jets are able to penetrate out of the star before the jet power declines significantly, and in the latter, the jet either does not have sufficient power or it is only powered for a duration shorter than needed for a successful breakout, causing it to choke inside a star. Below we quantify the different requirements to select cases that will yield a successful jet and a GRB. We find that a rapidly rotating stellar core, a sufficiently massive accretion disc, and a powerful, long-lived jet are all necessary conditions to successfully propagate through the progenitor star's envelope and produce a GRB without being choked.

3.3.1 Threshold jet power and breakout conditions

The propagation of the jet inside the star depends mainly on the engine activity time, t_{eng} , and the jet power, which must exceed a threshold value, L_j^{thr} . This threshold is governed by the condition that the velocity of the jet head must exceed the local sound speed at the distance from the central engine where the jet is injected (M. A. Aloy, C. Cuesta-Martínez & M. Obergaulinger 2018). The local sound speed depends on the ambient density ρ_a and pressure p_a , such that $c_{s,a} = \sqrt{\hat{\gamma} p_a / \rho_a}$, where $\hat{\gamma}$ is the adiabatic index. The velocity of the jet head is affected by the jet's interaction with the surrounding medium, that produces a forward shock which shock heats the ambient medium and a reverse shock which shock heats the relativistic jet and slows it down to at most mildly relativistic speeds. By balancing the momentum flux density in the frame of the jet head, its velocity can be derived to yield $v_h \approx c / (1 + \tilde{L}^{-1/2})$ (O. Bromberg et al. 2011; C. D. Matzner 2003), where $\tilde{L} = L_j / \Sigma_j \rho_a c^3$ gives the ratio of the jet energy density to that of the ambient medium. Here, $\Sigma_j = z_h^2 \Delta \Omega_j$ is the cross-sectional area of the jet head at a distance z_h away from the central engine along the jet axis, and $\Delta \Omega_j$ is the solid

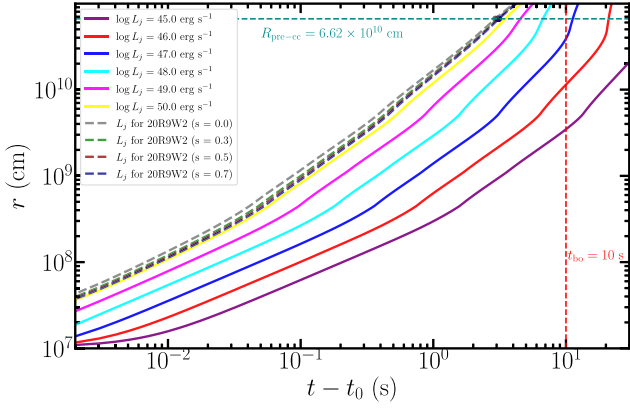


Figure 13. Temporal evolution of the jet head position z_h for the 20R9W2 model. The solid lines display the jet head trajectories for a range of constant L_j , spanning from $\log L_j = 45$ to 50 (in erg s^{-1}), computed within the framework of O. Bromberg et al. (2011). The stellar surface is shown by the horizontal line at $R_{\text{pre-cc}} = 6.62 \times 10^{10}$ cm, and the typical breakout time benchmark of $t_{\text{bo}} \lesssim 10$ s, denoted by the vertical line. The dashed lines represent the breakout times for the L_j curves shown in Fig. 12, corresponding to different values of the parameter s ($s = 0.0, 0.3, 0.5$, and 0.7). The jet breakout times for this model are $t_{\text{bo}} = (2.94, 3.05, 3.09, 3.12)$ s for $s = (0.0, 0.3, 0.5, 0.7)$, respectively. See the caption of Fig. 12 for more details.

angle subtended by it. Since we are interested in the conditions near the injection distance $z_h \approx z_{\text{inj}}$, $\Delta\Omega_j = \pi\theta_0^2$ for a conical jet with initial jet half-opening angle of $\theta_0 \ll 1$. By enforcing the injection condition, $v_h > c_{s,a}$, where the jet must move supersonically to avoid being choked, the threshold jet power can be obtained from (M. A. Aloy et al. 2018)

$$L_j^{\text{thr}} \approx 1.6 \times 10^{49} z_{\text{inj},9}^2 \theta_{0,-1}^2 p_{a,22} \text{ erg s}^{-1}. \quad (16)$$

For the 20R9W2 model, the threshold jet power is estimated as $L_j^{\text{thr}} \approx 10^{48} \text{ erg s}^{-1}$, where we have assumed an opening angle $\theta_0 = 0.1$ rad, an injection radius of $z_{\text{inj}} = 10r_g \approx 10^7$ cm, with $r_g \approx 10^6$ cm for a $7.5 M_\odot$ BH, and an ambient pressure of $p_a(z_{\text{inj}}) \simeq 6.5 \times 10^{24}$ dyne cm^{-2} from our model.

We ascertain the threshold jet power for the 20R9W2 model by analytically propagating the jet inside the star using the framework of O. Bromberg et al. (2011). We inject a relativistic, cold jet at the base of the flow, which is set to a distance $z_{\text{inj}} = 10R_g \approx 10^7$ cm along the jet symmetry axis. The jet is introduced with a bulk Lorentz factor of $\Gamma_{j0} = 10$ with a half-opening angle of $\theta_0 = 0.1$ rad, and it is powered with a time-dependent jet power $L_j(t)$ for an engine activity time t_{eng} . The adopted formalism includes the formation of forward and reverse shocks at the jet head, where the former shock heats the stellar material and the latter shock heats the jet. This inflates a pressurized cocoon of shock heated material surrounding the jet that collimates the jet head. Even though the jet is introduced moving relativistically, it slows down to mildly relativistic speeds due to its interaction with the stellar interior. After penetrating out of the stellar envelope, the jet material propagates relativistically.

Fig. 13 shows the temporal evolution of the jet head position z_h for the 20R9W2 model. We first test for the threshold jet power by injecting the jet with a constant power in the range $45 \leq \log L_j (\text{erg s}^{-1}) \leq 50$ and note the amount of time it takes for the jet to break out of the star, as shown by the solid lines in the figure. The goal here is to identify the value of L_j for which a hydrodynamic jet is able to break out of the star over a typical

breakout time of $t_{\text{bo}} \sim 10$ s (O. Bromberg, J. Granot & T. Piran 2015). As expected, more powerful jets break out over shorter times since their higher power drives a faster head, according to the relation $v_h \approx c/(1 + \tilde{L}^{-1/2})$. In this case, $\tilde{L} \propto L_j/\rho_a$ for a fixed size of the jet head, and since ρ_a decreases (equation 1), v_h increases as the jet advances, facilitating earlier breakout. As shown in the figure, jets with power $L_j < 10^{48} \text{ erg s}^{-1}$ take longer than the typical engine activity time to breakout for this particular stellar model and therefore may be choked. The argument presented here is not definitive proof that they are indeed choked, but their long breakout times are inconsistent with observations (O. Bromberg et al. 2015), and therefore offer an indirect way to disfavour them.

When we use the time-dependent jet power, as obtained from our accretion model and shown in Fig. 12, the jet is able to breakout over a much shorter break out time of $t_{\text{bo}} \approx 3$ s, consistent with recent GRMHD collapsar simulations (G. Urrutia, A. Janiuk & H. Olivares 2025). The propagation of the jet head is shown for the different values of the parameter s (0.0, 0.3, and 0.7) with negligible differences. These profiles reflect a more realistic scenario where the activity of the central engine decreases over time due to the reduction in material available for accretion.

The threshold jet power alone is not enough to guarantee a successful breakout. For that the jet must be powered for at least as long as the jet breakout time of the star, which can vary depending on the jet power, stellar radius, and whether the jet is hydrodynamic or Poynting flux dominated (O. Bromberg & A. Tchekhovskoy 2016). The typical jet breakout time from Wolf-Rayet progenitors that have $R_{\text{WR}} \sim 10^{11}$ cm is of the order of $t_{\text{bo}} \sim 10$ s. (O. Bromberg et al. 2015). If the jet is powered at L_j^{thr} over this period, then the total energy of bipolar jets is $E_j = 2L_j^{\text{thr}} t_{\text{bo}}$. A large fraction of this energy is deposited inside the star and goes into powering a quasi-spherical cocoon that also breaks out of the star. In order to produce the GRB the engine activity time must be larger than the breakout time, so that the minimum energy that is available for the GRB is around $E_{j,\text{min}} = 2L_j^{\text{thr}}(t_{\text{eng}} - t_{\text{bo}})$. The radiated γ -ray energy is some fraction of this, and the minimum isotropic-equivalent energy for a uniform jet with half-opening angle θ_j is then

$$E_{\gamma,\text{min}}^{\text{iso}} \approx \frac{\epsilon_\gamma 2E_{j,\text{min}}}{\theta_j^2} \approx \frac{4\epsilon_\gamma L_j^{\text{thr}}(t_{\text{eng}} - t_{\text{bo}})}{\theta_j^2} \quad (17)$$

$$\approx 4 \times 10^{50} (1+z)^{-1} \epsilon_{\gamma,-1} L_{j,48}^{\text{thr}} t_{\text{GRB},1} \theta_{j,-1}^{-2} \text{ erg}, \quad (18)$$

where we have assumed a γ -ray efficiency of $\epsilon_\gamma = 0.1 \epsilon_{\gamma,-1}$, $(t_{\text{eng}} - t_{\text{bo}}) = t_{z,\text{GRB}} = t_{\text{GRB}}/(1+z) = 10(1+z)^{-1} t_{\text{GRB},1}$ s, and $\theta_j = 0.1 \theta_{j,-1}$. The above estimate is in agreement with the low-end of the E_{γ}^{iso} distribution of long GRBs (e.g. S. Poolakkil et al. 2021). If stellar properties of Pop III progenitors are indeed similar to that of Pop I/II, then this observed limit should hold.

