



Old and Bright: The Remarkable Radio Brightening of the Engine-driven SN 2012au Several Years after Explosion Signals the Birth of a Pulsar Wind Nebula

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Abstract

We present the results from an extensive broadband (radio to X-rays) observing campaign of the engine-driven Type Ib SN 2012au in the first 13 yr of evolution. The early time ($\delta t \leq 190$ days) radio and X-ray evolution is well described by conventional models of a forward shock interacting with a wind-like circumstellar medium (CSM; $\rho_{\text{CSM}} \propto r^{-2}$). However, starting at $\delta t \approx 6.7$ yr, we detect a significant radio rebrightening. This late-time emission is dominated by a luminous component characterized by a broad and rapidly evolving spectral peak and a shallow optically thin spectral slope, $F_\nu \propto \nu^{-0.31 \pm 0.02}$. These properties imply a compact emitting region ($R \lesssim 10^{16}$ cm) expanding at a remarkably slow velocity ($v_s \lesssim 500$ km s⁻¹) into a high-density environment ($n_e \geq 10^4$ cm⁻³), accompanied by a hard electron power-law index $p \approx 1.6$. No soft or hard X-ray emission is detected at any epoch, indicating that high-energy radiation is either strongly absorbed or intrinsically absent. In the context of aspherical shock–CSM interaction models, these observations imply extreme properties of the CSM (geometry, density, and total mass) that lack clear astrophysical motivation. Instead, we show that the emergence of radiation from a newborn pulsar wind nebula (PWN) naturally explains the radio spectral evolution and high-energy limits, where the emission is governed by the adiabatic expansion of a relic pair plasma. We conclude that SN 2012au represents the most compelling candidate for a young, newborn PWN discovered to date, a scenario that can be directly tested with pending very long baseline interferometry observations.

Unified Astronomy Thesaurus concepts: Core-collapse supernovae (304); Supernovae (1668); Radio transient sources (2008); Pulsar wind nebulae (2215); Shocks (2086)

1. Introduction

Core-collapse supernovae (CCSNe) are known to produce synchrotron emission that is observable at radio frequencies. This emission typically originates from the shock interaction



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with the circumstellar medium (CSM), where high-velocity supernova (SN) ejecta plow through dense material, built up from mass lost by the stellar progenitor before explosion (e.g., R. A. Chevalier 1982a, 1982b, 1998; R. A. Chevalier & E. P. Liang 1989; K. W. Weiler et al. 2002; R. A. Chevalier & C. Fransson 2006; see R. A. Chevalier & C. Fransson 2017, for a review). Depending on the structure of the CSM, the radio emission in some cases may persist for long periods of time or rebrighten after many years, as observed in SN1987A (L. Staveley-Smith et al. 1993). Late-emerging radio emission from CCSNe has also been theorized as possible manifestation of off-axis gamma-ray burst jets²⁹ (e.g., J. E. Rhoads 1997; J. Granot et al. 2002), or the emergence of radiation powered by young pulsar wind nebulae (PWNe; e.g., S. P. Reynolds & R. A. Chevalier 1984; P. Slane 2017).

Historically, the literature on late-time SN radio emission has largely been dominated by single-object studies of notable explosions, including, e.g., SN 1986J (S. Sukumar & R. J. Allen 1989; M. F. Bietenholz & N. Bartel 2008, 2017a, 2017b; M. F. Bietenholz et al. 2010), SN 1987A (L. Staveley-Smith et al. 1993), and SN 1993J (C. Fransson et al. 1996; C. Fransson & C.-I. Björnsson 1998; M. A. Pérez-Torres et al. 2002; N. Bartel et al. 2007; I. Martí-Vidal et al. 2011; E. Kundu et al. 2019). Other approaches focused on following up samples of known radio SNe, like the recent works by M. F. Bietenholz et al. (2021), T. Matsuoka et al. (2025), and I. Sfaradi et al. (2025). These studies have found that detected radio SNe can typically be characterized by light curves with a monotonic rise and fall (as predicted by R. A. Chevalier 1998) and that Type Ib/c and Type II SNe have distinct inferred distributions of progenitor mass-loss rates. The Karl G. Jansky Very Large Array (VLA) Sky Survey (VLASS; M. Lacy et al. 2020) and the Variables and Slow Transients Pilot Survey for the Australian Square Kilometre Array Pathfinder (VAST; T. Murphy et al. 2013, 2021) provided the first unbiased radio surveys of the northern and (portions of the) southern sky, respectively, in the modern era. These surveys revealed a population of radio bright SNe at late times ($\delta t \gg 1$ yr), including SNe that were not radio detected at early times (M. C. Stroh et al. 2021; K. Rose et al. 2024). These studies revealed a population of SNe that display significantly brighter radio emission at late times than predicted by standard models. Here we present the study of one such SN, SN 2012au, which was independently rediscovered by VLASS (M. C. Stroh et al. 2021).

On 2012 March 14, SN 2012au was discovered by the Catalina Real-Time Transient Survey SNHunt project (S. Howerton et al. 2012). Optical, near-infrared (NIR), and ultraviolet (UV) follow-up of the source revealed Type Ib SN spectral features, alongside a luminosity in the top $\sim 10\%$ (M. R. Drout et al. 2011) of H-stripped CCSNe (D. Milisavljevic et al. 2013; K. Takaki et al. 2013). Modeling of the early time ($\delta t < 60$ days postexplosion) UV–optical–NIR light curve indicated a large kinetic energy $E_k \approx 10^{52}$ erg (D. Milisavljevic et al. 2013; K. Takaki et al. 2013), significantly larger than the $\approx 10^{51}$ erg that is expected from neutrino-driven explosions (A. Burrows 2000; T. Wang et al. 2022). This inference suggested the presence of a central engine driving the explosion. Spectroscopic observations of the event ~ 1 yr postexplosion further revealed a number of

extraordinary features: a “plateau” of iron emission lines, intermediate-width (~ 2000 km s^{−1}) O I $\lambda 7774$ emission, persistent P Cygni absorption features, in addition to a slow optical decay. These features, combined with the large E_k , were similar to those exhibited by superluminous SNe (SLSNe), such as SN 2007bi, as well as engine-driven “hypernovae,” such as SN 1998bw and SN 1997dq (D. Milisavljevic et al. 2013; K. Takaki et al. 2013). Despite this exceptional optical spectral and temporal behavior, the radio and X-ray properties of SN 2012au at early times were unremarkable and similar to those of ordinary CCSNe, consistent with a shock wave expanding through the progenitor’s stellar wind (A. Kamble et al. 2014).

Six years postexplosion, D. Milisavljevic et al. (2018) acquired spectroscopic observations of SN 2012au using the Magellan telescope. The mere persistence of detectable optical emission at this late epoch robustly ruled out radioactive decay as the primary power source. Moreover, the spectrum had evolved significantly since $\delta t \sim 1$ yr, transitioning to being heavily dominated by forbidden oxygen and sulfur emission lines. These features imply the presence of an ionizing source. Because the spectrum lacked narrow H Balmer lines, the common scenario of interaction with a hydrogen-rich CSM was disfavored. Consequently, D. Milisavljevic et al. (2018) proposed that SN 2012au is powered by a pulsar or magnetar wind nebula, though interaction with an H-poor CSM remained a viable alternative. This central-engine hypothesis has been supported by semianalytic (S. B. Pandey et al. 2021) and numerical (C. M. B. Omand & A. Jerkstrand 2023; L. Dessart 2024) modeling, which showed that the optical data are well reproduced by a nebula powered by a millisecond magnetar. Further supporting this interpretation are spectropolarimetric measurements tracking the SN’s evolution from early times into the nebular phase (S. DeSoto et al. 2025). These measurements showed strong polarization oriented along one axis during the photospheric phase, then a new dominant axis, orthogonal to the first, appearing during the nebular phase. S. DeSoto et al. (2025) suggested that this dual-axis structure was similar to that predicted for the ejecta of magnetar progenitors.

In this work we present the analysis of multifrequency radio (spanning from the meter to the millimeter wave bands), and X-ray (including soft and hard X-rays) emission of SN 2012au in the first 13 yr of evolution, and we complement this dataset with one epoch of optical photometry (Section 2). For consistency, we remodel the early time shock dynamics and the resulting synchrotron radiation (originally presented in A. Kamble et al. 2014) in Section 3. In Section 4, a model-agnostic fit reveals that the radio data are best described by two distinct emitting components. To interpret these physically, we explore models of CSM interaction (Section 5) and PWN (Section 6), before concluding and discussing future prospects in Section 7. Throughout this analysis, we draw comparisons to the recent results from very long baseline interferometry (VLBI) measurements of SN 2012au from M. Lazda et al. (2026); while M. Lazda et al. (2026) also favor a PWN model, they are unable to rule out the CSM-interaction scenario.

We follow D. Milisavljevic et al. (2013) and M. Lazda et al. (2026) and adopt 2012 March 3.5 (corresponding to MJD 55989.5) as time of explosion. Times are in the observer frame and are relative to the time of explosion unless otherwise stated. We adopt a distance of $D = 23.5 \pm 3.1$ Mpc, based on

²⁹ At the time of writing the afterglow of the neutron-star (NS) merger GW170817 is the best-observed example of off-axis jet emission entering the observer’s line of sight at late times, e.g., R. Margutti & R. Chornock (2021), their Figure 7.

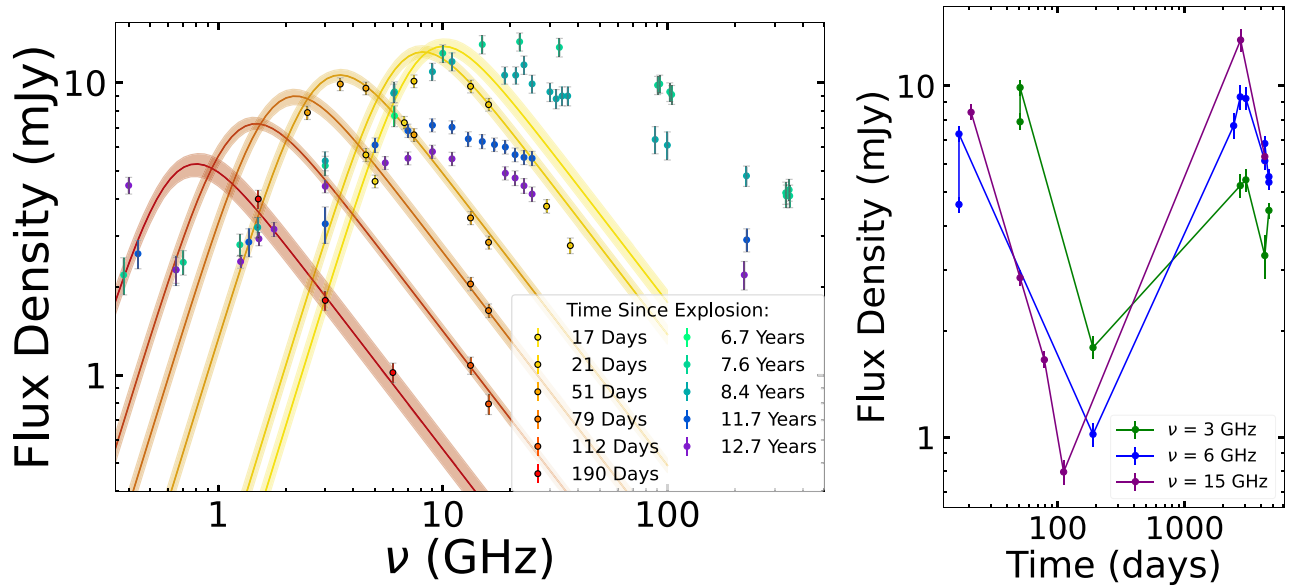


Figure 1. Left: complete SN 2012au radio (i.e., meter to millimeter) dataset spanning ≈ 13 yr of evolution. The early time data ($\lesssim 6$ months) originate from A. Kamble et al. (2014), to which we add one unpublished epoch at $\delta t = 190$ days. Solid lines (yellow to red): best-fitting model of synchrotron emission powered by the deceleration of the explosion’s forward shock (FS) into a broken power-law CSM-density model with a continuous transition between density indices, s_1 and s_2 , from our updated model described in Section 3.2.1. For each epoch, we show the 3σ credible interval as the shaded region around the best-fitting model. The peak of the radio emission at these early times clearly evolves to lower frequencies, as expected for a shock propagating through a medium with decreasing optical depth. The late-time data at $\delta t = 6.7$ – 12.7 yr that constitute the focus of this work notably deviate from this early time trend, indicating the onset of a new phase of radio evolution of SN 2012au, which is not a simple extrapolation of the previous behavior. The late-time deviation from expectations based on extrapolations can be seen clearly in Figure 2. Right: light curves of SN 2012au at selected well-sampled frequencies (3, 6, and 15 GHz). At early times, the SN 2012au follows the typical rise and fall behavior associated with synchrotron self-absorption (SSA). At late times, the source rebrightens, before dimming again.

association with the host galaxy NGC 4790 (G. Theureau et al. 2007).

2. Observations

Multiwavelength radio and X-ray observations of SN 2012au with the NSF’s VLA and the Neil Gehrels Swift Observatory X-Ray Telescope in the first 6 months of evolution were published by A. Kamble et al. (2014). Despite many unusual optical properties, the radio emission was consistent with synchrotron radiation from electrons accelerated by an explosion shock propagating into a typical wind-like density medium (i.e., $\rho_{\text{CSM}} \propto r^{-2}$), indicative of a steady mass-loss rate in the final years before core collapse. The early X-ray data were consistent with inverse-Compton emission from the same medium. We show the early time radio data in Figure 1, with one additional epoch at $\delta t = 190$ days that was not published by A. Kamble et al. (2014).

We started an extensive multiwavelength monitoring campaign of the late-time radio and X-ray emission from SN 2012au in 2018 November covering the time interval $\delta t = 2438$ – 4730 days. Our campaign includes observations acquired with the VLA (Section 2.1), the Atacama Large Millimeter/submillimeter Array (ALMA; Section 2.3), upgraded Giant Metrewave Radio Telescope (GMRT; Section 2.2), the Submillimeter Array (SMA; Section 2.4), the Northern Extended Millimeter Array (NOEMA; Section 2.5), the Chandra X-ray Observatory (CXO; Section 2.6), and the Nuclear Spectroscopic Telescope Array (NuSTAR; Section 2.7). The full late-time radio dataset, with the early data for comparison, is displayed in Figure 1, with flux densities listed in Table 3. Uncertainties presented in the table are the 1σ map rms values. We add 5% uncertainty in

quadrature to all radio observations at the time of modeling to capture residual systematic flux errors.

We note that there are a few additional sources of observation that we do not include in our dataset. Observations taken through the VLITE survey (W. Peters et al. 2019) have insufficient sensitivity to provide informative measurements at low frequency. While the VAST survey (T. Murphy et al. 2013, 2021) does achieve sufficient sensitivities, the angular resolution is insufficient to properly separate target emission from the host galaxy. Lastly, VLBI observations of the target systematically underestimate the flux density, most likely due to amplitude and phase errors driven by tropospheric variations (M. Lazda et al. 2026).

2.1. VLA (1–36 GHz)

We observed SN 2012au with the VLA over five epochs in 2018 November, 2019 September, 2020 July, 2023 December, and 2024 October, which spanned $\delta t = 2438$ – 4621 days (Figure 1). A log of our radio observations can be found in Table 3. When possible, we split the observed bandwidth into subbands to obtain higher spectral resolution. We used both 3C286 and 3C147 as bandpass and flux-density calibrators for different epochs and performed the observations in standard phase referencing mode with complex gain calibrators J1246-0730 (L , S , and C), J1239-1023 (S , C , and X), J1305-1033 (C , X , and Ku), J1305-1033 (Ku and K), J1256-0547 (K), and 3C279 (K and Ka). In addition to this dataset, we reduced one epoch of previously unpublished data from VLA/12A-458 (PI: Kamble) that was acquired on 2012 September 9 ($\delta t = 190$ days). 3C286 was used as a bandpass and flux-density calibrator, and J130533.0-103319 (C band) and J1246-0730 (L and S band) were used for complex gain calibration.

We calibrated both datasets using the VLA pipeline in the Common Astronomy Software Applications (CASA) package

(J. P. McMullin et al. 2007) using v.4.5.1 for the first four epochs, and v.6.5.4.9 for the last two epochs, with additional flagging. For imaging, we used the CLEAN algorithm in CASA using Briggs weighting with a robust parameter of one for the first four epochs and natural weighting for the last two epochs. Where necessary we performed phase-only self-calibration. We used Python Blob Detection and Source Measurement (PyBDSM; N. Mohan & D. Rafferty 2015) to fit the source in the image plane with a Gaussian fixed to the dimensions of the CLEAN beam.

We conducted these observations in the A and D array configurations. In A configuration, the SN was always clearly separated from the host; however, in D configuration at lower frequencies, the resolution was too low to fully avoid host contamination ($\approx 10''$ away from radio bright host emission). In these cases, we used our existing observations of the field to subtract the underlying host-galaxy flux.

2.2. GMRT (300–1300 MHz)

We observed SN 2012au with the GMRT over four epochs in 2019 December, 2020 July, 2023 December, and 2024 November ($\delta t = 2847.0$ – 4643.5 days). We used 3C286 or 3C147 as flux-density and bandpass calibrators, and QSO J1246-0730, J1248-199, or 3C275 as complex gain calibrators. The measured flux densities and other details of these observations are provided in Table 3.

We reduced and imaged the first two epochs of Band-3 (250–500 MHz), -4 (550–850 MHz) and -5 (1000–1400 MHz) observations in CASA v.6.1.2 following standard procedures. In each case, we performed a few rounds of phase-only self-calibration, followed by one round of amplitude and phase self-calibration. For imaging we used Briggs weighting with a robust parameter of one, and used two Taylor terms for the large fractional bandwidth. We measured the flux density in the image-plane using PyBDSM with an elliptical Gaussian fixed to the dimensions of the synthesized beam.

For Band-2 (120-250 MHz) data, direction-dependent calibration was necessary. We divided the Band-2 data into three subbands of 25 MHz, selecting only the usable parts of the recorded bandwidth. These subbands are centered on 148, 204, and 229 MHz. Each subband was processed independently with the SPAM pipeline (H. T. Intema et al. 2009, 2017) in the default mode, which includes direction-dependent calibration. The processed output visibilities of the pipeline for the three subbands were then combined to create a final combined image using WSClean (A. R. Offringa et al. 2014). We reduced and imaged the last two epochs of GMRT data in the Astronomical Image Processing System (AIPS; Associated Universities, I. 1999). We do not anticipate any systematic differences between reductions carried out in AIPS compared to those calibrated in CASA. We performed several rounds of phase-only self-calibration when needed. The imaging was carried out using a Gaussian fit with the task JMFIT. For all bands, the source positions in our GMRT and VLA images are consistent.

2.3. ALMA (90–350 GHz)

We observed SN 2012au with ALMA in 2019 October ($\delta t = 2796$ days). The observations were taken at 104.5 GHz (Band 3) and 350 GHz (Band 7). The data were calibrated using the ALMA pipeline in CASA v.5.6.1-8. For imaging, the data were divided into eight subbands, each of bandwidth of

2 GHz with central frequencies of 90.5, 92.4, 102.5, 104.5, 336.5, 338.4, 348.5, and 350.5 GHz. We imaged the data using the tclean algorithm in CASA v.4.5.1, using Briggs weighting with a robust parameter of one. For Band 3 we did phase-only self-calibration. Similarly to the VLA data, we fitted the source in the image plane with a Gaussian fixed to the dimensions of the clean beam with PyBDSM. The flux densities and observation details are shown in Table 3.

2.4. SMA (210–240 GHz)

SN 2012au was observed with the SMA on 2020 July 14 ($\delta t = 3055$ days), 2023 November 16 ($\delta t = 4275$ days), and 2025 February 14 ($\delta t = 4730$ days) under the SMA Large Program Pursuit of Extragalactic Transients with the SMA (POETS). The data were reduced and calibrated via two different paths which produced equivalent results, a standard user-driven calibration using the mid-infrared calibration suite found at <https://www.cfa.harvard.edu/~cqi/mircook.html>, and a new pipeline calibration process called COMPASS. Complex gains versus time were calibrated using the nearby blazar 3C279, which also provided a strong continuum source for bandpass calibration. The flux-density scale was set using observations of Callisto and Titan, using the brightness temperature models described in NRAO Memo 594.³⁰ The data were spectrally averaged to provide a mean continuum dataset for further analysis. The source flux density was measured using both visibility fitting (assuming a point source) and aperture fitting after imaging within AIPS and a proprietary imaging routine; all results were consistent with regard to position and flux density. The flux densities and observation details are shown in Table 3.

