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TESTING THE PREDICTIONS OF THE UNIVERSAL STRUCTURED GAMMA-RAY BURST JET MODEL

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ABSTRACT

The two leading models for the structure of gamma-ray burst (GRB) jets are the uniform jet model and the universal structured jet (USJ) model. In the latter, all GRB jets are intrinsically identical and the energy per solid angle drops as the inverse square of the angle from the jet axis. The simplicity of the USJ model gives it a strong predictive power, including a specific prediction for the observed GRB distribution as a function of both the redshift z and the viewing angle θ. We show that the current sample of GRBs with known z and estimated θ does not agree with the predictions of the USJ model. This can be best seen for a relatively narrow range in z, in which the USJ model predicts that most GRBs should be near the upper end of the observed range in θ, while in the observed sample most GRBs are near the lower end of that range. Since the current sample is very inhomogeneous (i.e., involves many different detectors), it should be taken with care and cannot be used to rule out the USJ model. Nevertheless, this sample strongly disfavors the USJ model. Comparing the prediction for the observed GRB distribution both in θ and in z, with a larger and more homogeneous GRB sample, like the one expected from *Swift*, would either clearly rule out the USJ model, or alternatively, provide a strong support for it. The test presented here is general and can be used to test any model that predicts both a luminosity function and a luminosity-angle relation.

Subject headings: gamma rays: bursts — ISM: jets and outflows — radiation mechanisms: nonthermal

On-line material: color figure

1. INTRODUCTION

Since the discovery of gamma-ray burst (GRB) afterglows in early 1997, several lines of evidence have emerged in support of collimated outflows, or jets. However, despite 6 years of extensive afterglow observations, the structure of the relativistic jets that produce GRBs is still unknown. The structure of GRB jets is a very fundamental and important property that affects the requirements from the source that accelerates and collimates the jets and has direct bearing on two of the most basic properties of any astrophysical radiation source: the rate and the total amount of energy output. Within the fireball model (for review see Piran 2000; Mészáros 2002), there are two very different jet structures that are compatible with the observations: (1) the uniform (or "top-hat") jet model (Rhoads 1997, 1999; Panaitescu & Mészáros 1999; Sari, Piran, & Halpern 1999; Kumar & Panaitescu 2000; Moderski, Sikora, & Bulik 2000; Granot et al. 2001, 2002), where the initial energy per solid angle ϵ and Lorentz factor Γ are uniform within some finite half-opening angle θ_j and sharply drop outside of θ_j, and (2) the universal structured jet (USJ) model (Lipunov, Postnov, & Prokhorov 2001; Rossi, Lazzati, & Rees 2002; Zhang & Mészáros 2002), with a standard jet structure for all GRBs where $\epsilon \propto \theta^{-2}$ (outside of some core angle). It is important to note that in the USJ model all GRB jets are intrinsically identical (both in their angular profile and total energy). Both jet structures can explain the observed correlation between $E_{\gamma,\mathrm{iso}}$ and the jet break time t_j in the optical afterglow light curve, $t_j \propto E_{\gamma,\mathrm{iso}}^{-1}$ (Frail et al. 2001; Bloom, Frail, & Kulkarni 2003). In the USJ model, this determines the jet structure ($\epsilon \propto \theta^{-2}$). In the top-hat model, t_j depends mainly on θ_j (Rhoads 1997, 1999; Sari et al. 1999), while in the USJ model it depends

mainly (and in a similar way) on the viewing angle θ_{obs} from the jet axis.