3.3.2 Accretion disc mass

Even with a disc in place, a minimum disc mass M_d is required to maintain accretion at the rates and duration needed for a GRB-producing jet. As was done above, we relate the mass accretion rate with the jet power, so that $\dot{M} = L_j/\eta_j c^2$. Since some of the disc mass can be lost to outflows, only a fraction η_d makes it to the BH over the duration the engine is active (t_{eng}), which yields the requisite disc mass to be $M_d = \dot{M} t_{\text{eng}}/\eta_d$. The engine must be active for longer than the jet break out time, so that $t_{\text{eng}} = t_{\text{bo}} + t_{z,\text{GRB}} = (1 + t_{\text{bo}}/t_{z,\text{GRB}}) t_{z,\text{GRB}} = \eta_t t_{z,\text{GRB}}$, with $1 \leq \eta_t \lesssim 2$, where the lower limit is valid for GRBs longer than the typically measured timescales of $t_{\text{GRB}} = (1+z)t_{z,\text{GRB}} \sim 10$ s and the upper limit applies when the

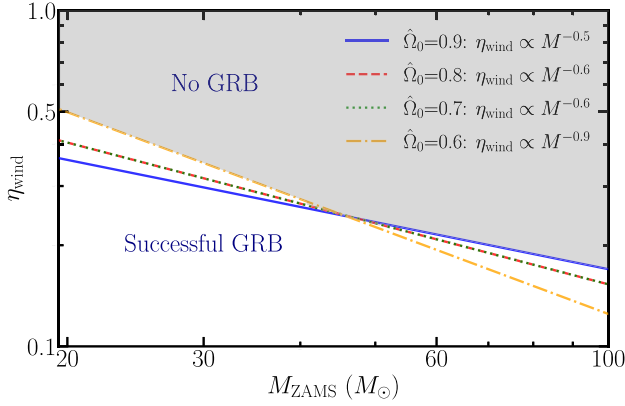


Figure 14. Phase diagram showing the conditions required for a successful gamma-ray burst (GRB) as a function of the initial mass and the wind mass-loss efficiency (η_{wind}). Each curve represents the critical boundary for a given initial rotation rate: $\hat{\Omega}_0 = 0.9$ (blue), 0.8 (red), 0.7 (green), and 0.6 (orange). The shaded region above each curve corresponds to parameter combinations that fail to produce a GRB: (i) $M_d \gtrsim 1 M_\odot$, (ii) $t_{\text{eng}} > t_{\text{bo}} \sim 10$ s, and (iii) $L_j > L_j^{\text{thr}}$. See Section 3.3 for a detailed discussion.

GRB is powered for at least as long as the breakout time, with $t_{\text{z,GRB}} \sim t_{\text{bo}}$. Combining this with equation (17), we get an estimate of the minimum disc mass,

$$\frac{M_d}{M_\odot} = \frac{\eta_t}{\eta_j} \frac{\theta_j^2}{4} \frac{E_\gamma^{\text{iso}}}{\epsilon_\gamma c^2} \approx 0.2 \eta_t \left(\frac{\eta_d}{0.7} \right)^{-1} \eta_{j,-3}^{-1} \theta_{j,-1}^2 \epsilon_{\gamma,-1}^{-1} E_{\gamma,52}^{\text{iso}}, \quad (19)$$

where $\eta_j = 10^{-3}$ when $a_* \sim 0.75$ and we have assumed that at most 30 per cent of the disc mass can be lost to outflows. This analytical estimate aligns with general expectations that a disc mass on the order of ~ 0.1 – $1 M_\odot$, depending on the jet power efficiency ($\eta_j \sim 10^{-3}$ – 10^{-2}), is necessary to sustain accretion and power a successful GRB (P. Kumar et al. 2008; A. I. MacFadyen & S. E. Woosley 1999). Such disc masses can liberate energies in excess of 10^{51} erg, sufficient for producing typical GRBs. Conversely, significantly smaller disc masses ($M_d \ll 0.1 M_\odot$) would be insufficient to sustain the necessary jet power long enough for jet breakout, leading to choked jets and a failed GRB (O. Bromberg et al. 2011).

3.3.3 Phase diagram for a successful GRB

The conditions for a successful jet in a collapsar are intricately linked. It requires a sufficiently rapidly rotating central engine and an accretion disc with enough mass to sustain an outflow with power greater than the threshold jet power, for timescales longer than the jet breakout time. To that end, we select the stellar models in our grid that satisfy the following essential requirements: (i) disc mass $M_d \gtrsim 1 M_\odot$, (ii) engine time $t_{\text{eng}} > t_{\text{bo}} \sim 10$ s, while maintaining (iii) the jet power above the threshold, with $L_j > L_j^{\text{thr}}$.

Fig. 14 combines all of the above requirements into a phase diagram over three fundamental qualities of the progenitor star, namely the initial mass (M_{ZAMS}), initial rotation ($\hat{\Omega}_0$), and wind mass-loss efficiency (η_{wind}). The lines correspond to different initial rotation rates $\hat{\Omega}_0$, where models lying in the shaded area above each curve fail to produce a GRB. For $M_{\text{ZAMS}} \lesssim 50 M_\odot$, having relatively lower initial rotation allows for a larger η_{wind} that could still produce a successful GRB. The trend reverse for $M_{\text{ZAMS}} \gtrsim 50 M_\odot$. For lower stellar masses, it is possible to have a higher η_{wind} . As a consequence, if the wind mass loss efficiency were to match that observed in the local Universe for massive stars, then that would restrict GRB

producing population in Pop III progenitors with $M_{\text{ZAMS}} \sim 10 M_\odot$ and $\hat{\Omega}_0 \sim 0.6$. In contrast, GRB progenitors on the high mass end in our grid require $\eta_{\text{wind}} \lesssim 0.2$.

4 THE GRB RATE AT HIGH REDSHIFTS

The intrinsic production rate of GRBs at any redshift, as given by the number of events per unit observer-frame time t_{obs} and per unit redshift, depends on the birthrate and rate of death of the progenitors. Since the lifespan of massive stars is much smaller and unable to cause any significant delay, the rate of GRBs is expected follow the formation rate of Pop III stars, such that

$$\Psi_{\text{GRB}}(z) \equiv \frac{d^2 N_{\text{GRB}}}{dt_{\text{obs}} dz} = \eta_{\text{GRB}}(z) \frac{\dot{\rho}_*(z)}{(1+z)} \frac{dV}{dz}, \quad (20)$$

where $\dot{\rho}_*(z)$ is the local star formation rate density (SFRD), given by stellar mass per unit comoving volume per unit comoving time t_{em} , and $\eta_{\text{GRB}}(z)$ is the efficiency of turning massive stars into GRBs. In general, η_{GRB} is redshift dependent; however, in what follows, we make the simplifying assumption that properties of the progenitor Pop III stars do not evolve significantly over redshift, which allows us to obtain a mean value for the GRB production efficiency. The cosmological time-dilation factor of $(1+z)^{-1}$ converts the rate from comoving time to observer-frame time $t_{\text{obs}} = (1+z)t_{\text{em}}$, and dV/dz is the comoving volume element per unit redshift,

$$\frac{dV}{dz} = 4\pi r^2(z) \left| \frac{dr}{dz} \right|, \quad (21)$$

where $r(z)$ is the comoving distance to redshift z , given by

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (22)$$

with c/H_0 giving the Hubble distance and $E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ when assuming a flat universe. In this case, the comoving distance has a simple relation with the luminosity distance, where $d_L(z) = (1+z)r(z)$.