2.5. NOEMA (88–100 GHz)

We observed the field of SN 2012au for a single epoch on 2020 November 17 ($\delta t = 3181$ days) with the NOEMA interferometer. The observations were carried out at a central frequency of 94 GHz. We reduced and measured flux-density values for the lower side band, which has a central frequency of 86 GHz and the upper side band, which has a central frequency of 100 GHz. We calibrated the data using CLIC within the GILDAS³¹ software package to perform bandpass, amplitude, and phase calibration. We measured the flux density of the target by performing a fit in the uv plane. With such high signal-to-noise ratio measurements, we report flux-density measurements in both subbands in Table 3.

2.6. CXO (0.3–10 keV)

A first deep epoch of CXO observations was obtained on 2018 August 2 ($\delta t = 2343$ days; observation ID 21660; PI: Patnaude) and published in D. Milisavljevic et al. (2018). We acquired two additional epochs of CXO ACIS-S observations covering the time period $\delta t = 3060$ – 3677 days. A log of the CXO observations can be found in Table 4. We analyzed the ACIS-S data following standard practice with CIAO v.4.15 and corresponding calibration files. We reanalyzed the first epoch of CXO data for consistency. No X-ray source is detected at the location of the SN in any epoch, either blindly or with a targeted detection algorithm. Assuming Poissonian

³⁰ Butler, B.J.; <https://library.nrao.edu/public/memos/alma/memo594.pdf>.

³¹ <https://www.iram.fr/IRAMFR/GILDAS>.

statistics, we inferred the 3σ count-rate limits reported in Table 4. We converted the count-rate limits into flux limits adopting an absorbed power-law spectrum with photon index $\Gamma = 2$ and a Galactic neutral hydrogen absorption column in the direction of the transient of $N_{\text{H,MW}} = 3.7 \times 10^{20} \text{ cm}^{-2}$ (HI4PI Collaboration et al. 2016). For each observation, a count-to-flux factor was derived by accounting for the instrumental response at the location of the transient.

2.7. NuSTAR (3–80 keV)

We obtained one epoch of hard X-ray observations with NuSTAR in coordination with the CXO at $\delta t \approx 3035$ days (Obs ID 90601521002; PI: Margutti). NuSTAR data were processed and analyzed with the `nupipeline` within the the NuSTAR Data Analysis Software (v.1.9.7) and corresponding calibration files. We removed the time intervals affected by high background with custom scripts. No source is detected at the location of the SN. We report in Table 4 the inferred count-rate limits in the 8–20 keV energy band (which corresponds to the peak of the NuSTAR effective area), combining the signal from both NuSTAR modules. As for the CXO observations, we assume a power-law spectrum with photon index $\Gamma = 2$ for the flux calibration.

2.8. Optical Photometry

We obtained optical imaging of SN 2012au on 2020 June 23 ($\delta t = 3034$ days), using the Low Resolution Imaging Spectrometer (J. B. Oke et al. 1995) mounted on the 10 m Keck I telescope (PI: Terreran, Program 2020A_O285). We used *BgRI* bands, exposing for 720 s per band. We reduced the images using standard overscan, bias, and flat-fielding within IRAF.³² We then extracted the photometry data using the SNOOPY³³ package. We performed point-spread function photometry with DAOPHOT (P. B. Stetson 1987). We estimated the zero-points and color term based on the magnitudes of field stars present in the Sloan Digital Sky Survey³⁴ (SDSS; D. G. York et al. 2000) catalog (Data Release 9). We converted the SDSS *ugriz* magnitudes to Johnson–Cousins *UBVRI* filters following T. S. Chonis & C. M. Gaskell (2008). We quantified the uncertainty on the instrumental magnitude injecting artificial stars (e.g., Y. Hu et al. 2011). The resulting uncertainty was then added in quadrature to the fit uncertainties returned by DAOPHOT and the uncertainties from the photometric calibration to obtain the total uncertainty on the photometry. We measure 23.0 ± 0.2 , 22.0 ± 0.2 , 21.6 ± 0.1 , and 21.2 ± 0.1 mag for *B*, *g*, *R*, and *I*, respectively (reported in Vega magnitudes).

3. Shock Dynamics and Deviation from a Pure Wind-like Density Model

3.1. Radio Emission from Supernovae in the Thin-shell Approximation

3.1.1. Dynamics

At early times ($\delta t \leq 190$ days), the radio spectral energy distributions (SEDs) follow a fairly standard evolution,

peaking at successively lower frequencies over time (Figure 1). A. Kamble et al. (2014) showed that the radio emission in this time range is well explained by synchrotron radiation from electrons accelerated at the FS expanding in a wind-like CSM with density $\rho_{\text{CSM}} \propto r^{-2}$, as expected from massive stars that have been losing mass at a constant $\dot{M}/v_w$ (where $\dot{M}$ is the mass-loss rate, and v_w is the wind velocity). In the following we generalize the approach of A. Kamble et al. (2014).

We first solve for the SN shock dynamics in a given CSM, and then predict the resulting synchrotron emission from the FS. In the thin-shell approximation (e.g., R. A. Chevalier 1982a), the conservation of momentum of the shocked shell of material of negligible thickness can be written as

$$M_s \frac{dv_s}{dt} = 4\pi R_s^2 [\rho_{\text{ej}}(v_{\text{ej}} - v_s)^2 - \rho_{\text{CSM}} v_s^2] \quad (1)$$

(e.g., R. A. Chevalier & C. Fransson 2017), where M_s is the total mass of the thin shell comprising both shocked ejecta and shocked CSM at the shock radius R_s , propagating at the shock velocity $v_s = dR_s/dt$; v_{ej} is the ejecta velocity at the reverse shock; and ρ_{ej} and ρ_{CSM} are the density profiles of the outer layers of the star and the CSM, respectively. We assume that the velocity of the CSM, v_w , is much smaller than v_s , such that $v_s - v_w \approx v_s$.

R. A. Chevalier & N. Soker (1989) derive a density profile for SN ejecta in homologous expansion. The expansion of the stellar envelope and corresponding rarefaction wave is modeled with a combination of self-similar solutions and numerical computation. The resulting profile comprises an outer steep density power law, and an inner flat density profile, with a transition velocity v_t marking the boundary between the two regimes:

$$\rho_{\text{ej}} = \begin{cases} A(t - t_0)^{-3}, & v_{\text{ej}} < v_t, \\ A(t - t_0)^{-3}(v_{\text{ej}}/v_t)^{-n}, & v_{\text{ej}} \geq v_t, \end{cases} \quad (2)$$

where

$$A \equiv \frac{1}{4\pi n} \frac{[3(n-3)M_{\text{ej}}]^{5/2}}{[10(n-5)E_k]^{3/2}}, \quad (3)$$

$$v_t \equiv \left[\frac{10(n-5)E_k}{3(n-3)M_{\text{ej}}} \right]^{1/2}, \quad (4)$$

and M_{ej} and E_k are the total explosion ejecta mass and kinetic energy, respectively. The outer ejecta density index, n , varies based on stellar progenitor. For Wolf–Rayet stars that are possible progenitors of H-stripped CCSNe, values of $n \approx 10$ are commonly used in the literature (R. A. Chevalier & C. Fransson 2006; Z.-W. Liu et al. 2015), based on analytic approximations of the simulations from C. D. Matzner & C. F. McKee (1999). Recent results by T. Matsuoka et al. (2025) that are derived from fitting radio SNe potentially indicate that lower values of $n \approx 5$ are possible and maybe even common. However, in our modeling n is degenerate with other parameters, and assumptions have to be made (see below). We adopt $n = 10$ in what follows, and we note that none of our major conclusions depends on this assumption.

Other radio-SN studies (e.g., T. Matsuoka et al. 2025; R. Soria et al. 2025, for recent examples) also used the

³² IRAF is distributed by the National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy (AURA) under a cooperative agreement with the National Science Foundation. <http://iraf.noao.edu/>.

³³ E. Cappellaro (2014). SNOOPY: a package for SN photometry, <http://sngroup.oapd.inaf.it/snoopy.html>.

³⁴ <http://www.sdss.org>

thin-shell approximation to model the dynamics of SN ejecta interacting with a dense CSM. In these studies, the R. A. Chevalier (1982a) self-similar solutions for a single power-law outer ejecta profile ($\rho_{\text{ej}} \propto r^{-n}$) interacting with a single power-law CSM profile ($\rho_{\text{CSM}} = Ar^{-s}$) are used. The solution for the temporal evolution of the shock radius is

$$R_s(t) = \left[\frac{(3-s)(4-s)}{(n-3)(n-4)} \frac{U_c^n}{A} \right]^{\frac{1}{n-3}} t^{\frac{n-3}{n-5}}, \quad (5)$$

where

$$U_c \equiv \left(\frac{3M_{\text{ej}}}{4\pi v_i^{3-n}} \frac{n-3}{n} \right)^{1/n}. \quad (6)$$

This solution is valid for $s < 3$ and $n > 5$. Scenarios with $s > 3$ in the thin-shell approximation would result in the unphysical situation of an accelerating contact discontinuity with nonaccelerating ejecta (a self-similar treatment of this problem can be found in E. Waxman & D. Shvarts 1993). Similarly, scenarios with $n < 5$ would result in a separation between the contact discontinuity and the outer shock wave (R. A. Chevalier 1982a). Within the regime of validity ($s < 3$ and $n > 5$), the solution of Equation (5) only applies when the two interacting density profiles are power laws. For the present study, we allow for the possibility of more complex CSM profiles by forgoing the analytic solution and instead integrating numerically Equation (1)—similar to the treatment of the X-ray analysis in A. L. Ibik et al. (2025). Given the explosion parameters and density profiles for the ejecta and CSM, we can numerically solve Equation (1) to compute the shock radius $R_s(t)$ and velocity $v_s(t)$ evolution.

3.1.2. Synchrotron Radiation

Following R. A. Chevalier & C. Fransson (2006), we assume that a fraction ϵ_B of the postshock internal energy density goes into the magnetic energy density:

$$\frac{B^2}{8\pi} = \epsilon_B \rho_{\text{CSM}} v_s^2, \quad (7)$$

where ρ_{CSM} is the unshocked CSM density. An additional parameter that will be relevant is ϵ_e , the fraction of the postshock energy density that goes into the energy density of relativistic electrons:

$$\int_{\gamma_{\min}}^{\gamma_{\max}} n(E) E dE = \epsilon_e \rho_{\text{CSM}} v_s^2, \quad (8)$$

where $n(E)$ is the number density of electrons per unit electron energy E above a minimum electron Lorentz factor $\gamma_{\min} = 1$, as in R. A. Chevalier (1998). We will assume $\gamma_{\max} = \infty$ unless otherwise specified. We define a system in equipartition when $\epsilon_B = \epsilon_e$, i.e., when relativistic electrons and magnetic field carry equal fractions of postshock energy density.

The combination of Equations (1)–(4) and Equation (7) thus give us the $R_s(t)$ and $B(t)$ evolution for a given $\rho_{\text{CSM}}(r)$ and explosion parameters E_k and M_{ej} , from which the synchrotron spectrum can be computed, given values for ϵ_B and ϵ_e and the distribution of accelerated electron energies. As the FS propagates through the CSM, electrons are accelerated at the shock front. Following the formalism of R. A. Chevalier (1998), we consider a power-law distribution of accelerated electron energies, $n(E) = n_0 E^{-p}$, as supported by particle-in-

cell simulations (e.g., D. Caprioli 2023, and references therein for a recent review). The spectral shape of the resulting emission will be determined by the relative values of ν_m , the characteristic synchrotron frequency, ν_{sa} , the SSA frequency, and ν_c , the cooling frequency. Different orderings of these break frequencies will result in different slopes and different locations of the spectral peak flux (J. Granot & R. Sari 2002). In order to model all these possible combinations of frequency orderings, we apply the methods described in Appendix C of I. Sfaradi et al. (2025), self-consistently calculating the break frequencies and their corresponding SEDs.

We find that the observed synchrotron emission of SN 2012au consistently falls into the regime of $\nu_m < \nu_{\text{sa}} < \nu_c$. In this regime, the flux density at the asymptotic intersection of the optically thin $F_\nu \propto \nu^{-(p-1)/2}$ and optically thick $F_\nu \propto \nu^{5/2}$ power-law segment is (R. A. Chevalier 1998)

$$F_{\nu, \text{brk}} = \frac{\pi R_s^2}{D^2} \frac{c_5}{c_6} (B \sin \theta)^{-1/2} \left(\frac{\nu_{\text{brk}}}{2c_1} \right)^{5/2}, \quad (9)$$

and the spectral peak occurs at $\nu \approx \nu_{\text{brk}}$:

$$\nu_{\text{brk}} = 2c_1 \left(R_s f \left(\frac{\epsilon_e}{\epsilon_B} \right) \frac{p-2}{6\pi} \right)^{\frac{2}{p+4}} \left(\frac{m_e}{c^2} \right)^{\frac{2(p-2)}{p+4}} B^{\frac{p+6}{p+4}} c_6^{\frac{2}{p+4}} (\sin \theta)^{\frac{p+2}{p+4}}. \quad (10)$$

Above, c_1 , $c_5(p)$, and $c_6(p)$ are functions of physical constants and parameter p . These functions are defined by A. G. Pacholczyk (1970) and fully described in Appendix D. f_V is the volumetric emission filling factor, defined as in R. A. Chevalier (1998) such that the total volume of the synchrotron emitting electrons is

$$V \equiv \frac{4\pi}{3} R_s^3 f, \quad (11)$$

and θ is the pitch angle of accelerated electrons. We note that the alternative choice of an isotropic distribution of pitch angles (as in R. A. Chevalier & C. Fransson 2017) would have no impact on our major conclusions. Following R. A. Chevalier (1998), we assume $\sin(\theta) = 1$. For a discussion of anisotropic pitch angle distributions, see Y.-P. Yang & B. Zhang (2018).

3.2. Application to SN 2012au

3.2.1. Early Time Radio Emission at $\delta t < 190$ days

We approximate the resulting synchrotron SEDs using a smoothed broken power-law model of the form

$$F_\nu = F_{\nu, \text{brk}} \left[\left(\frac{\nu}{\nu_{\text{brk}}} \right)^{-w(1-p)/2} + \left(\frac{\nu}{\nu_{\text{brk}}} \right)^{-w\alpha} \right]^{-1/w}, \quad (12)$$

where $F_{\nu, \text{brk}}$ and ν_{brk} are related to the shock properties via Equations (9) and (10), respectively; p is the power-law index for the distribution of emitting electron energies, $n(E) \propto E^{-p}$, α is the optically thick spectral index ($F_\nu \propto \nu^\alpha$), and w is a smoothing parameter for the transition between optically thick and optically thin power laws. $F_{\nu, \text{brk}}$, the flux at the intersection of the power laws, is related to the observed peak flux density in the light curve by the relation $F_{\nu, \text{pk}} = 2^{-1/w} F_{\nu, \text{brk}}$. Following J. Granot & R. Sari (2002), when the spectra fall in the

$\nu_m < \nu_{sa} < \nu_c$ regime that is relevant to our analysis below, we can parameterize w as a function of p :

$$w = 1.47 - 0.21p. \quad (13)$$

Next, Equation (1) depends on the total kinetic energy E_k and ejecta mass M_{ej} of the explosion. Ideally, one would either have these parameters as free in the radio modeling, or use independent estimates of these parameters from other wavelengths. For example, estimates of M_{ej} and E_k can be obtained from the UV–optical–NIR emission. D. Milisavljevic et al. (2013) model a “pseudobolometric” (i.e., UV + optical) light curve of SN 2012au and estimate $M_{ej} \approx (3\text{--}5) M_\odot$ and $E_k \approx 1 \times 10^{52}$ erg. K. Takaki et al. (2013) use spectroscopic measurements to estimate an expansion velocity v , and a rise-time t_r . Applying scaling relationships of these two parameters using another Type Ib SN 2008D as a template, K. Takaki et al. (2013) estimate $M_{ej} = (5 - 7) M_\odot$ and $E_k = (7 - 18) \times 10^{51}$ erg. S. B. Pandey et al. (2021) estimate $M_{ej} = (4.7\text{--}8.5) M_\odot$ and $E_k = (4.8\text{--}5.4) \times 10^{51}$ erg from the modeling of the bolometric light curve (where the NIR has been estimated based on the extrapolation of the observed optical emission).

In the radio modeling E_k and M_{ej} are highly degenerate. Both parameters affect the initial ejecta velocity and the transition velocity in the ejecta density profile, v_t . At early times, the outermost ejecta, which produce synchrotron emission, are expected to always be moving faster than v_t , and therefore, for the purpose of the early radio emission, M_{ej} and E_k only impact the initial velocity of the ejecta. Mathematically, M_{ej} and E_k only enter the calculation through Equations (2) and (3) as a ratio that determines the normalization of $\rho_{ej} \propto (M_{ej}^5/2)/E_k^{3/2}$. We will thus fix one of these parameters (M_{ej}) and fit for the other (E_k). We assume $M_{ej} = 5 M_\odot$ to remain within the inferred mass ranges of all three optical studies above.

For the early time ($\delta t \leq 190$ days) radio data, we further assume an ejecta density index of $n = 10$ and microphysical parameters of $\epsilon_e = \epsilon_B = 0.3$, to allow a direct comparison with the results from A. Kamble et al. (2014).³⁵ We find that there is large variance in the optically thin slope across epochs—corresponding to values of $p = 2.9\text{--}3.5$. Given that the value of p is poorly determined, we choose to assume $p = 3$, the standard value in synchrotron modeling of SNe (R. A. Chevalier 1998; R. A. Chevalier & C. Fransson 2006).

The free parameters of our model characterize the CSM-density profile, $\rho_{\text{CSM},0}(r)$. We first assumed a single power-law density profile, but failed to adequately fit the data. In particular, observations for the last early time epoch at $\delta t = 190$ days were consistently and significantly over-predicted by the model. We thus allow for an extra degree of freedom, and adopt a broken power-law CSM-density profile:

$$\rho_{\text{CSM},0}(r) = \begin{cases} \rho_0 \left(\frac{r}{R_{\text{brk}}} \right)^{-s_1}, & r < R_{\text{brk}}, \\ \rho_0 \left(\frac{r}{R_{\text{brk}}} \right)^{-s_2}, & r > R_{\text{brk}}, \end{cases} \quad (14)$$

³⁵ We note that while there are long-standing theoretical arguments for the equipartition assumption (G. R. Burbidge & E. M. Burbidge 1957), a number of observational studies (e.g., R. Ruiz-Carmona et al. (2022)) have found evidence of deviation from equipartition. That being said, our assumption of equipartition will not strongly impact the overall trends reported in this section, nor the extrapolations in Section 3.2.2.

Table 1
Early Radio Data ($\delta t < 190$ days): Best-fitting Parameters

Parameter	Priors	Best-fit Values
$\log_{10} \left(\frac{E_k}{\text{erg}} \right)$	[50, 53]	52.54 ± 0.01
$\log_{10} \left(\frac{\rho_0}{\text{g cm}^{-3}} \right)$	[−15, −23]	-22.11 ± 0.05
$\log_{10} \left(\frac{R_{\text{brk}}}{\text{cm}} \right)$	[16.5, 17]	16.76 ± 0.02
s_1	[1, 4]	1.85 ± 0.02
s_2	[1, 4]	2.25 ± 0.03

Note. Equipartition is assumed.

with free parameters ρ_0 , the normalization on the density, R_{brk} , a break radius where the density index may change, and s_1 and s_2 , the density indices at radii smaller and larger than R_{brk} , respectively. We use a Markov Chain Monte Carlo (MCMC) fit with the EMCEE Python package (D. Foreman-Mackey et al. 2013). We initialize 25 walkers, which each iterate the model calculations 1000 times. We discard the first 200 iterations for each walker to only consider samples after the burn-in phase. We report the results of convergence tests $\hat{R}$ and Effective Sample Size (ESS) statistics in Appendix A. We iterate over the parameters ρ_0 , E_k , and R_{brk} in logarithmic space, while we iterate over the parameters s_1 and s_2 in linear space. The initial parameter values and priors are summarized in Table 1. The prior on R_{brk} is informed by our single power-law CSM-density fit above. The corner plot of the resulting MCMC fit can be found in Appendix A, Figure 16, and the best-fitting parameters are reported in Table 1.

As seen in Figure 1, the early time radio data ($\delta t \leq 190$ days) are well reproduced by $\log_{10}(E_k/\text{erg}) = 52.54 \pm 0.01$, with a broken power-law CSM-density profile. The inferred explosion energy is a factor of a few higher than inferred by D. Milisavljevic et al. (2013), K. Takaki et al. (2013), and S. B. Pandey et al. (2021), which is not surprising given the orthogonal approaches, using completely different datasets. Our modeling results in a higher initial shock velocity than the radio analysis of A. Kamble et al. (2014). At $\delta t_0 = 25$ days (reference time of the analysis of A. Kamble et al. 2014), our shock radius is $R_s(\delta t_0) = 2.5 \pm 0.1 \times 10^{16}$ cm, corresponding to an average velocity of $\approx 0.38c$, compared to the velocity of $\approx 0.2c$ from A. Kamble et al. (2014). This discrepancy arises from the fact that our modeling self-consistently accounts for shock deceleration from CSM interaction, while A. Kamble et al. (2014) assume a constant shock velocity.