The simplicity of the USJ model gives it a strong predictive power. In a recent paper, Perna, Sari, & Frail (2003, hereafter PSF03) used this feature of the USJ model to predict the observed distribution of viewing angles, $n(\theta) = dn/d\theta$ (hereafter we use θ instead of θ_{obs} for brevity). They have shown that the current limited sample of 16 bursts with known θ and redshift z fits the predicted distribution very well. In this Letter, we extend the comparison between the USJ model predictions and observations into one more dimension—the redshift z. Namely, we use here the two-dimensional (2D) distribution $n(z, \theta) = dn/dz\, d\theta$ and compare it with the known θ and z of the observed sample. The 1D distribution that was used by PSF03 for comparison with the data is $n(\theta) = \int n(z, \theta)dz$. However, the known redshift of these GRBs is not used in the 1D analysis. Comparing the data both for z and for θ with the 2D distribution, $n(z, \theta)$, reveals that the agreement between the data and the 1D distribution, $n(\theta)$, that was found by PSF03 is accidental and arises because of the integration over z. The 2D data show a very poor agreement with the model, and the hypothesis that the data are drawn from the model is rejected at 99% significance by a 2D Kolmogorov-Smirnov (K-S) test. Thus, the agreement between the data and the 1D distribution is misleading and does not provide support for (and certainly does not prove) the USJ model. In order to test this statistically, while trying to minimize the possible selection effects as a function of z, it is most appropriate to compare $n(z, \theta)$ with the data over a relatively narrow range in z. Such a comparison has another important advantage: it depends only weakly on $R_{\mathrm{GRB}}(z)$—the GRB rate as a function of z—which is rather poorly known. A drawback of this test is that it requires a large number of data points at a given redshift. The narrowest range in z that contains a reasonably large number of the current data points (10 points) is $0.8 < z < 1.7$. In this range, the USJ model fails to explain the data with a significance of 99.8%.

We perform several tests in order to check the robustness of

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this result, and its sensitivity to various selection effects that we can quantify, and find it to be robust and significant. Thus, we find that the current data set strongly disfavors the USJ model. Nevertheless, it is still premature to draw a definite conclusion, mainly because many different detectors were involved in detecting the current sample. This situation is expected to improve in the near future with the launch of *Swift*. Once a homogeneous and large sample of bursts with measured z and estimated θ is available, applying the 2D test described here will result in either a definite rejection or a strong support for the USJ model. We stress that the specific test we carry out here is relevant only to a *universal* structured jet, i.e., with a universal profile of both $\epsilon(\theta)$ and the initial value of $\Gamma(\theta)$.[4] Finally, although in this Letter we apply the 2D test only to the USJ model, it can be easily generalized to any model that predicts a luminosity function and a luminosity-angle relation.

2. THEORY

Below we follow PSF03 and generalize their 1D distribution $n(\theta)$ to the 2D distribution $n(z, \theta)$. Equation (5) of PSF03 presents the photon peak luminosity in the energy range 50–300 keV, assuming that all GRBs are identical with a differential photon spectral index, $\alpha = 1$,

$$L_{\mathrm{ph}}(\theta, T) = 1.1 \times 10^{57} \tilde{T}^{-1} \theta^{-2} \text{ photons s}^{-1}, \quad (1)$$

where $T = \tilde{T}$ s is an "effective" duration that is given by the ratio of the (isotropic equivalent) energy output and peak luminosity [or equivalently $(1 + z)^{-1}$ times the ratio of the fluence and peak flux]. In practice, T changes from one burst to another, and we will denote its probability distribution by $P(T)$. For a detector with a given limiting flux for detection $F_{\mathrm{ph, lim}} = \tilde{F}_{\mathrm{ph, lim}}$ photons cm^{-2} s^{-1} and a burst at given θ and z, we can derive the maximal T for which this burst is detected, T_{max},

$$T_{\mathrm{max}} = 88(1 + z)^{-\alpha} D_{28}^{-2}(z) \tilde{F}_{\mathrm{ph, lim}}^{-1} \left(\frac{\theta}{0.1}\right)^{-2} \text{ s}, \quad (2)$$

where $D_{28}(z)$ is the comoving distance in units of 10^{28} cm. The total rate of bursts with an inferred viewing angle between θ and $\theta + d\theta$ and a redshift between z and $z + dz$ is then

$$n(z, \theta) = \frac{dn}{dz\, d\theta} = \sin\theta \frac{R_{\mathrm{GRB}}(z)}{1 + z} \frac{dV(z)}{dz} \int_0^{T_{\mathrm{max}}(\theta, z)} P(T) dT, \quad (3)$$

where $R_{\mathrm{GRB}}(z)$ is the GRB rate per unit comoving time per unit comoving volume $V(z)$. Equation (3) is similar to equation (11) of PSF03 but without integration over z. Thus, equation (3) describes the 2D distribution $n(z, \theta)$, while equation (11) of PSF03 describes the 1D distribution $n(\theta) = dn/d\theta$.