To calculate the SFR of Pop III stars we follow the treatment by A. Lazar & V. Bromm (2022) where they adopt the generalized SFRD from P. Madau & M. Dickinson (2014)

$$\frac{\dot{\rho}_*(z)}{M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3}} = \frac{a(1+z)^b}{1 + [(1+z)/c]^d}, \quad (23)$$

with $a = 7657$, $b = -5.92$, $c = 12.83$, and $d = -8.55$, as shown in Fig. 15 (dashed line). However, when comparing the recent estimates from O. Gupta et al. (2025), that uses deep observations made by the *James Webb Space Telescope* (JWST), we find that the A. Lazar & V. Bromm (2022) SFRD overproduces the rate of star formation beyond $z \simeq 10$. A. Lazar & V. Bromm (2022) used the fit to the cosmological numerical simulations of B. Liu & V. Bromm (2020), but raised the normalization by a factor of 10 at $z = 10$. Here, we use the original fit of B. Liu & V. Bromm (2020), with $a = 765.7$, that better agrees with limits from JWST. While the above is a measure of the rate at which stars are forming, the distribution in stellar mass of the number of formed stars is given by the initial mass function (IMF). Since there are no direct observations, the true IMF of Pop III stars is unknown. Following A. Lazar & V. Bromm (2022), here we consider a generalized IMF

$$\xi(m) \equiv \frac{dN}{dm} \propto m^{-\alpha} \exp \left[- \left(\frac{m}{m_{\text{char}}} \right)^{-\beta} \right], \quad (24)$$

that asymptotes to $m^{-\alpha}$ for large m and is suppressed exponentially below a characteristic mass m_{char} . There is large uncertainty regarding

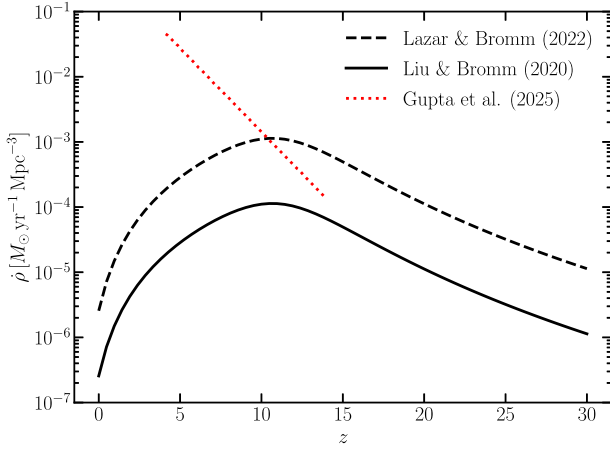


Figure 15. Star formation rate density of Pop III stars from A. Lazar & V. Bromm (2022) (dashed), same but with normalization reduced by a factor of 10 (solid) as in B. Liu & V. Bromm (2020), and from O. Gupta et al. (2025) (dotted) that uses recent results from *JWST* observations.

the power-law index of the Pop III IMF. The power-law index of the present day IMF is $\alpha = 2.35$ (E. E. Salpeter 1955) and it is bottom-heavy where the average mass,

$$\langle m \rangle = \frac{\int m \xi(m) dm}{\int \xi(m) dm}, \quad (25)$$

is dominated by low-mass stars with $\langle m \rangle \sim 0.5 M_\odot$ (V. Bromm 2013). Inferences regarding the IMF of Pop III stars have thus far been obtained from cosmological numerical simulations. Most of which generally find the IMF to be top-heavy, with recent simulations finding $\alpha \sim 1.13$ (e.g. A. Stacy, V. Bromm & A. T. Lee 2016; K. M. J. Wollenberg et al. 2020) which would yield larger average masses. Another important question is the range of masses that are produced in these simulations. Due to these being computationally intensive, they are limited to smaller runtimes, that probe the evolution of star-forming mini dark-matter halos to $\sim 10^3\text{--}5 \times 10^3$ yr with resolutions of few to several tens of astronomical units. Early works found the IMF dominated by very massive Pop III stars with $m \gtrsim 100 M_\odot$ (e.g. T. Abel et al. 2002; V. Bromm, P. S. Coppi & R. B. Larson 2002); however, more recent simulations show a greater diversity in stellar mass with $1 M_\odot \lesssim m \lesssim 10^3 M_\odot$ (e.g. S. Hirano et al. 2014; T. Hosokawa et al. 2016; A. Stacy et al. 2016).

Given the IMF, the fraction of stars in a given range of stellar mass, $m_{\text{low}} \leq m \leq m_{\text{high}}$, that will collapse to a rapidly rotating BH which is able to launch a jet and successfully produce a GRB can be calculated,

$$\eta_{\text{GRB}} = \frac{\int_{m_{\text{low}}}^{m_{\text{high}}(\hat{\Omega}_0)} \xi(m) P_{\text{rot}}(\hat{\Omega}_0) dm d\hat{\Omega}_0}{\int_{m_{\text{min}}}^{m_{\text{max}}} m \xi(m) dm}, \quad (26)$$

where η_{GRB} is the efficiency of producing a GRB per unit stellar mass. Here $m_{\text{min}} = 1.0 M_\odot$ and $m_{\text{max}} = 300 M_\odot$ are the minimum and maximum masses of Pop III stars (V. Bromm 2013; A. Lazar & V. Bromm 2022). We obtain the limits on the GRB progenitor stellar masses from the phase diagram in Fig. 14, that shows the range of initial masses that can yield a successful GRB for a range of wind efficiencies and initial rotation rates.

Since the distribution of initial rotation rates with stellar mass is not uniform, appropriate weights must be given to different stellar masses. From numerical simulations there is some evidence that Pop III stars may have been rapid rotators with surface angular speeds

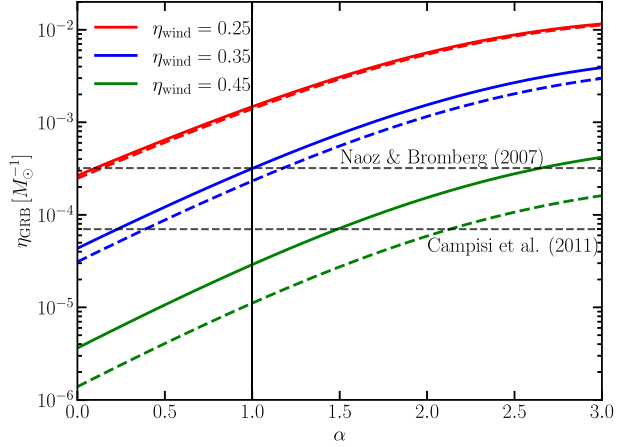


Figure 16. GRB efficiency for different power-law indices α of the generalized Pop III IMF, shown for different values of the wind efficiency η_{wind} when assuming $m_{\text{char}} = 20 M_\odot$. The solid curves assume a flat distribution in initial rotation, i.e. $P_{\text{rot}} \propto \hat{\Omega}_0^\delta$ with $\delta = 0$, and the dashed curves assume a steeper distribution with $\delta = 5$, so that 50 per cent of the distribution has $\hat{\Omega}_0 > 0.9$. The vertical line shows our fiducial value for the IMF that yields a top-heavy and flat distribution $m\xi(m) = dN/d \ln m$, in stellar mass. The horizontal dashed lines show the upper limits obtained in two separate works.

approaching the critical limit, such that $\hat{\Omega}_0 \sim 1$ (S. Hirano & V. Bromm 2018; A. Stacy, V. Bromm & A. Loeb 2011; A. Stacy et al. 2013). In general, the distribution of Pop III stars with $\hat{\Omega}_0$ can be expressed as

$$P_{\text{rot}}(\hat{\Omega}_0) \propto \hat{\Omega}_0^\delta, \quad \text{for } 0.6 \leq \hat{\Omega}_0 \leq 1, \quad (27)$$

where we have taken the limits on $\hat{\Omega}_0$ that are relevant for powering GRB jets. Due to resolution and computational time constraints in cosmological simulations, the exact distribution, and therefore the exact value of δ , is not clear. If critical rotation is indeed favoured then δ can be quite steep. Here we consider two value of $\delta = \{0, 5\}$, where $\delta = 0$ yields a flat distribution with 25 per cent of all Pop III stars at a given redshift born with $\hat{\Omega}_0 > 0.9$, while for $\delta = 5$ the same fraction rises to 50 per cent. As demonstrated in Fig. 14, the high-mass limit, $m_{\text{high}}(\hat{\Omega}_0)$, on the GRB progenitors is sensitive to the initial rotation rate, whereas the lower limit, $m_{\text{low}} = 20 M_\odot$, is insensitive and fixed to yield a BH remnant after core-collapse.

Fig. 16 shows the GRB production efficiency as a function of the IMF power-law index α for different wind efficiencies η_{wind} and initial rotation distributions, while assuming $m_{\text{char}} = 20 M_\odot$. An increasing value of η_{wind} leads to reduced efficiency due to a reduction in the range of masses that can produce a successful GRB, as can be seen in Fig. 14. Due to the same effect, giving more weight to very rapidly rotating progenitors, by having a steep distribution in $\hat{\Omega}_0$, further reduces the efficiency.