At $r < R_{\text{brk}}$, where $\log_{10}(R_{\text{brk}}/\text{cm}) = 16.76 \pm 0.02$, we find $\rho_{\text{CSM},0} \propto r^{-1.85 \pm 0.02}$. To match the emission at $\delta t = 190$ days, which was not included in A. Kamble et al. (2014), the CSM-density profile steepens to $\rho_{\text{CSM},0} \propto r^{-2.25 \pm 0.03}$ at $r > R_{\text{brk}}$. Our results are reasonably close to a wind-like density profile ($\rho_{\text{CSM},0} \propto r^{-2}$) that was assumed by A. Kamble et al. (2014), but indicate some important deviations. With this caveat in mind, our effective mass-loss rates range between $(4 - 5) \times 10^{-6} M_\odot \text{ yr}^{-1}$ over the period $\delta t = 17\text{--}190$ days, assuming $v_w = 1000 \text{ km s}^{-1}$ (typical of Wolf–Rayet progenitor stars). A. Kamble et al. (2014) reported a slightly lower constant mass-loss rate $\dot{M} = 3.6 \times 10^{-6} M_\odot \text{ yr}^{-1}$. Despite indicating higher mass-loss rates, the faster shock velocity of our best-fitting model leads to lower densities at a given time

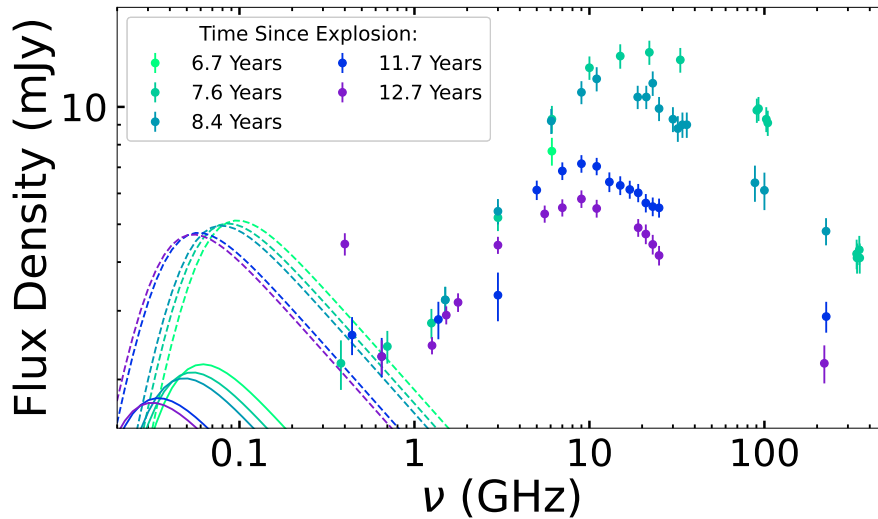


Figure 2. The extrapolation of our best-fitting model(s) of Section 3.2.2 to late times $\delta t \geq 6$ yr significantly underpredicts the detected emission (filled circles). Solid lines: extrapolation of the early time ($\delta t \leq 190$ days) best-fitting model using a standard wind CSM-density index $s = 2$, and electron energy distribution index $p = 3$. The dashed lines show how a model with shallower $p = 2.01$ and a slightly less steep density index $s = 1.75$ can match the flux density at low frequencies $\nu \lesssim 1$ GHz at late times, while still requiring some additional component to explain the dominant emission at higher frequencies ($\nu \geq 5$ GHz). This two-component model is explored in Section 4.

compared to, A. Kamble et al. (2014). For example, our best-fitting electron number density at t_0 , $n_e(t_0) \approx 200 \text{ cm}^{-3}$, compared with $n_e(t_0) \approx 900 \text{ cm}^{-3}$ of A. Kamble et al. (2014).³⁶ The combination of lower density but larger radii at a given time results in similar inferred B fields (Equation (7)): $B(t_0) = 0.49 \pm 0.02 \text{ G}$ for our analysis, roughly consistent with the reported $B(t_0) = 0.54 \text{ G}$ from A. Kamble et al. (2014), all assuming equipartition.

3.2.2. Late-time Radio Emission at $\delta t > 190$ days

We extrapolate our early time solution to later times. We assume the CSM-density index is a standard wind index $s = 2$ and compute the shock dynamics self-consistently with Equation (1); finally we predict the expected radio SEDs, using Equations (9)–(12) (thick solid lines in Figure 2). By computing the characteristic synchrotron frequencies at the times of interest, we verify that the break frequencies still satisfy $\nu_m < \nu_{sa} < \nu_c$, which implies that Equations (9) and (10) are still valid.

As shown by Figure 2, the extrapolation of the early time model significantly underpredicts the observed flux densities, and the overall SED peak is shifted to significantly lower frequencies. However, while the emission near the observed SED peak is completely unaccounted for by the extrapolated model, we note that flux densities at $\nu \lesssim 1$ GHz are better matched. With just a small deviation from the a wind density profile (from $\rho_{\text{CSM},0} \propto r^{-2}$ to $\rho_{\text{CSM},0} \propto r^{-1.75}$), and a decrease in the electron energy distribution index (from $p = 3$ to 2.01) the predicted flux density increases to a level comparable with the observed F_ν at $\nu \lesssim 5$ GHz—see the dashed line extrapolations in Figure 2.

This exercise suggests that slight modifications of the extended CSM-density profile at $r > 10^{16.8} \text{ cm}$, and some evolution of the shock parameters can reasonably account for the low-frequency portion of the late-time emission. However, the higher-frequency flux densities at $\gtrsim 5$ GHz indicate a clear

deviation from this simple scenario of continued shock interaction with the same CSM, suggesting the presence of a different component of emission. Thus motivated, we consider in Section 4 a multiple-component model to account for the bright peak of radio emission at $\delta t > 6$ yr.

4. A Two-component Model

We start with a few broad considerations about the shape of the spectral peak of the late-time SEDs. In addition to not being fully explained by extrapolations of the early time shock, the late-time radio emission from SN 2012au exhibits other notably unusual features: (i) The optically thick slope around ν_{pk} is unusually shallow: $F_\nu \propto \nu^{0.5}$,³⁷ compared to the expected $F_\nu \propto \nu^{5/2}$ or $F_\nu \propto \nu^2$ of the SSA regime (or even steeper for a free-free absorption (FFA) + SSA scenario of a single population of radiating electrons). (ii) The optically thin slope is also unusually shallow, scaling like $F_\nu \propto \nu^{-0.3}$, compared to a typical value for young SNe of $F_\nu \propto \nu^{-1}$ (R. A. Chevalier & C. Fransson 2006) or at the most extreme end $F_\nu \propto \nu^{-0.5}$ (D. Urošević et al. 2019). (iii) Overall, the radio SEDs are unusually broad.

The radio spectral shape of SN 2012au many years postexplosion bears many qualitative similarities with that of the late-time radio emission from two other SNe: SN 1986J and SN 2001em (see Figure 3). The SEDs of the three SNe observed at $\delta t > 12$ yr peak at $\nu \approx (10\text{--}20)$ GHz, and show an “inversion” of the optically thick slope around $\nu \approx$ a few gigahertz. They also have similarly shallow optically thick slopes close to the peak and, in some epochs, display similarly broad peaks. The analysis of VLBI observations of SN 1986J revealed that the observations are best explained by two distinct components contributing to the spectrum: a spatially extended component and a compact component dominating the low- and high-frequency range of emission, respectively (M. F. Bietenholz & N. Bartel 2017bb). A

³⁶ Electron number densities are derived adopting a mean molecular weight $\mu_e = 1$ for the CSM, which is typically assumed in the literature.

³⁷ There is some variance in the observed optically thick spectral index (between 0.4 and 0.7). The 0.5 number noted here is based on the 12.7 yr epoch for frequencies between 1 and 7 GHz.

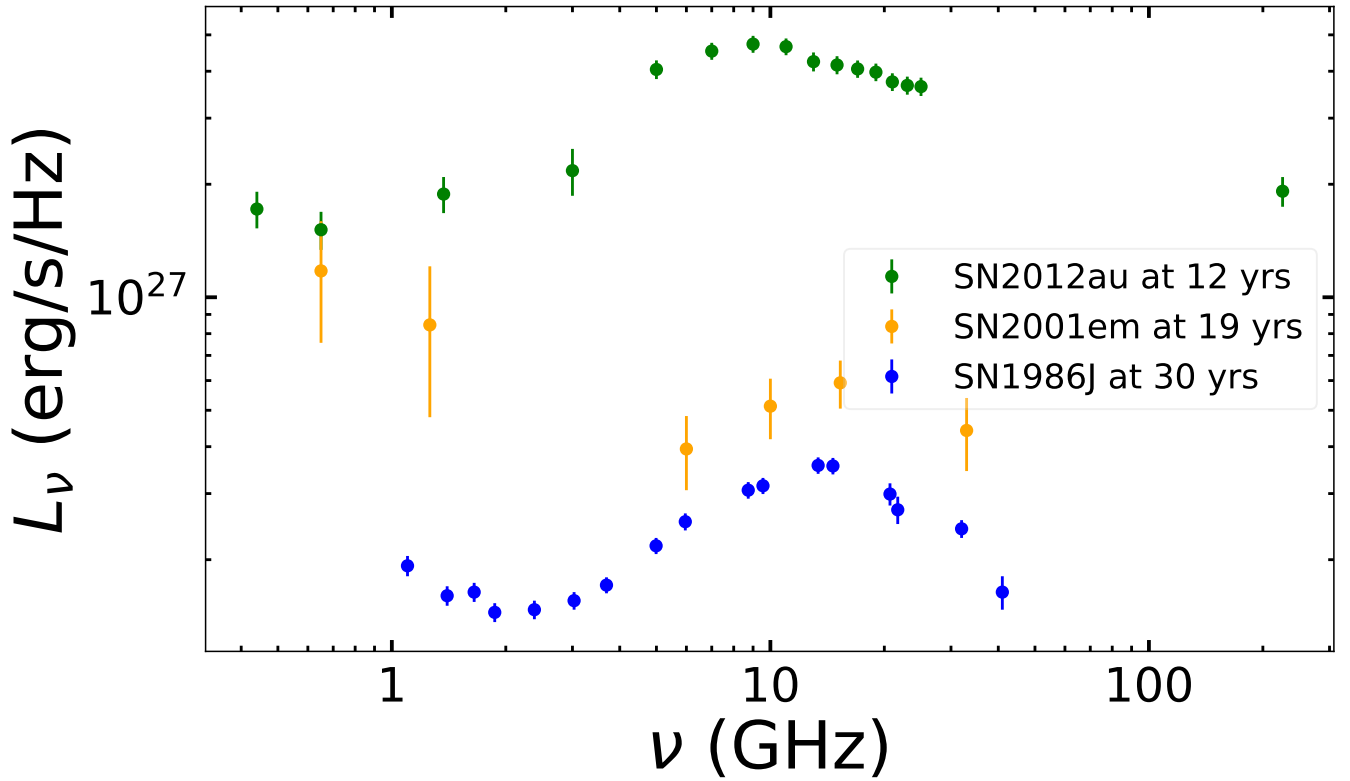


Figure 3. Late-time radio observations of SN 2012au (this paper), SN 1986J (M. F. Bietenholz & N. Bartel 2017bb), and SN 2001em (P. Chandra et al. 2020) show a qualitatively similar behavior, with an “inverted” SED at low frequencies.

similar explanation was favored for SN 2001em (P. Chandra et al. 2020), in large part due to its observed similarities with SN 1986J. VLBI observations of SN 2012au from M. Lazda et al. (2026) suggest a similar two-component scenario, which is directly supported by our radio SED modeling and which we explore in detail below.

4.1. A Model-agnostic Approach

First, we perform a two-component fit of the late-time radio SEDs adopting a model-agnostic perspective. The goal is to capture the key observed features of the SEDs, while making as few physical assumptions as possible. Our model has a low-frequency component that will capture the behavior of the expanding blast wave from early times, and a higher-frequency component that captures the remaining emission.

Based on our extrapolations shown in Figure 2, it is likely that our observations of the low-frequency component will fall entirely in the optically thin regime. As such, we parameterize this component as a single power-law spectrum. With observations showing very flat spectral indices, we fix the power law of this low-frequency component to $F_\nu \propto \nu^{-1/2}$, which is the shallowest spectral index allowable within the typical range of values for p : $2 < p < 4$ (e.g., R. A. Chevalier & C. Fransson 2006, 2017, and references therein) for young (i.e., preremnant phase) SNe. We allow the power-law normalization to vary freely between epochs.

We model the high-frequency component as a broken power law as in Equation (12). However, instead of assuming an optically thick slope based on SSA ($F_\nu \propto \nu^{5/2}$), we treat this slope as a free parameter, α . We do not assume any $w(p)$ smoothing parameterization, as we do not assume to know how the characteristic frequencies of the spectrum are ordered,

and therefore cannot use the formalism of J. Granot & R. Sari (2002) to describe the smoothing parameter of the broken power law. While w , α , and p will be able to vary freely, we will assume that they will stay constant across late-time epochs.

We find that the optically thin side of the high-frequency component is poorly fit by a single power-law segment, particularly at the $\delta t = 7.6$ yr epoch. To allow the model to better fit the observed SED, we add a cooling-break frequency, ν_c , at each epoch as a free parameter. At $\nu > \nu_c$, the spectral index steepens from $F_\nu \propto \nu^{-(p-1)/2}$ to $F_\nu \propto \nu^{-p/2}$, consistent with spectral steepening at an inverse-Compton or synchrotron cooling break (R. A. Chevalier & C. Fransson 2017). The change in slope at ν_c is the only physical prescription added to this model.

The results of this model-agnostic fit can be found in Figure 4, with the corresponding corner plots in Appendix A. We find that the best-fitting parameters are $p = 1.61^{+0.05}_{-0.04}$, $\alpha = 1.21^{+0.16}_{-0.13}$, and $w = 0.44^{+0.13}_{-0.10}$. $F_{\nu,\text{brk}}(t)$ and $\nu_{\text{brk}}(t)$ follow a consistent power-law evolution in time, as shown in Figure 5: $F_{\nu,\text{brk}} \propto t^{-1.98 \pm 0.17}$ and $\nu_{\text{brk}} \propto t^{-0.94 \pm 0.32}$. While fixing the low-frequency optically thin slope to $F_\nu \propto \nu^{-1/2}$ (compared to the early time value of $F_\nu \propto \nu^{-1}$) did produce a closer fit to the data, the inferred values of $F_{\nu,\text{brk}}(t)$ and $\nu_{\text{brk}}(t)$ are not highly sensitive to our specific choice of spectral slope of the low-frequency component.

We find statistical evidence for a cooling break in the radio data at $\delta t = 7.6$ yr, with $\log_{10}(\nu_c/\text{Hz}) = 11.21^{+0.06}_{-0.06}$. The broadband (radio to X-ray) SED at $\delta t = 8.4$ yr also implies the presence of a cooling break, with $\log_{10}(\nu_c/\text{Hz}) \approx 11.2$ required to maintain consistency with the optical data (Figure 4). The overestimated emission in the optical regime can also be explained by a distribution of electron energies that is not

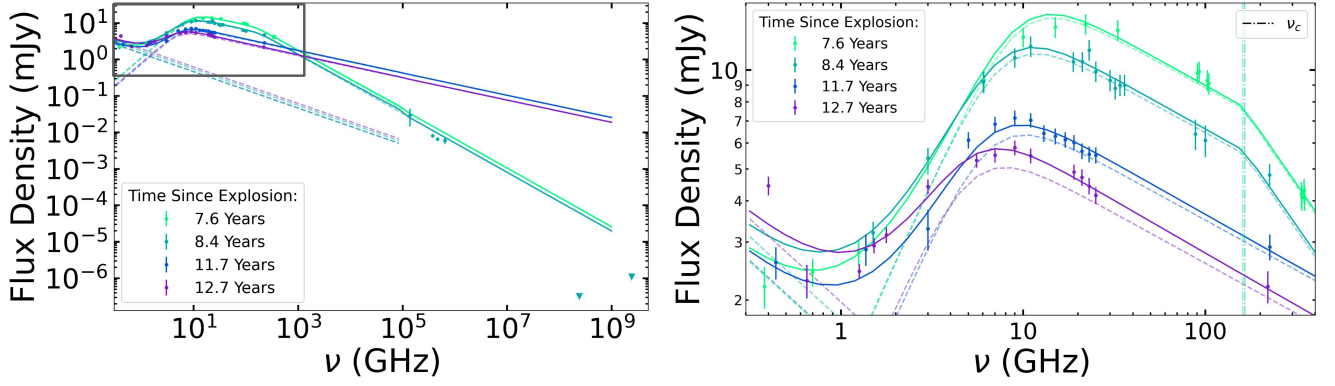


Figure 4. A model-agnostic, two-component fit to the late-time ($\delta t > 6$ yr) SN 2012au data. The model is sum of a low-frequency, single power-law component and a high-frequency, broken power-law component, with a spectral break that may represent a cooling break. Dashed lines: the best-fit individual components for each epoch. Solid lines: the total emission for each epoch. The low-frequency model is truncated at $\nu = 10^5$ GHz; while we cannot constrain any spectral break in this component, it is unlikely to extend this far without cooling effects. Left: the full radio to X-ray SED late-time dataset, including broadband radio data, optical data at $\delta t = 8.4$ yr, and X-ray upper limits at $\delta t = 8.4$ yr. Right: a zoom-in view of the broadband radio portion of the dataset—the boxed region in the left plot, with the cooling-break frequencies shown by vertical dotted–dashed lines for the 7.6 and 8.4 yr epochs.

characterized by a single power law and breaks to a steeper power law at high energies.

More interestingly, we find that even including a cooling break, the extrapolation of the radio SED leads to a significant overestimate of the X-ray flux at $\delta t = 8.4$ yr, implying photoelectric absorption of the X-rays. We derive a lower limit on the neutral hydrogen equivalent column density needed to reconcile the model predictions with the deep Chandra and NuSTAR nondetections of Sections 2.6 and 2.7 by considering the contributions from the high-frequency component only, where the high-frequency component has a cooling break. We consider two chemical compositions of the absorbing material: a solar-like composition and an “SN-ejecta-like” composition with $\approx 35\%$ of O by mass (e.g., N. N. Chugai & R. A. Chevalier 2006). We calculate the respective photoelectric cross sections with `vphabs` within `Xspec`. We find³⁸ $N_{\text{H,int}} > 2.1 \times 10^{22} \text{ cm}^{-2}$ for solar composition, and $N_{\text{H,int}} > 5.2 \times 10^{20} \text{ cm}^{-2}$ for the oxygen-rich, ejecta-like composition. These are conservative lower limits, as (i) we are not considering the possible contribution from the low-frequency radio component and (ii) we are not accounting for other sources of X-ray emission, like a reverse shock. This photoelectrically absorbed two-component model fits the data well, with a broken power law capturing the high-frequency emission, and an additional optically thin power law accounting for the low-frequency flux densities and hence the spectral index flattening toward lower frequencies.

4.2. Physical Inferences

Using the best-fitting values for the radio spectral peaks shown in Figure 5, and assuming the peaks are the result of SSA, we can infer some of the physical properties of the high-frequency component. Independently from the details of its astrophysical nature, we can apply Equations (7), (9), and (10) to determine the emitting radius, magnetic field, and CSM density for each epoch (Table 2).³⁹ We note that these values

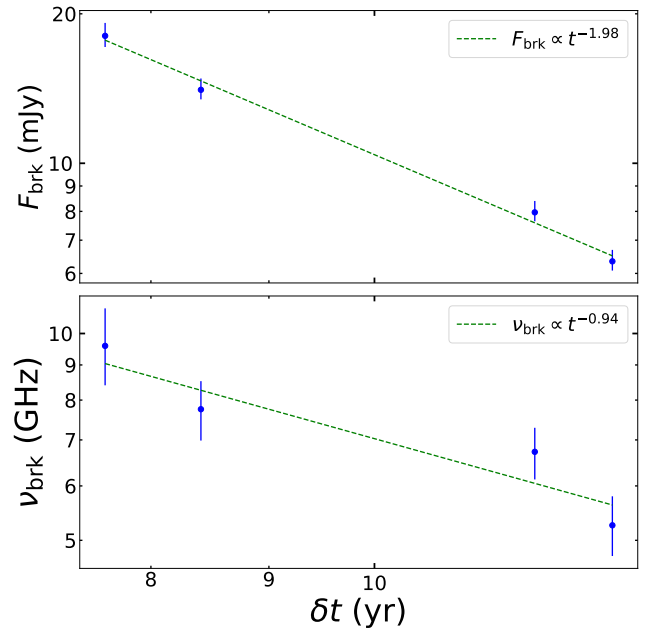


Figure 5. The best-fitting values for $F_{\nu_{\text{brk}}}$ and ν_{brk} of the high-frequency component as a function of time since explosion for the model-agnostic fit of Section 4.1. Both $F_{\nu_{\text{brk}}}$ and ν_{brk} follow consistent power-law decays with time.

are based on assumed microphysical parameters of $\epsilon_e = \epsilon_B = 0.1$; however, the inferred B fields imply values of ν_c that violate our assumed ordering $\nu_m < \nu_{\text{sa}} < \nu_c$. For our physical models of the next sections, we will require deviation from equipartition to ensure self-consistency. That being said, we can make several statements based on our model-agnostic fit that have very weak dependencies on microphysical parameters.