In order to calculate $n(z, \theta)$, one must assume some $R_{\mathrm{GRB}}(z)$ and $P(T)$. Below we consider the $R_{\mathrm{GRB}}(z)$ that PSF03 used as their "standard" model,[5] where for $z < 10$:

$$R_{\mathrm{GRB}}(z) \propto \begin{cases} 10^{0.75z}, & z < z_{\mathrm{peak}}, \\ 10^{0.75 z_{\mathrm{peak}}}, & z \geq z_{\mathrm{peak}}, \end{cases} \quad (4)$$

and $z_{\mathrm{peak}} = 2$. At $z > 10$, the rate declines rapidly. PSF03 first

considered a delta function in T, $P(T) = \delta(T - T_0)$, where T_0 is a free parameter that they used in order to get a good fit to the observed $dn/d\theta$. Using their standard $R_{\mathrm{GRB}}(z)$, they found a best-fit value of $T_0 = 8$ s. However, since $P(T)$ can be estimated pretty well from observations, we do not take it as a free function. In order to estimate $P(T)$, we used the flux table of the BATSE 4B Catalog. We have used the peak fluxes (in photons cm^{-2} s^{-1}), averaged over the 1024 ms BATSE trigger, and the fluences from the catalog in the energy range 50–300 keV. In order to convert the peak fluxes to ergs cm^{-2} s^{-1}, we used a Band spectrum (Band et al. 1993) for the energy distribution, with $\alpha = -1.0$, $\beta = -2.0$, and $E_0 = 100$ keV. Finally, T is approximated by the ratio of the fluence and the peak flux.[6] According to this estimation of T, the total distribution $P(T)$ is consistent with a lognormal distribution,

$$\frac{dP}{d\ln\tilde{T}} = \tilde{T}P(\tilde{T}) = \frac{1}{\sigma_{\ln T}\sqrt{2\pi}} \exp\left[-\frac{(\ln\tilde{T} - \mu)^2}{2\sigma_{\ln T}^2}\right], \quad (5)$$

with $\mu = 2.15$ and $\sigma_{\ln T} = 0.87$ ($T = 8.6^{+12}_{-5}$ s). Below we first use the delta function for $P(T)$ that was used by PSF03, and later we use our equation (5).

Next we consider measurement errors or intrinsic scatter in $\epsilon\theta^2$. The measurement errors in z are typically negligible. The error in θ, however, may be as large as tens of percent, and so is the intrinsic scatter in $\epsilon\theta^2$. In order to account for this scatter, and since it is more reasonable to assume a Gaussian scatter in $\ln\theta$ rather than in θ (both for error and intrinsic scatter), we change variables from θ to $\ln\theta$ and convolve $n(z, \ln\theta) = dn/dz\, d\ln\theta = \theta\, dn/dz\, d\theta = \theta n(z, \theta)$ along the $\ln\theta$ coordinate with a Gaussian of standard deviation $\sigma_{\ln\theta}$,

$$\tilde{n}(z, \ln\theta) = \frac{d\tilde{n}}{dz\, d\ln\theta}$$

$$= \frac{\int_0^{\ln(\pi/2)} n(z, \ln\theta') \exp\left[-(\ln\theta - \ln\theta')^2/2\sigma_{\ln\theta}^2\right]d(\ln\theta')}{\int_0^{\ln(\pi/2)} \exp\left[-(\ln\theta - \ln\theta')^2/2\sigma_{\ln\theta}^2\right]d(\ln\theta')}. \quad (6)$$

Now $\tilde{n}(z, \ln\theta)$ is a rate function that is smoothed along the $\ln\theta$ dimension by the typical scatter due to the bursts' intrinsic properties and θ measurement error. The total scatter cannot exceed the measured scatter in $\epsilon\theta^2$ that was found by Frail et al. (2001).

3. RESULTS

First we repeat the 1D analysis of PSF03 in two dimensions for their standard $R_{\mathrm{GRB}}(z)$ model (eq. [4]), using $P(T) = \delta(T - T_0)$ with $T_0 = 8$ s, for which they obtained the best fit to the data. We used the same parameters as PSF03: $F_{\mathrm{ph, lim}} = 0.424$ photons cm^{-2} s^{-1} (the threshold for the BATSE trigger on 1024 ms; Mallozzi, Pendleton, & Paciesas 1996), $\alpha = 1$, and the same cosmology: $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 71$ km s^{-1} Mpc^{-1}.