The intrinsic comoving rate density of GRBs as a function of redshift is shown in the top panel of Fig. 17. Using this we obtain the all-sky rate while accounting for beaming, flux sensitivity of the detector, and the luminosity function of GRBs, such that

$$\begin{aligned} \mathcal{R}_{\text{GRB}}(< z) &\equiv \frac{dN_{\text{GRB}}^{\text{obs}}}{dt_{\text{obs}}} \\ &= \eta_{\text{beam}} \int_0^z dz \Psi_{\text{GRB}}(z) \int_{L_{\text{lim}}(z)} \Phi(L) d \log L. \end{aligned} \quad (28)$$

The beaming fraction is given by $\eta_{\text{beam}} = \Delta\Omega_{\text{GRB}}/4\pi = 1 - \cos(\theta_{\text{max}}) \approx \theta_{\text{max}}^2/2$, where $\Delta\Omega_{\text{GRB}}$ is the solid angle into which emission from the bipolar jets is beamed. In general, GRB jets have

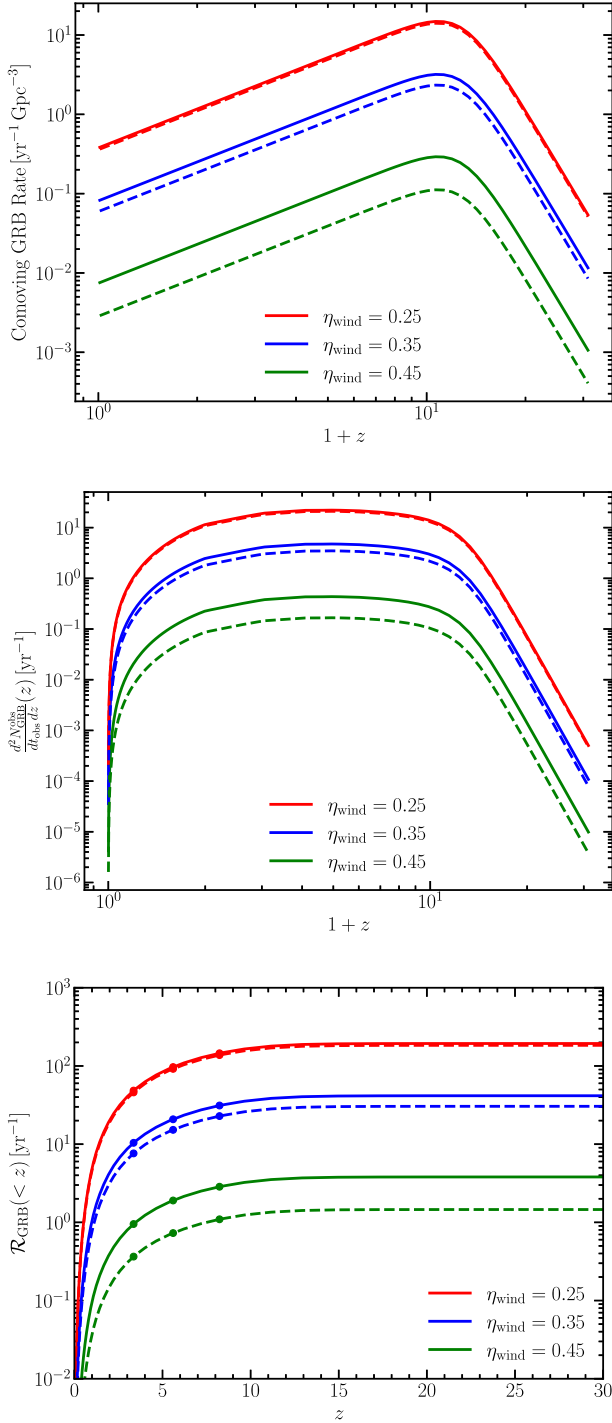


Figure 17. (Top) Intrinsic comoving GRB rate density, shown for a top-heavy IMF power-law index $\alpha = 1$, for different wind efficiency parameter η_{wind} values and initial rotation distribution, with $\delta = 0$ (solid) and $\delta = 5$ (dashed). All other parameters are the same as assumed for Fig. 16. (Middle) The observable all-sky Pop III GRB rate at any given redshift for *Swift*/BAT sensitivity. (Bottom) The integrated all-sky rate of detection by *Swift*/BAT up to a given z . The three dots show the redshifts that contribute 25 per cent, 50 per cent, and 75 per cent of the total rate.

angular structure comprising of a narrow energetic core of angular size θ_c and power-law wings at $\theta > \theta_c$, with θ measured from the jet symmetry axis. Unless the GRB is only several tens of Mpc away, one prime example being GW170817/GRB 170817A ($d_L \simeq 40$ Mpc), most GRBs are found at cosmological distances ($d_L \sim \text{Gpc}$) and so they can only be observed if they are viewed from within the core with viewing angle $\theta_{\text{obs}} \lesssim \theta_c$ (e.g. P. Beniamini & E. Nakar 2019; B. O’Connor, P. Beniamini & R. Gill 2024). In addition, the bulk Lorentz factor of the outflow $\Gamma \gtrsim 100$ as demanded by compactness constraints (T. Piran 1999), and therefore $\theta_{\text{max}} = \theta_c + \Gamma^{-1} \approx \theta_c$. The size of the jet core is most easily determined from observing jet breaks in the afterglow light curve, that occur when the angular size ($1/\Gamma$) of the observable region of the jet’s surface around the line-of-sight includes the edge of the jet core, yielding the condition $\Gamma(\theta_c - \theta_{\text{obs}}) = 1$. Canonical jet breaks are considered in the context of top-hat jets that feature sharp edges with vanishingly small energy outside of the jet core. This particular geometry is also the one that yields the sharpest jet breaks as compared to jets with shallower power-law wings. Not many GRBs show such sharp jet breaks but in a small fraction that do, jet core sizes of $\theta_c \sim 0.1$ rad have been inferred (D. A. Frail et al. 2001), with a mean beaming fraction $\langle \eta_{\text{beam}} \rangle \sim 1/500$.

The luminosity function of GRBs is given by D. Wanderman & T. Piran (2010), which yields the number of GRBs with isotropic-equivalent luminosities in the interval $\log L$ and $\log L + d \log L$,

$$\Phi(L) \equiv \frac{dN}{d \log L} = \begin{cases} \left(\frac{L}{L_*}\right)^{-0.2}, & L < L_* \\ \left(\frac{L}{L_*}\right)^{-1.4}, & L > L_* \end{cases}, \quad (29)$$

where $L_* = 10^{52.5} \text{ erg s}^{-1}$ is a characteristic isotropic-equivalent luminosity. This function has been demonstrated to hold for at least $z < 9$. To infer the limiting luminosity for a given detector, $L_{\text{lim}} = 4\pi d_L^2 F_{\text{lim}}$, we use the limiting bolometric flux, e.g. *Swift* has $F_{\text{lim}} = 1.2 \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}$ (L.-X. Li 2008).

The middle and bottom panels of Fig. 17 respectively show the differential observable all-sky equivalent rate for *Swift*/BAT for a given redshift and integrated rate up to a given redshift, for different wind efficiencies η_{wind} and initial stellar rotation distributions. For each curve shown in the bottom panel, the three dots show the range of redshifts, $z \simeq 3.3\text{--}8.2$, that contribute the central 50 per cent of the Pop III GRBs. The rate declines with increasing η_{wind} due to the corresponding decline in the m_{high} mass of the stellar progenitor for any given initial rotation rate. From Fig. 14 it is expected that if $\eta_{\text{wind}} \gtrsim 0.5$, there will be no massive Pop III stars capable of producing a GRB.

5 DISCUSSION & CONCLUSIONS

In this work, we obtain the radial makeup of Pop III stars just prior to core-collapse using a 1D stellar evolution code. The density and rotational profiles are used to calculate the initial mass and spin of the remnant black hole as well as the mass accretion rate onto it. The accretion rate is then converted to the jet power using a BH spin dependent efficiency, and the jet is propagated through the star in a semi-analytical calculation while keeping track of its collimation due to the pressurized cocoon. In the end, we constrain a parameter space comprising the initial properties of the progenitor, namely its ZAMS mass, rotation, and efficiency of mass loss during its CHE, that yields a successful GRB. It is this parameter space that ultimately determines the efficiency of GRB production from Pop

III progenitors given their initial mass function and initial rotation distribution at high redshifts.