- (i) The high-frequency radio component is associated with a small emitting volume, corresponding to an isotropic equivalent radius (defined in Equation (11)) of $\approx (f_V (\epsilon_B / \epsilon_e)^{1/16.2}) 10^{16} \text{ cm}$. This radius is much smaller than the inferred radius of the initial SN shock at this epoch, $\approx 10^{18} \text{ cm}$ (Section 3.2.2), thus providing

³⁸ The Chandra and the NuSTAR observations translate into similar constraints, and we list here the most constraining lower limit obtained between the two measurements.

³⁹ Performing these fits with a fixed optically thick spectral index of $\alpha = 5/2$, as appropriate for the SSA assumption, yields very similar values of F_{brk} and ν_{brk} .

(a) Asymmetric, compact CSM (e.g. “torus-like”) (b) Small clump of CSM

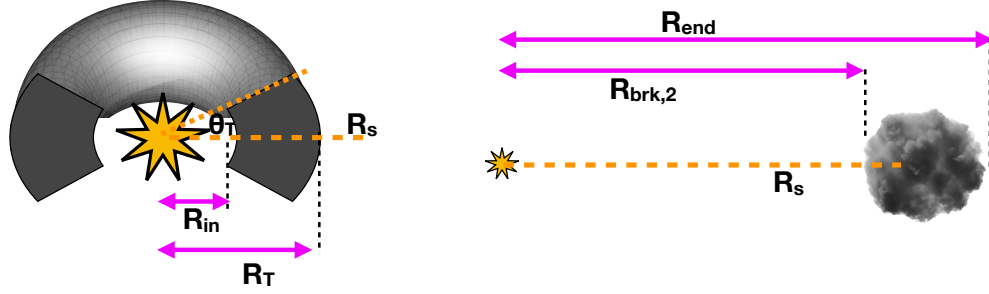


Figure 6. Cartoon showing two different CSM configurations. (a) Spherically asymmetric torus-like CSM extending from a distance $R = R_{in}$ to $R = R_T$ from the explosion site, with the shock interacting with a less dense medium at $r > R_T$. The half-opening angle of the torus θ_T is related to the volumetric filling factor in our modeling as $f_V = 3f_R \sin \theta_T$, where f_R is the thickness of the shocked region. (b) Clump of dense CSM at a distance $R_{brk,2}$ from the explosion site, extending to a distance of R_{end} . We do not assume a particular geometry for this scenario and instead fit for the projected area filling factor, f_A , across a range of volumetric filling factors, f_V .

Table 2

Fitted and Inferred Parameters for the High-frequency Component of the Two-component Model-agnostic Fit (Sections 4.1–4.2)

δt (yr)	F_{pk} (mJy)	ν_{pk} (GHz)	R (10^{16} cm)	B (G)	n_e (10^8 cm $^{-3}$)
7.64	$13.2^{+1.1}_{-0.9}$	$9.6^{+1.3}_{-1.2}$	$1.1^{+0.2}_{-0.1}$	$1.6^{+0.2}_{-0.2}$	$3.2^{+2.2}_{-1.4}$
8.41	$10.3^{+0.8}_{-0.6}$	$7.8^{+0.8}_{-0.8}$	$1.2^{+0.2}_{-0.1}$	$1.3^{+0.1}_{-0.1}$	$2.2^{+1.1}_{-0.8}$
11.74	$5.8^{+0.4}_{-0.3}$	$6.7^{+0.6}_{-0.6}$	$1.0^{+0.1}_{-0.1}$	$1.2^{+0.1}_{-0.1}$	$4.8^{+2.0}_{-1.7}$
12.68	$4.6^{+0.4}_{-0.3}$	$5.2^{+0.5}_{-0.5}$	$1.2^{+0.2}_{-0.1}$	$1.0^{+0.1}_{-0.1}$	$2.7^{+1.4}_{-1.0}$

Note. Inferred parameters are based on the assumptions of equipartition at $\epsilon_e = \epsilon_B = 0.1$, a characteristic frequencies ordering of $\nu_m < \nu_{sa} < \nu_c$, and a filling factor of $f_V = 0.5$. Note that these values are dependent on our best-fit value of the smoothing parameter, $w = 2.2$. For a more typical value of $w = 1$, the inferred R values are negligibly larger, B fields are a factor of ≈ 2 smaller, and n_e is on the order of 10^7 cm $^{-3}$ instead of 10^8 cm $^{-3}$.

additional evidence that this is an astrophysically distinct component. Importantly, this inference is consistent with the direct VLBI size constraints by M. Lazda et al. (2026).

- (ii) The small radius of the emitting region implies a low average expansion velocity ≈ 500 km s $^{-1}$ if interpreted as a shock radius, and a large number density of electrons ($\approx (\epsilon_B/0.1)^{-0.6} (\epsilon_e/0.1)^{0.4} 10^7$ – 10^8 cm $^{-3}$), the range being dependent on the smoothing parameter.
- (iii) The lack of detected soft and hard X-ray emission at $\delta t \approx 8.4$ yr implies that the high-frequency radio component is either photoelectrically absorbed, or that no X-ray emission is produced. Given the low shock velocities, diffusive shock acceleration (DSA) may not produce electrons of high enough energy to extend to the X-rays.
- (iv) If a distinct component is responsible for the high-frequency emission at late times, then it must be subdominant to the shock emission at $\delta t \leq 190$ days.
- (v) While the B field shows a clear, monotonic evolution with time ($B \propto t^{-0.7}$), the other quantities in Table 2 do not: the emitting radius is consistent with being constant within uncertainties, implying an expansion velocity $\lesssim$ a few hundred kilometers per second.
- (vi) If the ordering of characteristic frequencies is the typical $\nu_m < \nu_{sa} < \nu_c$, we require a deviation from equipartition of at least $\epsilon_B/\epsilon_e \leq 0.1$.

Any physical model proposed to explain the observations of SN 2012au must meet all the criteria above. In Sections 5 and 6, we explore some of such models.

5. Circumstellar Medium Interaction Models

We first consider the possibility that the high-frequency radio component is the result of the SN shock interaction with an aspherical CSM. We explore two scenarios, shown in Figure 6: (a) dense CSM subtending a small solid angle (e.g., “torus-like” CSM, Section 5.1) and (b) a small clump of dense CSM, placed at some large radius from the explosion site (Section 5.2).

5.1. A Torus-like Dense Circumstellar Medium

We consider a scenario where high-density CSM is confined to a small solid angle, a geometry motivated by the equatorial tori of material frequently observed around massive stars. The solid angle Ω_T subtended by the torus is related to the half-opening angle θ_T of Figure 6 as $\Omega_T = 4\pi \sin^2(\theta_T/2)$. In this configuration, the early radio emission from SN 2012au would result from the shock interaction with the lower-density material (for example, along the poles), with density as inferred in Section 3.2.1, while the late-time radio emission originates from the interaction with the high-density CSM in the torus. Qualitatively, the dense CSM of the torus could decelerate the shock significantly, explaining the small inferred emitting volume, the low observed shock velocity, and the late emergence of the radio emission. Because of the high density, we expect the early time emission from this component to be suppressed by FFA, which would explain why it did not appear in the initial dataset. We model the free-free optical depth as in B. Rybicki & A. Lightman (1985):

$$\tau_{ff}(R, \nu) = \int_R^\infty 0.018 T_e^{-3/2} Z^2 \nu^{-2} \bar{g}_{ff} n_e(r) n_i(r) dr, \quad (15)$$

where all quantities are in cgs units. We assume an electron temperature $T_e = 10^5$ K, a Gaunt factor $\bar{g}_{ff} = 5$, a hydrogen CSM composition with $Z = 1$, and full ionization such that n_e , the number density of electrons, is equal to n_i , the number density of ions: $n_e = n_i$. A similar scenario was invoked to explain the broadband X-ray and radio emission from the strongly interacting SN 2014C (D. Brethauer et al. 2022;

B. P. Thomas et al. 2022). We quantitatively explore this model below.

We consider the late-time radio epochs at $\delta t > 6$ yr, selecting data at $\nu \geq 5$ GHz to minimize the contribution from the low-frequency component, while still having sufficient spectral coverage to characterize the spectral peak at each epoch. We model the shock dynamics as described in Section 3.1, adopting a broken power-law model for the dense CSM with break radii, R_{in} and R_{T} , describing the inner radius and transition radius of the dense torus, respectively. At $r < R_{\text{in}}$, we assume that the early time density profile inferred in Section 3.2.1 ($\rho_{\text{CSM},0}(r)$, Equation (14)) applies. The profile at $R_{\text{in}} < r < R_{\text{T}}$ is poorly constrained, with the only constraint being that this component is not detected at $\delta t < 6$ months. Consequently, the CSM-density index and normalization at $r < R_{\text{T}}$ are degenerate. We thus assume an inner density profile that is wind-like without any loss of generality of our argument. We also find that the density required to decelerate the ejecta to slow velocities of a few hundreds of kilometers per second far exceeds the density required to produce the late-time radio emission, so we allow for a discontinuity at R_{T} , where the density can drop significantly. The torus density profile is therefore described as

$$\rho_{\text{T}}(r) = \begin{cases} \rho_{\text{CSM},0}(r), & r < R_{\text{in}}, \\ \rho_1 \left(\frac{r}{R_{\text{T}}} \right)^{-2}, & R_{\text{in}} < r < R_{\text{T}}, \\ \rho_2 \left(\frac{r}{R_{\text{T}}} \right)^{-s}, & r > R_{\text{T}}. \end{cases} \quad (16)$$

The initial velocity of the ejecta is also poorly constrained, thus we adopt $M_{\text{ej}} = 5 M_{\odot}$ and $E_{\text{k}} = 10^{52.5} \text{ erg}$.⁴⁰ From these initial conditions and the torus CSM-density profile in Equation (16), we calculate the radius $R_{\text{s}}(t)$ and velocity $v_{\text{s}}(t)$ of the shock as it plows through the torus from numerical integration of Equation (1), and $B(t)$ through Equation (7). We fix $\epsilon_e = 0.1$ and fit for ϵ_B and p . We do not assume any specific ordering of the synchrotron break frequencies, and instead we self-consistently calculate the characteristic break frequencies and their corresponding SEDs using the method described in Appendix C of I. Sfaradi et al. (2025). Because we are considering values of $p < 2$, we need to specify a maximum value for the accelerated electron energy, γ_{max} , to prevent divergent solutions. In this model, we fix $\gamma_{\text{max}} = 10^7$.⁴¹

In our modeling, we adopt a “wedge-like” cross section for the CSM (as shown in Figure 6) for simplicity. We account for the small solid angle by prescribing a volumetric filling factor f_V and projected area filling factor f_A . To calculate these values, we define the parameter $f_R \equiv \Delta R/R_{\text{s}}$, where ΔR is the thickness of the shell of shocked material. Under this prescription, we can approximate $f_V = 3f_R \sin(\theta_{\text{T}})$, which is valid for small angles of θ_{T} . We assume a value of $f_R = 0.1$, in line with typical values derived from self-similar solutions (R. A. Chevalier 1982a). The value of f_A is viewing angle dependent, so we fit models for the two extreme cases: (i) viewing the torus edge on, such that the torus appears as a thin strip, and (ii) viewing the torus face on, such that the torus appears as a ring.

⁴⁰ We note that adopting initial values to within a factor of a few from those assumed does not change the major conclusions from this section.

⁴¹ Our solutions are not sensitive to this choice of γ_{max} .

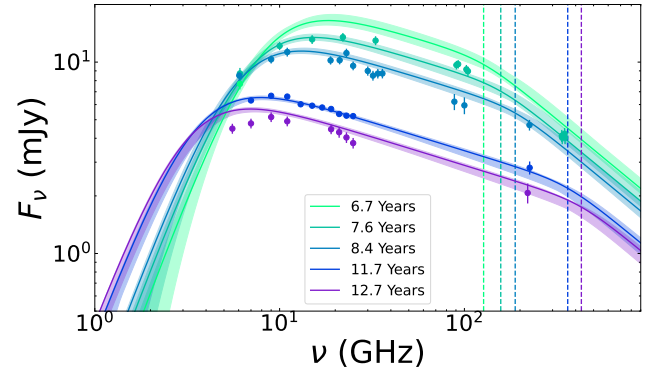


Figure 7. Best-fitting model for a torus-like dense CSM viewed edge on. For each epoch, we show the 3σ credible interval as the shaded region around the best-fitting model. While the model captures the overall SED behavior at $\delta t = 8.4$ and 11.7 yr reasonably well, it underpredicts the $\delta t = 7.6$ yr epoch, while overpredicting the $\delta t = 12.7$ yr epoch. The cooling breaks, visualized as dashed lines, line up well with the breaks required in our model-agnostic fits at $\delta t = 7.6$ and 8.4 yr.

5.1.1. Edge-on Torus Viewing Angle

For an edge-on torus viewing angle, we consider a projected area filling factor $f_A = 4/\pi \sin(\theta_{\text{T}})$. At this orientation, the early time emission from the torus is not detected because it is entirely free-free absorbed. Without any constraint on R_{in} , we assume that the torus begins very close to the surface of the progenitor, taking $R_{\text{in}} = R_{\text{star}}$, where $R_{\text{star}} \approx R_{\odot}$ is the assumed radius of the progenitor at time of explosion (as appropriate for Wolf-Rayet stars).

We compute the best-fitting values for the parameters that characterize the density profile: ρ_1 , ρ_2 , R_{T} , and s , in addition to the following parameters that impact the emission calculation: ϵ_B , p , and θ_{T} . We employ MCMC to fit, with 50 walkers iterating the model calculations 10,000 times and discarding the first 1000 iterations for each walker. The corner plots for this fits can be found in Appendix E. We plot the best-fitting model in Figure 7. The model fits the data well at $\delta t = 8.4$ and 11.7 yr, while underpredicting and overpredicting the data at $\delta t = 7.6$ and 12.7 yr, respectively. The cooling-break frequencies, represented as dashed vertical lines in Figure 7, agree with those predicted in Section 4.1.

The resulting best-fitting shock dynamics are shown in Figure 8. At $r > R_{\text{T}}$ the shock velocity plateaus to a roughly constant value $\approx 400\text{--}500 \text{ km s}^{-1}$, as the shock expands past the dense region of the torus. We note that our best-fits indicate steep values of $s > 3$, which should result in shock acceleration (E. Waxman & D. Shvarts 1993). Instead we assume a constant shock velocity. We acknowledge that this is a current limitation of our model; however, we do not expect this simplified dynamics to change the major conclusions from this study.

In Figure 9, we plot the best-fitting torus CSM number density of electrons (n_e), the magnetic field strength (B), and the internal energy of the shock (U) as a function of shock radius, where U is calculated as

$$U = \frac{B^2}{8\pi\epsilon_B} \left(\frac{4}{3} \pi R_{\text{s}}^3 f_V \right) \quad (17)$$

We emphasize that all these quantities apply to the shock interaction with the torus (i.e., the shock propagating along the low-density poles will have markedly different physical

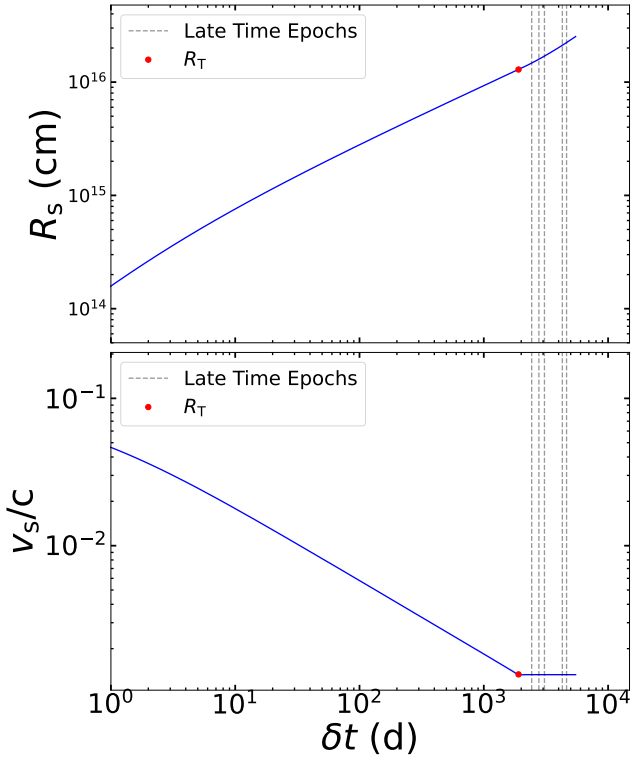


Figure 8. Dynamical evolution of the shock (radius and velocity) expanding in a torus-like dense-CSM scenario (Section 5.1). The vertical gray dashed lines mark the epochs of our late-time radio observations. The red dot in each curve marks the transition radius, R_T .

properties). Notably, the magnetic field strength shown in Figure 9 differs significantly from those in the model-agnostic fit (reported in Table 2). This is a result of FFA affecting the observed fits in this model compared to the assumption of SSA in the model-agnostic fit. In the edge-on torus case, the emission is free-free absorbed and the intrinsic SSA peak is located at a lower ν_{pk} . While these inferences depend on the shock microphysical parameters, they are primarily dependent on their ratio, i.e., ϵ_e/ϵ_B . Since we are fitting for ϵ_B , a different choice of fixed ϵ_e would have little impact on inferred CSM mass.

A few considerations follow. First, very large torus densities are required at $r < R_T$ to sufficiently decelerate the shock, corresponding to effective mass-loss rates $\dot{M} \gtrsim 2.4 M_\odot \text{ yr}^{-1}$ for wind velocity $v_w = 1000 \text{ km s}^{-1}$ (Figure 9). These large densities also strongly free-free absorb the early radio flux, consistent with the early time radio observations. The associated $B > 100 \text{ G}$ would also be problematic. We note that by setting R_{in} as close as possible to the explosion center, we are minimizing the required densities for deceleration and they are still extremely high. At $r > R_T$, we find that the CSM density must decline much more steeply than a wind-like scenario, with $s = 3.8$. This differs from the model-agnostic fit because this density profile has to account for the effects of FFA, in addition to the effects of density on the total amount of emitting electrons.

Second, the shock must propagate through a large column density of CSM in order to decelerate to the low inferred velocities. The total torus CSM mass is calculated as

$$M_{\text{CSM}} = \int_{R_{\text{star}}}^{R_s(12.7\text{yr})} f_V \times \rho_T(r) \times 4\pi r^2 dr. \quad (18)$$

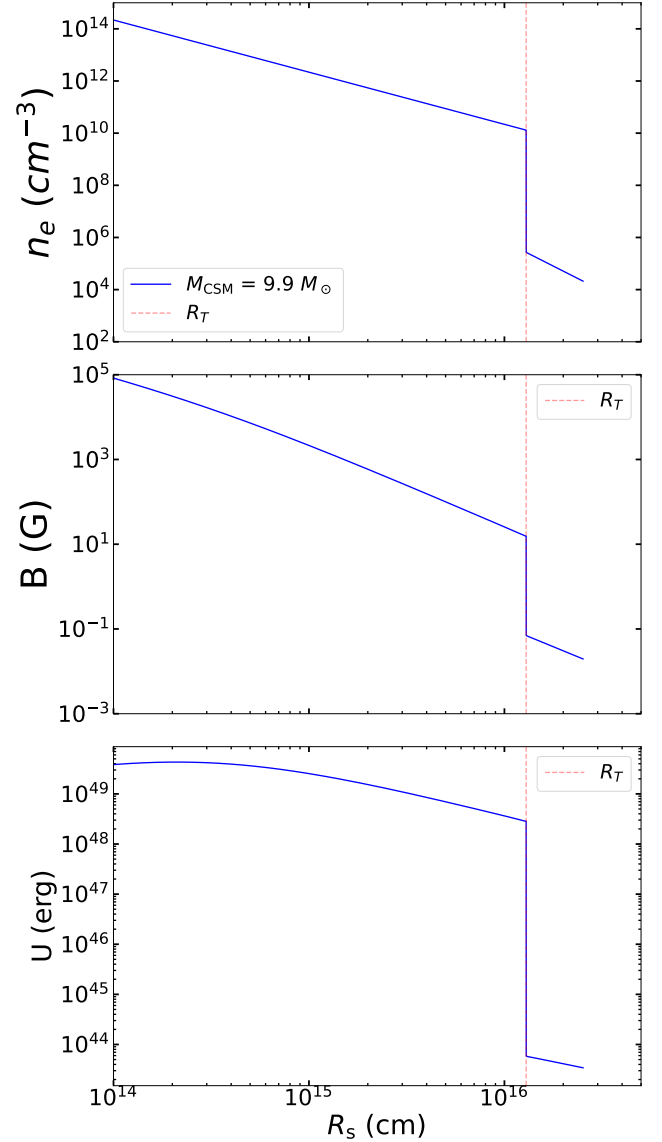


Figure 9. Best-fitting torus CSM density (n_e) and shock properties (B and U) as a function of R_s in the context of the torus-like dense-CSM scenario viewed edge on (Section 5.1.1). The sharp discontinuities in each curve correspond to when the shock reaches the discontinuity in the CSM-density profile at R_T . The requirement of a small emitting region translates into extremely large densities and magnetic field strengths. We infer a total CSM mass in the torus of $M_{\text{CSM}} \approx 10 M_\odot$, based on integration of the displayed profile up to the time of the most recent radio observation (Equation (18)).