Figure 1a depicts the 2D distribution $n(z, \ln\theta)$, and the circles mark the 16 GRBs of the Bloom et al. (2003) sample, which

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[4] If the initial Γ profile is not universal, then different jets may produce γ-rays within different solid angles.

[5] This model is the Rowan-Robinson (1999) star formation rate with a cutoff at $z > 10$ as seen in the numerical simulation of Gnedin & Ostriker (1997).

[6] This ratio includes the effect of cosmological time dilation and is therefore a factor of $(1 + z)$ larger than the actual value of T, which is measured at the cosmological frame of the GRB. Therefore, we overestimate both $\langle T \rangle$ (by a factor of ~2–3) and $\sigma_{\ln T}$ [as part of the observed scatter is due to the scatter in $(1 + z)$ between different GRBs]. Both effects worsen the fit between the USJ model and the data.
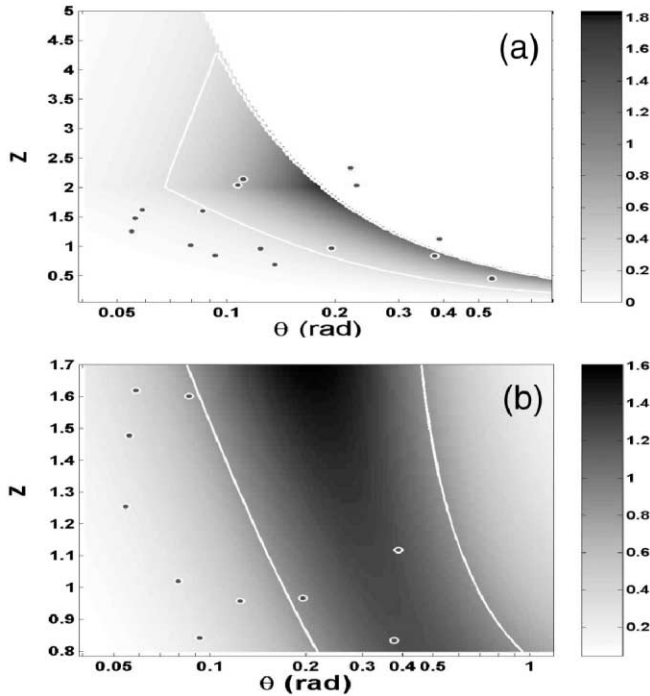
Fig. 1.—The 2D distribution density, $n(z, \ln \theta)$, of the GRB rate as a function of redshift z and viewing angle θ, as predicted by the USJ model. The white contour lines confine the minimal area that contains 1 σ of the total probability. The circles denote the 16 bursts with known z and θ from the sample of Bloom et al. (2003). (a) The parameters of the models are similar to those of PSF03. This figure is the 2D realization of their Fig. 1. (b) Here we use a limited range in redshift, $0.8 < z < 1.7$ (containing 10 out of the 16 data points), in order to minimize redshift selection effects and reduce the sensitivity of the results to the unknown GRB rate. We take into account 20% measurement errors in $\ln \theta$ ($\sigma_{\ln \theta} = 0.2$) and a lognormal distribution in T that we deduced from observations (eq. [5]). [See the electronic edition of the Journal for a color version of this figure.]

were used by PSF03. This figure is a 2D representation of Figure 1 of PSF03. When integrating our Figure 1a over the z dimension, we reproduce Figure 1 of PSF03 (model 1). Figure 1a shows that while the 1D distribution, $n(\theta)$, provides a good fit to the data, the 2D distribution, $n(z, \theta)$, does not agree with the data. The 2D K-S test rejects the null hypothesis that the data are drawn from the model with a confidence of 99%. The reason for the striking difference between the 2D analysis and the 1D analysis is that the data disagree with the 2D model in two ways that roughly cancel out when integrated over redshift: (1) at high z ($\gtrsim 2$), there are not enough bursts with low θ, and (2) at low z, there are too many bursts with low θ compared to the number of bursts with high θ ($\gtrsim 0.2$ rad). When one integrates over the redshift, these two shortcomings roughly cancel each other out. The fact that the 1D distribution of PSF03 peaks at $\theta \sim 0.12$ rad arises from the contribution of predicted bursts at high z and low θ, which are not present in the observational sample but are compensated for by the overabundance of bursts at low z and low θ.