We find that the phase space to produce a successful GRB is very sensitive to the efficiency of mass-loss due to line-driven winds. Even though Pop III stars begin with having no metals in their envelopes, rotationally driven CHE mixes nuclear products from the core into the outer stellar layers. As the efficiency of mass loss is increased, it starts to limit the maximum stellar mass capable of producing a successful GRB, until an initial rotation-dependent $\eta_{\text{wind}}(\hat{\Omega}_0)$. For any higher efficiency, the stars start to lose more than the requisite angular momentum that is needed to launch and breakout jets from the stellar envelope. Low wind efficiency in the range of $0.1 \lesssim \eta_{\text{wind}} \lesssim 0.3$ has been employed in earlier works dealing with Pop I/II (e.g. S. E. Woosley & A. Heger 2006) and Pop III (S.-C. Yoon et al. 2012) progenitors for the same reason. While this range of η_{wind} still yields successful GRBs for a broad range in initial rotation rates, albeit over a narrow range of M_{ZAMS} , only slower rotators with $\hat{\Omega}_0 < 0.9$ may be able to produce GRBs in an even narrower range of stellar masses near $M_{\text{ZAMS}} \sim 20M_{\odot}$ when $\eta_{\text{wind}} \gtrsim 0.5$. Therefore, the want for larger wind efficiency restricts the stellar models to slow rotators and masses smaller than $20M_{\odot}$, where the latter condition becomes prohibitive for collapsar models that require a BH remnant.

The true value of the wind mass-loss efficiency in Pop III stars is still very uncertain. It may also vary with the particular mass-loss scheme used in stellar evolution codes. In particular, our study used the *Dutch* scheme that has been empirically calibrated using observations of local stars (Pop I and II). Within this scheme, the mass-loss rate in Pop III stars is initially negligible. When the star evolves off the main sequence, rotational mixing transports carbon from the core to the surface that increases mass loss, thereby causing loss of angular momentum. However, recent works from optically thick winds (e.g. A. A. C. Sander & J. S. Vink 2020) have shown that the surface abundance of iron (not the total metallicity) is what mostly drives line-driven winds in massive stars. This implies that the empirical recipes used may overestimate mass-loss, since the surface abundance of iron remains essentially constant during stellar evolution. Building on this, S. K. Jeena et al. (2023) provide an updated discussion of modern mass loss recipes and their limitations in very low metallicity regimes. In light of this, we acknowledge the limitations of the mass-loss scheme used in this work, and note that a more detailed analysis of alternative mass-loss schemes remains out of the scope of this work.

Taking $\eta_{\text{wind}} = 0.45$ to be the fiducial value, we find the GRB production efficiency to be $\eta_{\text{GRB}} \sim 10^{-5} M_{\odot}^{-1}$ when assuming a flat distribution in $dN/d \ln m$ of the Pop III IMF. This estimate is in agreement with earlier upper limits (e.g. M. A. Campisi et al. 2011; S. Naoz & O. Bromberg 2007), and it would yield an all-sky detectable rate of ~ 2 Pop III GRBs per year for an instrument with *Swift*/BAT sensitivity. Since *Swift*/BAT has a field of view of $\text{FOV} = 1.4 \text{ str}$, the detection rate is reduced by $\text{FOV}/4\pi \approx 0.11$, which yields a total of $\sim 4 - 5$ Pop III GRBs over the past 20 years of its present operational lifetime.

So far, two very high- z long-duration GRBs with $z = 8.2$ and 9.4 have been observed, where the redshift of the latter was determined using the non-observations of the host galaxy. For that GRB it is not clear if the stellar progenitor is a Pop III star, although no notable differences in its emission were seen (A. Cucchiara et al. 2011). In fact, below $z \sim 15$ more metal rich progenitors start to dominate the SFR, making it less likely for this GRB to have come from a Pop III star (K. Belczynski et al. 2010). One possible way to infer if the progenitor is a Pop III star is through high-resolution and IR spectroscopy of the afterglow. If the afterglow spectrum shows no

absorption from iron-group elements, which would not be present if earlier SNe and stellar winds from the star itself had not polluted the surrounding external medium with metals, then a Pop III progenitor becomes a possibility (K. Toma et al. 2016). GRBs from very massive Pop III stars ($M_{\text{ZAMS}} \gtrsim 300 M_{\odot}$), which could have produced longer lasting and more energetic bursts, may be ruled out (S.-C. Yoon et al. 2015). The properties of the bursts produced by stars in our grid are similar to the more routinely detected bursts. This similarity makes it very challenging to distinguish between Pop I/II and Pop III GRBs.

At high- z it becomes challenging to detect the host galaxy even for well localized bursts, which means no redshift information. At the time of writing, the fraction of GRBs that do not have redshifts is 77 per cent¹. These include a total of 1962 detected GRBs by any space-based observatory operating since the year 2004, out of which only 453 bursts have a measured redshift. A fraction of the GRBs without redshifts may have Pop III progenitors, assuming that such bursts are not discernible from the more commonly detected ones. This fraction is not known and the possibility of it being negligible cannot be ruled out based on existing observations.

If the actual all-sky detection rate of Pop III GRBs is significantly smaller than $\sim 1 \text{ yr}^{-1}$ for an instrument similar to *Swift*/BAT, then two conclusions can be drawn. First, it can be argued that the wind efficiency must remain higher than $\eta_{\text{wind}} \approx 0.5$ for Pop III stars, and due to which they lose significant angular momentum, which ultimately renders them unable to launch relativistic jets and power GRBs. This scenario assumes that these stars were born with high initial surface angular velocities ($\hat{\Omega}_0 > 0.6$). Secondly, it might also mean that the assumption/inference of rapid rotation is incorrect, and that these stars did not have the requisite angular momentum to begin with.

An alternative possibility is that of a Pop III binary in which additional angular momentum can be gained from a donor companion either by Roche lobe overflow or common envelope evolution (K. Belczynski et al. 2007; V. Bromm & A. Loeb 2006; T. Kinugawa et al. 2019; N. Lloyd-Ronning 2022). If the GRB progenitor is spun up by binary evolution, then the detection rate for *Swift* has been estimated to be rather low with $\sim 0.01 - 0.1 \text{ yr}^{-1}$ (K. Belczynski et al. 2007), where the latter value is for the most optimistic scenario. Several works (though not necessarily for Pop III stars) have explored this scenario (e.g. M. Cantiello et al. 2007; I. Mandel & S. E. de Mink 2016; P. Marchant & J. Bodensteiner 2024; H. Sana et al. 2012) and have shown that binary interactions can induce chemically homogeneous evolution or envelope stripping, both of which favour the formation of rapidly rotating, compact cores. Modelling this scenario lies beyond the scope of this work.

Hopes are now pinned on future more sensitive missions, e.g. HiZ-GUNDAM, Gamow, THESEUS, that will be capable of detecting the most distant GRBs. The challenge with identifying the host galaxy and obtaining a reliable redshift estimate, that can be complicated due to dust extinction (H. M. Fausey et al. 2023), will still need overcoming.

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¹ GRB data obtained from https://swift.gsfc.nasa.gov/archive/grb_table

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DATA AVAILABILITY

The data will be shared on reasonable request to the corresponding authors.

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APPENDIX A: SOME EXTRA MATERIAL

Table A1. Final stellar parameters for models with $\eta_{\text{wind}} = 0.2$ and different initial rotation rates $\dot{\Omega}_0$. Each row corresponds to a different initial mass M_{ZAMS} (a). The values reported are taken at the pre-core-collapse stage. The columns show (b) the final stellar mass $M_{\text{pre-cc}}$, (c) the specific angular momentum at a mass coordinate of $3 M_{\odot}$, (d) the compactness parameter $\xi_{2.5}$, (e) the carbon-oxygen core mass M_{CO} , (f) the initial black hole mass $M_{\text{BH},0}$, (g) the dimensionless spin parameter a_0 , and (h) the disc mass M_{disc} .