The CSM mass for our best-fitting parameters is $M_{\text{CSM}} = 10 M_\odot$. This CSM material is within a small radius $R \approx 10^{16} \text{ cm}$, therefore requiring that a large fraction of progenitor mass was lost within a short time period prior to explosion. The small inferred filling factor $\log_{10} f_V = -1.48^{+0.48}_{-0.50}$ implies a very small half-opening angle of $\theta_T \leq 5^\circ$. There is no clear physical mechanism for such collimated deposition of mass into the CSM.

Third, for the model we compute the torus CSM column density beyond the shock radius at 8.4 yr, and we express the result in terms of neutral hydrogen equivalent column density for solar-like and ejecta-like composition defined as in Section 4.1. We find that this model is in tension with the lack of X-ray detections for a solar-like composition. The best-

fitting model for CSM density predicts $N_{\text{H,int}} = 5.8 \times 10^{20} \text{ cm}^{-2}$. This prediction is lower than the minimum $N_{\text{H,int}}$ needed to suppress the emission derived in Section 4.1— $N_{\text{H,int}} = 2.1 \times 10^{22} \text{ cm}^{-2}$ for a solar-like composition, but consistent with $N_{\text{H,int}} > 5 \times 10^{20} \text{ cm}^{-2}$ for an ejecta-like composition. We note, however, that this result is limited by our 1D treatment of the shock dynamics. In a multidimensional treatment, fast moving ejecta close to the torus would wrap around it, increasing the density beyond the shock radius and potentially providing enough interceding material to suppress the X-rays.

5.1.2. Face-on Torus Viewing Angle

For a face-on torus viewing angle, we define $f_A = 2f_R$. In this scenario, the medium between the emitting region and the observer is defined by the inferred density profile in Section 3.2.1. This inferred profile is much less dense than the torus itself and will not result in any significant FFA at early times. In order to maintain consistency with the nondetections of this component in the early time data, we set $R_{\text{in}} = 10^{17.1} \text{ cm}$. This is the smallest radius we can prescribe without observing interaction with the dense torus at our last early time epoch ($\delta t = 190$ days). Such a structure of a ring of CSM has been observed directly in SN 1987A (P. C. Plait et al. 1995).

We perform an MCMC fit, with the same hyperparameters as in Section 5.1.1. This model fails to converge to any solution that well characterizes both the optically thick and optically thin sides of the SEDs. We attempt fits fixing $p = 1.6$, varying between $f_R = 0.1$ and 0.01 , and varying across different fixed f_V values, but consistently only find solutions that match the data on the optically thin side. One such solution is plotted in Appendix E. Without strong FFA shifting the observed peak to a lower F_{pk} and higher ν_{pk} (which is present in the edge-on case), there does not appear to be a density profile for the torus that can result in the observed emission.

We note that, for the range of solutions between the two extremes of viewing angle (edge on and face on), the suppression of the emission by FFA will lie between the highly absorbed case of the edge-on viewing angle and the lack of significant absorption in the face-on case. For these intermediate viewing angles, if a solution does exist, the intrinsic SSA peak will be at lower F_{pk} and higher ν_{pk} than the edge-on case, resulting in even more extreme values of B -field strength and n_e at $r < R_T$.

In summary, a high-density CSM with a very small net cross section could explain the radio emission we observe in SN 2012au, but the physical setup for this scenario appears highly contrived. The required filling factor of $f_V \leq 4 \times 10^{-2}$, corresponds to a solid angle of $\approx 2700 \text{ deg}^2$ or a half-opening angle of $\theta_T \approx 5^\circ$. Within that small solid angle must reside CSM of extremely high density to decelerate the shock containing solar masses of material within $R_T \sim 10^{16} \text{ cm}$, equivalent to far greater mass loss than typically observed in SNe, yet leaving no imprint on the early SN optical spectroscopy. Furthermore, even in this contrived scenario, the inferred density profile is potentially inconsistent with the X-ray nondetections at $\delta t = 8.4 \text{ yr}$.

5.2. Small Clump of Dense Circumstellar Material

The small cross-section scenario above requires such extreme densities at $r < R_T$ because we are requiring the shock radius to be small for an extended period of time. However, the observed F_{pk} and ν_{pk} of the high-frequency component in the late-time SEDs only imply a small emitting volume, not a small shock radius. The shock could propagate out to a large radius before encountering a clump of overdense CSM, where strong interaction occurs. In this scenario, this component would be radio faint at early times not because of absorption, but by the lack of radiating electrons (i.e., the shock has not yet reached a dense region of CSM).

To model this scenario, we assume that the shock at early times follows the dynamics inferred in Section 3.2.1. As in Section 3.2.1, we use $M_{\text{ej}} = 5 M_\odot$ and we adopt the inferred $E_k = 10^{52.5} \text{ erg}$. At $\delta t < 190$ days, we fix the density profile to the broken power law of Equation (14) with the best-fitting parameters in Table 1. The CSM density extends to larger radii as $\rho_{\text{CSM},0}(r) \propto r^{-s_2}$ until it reaches a second break radius $R_{\text{brk},2}$, where the shock encounters a clump of uniform density, ρ_c . The clump covers a small fraction of a sphere of radius $R_{\text{brk},2}$, and it is described by a volumetric filling factor f_V , and a projected area filling factor, f_A . In order to avoid assuming a particular geometry for this clump, we fit for values of f_A over a range of values for f_V . We also fit for the radius at which the shock stops interacting with the clump, R_{end} . This end radius is not important for the dynamics of the model, but will determine the contribution of FFA and photoelectric absorption to the radio and X-rays, respectively, from material in front of the shock.⁴²

For this scenario Equation (15) takes the form

$$\tau_{\text{ff}} = 0.018 T_e^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{\text{ff}} (R_{\text{end}} - R_s). \quad (19)$$

For simplicity, we do not consider any density contribution beyond R_{end} . The equation above also assumes that the radiation produced at R_s travels through a distance of the order $\approx (R_{\text{end}} - R_s)$ within the clump. The exact value clearly depends on the observer's line of sight and the actual 3D geometry of the clump.

As before, we fit the late-time observations at $\nu > 5 \text{ GHz}$, where no significant contributions from the low-frequency component are present. We are seeking clumps located at a large shock radius ($\approx 10^{18} \text{ cm}$), but that still occupy a small emitting volume. In this setup, the interaction will be delayed as the shock takes time to reach the clump from the explosion center, but the number of emitting electrons is still small. We find that to satisfy these requirements we are forced to consider a range of very small volumetric filling factors: $f_V = [5 \times 10^{-6}, 5 \times 10^{-7}, 5 \times 10^{-8}]$. These filling factors quantify the portion of the sphere at $R_{\text{brk},2}$ that is covered by the clump, and thus describe the region where the shock interacts with the clump; instead, the early time filling factors before the interaction with the clump remain as in Section 3.1. In order to limit our number of free parameters, we fix $p = 1.6$, the value observed in all other fits. As in the torus scenario, we fix $\gamma_{\text{max}} = 10^7$.

We again employ MCMC, using 25 walkers iterating the model calculations 1000 times and discarding the first 200

⁴² We expect that since the interaction region resides at larger radii, the effects of ejecta wrapping around the clump will be minor in a multidimensional treatment.

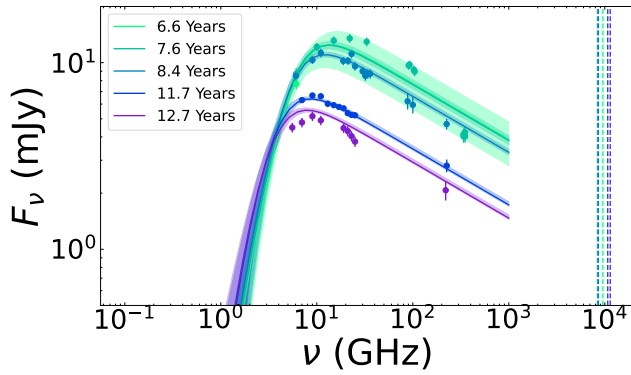


Figure 10. Best-fitting models for a small clump of dense CSM with filling factor $f_V = 5 \times 10^{-8}$ at $R_{\text{brk},2} = 4.7 \times 10^{17}$ cm through $R_{\text{end}} = 1.2 \times 10^{18}$ cm. For each epoch, we show the 3σ credible interval as the shaded region around the best-fitting model. Like the edge-on torus model, the data are well fit at $\delta t = 8.4$ and 11.7 yr, while being underpredicted at $\delta t = 7.6$ yr epoch and overpredicted at $\delta t = 12.7$ yr. Unlike in the torus model, the cooling breaks, represented by dashed vertical lines, lie outside the range of observed frequencies, failing to reproduce the observed spectral break at $\delta t = 7.6$ yr.

iterations for each walker. The resulting corner plots can be found in Appendix E. An example of one of the fits is shown in Figure 10 for $f_V = 5 \times 10^{-8}$, which is the model with the lowest ν_c (i.e., the closest to the data) in this class. While much of the broad evolution is well captured in these models, this class of models fail to self-consistently account for the location of ν_c (dashed vertical lines in the figure).

The best-fitting dynamical evolution for each filling factor can be found in Figure 11. We plot the evolution of the shock radius R_s , as before, but also include R_c , which represents the radius of a sphere with the same volume as the emitting volume, computed as

$$V \equiv \frac{4}{3}\pi R_c^3 = \frac{4}{3}\pi f_V (R_s^3 - R_{\text{brk},2}^3), \quad (20)$$

which implicitly defines

$$R_c \equiv f_V^{1/3} (R_s^3 - R_{\text{brk},2}^3)^{1/3}. \quad (21)$$

The 1D shock dynamics for each filling factor are almost exactly the same. Higher filling factors result in larger emitting volumes, but these larger volumes are precisely offset by lower values of ϵ_B . Notably, this precise balance is only possible because these models violate the spectral break constraint at ν_c . In contrast, in the torus-like model fit of Section 5.1, ϵ_B cannot compensate for the filling factor because each model requires approximately the same magnetic field strength at $\delta t = 7.6$ yr to account for the same cooling-break frequency at that epoch.

With $R_c < 10^{16}$ cm, clump models satisfy the compact emitting region requirement by requiring large deviations from equipartition: $\epsilon_B \ll \epsilon_e$. After encountering the clump, the velocity of the shock inside the clump sharply decreases, leveling out as the denser parts of the ejecta profile reach the clump, then smoothly decreases.

For each f_V , we also plot the clump n_e , B , and U as a function of R_s in Figure 12. Since the 1D dynamics for each scenario are nearly identical, the density profiles are all almost identical as well, reaching the clump at $R_{\text{brk},2} \approx 10^{17.7}$ cm with density $\rho_c \approx 10^{-19.4} \text{ g cm}^{-3}$ ($n_e = 2.3 \times 10^4 \text{ cm}^{-3}$). The models all converge to $10^{-5} < f_A < 10^{-4}$, with f_A increasing for decreasing f_V . The required CSM masses reported are all

$< 2 \times 10^{-4} M_\odot$, notably much lower than the CSM mass required in the edge-on torus scenario. The large CSM mass in the torus case was required to slow the shock down and keep it contained within a small radius. In the clump scenario, this requirement is bypassed by having the interaction occur at a separated distance from the explosion site. A large CSM mass is not required for the interaction that produces the observed emission.

The models all account for the same level of FFA, modeling by an end radius of $\log(R_{\text{end}}/\text{cm}) = 18.09^{+0.22}_{-0.19}$. Any larger R_{end} would result in absorption of the observed flux at lower frequencies. The magnetic field strength and shock energy are both largest for the smallest filling factor, in order to match the observed luminosity within a smaller emitting volume. These two values also exhibit a spike at the density discontinuity; this should be regarded as a numerical artifact and not physical. For these models, $R_{\text{end}} \approx 10^{18.1} \text{ cm}$ implies $N_{\text{H,int}} \geq 2 \times 10^{22} \text{ cm}^{-2}$, which is consistent with the requirements to suppress the X-ray emission from synchrotron radiation (Section 4.1) even in the case of solar composition.

In summary, a small, disconnected clump of dense CSM can match some of the observed late-time properties of SN 2012au. However, it fails to properly account for the observed spectral cooling break at ν_c in our $\delta t = 7.6$ yr epoch, and overall struggles to reproduce the flux temporal evolution. Furthermore, we also lack a strong astrophysical motivation for the existence of dense clump (which would have a density contrast of a factor $\approx 10^3$ compared to its surroundings), especially one that does not show strong hydrogen features in the optical spectra (D. Milisavljevic et al. 2018).

We note that while keeping f_A as a free parameter does allow us to not make any assumptions about the geometry of the clump or how it is distributed, we still assume that the material is of uniform density and placed at a specific distribution of radii. This problem could instead be treated as inhomogeneities in the CSM with variations in density, distribution of relativistic electrons, and magnetic field strength as described in C.-I. Björnsson (2013, 2024) and C.-I. Björnsson & S. T. Keshavarzi (2017). CSM inhomogeneities have been suggested as explanations for broad synchrotron SED peaks and flatter than typical spectral indices, as observed in SN 2012au (F. J. Cowie & R. P. Fender 2026; F. De Colle et al., in preparation). These inhomogeneities can be modeled by assuming a probability distribution of magnetic field strength, as done for SN 2019oys (I. Sfaradi et al. 2024) and PSNJ1404 (P. Chandra et al. 2019). Such a treatment is beyond the scope of this paper, but we note that, based on the radio data, we cannot entirely rule out the possibility of a inhomogeneous CSM for SN 2012au.

6. A Nascent Pulsar Wind Nebula

Motivated by the lack of a clearly viable CSM-interaction interpretation and the conclusions of D. Milisavljevic et al. (2018), S. B. Pandey et al. (2021), and M. Lazda et al. (2026), we now consider the possibility that the high-frequency radio component is the result of an emerging PWN. In a PWN, a central rapidly rotating NS injects energy into the surrounding ejecta via magnetic winds of charged particles. This central engine has often been proposed as a powering mechanism for SLSNe (e.g., D. Kasen & L. Bildsten 2010; L. Dessart et al. 2012; B. D. Metzger et al. 2015; K. Kashiyama et al. 2016; K. Murase et al. 2016, 2021; T. Sukhbold & S. E. Woosley 2016;

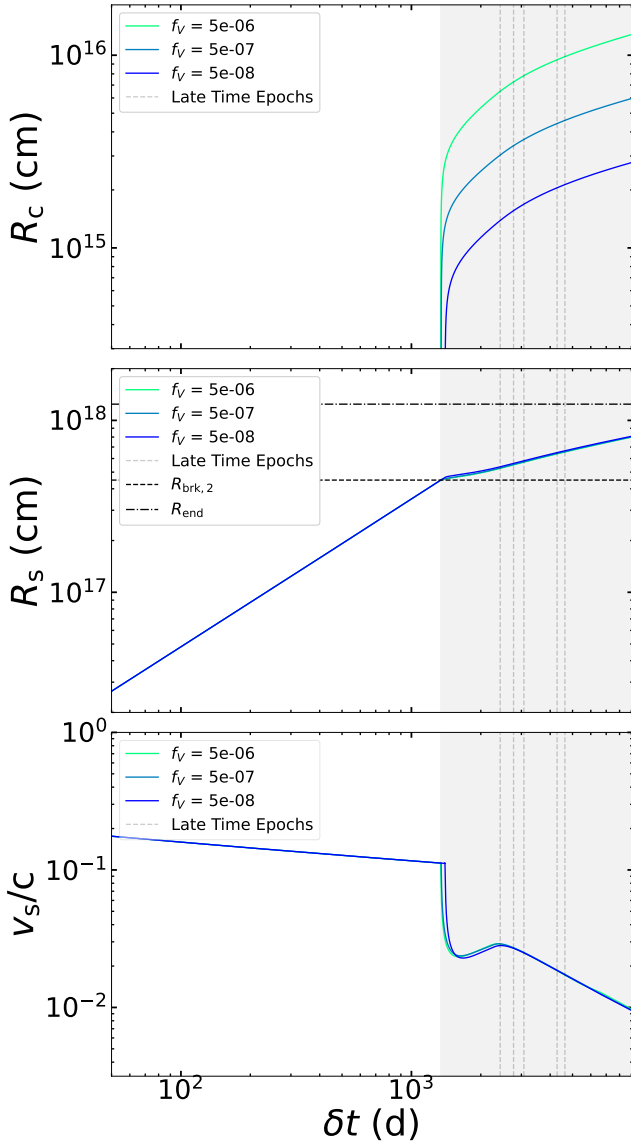


Figure 11. The dynamical evolution of the shock radius (R_c), shock velocity (v_s), and radius of the emitting region (R_s) for the CSM clump scenario. The different colored lines correspond to different volumetric filling factors of the dense material. The gray dashed lines correspond to the epochs at which we have late-time observations of the event. The dashed and dotted-dashed black horizontal lines in the R_s plot mark $R_{\text{brk},2}$ and R_{end} , respectively. For the times after the shock encounters the clump at $R_{\text{brk},2}$ (marked as the gray region), the dynamics reported only reflect the portion of the shock interacting with the clump. This portion does not reach 10^{18} cm at the times of our observations like we expect for the extrapolations of the early shock because it is significantly decelerated by the dense clump.

M. Nicholl et al. 2017; L. Dessart 2018; K. Eiden & D. Kasen 2026), but could also be relevant in broad-line and regular H-stripped CCSNe (e.g., K. Maeda et al. 2007; S. E. Woosley 2010; K. Kashiyama et al. 2016; F. Taddia et al. 2019; W. L. Lin et al. 2020; Ó. Rodríguez et al. 2024) because of the large inferred ^{56}Ni mass and explosion energy compared to their H-rich explosion analogs.