This disagreement with the data, however, cannot be used to draw strong conclusions. The reason is that the current sample suffers from numerous selection effects, mainly in redshift. The selection effects in z can be minimized by testing the θ distribution for a given z (i.e., $dn/d\theta$ for a given z). This test has another important advantage: it depends only weakly on the poorly known GRB rate $R_{GRB}(z)$. The main disadvantage of this test is that the size of the data sample is reduced. We therefore take a slice in redshift of $0.8 < z < 1.7$, which contains

10 of the 16 bursts in the current sample. We use $R_{GRB}(z)$ and $P(T)$ from equations (4) and (5). We account for a 20% scatter in $\ln \theta$ by using $\tilde{n}(z, \ln \theta)$ from equation (6) with $\sigma_{\ln \theta} = 0.2$. Figure 1b shows the expected distribution $\tilde{n}(z, \ln \theta)$ in this redshift range. Here the paucity of bursts with large θ, and overabundance of bursts with small θ, is clear. The concentration of bursts at $\theta < 0.1$ while $\theta_{max}(z) \sim 0.25$–0.4 in this z-range contradicts the predictions of the USJ model—$n(\theta) \propto \sin \theta$, for $\theta < \theta_{max}$. The 2D K-S test rejects the model with a confidence level of 99.8%.

In order to check the reliability and robustness of our results, we consider below the sensitivity of the results to our assumptions and to the values of the different parameters (z_{peak}, $F_{ph, lim}$, etc.). We also consider different observational selection effects and carry out additional tests to estimate their influence on the results. First, we varied the value of z_{peak} (eq. [4]) and found that it has almost no influence on the results. We considered also the possibility of a larger $\sigma_{\ln \theta}$.[7] Bloom et al. (2003) obtain a factor of 2.2 scatter in $\epsilon \theta^2$, implying $\sigma_{\ln \theta} \lesssim 0.4$. Repeating our analysis with $\sigma_{\ln \theta} = 0.4$, the USJ model is rejected at 98.7% confidence.

Next we consider the dependence of our result on the value of $F_{ph, lim}$. It is important to note that we consider here a stiff threshold (i.e., constant $F_{ph, lim}$). In reality, it is not the case (both the detection threshold of the detector and the level of the background vary). This effect may result in an underestimate of the number of weak events. One way to overcome this obstacle is by choosing a relatively bright threshold that is above the detection threshold at any time. Unfortunately, this significantly reduces the size of the observed sample, preventing us from applying this method here (it may be applied to a future larger sample, e.g., Swift). The only test we can do with the current sample is to check the effect of a larger (stiff) threshold. Lower sensitivity (larger $F_{ph, lim}$) reduces $\theta_{max}(z)$. In order for the model to accommodate the data concentration at $\theta < 0.1$ rad and $0.8 < z < 1.7$, $F_{ph, lim}$ needs to be increased by a factor of ~ 5. However, this would imply $\theta_{max}(z = 2) \approx 0.06$ rad, which is significantly inconsistent with the observational values of θ at this redshift (two bursts with $\theta \approx 0.12$ and two with $\theta \approx 0.22$). Another threshold-related selection effect is the low sensitivity of BATSE in the X-ray. This effect is important because of the correlation $E_p \propto \epsilon^{1/2}$ (Amati et al. 2002), which in our context implies $E_p \propto \theta^{-1}$. Thus, BATSE is less sensitive to bursts with large θ (low E_p). Five bursts from our specific sample are also used by Amati et al. (2002): four with $\theta < 0.1$ rad and $E_p \gtrsim 400$ keV and one (GRB 970508) with $\theta = 0.38$ rad and an intrinsic $E_p \sim 150$ keV. This subsample roughly follows the relation $E_p \propto \theta^{-1}$ and demonstrates that in the range of θ where there is a deficit of observed bursts, ~ 0.2–0.3 rad, the expected intrinsic E_p is ~ 200 keV (observed $E_p \sim 100$ keV), which is well within the range of BATSE. Therefore, although we cannot quantify this effect accurately, it should not strongly affect our sample with limited z-range.