| $M_{\text{ZAMS}}^{\text{a}}$ | $\dot{\Omega}_0 = 0.6$ | | | | | | | | $\dot{\Omega}_0 = 0.7$ | | | | | | | | $\dot{\Omega}_0 = 0.8$ | | | | | | | | $\dot{\Omega}_0 = 0.9$ | | | | | | | |
|------------------------------|----------------------------------|-------------------------------|------------------------|----------------------------|------------------------------|------------------|------------------------------|--------------------------------|----------------------------------|------------------------|----------------------------|------------------------------|------------------|------------------------------|--------------------------------|----------------------------------|------------------------|----------------------------|------------------------------|------------------|------------------------------|--------------------------------|----------------------------------|------------------------|----------------------------|------------------------------|------------------|------------------------------|--|--|--|--|
| | $M_{\text{pre-cc}}^{\text{b}}$ | $\log i/M_{\odot}^{\text{c}}$ | $\xi_{2.5}^{\text{d}}$ | M_{CO}^{e} | $M_{\text{BH},0}^{\text{f}}$ | a_0^{g} | $M_{\text{disc}}^{\text{h}}$ | $M_{\text{pre-cc}}^{\text{b}}$ | $\log i/M_{\odot}^{\text{c}}$ | $\xi_{2.5}^{\text{d}}$ | M_{CO}^{e} | $M_{\text{BH},0}^{\text{f}}$ | a_0^{g} | $M_{\text{disc}}^{\text{h}}$ | $M_{\text{pre-cc}}^{\text{b}}$ | $\log i/M_{\odot}^{\text{c}}$ | $\xi_{2.5}^{\text{d}}$ | M_{CO}^{e} | $M_{\text{BH},0}^{\text{f}}$ | a_0^{g} | $M_{\text{disc}}^{\text{h}}$ | $M_{\text{pre-cc}}^{\text{b}}$ | $\log i/M_{\odot}^{\text{c}}$ | $\xi_{2.5}^{\text{d}}$ | M_{CO}^{e} | $M_{\text{BH},0}^{\text{f}}$ | a_0^{g} | $M_{\text{disc}}^{\text{h}}$ | | | | |
| (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | | | | |
| 20 | 19.94 | 16.15 | 0.27 | 6.96 | 10.60 | 0.59 | 9.34 | 19.21 | 16.18 | 0.32 | 9.97 | 6.34 | 0.79 | 12.87 | 14.50 | 16.24 | 0.37 | 14.40 | 7.46 | 0.69 | 7.04 | 14.57 | 16.24 | 0.37 | 14.42 | 7.49 | 0.72 | 7.08 | | | | |
| 25 | 18.08 | 16.23 | 0.38 | 17.88 | 10.93 | 0.67 | 7.15 | 17.74 | 16.24 | 0.34 | 17.56 | 9.94 | 0.69 | 7.80 | 17.80 | 16.25 | 0.34 | 17.58 | 10.06 | 0.70 | 7.74 | 20.65 | 16.19 | 0.34 | 20.45 | 12.94 | 0.66 | 7.71 | | | | |
| 30 | 26.15 | 16.05 | 0.30 | 16.34 | 24.66 | 0.47 | 1.49 | 20.68 | 16.10 | 0.34 | 20.48 | 13.79 | 0.63 | 6.89 | 20.25 | 16.14 | 0.43 | 20.27 | 13.07 | 0.64 | 7.18 | 20.89 | 16.14 | 0.39 | 20.65 | 13.52 | 0.63 | 7.37 | | | | |
| 35 | 25.18 | 15.99 | 0.41 | 24.97 | 23.55 | 0.58 | 1.63 | 23.67 | 16.06 | 0.34 | 23.34 | 16.91 | 0.61 | 6.76 | 23.69 | 16.04 | 0.35 | 23.23 | 17.18 | 0.60 | 6.51 | 24.24 | 16.06 | 0.34 | 23.98 | 16.74 | 0.64 | 7.50 | | | | |
| 40 | 26.75 | 16.00 | 0.42 | 26.51 | 24.99 | 0.56 | 1.76 | 26.63 | 15.97 | 0.35 | 26.37 | 24.79 | 0.60 | 1.84 | 26.42 | 15.98 | 0.34 | 26.17 | 24.46 | 0.59 | 1.96 | 26.25 | 15.99 | 0.41 | 26.02 | 24.41 | 0.59 | 1.84 | | | | |
| 45 | 29.69 | 15.95 | 0.37 | 29.42 | 28.02 | 0.56 | 1.67 | 29.25 | 15.94 | 0.35 | 28.98 | 27.34 | 0.54 | 1.91 | 29.52 | 15.92 | 0.35 | 29.23 | 27.69 | 0.56 | 1.83 | 29.23 | 15.95 | 0.36 | 28.95 | 27.47 | 0.56 | 1.76 | | | | |
| 50 | 32.75 | 15.89 | 0.32 | 32.44 | 31.42 | 0.50 | 1.33 | 32.19 | 15.90 | 0.35 | 31.84 | 30.85 | 0.53 | 1.34 | 32.01 | 15.91 | 0.33 | 31.71 | 30.73 | 0.53 | 1.28 | 31.94 | 15.89 | 0.34 | 31.66 | 30.61 | 0.53 | 1.33 | | | | |
| 55 | 35.62 | 15.94 | 0.26 | 35.22 | 34.74 | 0.47 | 0.88 | 35.18 | 15.97 | 0.28 | 34.85 | 33.91 | 0.50 | 1.27 | 34.80 | 16.00 | 0.27 | 34.48 | 33.72 | 0.51 | 1.08 | 34.84 | 15.98 | 0.28 | 34.49 | 33.59 | 0.51 | 1.25 | | | | |
| 60 | 38.06 | 15.94 | 0.20 | 37.71 | 37.07 | 0.48 | 0.99 | 37.82 | 15.96 | 0.21 | 37.22 | 37.08 | 0.50 | 0.74 | 37.55 | 15.96 | 0.19 | 37.22 | 36.83 | 0.49 | 0.72 | 37.52 | 15.96 | 0.22 | 37.10 | 36.48 | 0.48 | 1.04 | | | | |
| 65 | 42.48 | 15.90 | 0.23 | 40.73 | 42.48 | 0.44 | 0.00 | 40.61 | 15.93 | 0.23 | 38.86 | 40.57 | 0.49 | 0.04 | 40.24 | 15.94 | 0.20 | 39.40 | 39.61 | 0.48 | 0.63 | 40.15 | 15.95 | 0.22 | 39.34 | 39.44 | 0.49 | 0.71 | | | | |
| 70 | 43.34 | 15.91 | 0.24 | 41.68 | 43.33 | 0.46 | 0.01 | 43.32 | 15.92 | 0.21 | 41.21 | 43.32 | 0.47 | 0.00 | 43.19 | 15.92 | 0.23 | 40.61 | 43.19 | 0.47 | 0.00 | 42.67 | 15.92 | 0.23 | 40.93 | 42.67 | 0.47 | 0.00 | | | | |
| 75 | 74.04 | 17.04 | 0.01 | 0.00 | - | - | - | 45.77 | 15.90 | 0.23 | 42.82 | 45.77 | 0.44 | 0.00 | 45.66 | 15.90 | 0.20 | 42.44 | 45.66 | 0.44 | 0.00 | 45.45 | 15.90 | 0.23 | 42.90 | 45.44 | 0.44 | 0.01 | | | | |
| 80 | 55.59 | 15.90 | 0.28 | 53.22 | 54.72 | 0.41 | 0.87 | 48.57 | 15.88 | 0.21 | 45.31 | 48.57 | 0.42 | 0.00 | 48.17 | 15.90 | 0.24 | 45.07 | 48.17 | 0.43 | 0.00 | 48.32 | 15.89 | 0.24 | 44.95 | 48.32 | 0.43 | 0.00 | | | | |
| 85 | 57.55 | 15.87 | 0.04 | 52.82 | 57.55 | 0.39 | 0.00 | 51.24 | 15.87 | 0.26 | 48.54 | 51.24 | 0.41 | 0.00 | 50.83 | 15.88 | 0.04 | 47.03 | 50.82 | 0.41 | 0.01 | 50.69 | 15.89 | 0.05 | 46.87 | 50.69 | 0.41 | 0.00 | | | | |
| 90 | 54.15 | 15.86 | 0.04 | 49.72 | 54.15 | 0.38 | 0.00 | 53.95 | 15.86 | 0.24 | 50.47 | 53.95 | 0.38 | 0.00 | 53.66 | 15.87 | 0.04 | 49.20 | 53.66 | 0.38 | 0.00 | 53.95 | 15.86 | 0.26 | 50.79 | 53.95 | 0.39 | 0.00 | | | | |
| 95 | 57.71 | 15.84 | 0.27 | 54.82 | 57.71 | 0.35 | 0.00 | 56.54 | 15.86 | 0.04 | 52.01 | 56.54 | 0.38 | 0.00 | 56.07 | 15.86 | 0.28 | 53.57 | 56.07 | 0.38 | 0.00 | 56.03 | 15.85 | 0.05 | 51.41 | 56.03 | 0.37 | 0.00 | | | | |
| 100 | 77.22 | 15.94 | 0.34 | 75.79 | 77.22 | 0.50 | 0.00 | 60.45 | 15.82 | 0.26 | 57.21 | 60.45 | 0.34 | 0.00 | 58.67 | 15.83 | 0.28 | 56.05 | 58.67 | 0.35 | 0.00 | 58.76 | 15.84 | 0.04 | 54.00 | 58.76 | 0.36 | 0.00 | | | | |

Table A2. Stellar parameters for $\eta_{\text{wind}} = 0.5$, different $\hat{\Omega}_0$ values.