Because of degeneracies in the modeling of optical SN light curves and the faintness of late-time optical spectra, the nonthermal emission can be a better probe of the suspected central engine (e.g., K. Kotera et al. 2013; B. D. Metzger et al. 2014; K. Murase et al. 2015, 2016; K. Kashiyama et al. 2016; C. M. B. Omand et al. 2018). PWN synchrotron emission

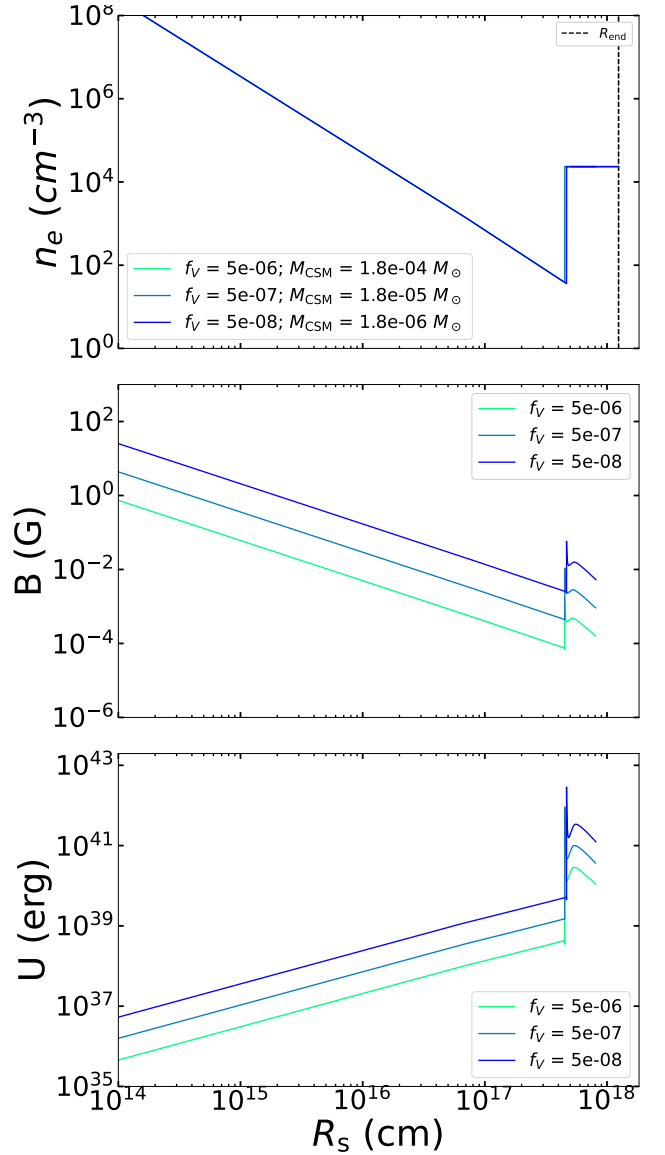


Figure 12. The CSM density (n_e) and shock properties (B and U) as a function of R_s for the dense-CSM clump scenario. The different colored lines correspond to different volumetric filling factors composed of this dense material.

results from wind interaction with the innermost ejecta, offering a natural explanation of the small inferred emitting volume and large density of electrons and positrons of SN 2012au. The high-density ejecta surrounding the PWN may also explain the lack of observed radio emission from this component at early times and the continued lack of X-ray detections until late times (see Sections 6.3.1 and 6.3.2). In particular, high-frequency radio and millimeter emission in the ~ 10 – 100 GHz range has been proposed as a powerful probe of nascent PWN emission from pulsar- or magnetar-powered SNe, because synchrotron radiation from freshly injected and accumulated electron–positron pairs can emerge once SSA and FFA in the ejecta become sufficiently weak (K. Murase et al. 2016, 2021; C. M. B. Omand et al. 2018). Parameter inference with a full, realistic PWN model would involve time-dependent numerical treatments that are beyond the scope of this paper. Instead, in this section, we investigate the feasibility of a PWN model through (i) observational comparisons to

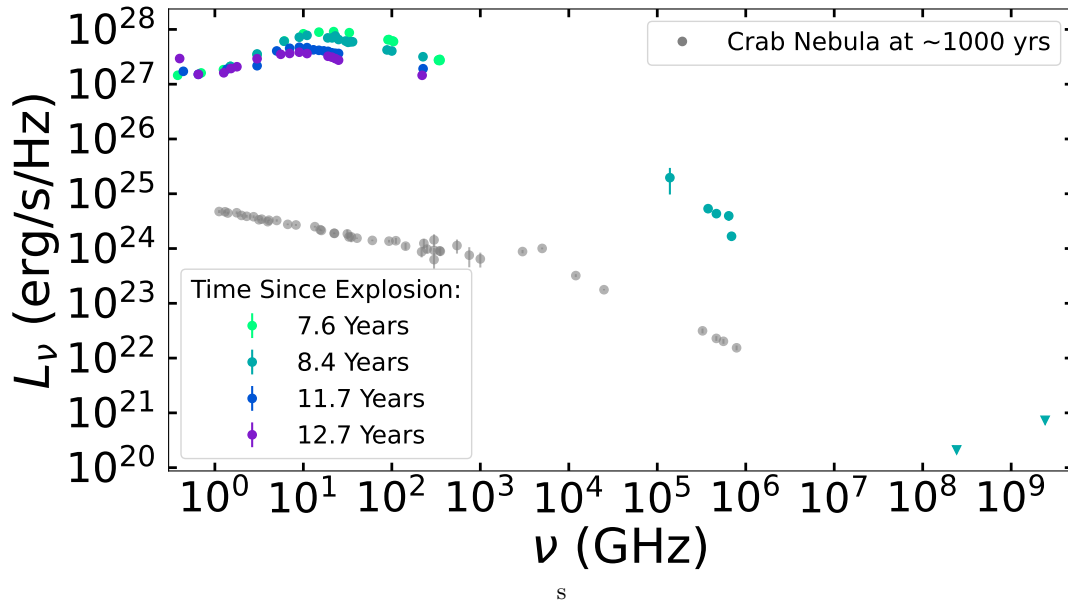


Figure 13. Late-time ($\delta t > 6$ yr) broadband SED of SN 2012au, compared to the Crab Nebula at $\delta t \sim 1000$ yr (using data compiled in J. F. Macías-Pérez et al. 2010). Both objects exhibit a shallow spectral index at radio frequencies, which steepens at some point in the millimeter–optical regime. The Crab Nebula is significantly less luminous than SN 2012au, as expected for a much older source.

known PWNe, (ii) comparison to existing numerical models, and (iii) analytic estimates based on known properties of SN 2012au.

6.1. Observational Comparisons

We begin by considering observational comparisons to known PWNe. Typically, the radio spectra of PWNe are characterized by flat spectral indices: $0.0 < \alpha < -0.3$ (e.g., P. Slane 2017; B. M. Gaensler & P. O. Slane 2006). The observed optically thin radio spectral index of SN 2012au ($F_\nu \propto \nu^{-0.31 \pm 0.02}$) falls within this range of PWNe observations. Explosive transients, on the other hand, are expected to have $\alpha < -0.5$, as DSA models predict $\alpha = -0.5$ and nonlinear corrections only steepen the spectrum (D. Urošević et al. 2019). The broadband spectra of PWNe typically exhibit a spectral steepening in the millimeter or optical bands (P. Slane 2017). This steepening is generally greater than $\Delta\alpha = -0.5$, suggesting that a simple cooling break applied to a single power-law distribution of electrons is insufficient to describe the emission (B. M. Gaensler & P. O. Slane 2006). This steepening is also observed in our data at $\delta t = 8.4$ yr (see Figure 4).

In Figure 13, we plot SN 2012au, alongside the Crab Nebula, the best-observed PWN. The Crab Nebula exhibits a similarly shallow spectral index in the radio regime, which steepens in the infrared. Notably, the Crab is 2–3 orders of magnitude less luminous than SN 2012au; however, this would be unsurprising given the Crab is a factor of ≈ 100 times older than SN 2012au and PWNe are expected to fade over time (B. M. Gaensler & P. O. Slane 2006). Theoretical work also predicts that young PWNe will have luminosities 10–1000 times greater than the present-day Crab Nebula (R. Bandiera et al. 1984), in line with our observations of SN 2012au.

While direct observations can provide useful comparisons, we lack any confirmed detections of PWNe as young as our object. The youngest confirmed Galactic PWN has an inferred age 70–500 yr (Z. J. Smeaton et al. 2024). The emergence of

radiation from a PWN has been proposed in some SNe: SN 1979C (D. J. Patnaude et al. 2011; J. Ahlwind et al. 2026; P. Lundqvist et al. 2026), SN 1986J (M. F. Bietenholz & N. Bartel 2017bb; S. J. Tanaka & K. Kashiyama 2023), SN 1987A (C. Fransson et al. 2024), PTF10hgi (T. Eftekhari et al. 2019), and VT 1137-0337 (D. Z. Dong & G. Hallinan 2023). Among these, SN 1979C has been observed to have progressively steepening spectral indices, in contradiction with expectations for a central engine (P. Lundqvist et al. 2026); SN 1986J has a central radio-emitting component that is consistent with shock interaction with a dense and highly structured CSM (M. F. Bietenholz & N. Bartel 2017bb); PTF10hgi has shown evidence for late-time appearance of hydrogen in the spectra, suggesting that CSM interaction is a more viable explanation of its late-emerging radio emission (R. M. Quimby et al. 2018); and for VT 1137-0337, a CSM-interaction model cannot be ruled out (D. Z. Dong & G. Hallinan 2023). SN 1987A has the strongest evidence for a PWN, with observation of accompanying neutrinos at the time of explosion indicating compact object formation and infrared spectroscopy revealing emission features consistent with PWN photoionization (C. Fransson et al. 2024).⁴³ However, SN 1987A lacks a comparable radio signal⁴⁴ and originates from a lower energy explosion ($\sim 10^{51}$ erg; S. E. Woosley et al. 1988) of a completely different progenitor—a blue supergiant star. Given this sample, we lack any direct observational analogs for SN 2012au at early times.

SN 2012au, however, clearly presents many of the observational traits of more-evolved, observed PWNe. In order to compare to early time PWN emission, we now turn to the predictions of analytical and numerical models.

⁴³ See E. Greco et al. (2021, 2022) for the association of hard X-ray emission and a PWN in SN 1987A.

⁴⁴ The compact, faint ALMA radio source is possibly the only evidence for a PWN in the radio bands (P. Cigan et al. 2019).

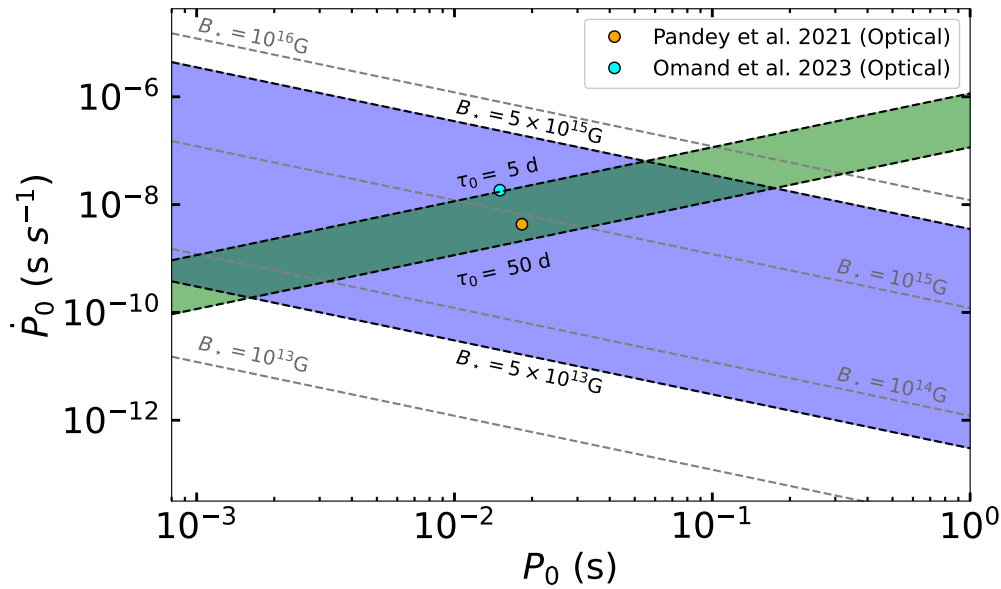


Figure 14. Motivated parameter space for the initial spin period, P_0 , and period derivative, $\dot{P}_0$, in the PWN emission scenario. The green region shows the reference range motivated by the optical rise time if the central engine contributed significantly to the optical luminosity. The blue region shows the parameter space inferred from the observed radio luminosity using an effective radio efficiency range $\eta_{r,\text{eff}} = [1, 10^{-4}]$. These regions should be interpreted as illustrative rather than model-independent constraints. The parameters inferred in the optical studies by S. B. Pandey et al. (2021) and C. M. B. Omand & A. Jerkstrand (2023) are also plotted as an orange and cyan point, respectively. The gray dashed lines represent lines of constant surface dipole magnetic field strength, B_* . The allowed region (i.e., overlap) includes many combinations of P_0 and $\dot{P}_0$ that correspond to magnetar engines.

6.2. Comparison to Numerical Models

J. D. Gelfand et al. (2009) present a semianalytic model of the dynamical and radiative evolution of PWNe that explores epochs as early as 10 yr postexplosion. The dynamics are computed assuming that the PWN is surrounded by a thin shell of material, and the setup is applicable to the preradiative evolutionary phases of the object’s evolution (we are fully within this regime at $\delta t \sim 10$ yr). The PWN is powered by the NS rotational energy, and a simple power-law injection of particle energies is assumed. The radio spectral index is found to be $\alpha \sim -0.3$ for the PWN evolution at late times (10–10,000 yr). Furthermore, these authors find a PWN radius at $\delta t \approx 10$ yr of $R_{\text{pwn}} \approx 10^{16}$ cm and ejecta velocities on the order of 10^2 km s^{-1} , comparable to our inferred radius and velocity for a spherically symmetric geometry in equipartition in Section 4.2. We caution, however, that the PWN radius, the swept-up shell radius, and the equipartition radius inferred from the radio SED need not be identical. If we do assume that the equipartition parameters in Table 2 are representative of the parameters of a PWN, it would imply levels of magnetic pressure that would accelerate the inner ejecta to $v_s > 500 \text{ km s}^{-1}$. In this scenario, it would be necessary to invoke asymmetries to explain the inferred emitting region. Overall, the observed and inferred properties of SN 2012au are order-of-magnitude consistent with numerical estimates for a young PWN simulated by J. D. Gelfand et al. (2009), although the visibility of early time radio emission through the young SN ejecta is more directly affected by the radio transparency (discussed in Section 6.3.1).

The possibility of detecting nonthermal radio and millimeter emission from a nascent pulsar or magnetar wind nebula embedded in SN ejecta has been further explored in the context of pulsar-driven SNe, SLSNe, and fast-radio-burst-related scenarios. K. Murase et al. (2016) showed that persistent synchrotron emission from electron–positron pairs

in young PWNe can become detectable once the surrounding ejecta become sufficiently transparent, with high-frequency radio and millimeter bands being especially promising at early epochs. The radio spectral index transits from $\alpha \approx -0.8$ in the fast cooling case to $\alpha \approx -0.3$ in the slow cooling case. This framework was further developed for embryonic SLSN remnants by C. M. B. Omand et al. (2018), and was later compared with ALMA and NOEMA observations of SLSNe by K. Murase et al. (2021).

C. M. B. Omand & A. Jerkstrand (2023) utilize this model for SN 2012au with the goal of explaining the optical spectra and light curves of D. Milisavljevic et al. (2013, 2018), with non-local thermodynamic equilibrium spectral synthesis computations. The inferred values of surface dipole B field ($B_* = 3\text{--}4 \times 10^{14} \text{ G}$) and initial pulsar period ($P_0 = 15 \text{ ms}$) are comparable to those of S. B. Pandey et al. (2021; $P_0 = 18 \text{ ms}$ and $B_* = 8 \times 10^{14} \text{ G}$) and fall within our allowed region (see Section 6.3.3, Figure 14). C. M. B. Omand & A. Jerkstrand (2023) also offer a prediction of the radio evolution of SN 2012au based on the best-fitting parameters from their optical spectra modeling. Comparing to our data, we find that the predicted optically thin radio emission at $\delta t = 10$ yr is a good representation of the observations both in terms of flux density and spectral index. However, the predicted peak frequency is significantly lower than observed, by a factor of ≈ 5 . The theoretical optically thick spectrum is also very steep, (presumably dominated by FFA), compared to the observed $F_\nu \propto \nu^{0.5}$. This discrepancy can be partially attributed to the additional contribution of the low-frequency component in the observed spectrum. As noted in C. M. B. Omand & A. Jerkstrand (2023), clumping in the ejecta may also have significant, unaccounted for effects on the spectrum. Nevertheless, based on these numerical models a PWN explanation remains plausible for our observations. In Section 6.3.3, we will compare the PWN parameters inferred

from these models to constraints we place based on the radio data.

We end by noting that the large P_0 inferred by S. B. Pandey et al. (2021) and C. M. B. Omand & A. Jerkstrand (2023) imply small rotational kinetic energy $\approx 6 \times 10^{50}$ erg, which is much less than the E_k carried by the ejecta. A different energy source is needed to explain the large E_k of SN 2012au in these models.

6.3. Analytical Estimates and Considerations

6.3.1. Radio Detectability of the Pulsar Wind Nebula at $\delta t \approx 10$ yr

We first investigate whether the radio emission from a central PWN would be detectable on our observed timescales. To compute the timescale for our object, we employ the analytic formalism used in the transparency calculation of B. D. Metzger et al. (2017). Following their Equation (11), we expect oxygen-dominated ($Z=8$, $A=16$) ejecta to be transparent to FFA at the following time postexplosion:

$$t_{\text{ff}} = 6.25 \bar{g}_{\text{ff}}^{1/5} f_{\text{ion}}^{2/5} \left(\frac{T_{\text{ej}}}{10^4 \text{ K}} \right)^{-3/10} \left(\frac{M_{\text{ej}}}{5 M_{\odot}} \right)^{2/5} \times \left(\frac{v_{\text{ej}}}{0.1c} \right)^{-1} \left(\frac{\nu}{1 \text{ GHz}} \right)^{-2/5} \text{ yr}, \quad (22)$$

where $\bar{g}_{\text{ff}} = 5$ is the Gaunt factor at radio frequencies, f_{ion} is the ionization fraction of the ejecta, T_{ej} is the ejecta electron temperature, M_{ej} is the ejecta mass, and v_{ej} is the ejecta velocity. For SN 2012au, we assume $T_{\text{ej}} = 10^4$ K, $M_{\text{ej}} = 5 M_{\odot}$, and use the inferred total kinetic energy from Section 3.2.1, $E_k = 10^{52.5}$ erg, to calculate the ejecta velocity ($E_k = 1/2 M_{\text{ej}} v_{\text{ej}}^2$). We first verify that PWN emission would be completely free-free absorbed during the early time observations ($\delta t \leq 190$ days). To do this, we calculate t_{ff} over a range of ionization fractions $f_{\text{ion}} = [0.1, 1]$ for $\nu = 37$ GHz, the maximum observed frequency at early times (i.e., the frequency that will see the smaller FFA). We find that $t_{\text{ff}} > 190$ days over the range of computed ionization fractions. While emission at this frequency may not be absorbed for $f_{\text{ion}} < 0.1$, we know that there is some degree of ionization because of the ionized emission features in the early time optical spectra (D. Milisavljevic et al. 2013). Higher values of ejecta temperature may also lower t_{ff} ; however, at $T_{\text{ej}} = 10^5$ K, $t_{\text{ff}} > 190$ days for all $f_{\text{ion}} > 0.25$. Next, we calculate whether the ejecta are transparent to FFA during the late-time observations ($\delta t \geq 6.7$ yr). We compute t_{ff} over the same range of ionization fractions for $\nu = 16$ GHz, the observing frequency of the earliest late-time epoch. We find that $t_{\text{ff}} < 6.7$ yr across all ionization fractions, even for a low ejecta temperature of $T_{\text{ej}} = 10^3$ K. We therefore conclude that the emergence of the high-frequency component in radio on our observed timescales is consistent with a PWN interpretation.

Our conclusion is strengthened by previous theoretical and numerical treatments of nascent PWN radio emission. K. Murase et al. (2016) calculated the attenuation of radio waves by SSA in the PWN and FFA in the surrounding ejecta, and showed that higher-frequency radio waves can escape more readily than gigahertz emission and can be used as

signatures of magnetar-driven SNe including SLSNe. C. M. B. Omand et al. (2018) find a timescale of ~ 10 – 100 yr for transparency to 1 GHz emission for SLSNe while the ejecta become transparent earlier at higher frequencies, on timescales of order ~ 1 – 10 yr at 100 GHz (see also K. Murase et al. 2021). B. Margalit et al. (2018) present a numerical calculation of the ionization states of the SN ejecta using the photoionization code CLOUDY. They also find that for a sample of SLSNe, the expected timescale for ejecta transparency to FFA is on the order ≈ 10 – 100 yr. These SNe have typical explosion energies of $E_k \sim 10^{52}$ erg, comparable to SN 2012au, while having larger $M_{\text{ej}} \sim 10 M_{\odot}$. This larger M_{ej} will only increase t_{ff} , making these values an overestimate for our source.

6.3.2. X-Ray Suppression of the Pulsar Wind Nebula at $\delta t \approx 10$ yr

In the PWN scenario, the lack of detectable X-ray emission at late times can be due to (i) photoelectric absorption by the SN ejecta or (ii) intrinsic faintness, i.e., a spectral break between the optical and the X-ray regime, which is often observed in PWNe and the nature of which is still a matter of debate (e.g., B. M. Gaensler & P. O. Slane 2006). In the following we discuss the X-ray absorption scenario. Radio transparency does not necessarily imply X-ray transparency, because FFA depends on the ionized gas density whereas soft X-ray photoelectric absorption is controlled by the ejecta column density and ionization state of the ejecta. Thus, a PWN can become visible at the high-frequency radio band while its soft X-ray emission remains attenuated by the ejecta.