Next we consider the selection effects in θ. A very small θ implies a very early jet break time t_j, which may be before the first optical detection and thus result only in an upper limit on θ. A large θ, on the other hand, results in a late jet break time t_j, which occurs when the optical afterglow is too dim for detection (it can be dimmer than either the detection threshold or its host galaxy). This would result in only a lower limit on θ. The sample of Bloom et al. (2003) shows both effects: in

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[7] As discussed above, it accounts for both the intrinsic scatter and measurement errors.

the redshift range $0.8 < z < 1.7$, there is one burst with an upper limit on θ and two with lower limits on θ. In order to account for the above selection effects, we added the latter three bursts in the most favorable way for the USJ model. Namely, we assigned to each of the three bursts the value of θ within the allowed range where $\tilde{n}(\theta, z)$ assumes its maximal value. Even after adding these three data points in this way, the $10 + 3$ data set rejects the USJ model with a confidence level of 99.8%.

Another selection effect in θ may result from "dark" bursts. These are bursts with an observed X-ray afterglow in which an optical afterglow was not detected despite deep and rapid follow-up observations. De Pasquale et al. (2003) argue that a large fraction of the dark bursts are intrinsically dim, by showing that they have, on average, dimmer X-ray afterglows. In the USJ model, the intrinsically dim bursts may be interpreted as bursts with very large θ that are barely detected in γ-rays and X-rays. Since about 50% of the bursts with observed X-ray afterglows have no detected optical afterglow, we will make the extreme assumption that all of these bursts are intrinsically dim (i.e., have a large θ). We estimate the effect of this assumption by adding 13 fictitious points (to the $10 + 3$ original data points) in the most favorable way for the model. Namely, we generate z according to the model z distribution, and then for each point we choose θ where $\tilde{n}(\theta, z)$ is maximal (at the given z of that point). Even after making these extreme assumptions, the 26 "data" points reject the model with confidence level larger than 90%. We conclude that our result, that the USJ model is incompatible with the current data, is robust and significant.

4. DISCUSSION

We have shown that the strong predictive power of the USJ model enables a determination of the expected distribution of observed GRB rate as a function of both redshift z and viewing angle θ. We have compared this predicted 2D distribution to current observations and found a very poor agreement. This is in contrast with the result of PSF03, which compared only the observed distribution of θ to the 1D prediction of the USJ model (that is obtained from the 2D prediction by integrating over z) and found a good agreement. Our analysis shows that this agreement is accidental (resulting from the integration over z) and does not support the USJ model in any way.

However, the poor agreement between the data and the 2D distribution should be considered with care and may not be used to draw definite conclusions. This is since the current GRB sample is highly nonhomogeneous. It involves many different instruments and is likely affected by various selection effects, which are hard to quantify very accurately. A larger and much more homogeneous sample of GRBs with known z and θ is expected with the upcoming launch of *Swift*, which would enable much stronger and clearer conclusions to be drawn from a similar statistical analysis as was done in this work.[8] This would either clearly rule out or strongly support the USJ model. Nevertheless, we point out that at least some of the selection effects may be overcome by restricting the analysis to a relatively narrow range in z. This significantly reduces the redshift selection effects and the uncertainty that is introduced by the assumption that has to be made about the poorly known GRB rate $R_{\mathrm{GRB}}(z)$. The main drawback of this second method is that in order to obtain a statistically significant sample in a very narrow range in z, many GRBs (many more than in the current sample) are required. Again, such a sample is expected to become available with *Swift*.

In the current sample, the narrowest redshift range that still contains enough data points for the purpose of statistical analysis is $0.8 < z < 1.7$ (it contains 10 out of the 16 points). In this range, the current data are in complete disagreement with the predictions of the USJ model, with a significance of 99.8% according to the 2D K-S test. We checked the robustness of this result by varying our assumptions and by examining a few possible selection effects in the viewing angle θ and found it to be robust. Thus, although the relatively small size and the inhomogeneity of the current sample prevent us from drawing a definite conclusion at this stage, we find that the current data disfavor the USJ model.

[8] Note that the effect of the variable detection threshold and the E_p-ϵ correlation should be considered carefully in the analysis of *Swift* results as well.

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