| M_{ZAMS}^a | $\hat{\Omega}_0 = 0.6$ | | | | | | | | | | $\hat{\Omega}_0 = 0.7$ | | | | | | | | | | $\hat{\Omega}_0 = 0.8$ | | | | | | | | | | $\hat{\Omega}_0 = 0.9$ | | | | | | | | | |
|---------------------|------------------------|----------------------------------|---------------|-------------------|---------------------|---------|---------------------|-----------------------|----------------------------------|---------------|------------------------|---------------------|---------|---------------------|-----------------------|----------------------------------|---------------|-------------------|---------------------|---------|------------------------|-----------------------|----------------------------------|---------------|-------------------|---------------------|---------|---------------------|-----------------------|----------------------------------|------------------------|-------------------|---------------------|---------|---------------------|--|--|--|--|--|
| | $M_{\text{pre-cc}}^b$ | $\log \dot{M}_{\text{CO}}^c$ | $\xi_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | $M_{\text{pre-cc}}^b$ | $\log \dot{M}_{\text{CO}}^c$ | $\xi_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | $M_{\text{pre-cc}}^b$ | $\log \dot{M}_{\text{CO}}^c$ | $\xi_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | $M_{\text{pre-cc}}^b$ | $\log \dot{M}_{\text{CO}}^c$ | $\xi_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | $M_{\text{pre-cc}}^b$ | $\log \dot{M}_{\text{CO}}^c$ | $\xi_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | | | | | |
| (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | (M_{\odot}) | ($\text{cm}^2 \text{ s}^{-1}$) | | (M_{\odot}) | (M_{\odot}) | | (M_{\odot}) | | | | | |
| 20 | 19.75 | 16.17 | 0.29 | 6.67 | 10.34 | 0.60 | 9.41 | 14.46 | 15.93 | 0.32 | 10.93 | 13.67 | 0.54 | 0.79 | 10.32 | 15.84 | 0.33 | 7.90 | 10.04 | 0.45 | 0.28 | 10.29 | 15.88 | 0.36 | 7.91 | 9.87 | 0.45 | 0.42 | | | | | | | | | | | | |
| 25 | 12.23 | 15.98 | 0.34 | 9.63 | 11.98 | 0.43 | 0.25 | 12.17 | 15.94 | 0.36 | 9.51 | 11.92 | 0.43 | 0.25 | 12.01 | 15.97 | 0.32 | 9.50 | 11.78 | 0.43 | 0.23 | 12.45 | 15.90 | 0.36 | 9.74 | 12.04 | 0.42 | 0.41 | | | | | | | | | | | | |
| 30 | 20.70 | 15.86 | 0.34 | 20.50 | 20.04 | 0.48 | 0.66 | 14.09 | 15.89 | 0.35 | 13.95 | 13.64 | 0.39 | 0.45 | 13.85 | 15.91 | 0.33 | 13.72 | 13.63 | 0.41 | 0.22 | 13.97 | 15.88 | 0.35 | 13.82 | 13.73 | 0.40 | 0.24 | | | | | | | | | | | | |
| 35 | 16.89 | 15.83 | 0.33 | 16.71 | 16.72 | 0.38 | 0.17 | 15.78 | 15.83 | 0.38 | 15.64 | 15.59 | 0.38 | 0.19 | 15.61 | 15.86 | 0.36 | 15.48 | 15.41 | 0.39 | 0.20 | 15.74 | 15.83 | 0.37 | 15.61 | 15.51 | 0.38 | 0.23 | | | | | | | | | | | | |
| 40 | 17.59 | 15.77 | 0.39 | 17.43 | 17.27 | 0.36 | 0.32 | 17.36 | 15.79 | 0.37 | 17.28 | 17.25 | 0.37 | 0.11 | 17.33 | 15.77 | 0.37 | 17.16 | 17.16 | 0.37 | 0.17 | 17.40 | 15.77 | 0.39 | 17.21 | 17.22 | 0.37 | 0.18 | | | | | | | | | | | | |
| 45 | 19.24 | 15.76 | 0.40 | 19.06 | 19.12 | 0.36 | 0.12 | 19.07 | 15.74 | 0.40 | 18.90 | 18.97 | 0.36 | 0.10 | 18.75 | 15.76 | 0.39 | 18.68 | 18.72 | 0.36 | 0.03 | 18.88 | 15.77 | 0.36 | 18.70 | 18.79 | 0.36 | 0.09 | | | | | | | | | | | | |
| 50 | 21.40 | 15.73 | 0.42 | 21.23 | 21.32 | 0.34 | 0.08 | 20.82 | 15.71 | 0.41 | 20.65 | 20.64 | 0.33 | 0.18 | 20.44 | 15.72 | 0.42 | 20.47 | 20.44 | 0.35 | 0.00 | 20.65 | 15.74 | 0.36 | 20.43 | 20.55 | 0.35 | 0.10 | | | | | | | | | | | | |
| 55 | 23.71 | 15.66 | 0.42 | 23.50 | 23.58 | 0.31 | 0.13 | 22.27 | 15.71 | 0.40 | 22.06 | 22.04 | 0.32 | 0.23 | 22.32 | 15.72 | 0.41 | 22.11 | 22.31 | 0.33 | 0.01 | 22.23 | 15.70 | 0.35 | 22.00 | 22.17 | 0.34 | 0.06 | | | | | | | | | | | | |
| 60 | 24.24 | 15.71 | 0.37 | 24.01 | 24.16 | 0.32 | 0.08 | 23.97 | 15.69 | 0.41 | 23.77 | 23.97 | 0.32 | 0.00 | 23.87 | 15.66 | 0.36 | 23.60 | 23.81 | 0.32 | 0.06 | 23.69 | 15.70 | 0.36 | 23.44 | 23.65 | 0.32 | 0.04 | | | | | | | | | | | | |
| 65 | 28.17 | 15.73 | 0.27 | 27.82 | 28.17 | 0.31 | 0.00 | 25.53 | 15.68 | 0.37 | 25.30 | 25.53 | 0.31 | 0.00 | 25.26 | 15.69 | 0.35 | 25.02 | 25.26 | 0.31 | 0.00 | 25.34 | 15.67 | 0.35 | 25.09 | 25.29 | 0.31 | 0.05 | | | | | | | | | | | | |
| 70 | 27.16 | 15.59 | 0.34 | 26.90 | 27.04 | 0.28 | 0.12 | 27.03 | 15.68 | 0.36 | 26.75 | 27.03 | 0.30 | 0.00 | 26.81 | 15.64 | 0.35 | 26.54 | 26.67 | 0.29 | 0.14 | 26.78 | 15.65 | 0.35 | 26.50 | 26.69 | 0.29 | 0.09 | | | | | | | | | | | | |
| 75 | 32.01 | 15.67 | 0.29 | 31.67 | 32.01 | 0.27 | 0.00 | 28.43 | 15.65 | 0.35 | 28.16 | 28.43 | 0.29 | 0.00 | 28.38 | 15.61 | 0.36 | 28.05 | 28.32 | 0.29 | 0.06 | 28.15 | 15.63 | 0.35 | 27.86 | 28.15 | 0.28 | 0.00 | | | | | | | | | | | | |
| 80 | 36.20 | 15.67 | 0.18 | 35.06 | 36.20 | 0.26 | 0.00 | 29.93 | 15.60 | 0.31 | 29.62 | 29.82 | 0.27 | 0.11 | 29.98 | 15.65 | 0.29 | 29.54 | 29.75 | 0.28 | 0.23 | 29.66 | 15.59 | 0.33 | 29.40 | 29.66 | 0.28 | 0.00 | | | | | | | | | | | | |
| 85 | 39.16 | 15.66 | 0.23 | 36.94 | 39.16 | 0.25 | 0.00 | 31.55 | 15.66 | 0.26 | 31.24 | 31.55 | 0.27 | 0.00 | 31.24 | 15.62 | 0.25 | 30.87 | 31.24 | 0.26 | 0.00 | 30.98 | 15.62 | 0.25 | 30.71 | 30.98 | 0.27 | 0.00 | | | | | | | | | | | | |
| 90 | 33.21 | 15.65 | 0.24 | 32.84 | 33.21 | 0.25 | 0.00 | 32.69 | 15.64 | 0.24 | 32.36 | 32.69 | 0.25 | 0.00 | 32.73 | 15.65 | 0.27 | 32.45 | 32.73 | 0.26 | 0.00 | 32.51 | 15.66 | 0.19 | 31.20 | 32.51 | 0.26 | 0.00 | | | | | | | | | | | | |
| 95 | 34.60 | 15.63 | 0.23 | 34.28 | 34.60 | 0.24 | 0.00 | 34.08 | 15.64 | 0.22 | 33.54 | 34.08 | 0.24 | 0.00 | 34.26 | 15.64 | 0.20 | 33.79 | 34.26 | 0.25 | 0.00 | 33.98 | 15.66 | 0.29 | 33.64 | 33.98 | 0.24 | 0.00 | | | | | | | | | | | | |
| 100 | 56.56 | 15.72 | 0.28 | 54.74 | 56.56 | 0.28 | 0.00 | 57.61 | 15.60 | 0.23 | 56.22 | 57.61 | 0.22 | 0.00 | 55.67 | 15.63 | 0.16 | 55.21 | 55.66 | 0.24 | 0.01 | 55.24 | 15.64 | 0.17 | 54.81 | 55.24 | 0.24 | 0.00 | | | | | | | | | | | | |

Table A3. Stellar parameters for $\eta_{\text{wind}} = 1.0$, different $\hat{\Omega}_0$ values.