From Section 4.1, for ejecta-like composition, a neutral hydrogen column density $N_{\text{H,int}} > 6.2 \times 10^{20} \text{ cm}^{-2}$ is needed to suppress the X-ray emission below detectability, which is easily achieved by neutral SN ejecta at ≈ 3000 days. However, the pulsar wind can lead to high-energy X-ray photons (e.g., the wind-injected electron-positron pairs cool via synchrotron and inverse-Compton emission) that can ionize the SN ejecta from the inside.⁴⁵ The result is a time- and spatially dependent ionization state of the ejecta, which directly impacts the X-ray opacity (with ionized ejecta having lower X-ray opacity). The problem has been treated by different authors in the literature (K. Kotera et al. 2013; B. D. Metzger et al. 2014; K. Murase et al. 2015) and an ionization breakout has been invoked to explain the late UV brightening of ASASSN-15lh (R. Margutti et al. 2017). Here we use the results from the numerical simulations of B. Margalit et al. (2018), which self-consistently track the ejecta ionization state with time as the spin-down power declines.

B. Margalit et al. (2018) concluded that for oxygen-dominated composition and ionizing luminosity $\propto t^{-2}$ that is relevant to our problem (see Section 6.3.3), the ejecta are either ionized during a spin-down timescale τ_0 , or never achieve a state of complete ionization (and hence transparency to the X-rays). For efficient deposition of energy into the ejecta τ_0 needs to be comparable to the timescale of evolution of the SN (e.g., $\tau_0 \approx t_{\text{rise}}$), and thus $\ll 10$ yr. This is consistent with the value inferred from the optical light curves in S. B. Pandey et al. (2021): $\tau_0 = 24.49 \pm 2.12$ days.

⁴⁵ We note that the SN ejecta will be eventually collisionally ionized by the passage of the reverse shock. However, this happens on a Sedov–Taylor timescale of hundreds of years (e.g., S. P. Reynolds & R. A. Chevalier 1984; J. K. Truelove & C. F. McKee 1999), which is much larger than the timescales considered in this paper, and thus not relevant.

For a ratio of absorptive to scattering opacity in the ejecta $\eta_{\text{thr}} > 1$ that applies here (Equation (A7) of B. D. Metzger et al. 2014), the ionization timescale of the SN ejecta is

$$t_{\text{ion}} \approx 340 \text{ days} \left(\frac{M_{\text{ej}}}{5 M_{\odot}} \right) \left(\frac{v_{\text{ej}}}{10,000 \text{ km s}^{-1}} \right)^{-3/2} \left(\frac{T}{10^5 \text{ K}} \right)^{-0.4} \\ \times \left(\frac{X_Z}{0.35} \right)^{1/2} \left(\frac{E_{\text{ion}}}{10^{52} \text{ erg}} \right)^{-1/2} \times Z_8^{3/2},$$

where T is the temperature of electrons in the recombination layer, and X_Z is the mass fraction of elements with atomic number $Z = 8Z_8$ in the ejecta expanding with velocity v_{ej} . The calculation above suggests that $t_{\text{ion}} > \tau_0$, even assuming an ionizing energy $E_{\text{ion}} = L_{\text{ion}} \times t$ comparable to the total ejecta kinetic energy (where L_{ion} is some fraction of the spin-down luminosity). It is thus unlikely to achieve complete ionization of the SN ejecta on a spin-down timescale, from which we conclude that it is plausible that the lack of X-ray detection at ≈ 10 yr is due to photoelectric absorption.

6.3.3. Analytical Inferences on the Pulsar Wind Nebula Parameters

Energy injection from a central NS may come from two sources: rotational energy, in the form of magnetized winds, or magnetic energy from the interior magnetic field, in the form of magnetar giant flares. In this analysis, we focus on rotationally powered energy injection because of the larger energy budget (up to a few $\times 10^{52}$ erg for millisecond period magnetars, compared to the more modest $\sim 10^{50}$ erg of magnetic energy for a strong B field on the order $\sim 10^{16}$ G) and because this scenario is expected to dominate at early times (B. Margalit et al. 2018). In this case, the PWN luminosity is related to the evolution of the spin-down luminosity of the pulsar, which evolves as (B. M. Gaensler & P. O. Slane 2006)

$$L_{\text{sd}} = \dot{E}_0 \left(1 + \frac{t}{\tau_0} \right)^{-\frac{n+1}{n-1}}, \quad (23)$$

where $\dot{E}_0$ is the initial spin-down luminosity at time t_0 , n is the braking index of the pulsar with angular frequency Ω (such that $\dot{\Omega} \propto \Omega^{-n}$), and τ_0 is the initial spin-down timescale (B. M. Gaensler & P. O. Slane 2006)

$$\tau_0 = \frac{P_0}{(n-1)\dot{P}_0}, \quad (24)$$

where P_0 is the initial pulsar period, and $\dot{P}_0$ is the initial period derivative. For $t \ll \tau_0$, L_{sd} is constant in time, whereas for $t \gg \tau_0$

$$L_{\text{sd}} \approx \dot{E}_0 \left(\frac{t}{\tau_0} \right)^{-\frac{n+1}{n-1}}. \quad (25)$$

The spin-down timescale can be related to the surface dipole magnetic field strength, B_* , as in D. Kasen & L. Bildsten (2010):

$$\tau_0 = 1.3 \left(\frac{B_*}{10^{14} \text{ G}} \right)^{-2} \left(\frac{P_0}{10 \text{ ms}} \right)^2 \text{ yr}, \quad (26)$$

where it is assumed $\sin^2 \alpha = 1/2$ (α being the angle between the rotational axis and the magnetic dipole axis), B_* is the NS dipole magnetic field at the poles, the NS moment of inertia is $I \approx 10^{45} \text{ g cm}^2$, and the NS radius is $R_{\text{NS}} = 10 \text{ km}$. B_* differs from the magnetic field that powers the synchrotron emission of the nebula B_n , which was probed in our model-agnostic fit (Section 4.1). From Section 4.1 we find that B_n decreases monotonically with time, as expected from the adiabatic expansion of the nebula and/or the rapid decay of injected energy with time. The resulting synchrotron emission can either trace in time the energy injection, or not. We therefore distinguish between B_* , which controls the spin-down power of the NS, and B_n , which controls the synchrotron emission from the nebula. The two quantities are not directly interchangeable, because B_n depends on the magnetization of the wind, the integrated injection history, and the subsequent expansion of the nebula.

We compute the optically thin luminosity, νL_{ν} , and find that it scales approximately as $L(t) \propto t^{-2}$ thus implying that, if the observed luminosity reflects L_{sd} , we are in the regime $t \gg \tau_0$ and $n \approx 3$ (Equation (25)). The $n = 3$ braking index is typically used in the literature for magnetar spin-down (J. P. Ostriker & J. E. Gunn 1969; D. Kasen & L. Bildsten 2010), as it corresponds to the expected braking in the case of magnetic dipole radiation (J. P. Ostriker & J. E. Gunn 1969; A. Spitkovsky 2006). This reasoning is consistent with (but cannot be considered a proof of) our assumption that the PWN is primarily rotation powered as the observed radio luminosity does not have to mirror the spin-down luminosity (see below).

For an independent probe of τ_0 , we turn to the observed optical evolution of the source. If the same central engine contributed significantly to the bright optical luminosity, one expects τ_0 to be comparable to the photon diffusion time, t_{diff} , which is observationally related to the SN rise time. From D. Milisavljevic et al. (2013), $t_{\text{rise}} = 16.5 \pm 1.0$ days, which is much less than all our late-time epochs—independently indicating that we are in the $t \gg \tau_0$ regime.

For comparison with simple spin-down estimates, we define an effective radio efficiency, $\eta_{\text{r,eff}} \equiv L_{\text{obs}}/L_{\text{sd}}$. This quantity should not be interpreted as a constant bolometric conversion efficiency, because in time-dependent PWN calculations, the observed radio emission depends on the accumulated population of relativistic pairs, the nebular magnetic field, adiabatic and radiative losses, SSA, and FFA in the surrounding ejecta (e.g., K. Murase et al. 2016, 2021; C. M. B. Omand et al. 2018; I. Vurm & B. D. Metzger 2021). This is particularly relevant when most of the rotational energy is injected at $t \ll t_{\text{obs}}$, as expected for the regime of interest $\tau_0 \ll t_{\text{obs}}$. In this regime, the nebular magnetic energy may evolve approximately as $\epsilon_B \Delta \mathcal{E}_{\text{rot}} \sim V_n B_n^2 / 8\pi$, where $\Delta \mathcal{E}_{\text{rot}} \sim \dot{E}_0 \tau_0$ is the total energy injected at early times and $V_n \propto R_n^3$. For approximately homologous expansion with $R_n \propto t$, this gives $B_n \propto t^{-3/2}$, ignoring more details such as decay. Then, if the number of radio-emitting pairs is approximately conserved (see Section 6.3.4), the optically thin synchrotron flux scales as $F_{\nu} \propto N_e B_n^{(p+1)/2}$, which can naturally produce a decline close to t^{-2} for $p = 1.6$, as observed. We note that this analysis does not consider adiabatic cooling of the lepton population, which, if relevant, would result in a steeper decline of B_n and may predict a faster luminosity decay than we observe.

When $t \gg \tau_0$ we cannot place valuable constraints on P_0 and $\dot{P}_0$ based purely on the evolution of luminosity, as the normalization of L_{sd} constrains a product of P_0 , $\dot{P}_0$, and $\dot{E}_0$, but not the individual terms. To break this degeneracy, we conservatively adopt $5 < \tau_0 < 50$ days as a motivated reference range based on the rise time from D. Milisavljevic et al. (2013), rather than as a model-independent constraint. This τ_0 range translates into an allowed range of $P_0/\dot{P}_0$ ratios via Equation (24) (green-shaded region in Figure 14).

These parameters are further constrained by comparing L_{sd} with the observed luminosity L_{obs} . Since in our physical scenario (B. M. Gaensler & P. O. Slane 2006)

$$L_{\text{sd}} = \dot{E}_{\text{rot}} = 4\pi^2 I \frac{\dot{P}}{P^3}, \quad (27)$$

where I is the moment of inertia of the pulsar and $I \approx 10^{45} \text{ g cm}^2$ for standard values of $M = 1.4 M_\odot$ and $R = 10 \text{ km}$, and $L_{\text{obs}} \equiv \eta_{\text{r,eff}} L_{\text{sd}}$ with $\eta < 1$, it follows that for $n \approx 3$ and $t \gg \tau_0$

$$L_{\text{obs}} \equiv \eta_{\text{r,eff}} L_{\text{sd}} \approx \pi^2 I \eta (P_0 \dot{P}_0)^{-1} t^{-2}. \quad (28)$$

Following D. Z. Dong & G. Hallinan (2023), a lower limit on $\eta_{\text{r,eff}}$ can be placed by observational surveys. Radio observations of several thousands of year old PWNe (D. A. Frail & B. R. Scharinghausen 1997) imply $\eta_{\text{r,eff}}^{\text{min}} \sim 10^{-4}$ among the population of detected pulsars. Since $\eta_{\text{r,eff}}$ is expected to decrease over time, we adopt $\eta_{\text{r,eff}}^{\text{min}} > 10^{-4}$, which leads us to conservatively consider the range of efficiencies $1 \geq \eta \geq 10^{-4}$. The observed radio luminosity⁴⁶ and Equation (28) constrain the product of $P_0 \dot{P}_0$ as in Figure 14, blue-shaded region. Lines of constant B_* are depicted in gray, computed by combining Equations (24) and (26). Interestingly, the PWN parameters inferred by S. B. Pandey et al. (2021) and C. M. B. Omand & A. Jerkstrand (2023; orange and cyan filled circles, respectively, in Figure 14) fall within the overlapping region of allowed parameter space. With $B_* = (0.5\text{--}50) \times 10^{14} \text{ G}$, our central source likely qualifies as a magnetar. We note that this result does differ from the corresponding figure in M. Lazda et al. (2026), which instead focuses on the scenario where $t < \tau_0$. Our allowed regions fall within the “unconstrained” region of their Figure 4, which they do not exclude, but rather defer to future works.

To place our source in the context of detailed nascent PWN calculations, we compare the observed radio SED with a representative time-dependent PWN model based on the framework of K. Murase et al. (2016, 2021) and C. M. B. Omand et al. (2018). A selected model with pulsar parameters is plotted in Figure 15, with parameters described in the caption. The comparison is not intended as a formal parameter fit, but it illustrates that a young PWN can reproduce high-frequency radio emission of the observed data. However, we emphasize that the representative PWN model does not explain the entire radio spectrum, especially at low frequencies. This is not surprising because the low-frequency side is particularly sensitive to SSA, FFA, ejecta clumping, asphericity, inhomogeneous ionization, and possible contamination from a separate CSM-interaction component. These effects can

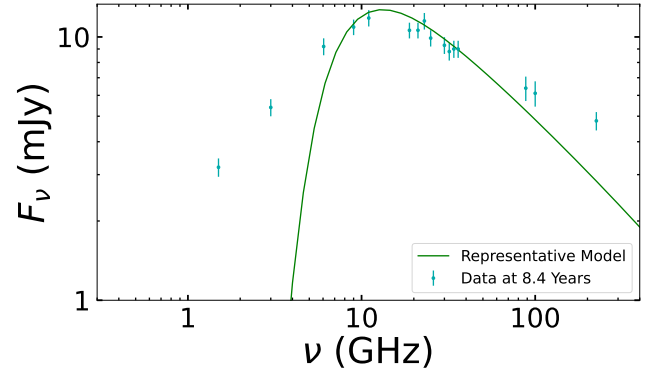


Figure 15. Observed late-time radio SED of SN 2012au compared with a representative nascent PWN synchrotron model (K. Murase et al. 2016). The model is not a formal fit, but demonstrates that high-frequency radio emission from a young PWN can reach the observed luminosity scale once absorption in the nebula and ejecta becomes sufficiently weak. This model utilizes a pulsar with $P_0 = 15 \text{ ms}$ and $B_* = 10^{14} \text{ G}$, and explosion parameters of $M_{\text{ej}} = 7 M_\odot$ and $E = 10^{52} \text{ erg}$ with $p = 1.6$, $\gamma_b = 3 \times 10^4$, $\epsilon_B = 0.1$, and $f_{\text{ion}} = 0.5$.

shift or broaden the apparent spectral turnover, while leaving the optically thin high-frequency component largely consistent with a nascent PWN interpretation. A quantitative fit including these effects is beyond the scope of this work, and we therefore use the comparison only to demonstrate that a young PWN can plausibly produce the observed high-frequency radio emission.⁴⁷

6.3.4. Source of Injected Electron–Positron Pairs

A key element of the PWN model to address is what process produces the radio-emitting electrons (or electron–positron pairs). We start with an estimate of the total number of emitting leptons in the nebula N_e at the time of our observations. Using the equation for optically thin synchrotron flux from R. A. Chevalier (1998)

$$F_\nu = \frac{4\pi f_\nu R^3}{3D^2} c_5 n_0 B^{\frac{p+1}{2}} \left(\frac{\nu}{2c_1} \right)^{\frac{1-p}{2}}, \quad (29)$$

where $B \equiv B_n$ is the nebula field and n_0 is a normalization on the density of relativistic electrons per unit energy, such that

$$\frac{dn}{dE} = n_0 E^{-p}, \quad (30)$$

extending between E_{min} and E_{max} . It follows that

$$N_e = \frac{F_\nu D^2}{c_5(p)} B_n^{-(1+p)/2} \left(\frac{2c_1}{\nu} \right)^{(1-p)/2} \frac{(E_{\text{max}}^{1-p} - E_{\text{min}}^{1-p})}{1-p}. \quad (31)$$

We consider a range of energies $(\gamma_{\text{min}} - \gamma_{\text{max}}) m_e c^2$. The computed values have a very weak dependence on γ_{max} , provided we consider $\gamma_{\text{max}} > 10^4$. We select a representative optically thin spectrum at $\delta t = 8.4 \text{ yr}$ and

⁴⁶ The radio luminosity is calculated by integrating the higher-frequency component of Section 4.1 over the typical range of 100 MHz to 100 GHz (B. M. Gaensler & P. O. Slane 2006).

⁴⁷ We similarly attempt to find a representative time-dependent model for an electron–ion nebula, using the formalism of B. Margalit & B. D. Metzger (2018). We find that while we are able to match the evolution of the peak flux and frequency, both the optically thick and optically thin slopes are far too steep to match the data.

$F_\nu(\nu = 30 \text{ GHz}) = 9.3 \text{ mJy}$ with $p = 1.6$, and infer

$$N_e \approx 7 \times 10^{52} \left(\frac{B_n}{0.1 \text{ G}} \right)^{-1.3} \left(\frac{\gamma_{\min}}{1.0} \right)^{-0.6}, \quad (32)$$

corresponding to a number density of radiating leptons

$$n_e \approx 3 \times 10^4 \left(\frac{f_V}{0.5} \right)^{-1} \left(\frac{R_n}{10^{16} \text{ cm}} \right)^{-3} \left(\frac{B_n}{0.1 \text{ G}} \right)^{-1.3} \times \left(\frac{\gamma_{\min}}{1.0} \right)^{-0.6} \text{ cm}^{-3}. \quad (33)$$

A more conservative estimate would consider only electron energies that we directly observe, setting $\gamma_{\min} \approx 60$ for $\nu_{\min} = 1 \text{ GHz}$ and $B = 0.1 \text{ G}$. For this conservative estimate, we find $N_e \approx 5 \times 10^{51}$ and $n_e = 2 \times 10^3 \text{ cm}^{-3}$.

We compare these estimates to the “classic” picture of direct pulsar lepton injection at multiplicity $\mu_{\pm}$, i.e., the Goldreich–Julian (GJ) injection rate $\dot{N}_{\text{GJ}}$ (P. Goldreich & W. H. Julian 1969), where $\mu_{\pm}$ is the number of leptons per primary GJ particle. Assuming an outflow speed of $\approx c$ of GJ particles from a polar cap area A_{pc} , we estimate

$$\dot{N}_{\text{GJ}} = \frac{\rho_{\text{GJ}}}{e} A_{\text{pc}} c \approx \frac{B_* R_{\text{NS}}^3 \Omega^2}{2ec}, \quad (34)$$

where e is the electron charge. We can relate this quantity to L_{sd} expressed in terms of magnetic dipole luminosity as in D. Kasen & L. Bildsten (2010):

$$L_{\text{sd}} = \frac{B^2 R_{\text{NS}}^6 \Omega^4}{12c^3}, \quad (35)$$

therefore

$$\dot{N}_{\text{GJ}} \approx L_{\text{sd}}^{1/2} \frac{\sqrt{3} c^{1/2}}{e}. \quad (36)$$

$\dot{N}_{\text{GJ}}$ is related to the lepton injection rate $\dot{N}_{\pm}^{\text{inj}}$ by the multiplicity factor, $\mu_{\pm}$, such that

$$\dot{N}_{\pm}^{\text{inj}} = \mu_{\pm} \dot{N}_{\text{GJ}}. \quad (37)$$

Over time, $\dot{N}_{\pm}^{\text{inj}}$ will evolve like $\dot{N}_{\pm}^{\text{inj}} \propto L_{\text{sd}}^{1/2}$:

$$\dot{N}_{\pm}^{\text{inj}}(t) = \dot{N}_{\pm,0}^{\text{inj}} \left(1 + \frac{t}{\tau_0} \right)^{-1}. \quad (38)$$

We compute the maximum E_0 allowed in the overlapping region of Figure 14 and find $\dot{E}_0^{\text{max}} \approx 1 \times 10^{46} \text{ erg s}^{-1}$, from which we derive

$$\dot{N}_{\pm,0}^{\text{inj}} \leq 6 \times 10^{42} \left(\frac{\mu_{\pm}}{10^5} \right) \text{ s}^{-1}. \quad (39)$$

We normalize by $\mu_{\pm} = 10^5$, the order of magnitude of the theoretical maximum multiplicity (A. N. Timokhin & A. K. Harding 2019) and observational measurements of the Crab Pulsar (J. A. Hirschman & J. Arons 2001). Integrating Equation (38) from $t = 0$ to 8.4 yr for $\tau_0 \approx 24.5 \text{ days}$, the total number of injected leptons is

$$N_{\pm}^{\text{inj}} \leq 6 \times 10^{49} \left(\frac{\mu_{\pm}}{10^5} \right). \quad (40)$$

We conclude that even when we maximize the L_{sd} and $\mu_{\pm}$, the directly injected particle count is a factor $\sim 10^3$ below the

$N_e \approx 10^{53}$ radiating leptons number required by observations (Equation (32)). We thus consider alternative scenarios, starting with giant magnetar flares.