| M_{ZAMS}^a | $\hat{\Omega}_0 = 0.6$ | | | | | $\hat{\Omega}_0 = 0.7$ | | | | | $\hat{\Omega}_0 = 0.8$ | | | | | $\hat{\Omega}_0 = 0.9$ | | | | | | | | | | | | |
|---------------------|------------------------|----------------------------------|--------------------|-------------------|---------------------|------------------------|---------------------|-----------------------|----------------------------------|--------------------|------------------------|---------------------|---------------|---------------------|-----------------------|----------------------------------|--------------------|-------------------|---------------------|---------------|---------------------|---------------|-------|------|-------|-------|------|------|
| | $M_{\text{pre-cc}}^b$ | $\log j_{\text{M}\odot}^c$ | $\epsilon_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | $M_{\text{pre-cc}}^b$ | $\log j_{\text{M}\odot}^c$ | $\epsilon_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | $M_{\text{pre-cc}}^b$ | $\log j_{\text{M}\odot}^c$ | $\epsilon_{2.5}^d$ | M_{CO}^e | $M_{\text{BH},0}^f$ | a_0^g | M_{disc}^h | | | | | | | |
| | (M_\odot) | ($\text{cm}^2 \text{ s}^{-1}$) | | | (M_\odot) | (M_\odot) | (M_\odot) | (M_\odot) | ($\text{cm}^2 \text{ s}^{-1}$) | | | (M_\odot) | (M_\odot) | (M_\odot) | (M_\odot) | ($\text{cm}^2 \text{ s}^{-1}$) | | | (M_\odot) | (M_\odot) | (M_\odot) | (M_\odot) | | | | | | |
| 20 | 20.22 | 16.16 | 0.26 | 6.94 | 10.93 | 0.58 | 9.29 | 19.52 | 16.17 | 0.30 | 7.91 | 12.59 | 0.58 | 6.93 | 7.12 | 15.83 | 0.28 | 5.47 | 7.04 | 0.41 | 0.08 | 6.95 | 15.77 | 0.35 | 5.29 | 6.82 | 0.41 | 0.13 |
| 25 | 8.10 | 15.78 | 0.35 | 6.29 | 8.01 | 0.39 | 0.09 | 8.01 | 15.65 | 0.35 | 6.18 | 7.84 | 0.39 | 0.17 | 8.07 | 15.64 | 0.31 | 6.19 | 7.95 | 0.39 | 0.12 | 10.29 | 15.64 | 0.35 | 10.20 | 10.16 | 0.38 | 0.13 |
| 30 | 17.35 | 15.78 | 0.36 | 17.27 | 17.26 | 0.38 | 0.09 | 9.07 | 15.78 | 0.30 | 8.95 | 8.92 | 0.36 | 0.15 | 9.02 | 15.78 | 0.37 | 8.93 | 8.94 | 0.38 | 0.08 | 9.04 | 15.70 | 0.36 | 8.93 | 8.87 | 0.37 | 0.17 |
| 35 | 10.87 | 15.83 | 0.44 | 10.78 | 10.81 | 0.35 | 0.06 | 9.92 | 15.90 | 0.37 | 9.81 | 9.83 | 0.36 | 0.09 | 9.91 | 15.42 | 0.37 | 9.81 | 9.82 | 0.36 | 0.09 | 9.94 | 15.68 | 0.35 | 9.85 | 9.79 | 0.35 | 0.15 |
| 40 | 10.99 | 15.82 | 0.38 | 10.93 | 10.99 | 0.34 | 0.00 | 10.76 | 15.49 | 0.32 | 10.71 | 10.75 | 0.34 | 0.01 | 10.77 | 15.89 | 0.36 | 10.66 | 10.71 | 0.34 | 0.06 | 10.69 | 15.71 | 0.32 | 10.67 | 10.69 | 0.34 | 0.00 |
| 45 | 11.73 | 15.60 | 0.37 | 11.60 | 11.15 | 0.30 | 0.58 | 11.54 | 15.67 | 0.34 | 11.51 | 11.51 | 0.32 | 0.03 | 11.62 | 15.57 | 0.38 | 11.44 | 11.47 | 0.32 | 0.15 | 11.51 | 15.82 | 0.36 | 11.45 | 11.49 | 0.33 | 0.02 |
| 50 | 13.04 | 15.74 | 0.32 | 12.95 | 13.04 | 0.30 | 0.00 | 12.48 | 15.62 | 0.35 | 12.38 | 12.41 | 0.30 | 0.07 | 12.42 | 15.72 | 0.32 | 12.30 | 12.35 | 0.30 | 0.07 | 12.42 | 15.74 | 0.37 | 12.30 | 12.35 | 0.31 | 0.07 |
| 55 | 13.74 | 15.49 | 0.33 | 13.60 | 13.62 | 0.27 | 0.12 | 13.23 | 15.68 | 0.36 | 13.08 | 13.16 | 0.29 | 0.07 | 13.12 | 15.70 | 0.36 | 12.99 | 13.06 | 0.29 | 0.06 | 13.10 | 15.70 | 0.36 | 13.02 | 13.10 | 0.31 | 0.00 |
| 60 | 14.00 | 15.56 | 0.37 | 13.88 | 13.93 | 0.27 | 0.07 | 13.95 | 15.70 | 0.33 | 13.82 | 13.87 | 0.28 | 0.08 | 13.89 | 15.62 | 0.36 | 13.77 | 13.86 | 0.28 | 0.03 | 13.85 | 15.61 | 0.38 | 13.71 | 13.83 | 0.28 | 0.02 |
| 65 | 15.39 | 15.60 | 0.35 | 15.24 | 15.37 | 0.26 | 0.02 | 14.69 | 15.63 | 0.37 | 14.55 | 14.69 | 0.27 | 0.00 | 14.49 | 15.57 | 0.37 | 14.35 | 14.42 | 0.26 | 0.07 | 14.50 | 15.63 | 0.37 | 14.37 | 14.47 | 0.26 | 0.03 |
| 70 | 15.30 | 15.59 | 0.36 | 15.16 | 15.28 | 0.25 | 0.02 | 15.26 | 15.53 | 0.35 | 15.12 | 15.22 | 0.25 | 0.04 | 15.22 | 15.61 | 0.37 | 15.07 | 15.15 | 0.25 | 0.07 | 15.26 | 15.56 | 0.37 | 15.02 | 15.17 | 0.25 | 0.09 |
| 75 | 21.21 | 15.51 | 0.33 | 21.03 | 21.21 | 0.22 | 0.00 | 15.90 | 15.49 | 0.37 | 15.75 | 15.84 | 0.23 | 0.06 | 15.84 | 15.49 | 0.38 | 15.70 | 15.83 | 0.24 | 0.01 | 15.85 | 15.56 | 0.37 | 15.69 | 15.83 | 0.25 | 0.02 |
| 80 | 25.24 | 15.49 | 0.35 | 24.99 | 25.24 | 0.23 | 0.00 | 16.54 | 15.44 | 0.34 | 16.39 | 16.53 | 0.22 | 0.01 | 16.44 | 15.52 | 0.38 | 16.28 | 16.44 | 0.23 | 0.00 | 16.41 | 15.55 | 0.36 | 16.28 | 16.40 | 0.23 | 0.01 |
| 85 | 26.96 | 15.53 | 0.33 | 26.69 | 26.96 | 0.24 | 0.00 | 17.13 | 15.50 | 0.39 | 16.97 | 17.13 | 0.21 | 0.00 | 17.03 | 15.51 | 0.36 | 16.86 | 17.03 | 0.22 | 0.00 | 16.93 | 15.36 | 0.37 | 16.76 | 16.93 | 0.21 | 0.00 |
| 90 | 17.84 | 15.49 | 0.34 | 17.69 | 17.84 | 0.20 | 0.00 | 17.66 | 15.49 | 0.39 | 17.48 | 17.62 | 0.21 | 0.04 | 17.62 | 15.49 | 0.39 | 17.44 | 17.62 | 0.20 | 0.00 | 17.53 | 15.55 | 0.40 | 17.37 | 17.53 | 0.22 | 0.00 |
| 95 | 20.02 | 15.46 | 0.31 | 19.82 | 20.02 | 0.18 | 0.00 | 18.21 | 15.43 | 0.45 | 18.07 | 18.21 | 0.20 | 0.00 | 18.24 | 15.44 | 0.35 | 18.04 | 18.24 | 0.19 | 0.00 | 18.04 | 15.40 | 0.40 | 17.93 | 18.04 | 0.20 | 0.00 |
| 100 | 29.51 | 15.53 | 0.28 | 29.19 | 29.51 | 0.19 | 0.00 | 20.42 | 15.41 | 0.41 | 20.20 | 20.42 | 0.18 | 0.00 | 18.83 | 15.42 | 0.33 | 18.64 | 18.83 | 0.19 | 0.00 | 18.70 | 15.40 | 0.39 | 18.53 | 18.70 | 0.19 | 0.00 |

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