A magnetar giant flare has an associated baryonic mass ejection into the nebula that could supply the needed electrons. Relativistic hydrodynamical simulations imply that these flares eject crustal material characterized by an electron fraction of $Y_e \approx 0.4$ (J. Cehula et al. 2024). Recent work has inferred a baryonic ejecta mass of $M_b \sim 10^{-6} M_{\odot}$ from the SGR 1806-20 magnetar giant flare (A. Patel et al. 2025). Normalizing based on these parameter, the total number of electrons from a magnetar flare is

$$N_{e,\text{flare}} \approx 5 \times 10^{50} \left(\frac{Y_e}{0.4} \right) \left(\frac{M_b}{10^{-6} M_{\odot}} \right), \quad (41)$$

therefore, giant flares appear to be insufficient to account for the total number of required electrons.

Next we consider the scenario proposed in B. D. Metzger & A. L. Piro (2014; see also I. Vurm & B. D. Metzger 2021), whereby the enormous energy density in the magnetic fields and radiation in the early time nebula generates high-energy photons that produce a high abundance of electron–positron pairs. A key parameter is the pair multiplicity factor, $Y \equiv L_{\text{sd}}/(m_e c^2)$, defined as the fraction of the spin-down power placed into electron (or positron) rest mass. We take $Y = 0.1$, corresponding to an estimate of maximum pair yield for a “saturated” cascade R. Svensson (1987). We begin by estimating a freeze-out timescale, t_{fo} after which the number of pairs created through this cascade ceases to change significantly as a result of the nebula expansion. At t_{fo} the equilibrium timescale of pair annihilation and creation (t_{eq}) is comparable to the nebula expansion timescale: $t_{\text{eq}}(t_{\text{fo}}) \approx R_{\text{fo}}/v$. Following B. D. Metzger & A. L. Piro (2014), the optical depth of pairs through the nebula at freeze out is

$$\tau_{\text{es}}^n(t_{\text{fo}}) = \frac{16}{3} \beta, \quad (42)$$

where $\beta \equiv v/c$ and $\tau_{\text{es}}^n(t)$ is determined by the balance between pair creation and annihilation (B. D. Metzger & A. L. Piro 2014, their Equation (14)). We consider $\tau_{\text{es}}^n(t)$ in the limit $t \gg t_{\text{sd}}$ as $t_{\text{fo}} > t_{\text{sd}}$ for our relevant range of parameters. Setting Equation (42) equal to Equation (14) in B. D. Metzger & A. L. Piro (2014)⁴⁸ we find

$$t_{\text{fo}} = 760 \text{ days} \left(\frac{\beta}{1.67 \times 10^{-3}} \right)^{-1} \left(\frac{B_*}{5 \times 10^{14} \text{ G}} \right)^{-2/3}, \quad (43)$$

where we normalize to our average value of B_* and a nebula expansion velocity of $v = 500 \text{ km s}^{-1}$ as inferred from observations. The total number of pairs at freeze out is then

$$N_{\pm}^{\text{fo}} = n_{\pm}^{\text{fo}} V^{\text{fo}} = \frac{\tau_{\text{es}}^n(t_{\text{fo}})}{\sigma_T R_{\text{fo}}} \frac{4\pi R_{\text{fo}}^3}{3}. \quad (44)$$

Using $R_{\text{fo}} = \beta c t_{\text{fo}}$, leads to

$$N_{\pm}^{\text{fo}} \approx 6 \times 10^{53} \left(\frac{\beta}{1.67 \times 10^{-3}} \right) \left(\frac{B_*}{5 \times 10^{14} \text{ G}} \right)^{-4/3}, \quad (45)$$

⁴⁸ With normalization revised for self-consistency with our spin-down timescale definition of Equation (26).

which is comparable with the observational constraint of Equation (32). While we acknowledge that residual e^+e^- annihilation might still play a role after formal freeze out, the early pair cascade thus provides a plausible source of radiating leptons in the PWN scenario.

We conclude that the late-time radio observations are consistent with most of the emitting electrons originating from pair creation in the first ≈ 2 yr after SN explosion. In this scenario, the observed decline in radio luminosity is accounted for by the adiabatic expansion of the nebula, rather than mirroring the spin-down luminosity of the pulsar. We note that this is a simple treatment, which only explains the total number of emitting leptons and not the lepton energies needed to produce the observed emission.

7. Conclusion

In this work, we have presented the broadband evolution of SN 2012au across 13 yr of evolution. While the early radio and X-ray emission of SN 2012au at $\delta t < 190$ days is consistent with a standard model of a shock propagating through a wind-like density medium, the late-time evolution ($\delta t \geq 6.7$ yr) markedly deviates from this model. Our monitoring campaign reveals the onset of a major radio rebrightening that dominates at $\nu \gtrsim 5$ GHz. This luminous emission component is observationally characterized by a broad and rapidly evolving spectral peak, with a shallow optically thin slope ($F_\nu \propto \nu^{-0.31 \pm 0.02}$) and an optically thick spectral slope that is shallower than SSA from a single population of electrons. Modeling the broadband X-ray to radio spectrum we infer the following properties.

1. The late-time millimeter to radio data are consistent with a two-component synchrotron emission model, with the lower-frequency component being roughly consistent with extrapolations of the emission expected from the initially observed shock.
2. The component responsible for the higher-frequency emission must be subdominant to the initial shock emission at $\delta t < 190$ days.
3. The higher-frequency component is associated with a small emitting volume (with an isotropic equivalent radius $R \approx 10^{16}$ cm), low average expansion velocities ($v \approx 500$ km s $^{-1}$), and high number densities of electrons ($n_e \geq 10^4$ cm $^{-3}$).
4. The higher-frequency component is characterized by a hard electron power-law index of $p \approx 1.6$.
5. No high-energy emission is detected at any epoch. The late-time soft and hard X-ray emission is either photoelectrically absorbed or intrinsically faint.

We interpret these findings in the context of two families of models: shock–CSM interaction with an asymmetric medium, and the emergence of radiation from a PWN. Through our analysis of these models, we have found the following.

1. A torus-like CSM-interaction geometry is able to reproduce many of the observational features of the late-time SEDs; however, the parameters required are extreme (e.g., a large CSM mass of $10 M_\odot$, within a radius of $\sim 10^{16}$ cm with half-opening angle of $\theta < 5^\circ$ and lack viable stellar evolutionary motivation).
2. Models invoking the shock interaction with an isolated small clump of CSM at a large radius can account for the

delayed onset, but fail to reproduce some key observational features of the late-time SEDs, like the observed spectral cooling breaks. While the parameters in this model are not unphysical, the astrophysical setup is highly contrived.

3. The late-time broadband SEDs of SN 2012au bear many similarities to those of evolved PWNe. In particular, our observed optically thin slope is consistent with the flat radio spectral indices of PWNe and inconsistent with standard predictions of DSA in an explosive event. Additionally, we find that the high-energy limits are naturally accounted for by photoelectric absorption within the expanding, oxygen-dominated ejecta. By contrast, the ejecta are transparent to radio photons because of the significantly lower free–free optical depth, allowing the emergence of the new radio component (K. Murase et al. 2016). Within this model we find that the radio evolution is consistent with the adiabatic expansion of a relic saturated electron–positron pair-cascade plasma injected by a central engine posited by B. D. Metzger & A. L. Piro (2014), providing new insights into young pulsar physics.⁴⁹

Based on this evidence, we conclude that a PWN represents the most likely explanation of the remarkable radio rebrightening of SN 2012au. While this conclusion is based purely on the data presented in this paper, this model is further supported by the late-time optical–NIR data presented in D. Milisavljevic et al. (2018) and S. B. Pandey et al. (2021), the VLBI data of M. Lazda et al. (2026), the spectropolarimetric observations of S. DeSoto et al. (2025), and the modeling efforts of S. B. Pandey et al. (2021), C. M. B. Omand & A. Jerkstrand (2023), and L. Dessart (2024). Taken together these studies establish SN 2012au as the most compelling candidate for an unprecedentedly young extragalactic PWN. Future observations may offer further support for the PWN interpretation. Late-time polarization measurements may reveal a departure from the low ($< 1\%$) fractional polarization observed in typical radio SNe (A. M. Soderberg et al. 2006) to the significantly higher levels associated with known PWNe (S. P. Reynolds et al. 2012). As described in M. Lazda et al. (2026), planned future VLBI observations will allow us to conclusively rule out CSM-interaction scenarios and directly probe the physical properties of an emerging PWN.

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⁴⁹ We note that numerical modeling applied to the radio data can reproduce the high-frequency emission, while discrepancies at lower frequencies can be explained by inhomogeneities, ejecta clumping, and contamination from the separate CSM-interaction component.

of Korea), in cooperation with the Republic of Chile. The Joint ALMA Observatory is operated by ESO, AUI/NRAO, and NAOJ. We thank the staff of the GMRT that made these observations possible. The GMRT is run by the National Centre for Radio Astrophysics of the Tata Institute of Fundamental Research. The Submillimeter Array is a joint project between the Smithsonian Astrophysical Observatory and the Academia Sinica Institute of Astronomy and Astrophysics and is funded by the Smithsonian Institution and the Academia Sinica. We recognize that Maunakea is a culturally important site for the indigenous Hawaiian people; we are privileged to study the cosmos from its summit. This work is based on observations carried out under project number D20AE with the IRAM NOEMA Interferometer. IRAM is supported by INSU/CNRS (France), MPG (Germany), and IGN (Spain). This research has made use of the NuSTAR Data Analysis Software (NuSTARDAS) jointly developed by the ASI Space Science Data Center (SSDC; Italy) and the California Institute of Technology (Caltech; USA). This paper employs a list of Chandra datasets, obtained by the Chandra X-ray Observatory, contained in the Chandra Data Collection (CDC) 607 DOI: [10.25574/cdc.607](https://doi.org/10.25574/cdc.607). Some of the data presented herein were obtained at Keck Observatory, which is a private 501(c)3 nonprofit organization operated as a scientific partnership among the California Institute of Technology, the University of California, and the National Aeronautics and Space Administration. The Observatory was made possible by the generous financial support of the W. M. Keck Foundation. The authors wish to recognize and acknowledge the very significant cultural role and reverence that the summit of Maunakea has always had within the Native

Hawaiian community. We are most fortunate to have the opportunity to conduct observations from this mountain.

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Software: astropy (Astropy Collaboration et al. 2022), EMCEE (D. Foreman-Mackey et al. 2013), CASA (J. P. McMullin et al. 2007), AIPS (Associated Universities, I. 1999), IRAF (D. Tody 1986), SNOOPY (C. R. Burns et al. 2015), DAOPHOT (P. B. Stetson 1987) NuSTARDAS (F. A. Harrison et al. 2013) CIAO (A. Fruscione et al. 2006).

Appendix A

Early Time Fit Corner Plot

Here we present the corner plot (Figure 16) for the early time radio data fit described in Section 3.2.1.

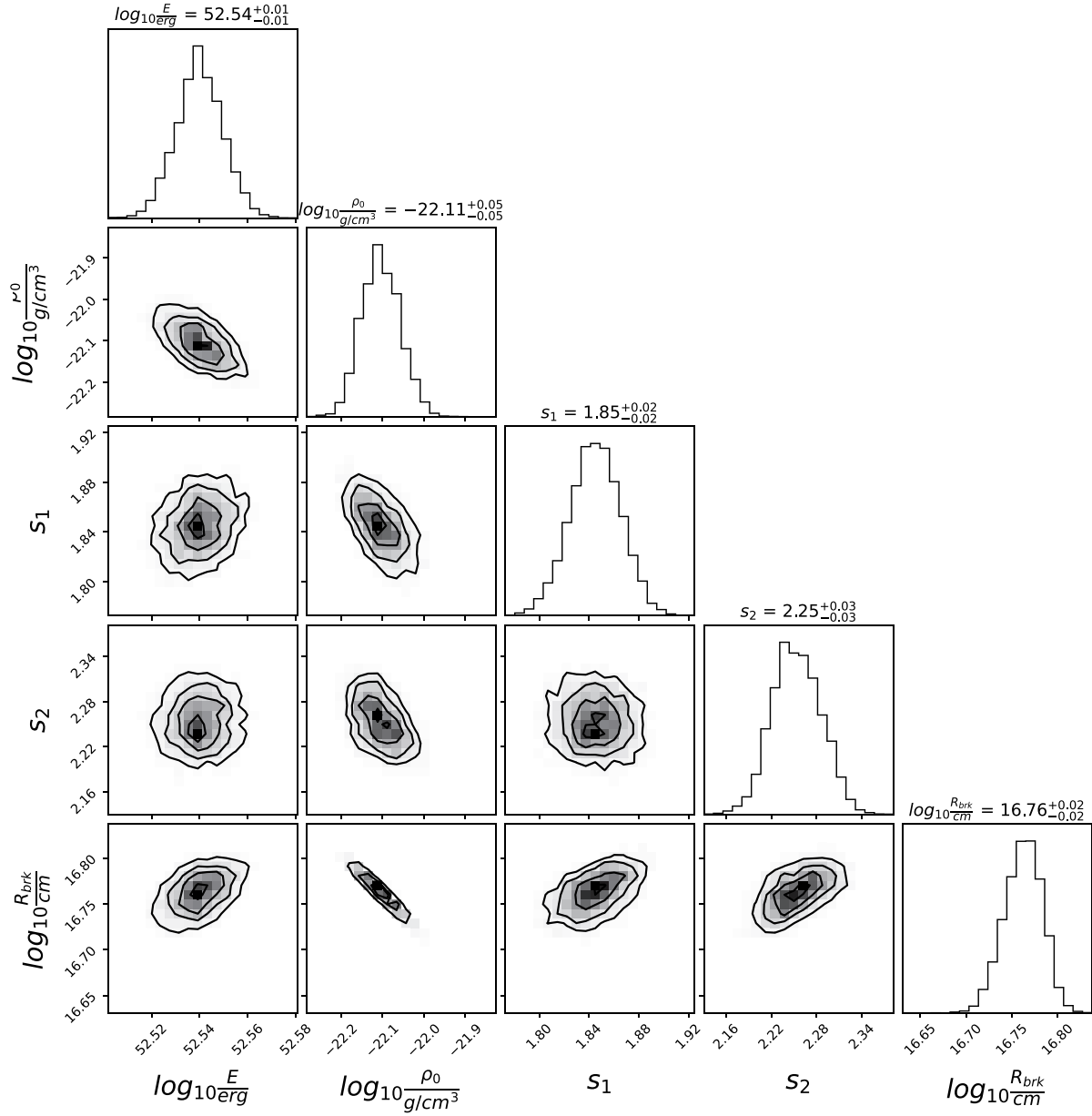


Figure 16. Corner plots for the posteriors of the fit to the early time data $\delta t \leq 190$ days described in Section 3.2.1. The data are well modeled by synchrotron emission originating from a blast wave propagating into a broken power-law density CSM, with a change in density indexes from $s_1 = 1.85 \pm 0.02$ to $s_2 = 2.25 \pm 0.03$. Chains have converged with all parameters' ESS > 300 and $\hat{R} \approx 1$.

Appendix B

Model-agnostic Fit Corner Plot

Here we present the corner plot (Figure 17) for the model-agnostic two-component fit described in Section 2.

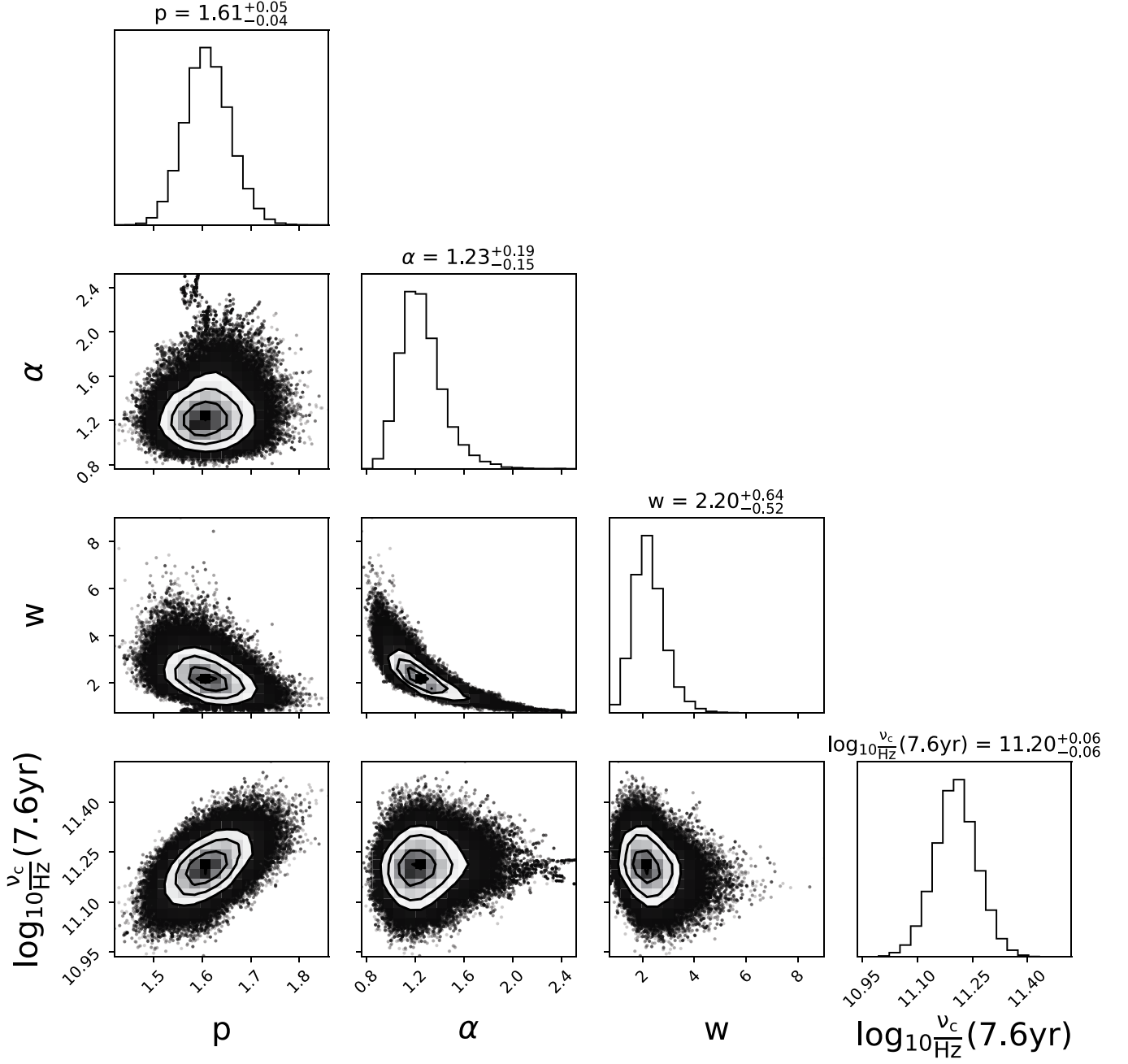


Figure 17. Corner plot for the posteriors of key parameters in the model-agnostic fit of the late-time data shown in Figure 4 and described in Section 4.1. Additional fitted parameters include the F_{brk} and ν_{brk} , shown in Figure 5, and the normalization on the optically thin, low-frequency component. Chains have converged with all parameters' ESS > 600 and $\hat{R} \approx 1$.

Appendix C

Radio and X-Ray Data

Here we report a log of our broadband late-time radio monitoring campaign of SN 2012au, in addition to the additional epoch at $\delta t = 190 d$ that was not published by A. Kamble et al. (2014) in Table 3. The error bars on the

radio flux density measurements reported are the image rms. At the time of modeling the radio data, we add 5% systematic flux calibration uncertainty in quadrature to this reported rms. In Table 4, we report a log of our CXO and NuSTAR X-ray observations.

Table 3
Radio Observations of SN 2012au

Start Date (UT) (MM/DD/YYYY)	Time ^a (days)	Frequency (GHz)	Flux Density ^b (mJy)	Telescope	Array Config.	Project Code	PI
9/9/2012	190.0	1.51	4 ± 0.20	VLA	BnA	12A-458	Soderberg
9/9/2012	190.0	3.0	1.8 ± 0.10	VLA	BnA	12A-458	Soderberg
9/9/2012	190.0	6.0	1.02 ± 0.06	VLA	BnA	12A-458	Soderberg
11/5/2018	2438.0	6.1	7.7 ± 0.50	VLA	D	18A-119	Margutti
9/12/2019	2749.0	1.5	3.2 ± 0.20	VLA	A	SC1068	Margutti
9/12/2019	2749.0	3.0	5.2 ± 0.30	VLA	A	SC1068	Margutti
9/12/2019	2749.0	6.1	9.3 ± 0.60	VLA	A	SC1068	Margutti
9/28/2019	2765.0	10.0	12.6 ± 0.60	VLA	A	SC1068	Margutti
9/28/2019	2765.0	15.0	13.5 ± 0.70	VLA	A	SC1068	Margutti
9/28/2019	2765.0	22.0	13.8 ± 0.70	VLA	A	SC1068	Margutti
10/12/2019	2779.0	33.06	13.2 ± 0.70	VLA	A	SC1068	Margutti
10/29/2019	2796.0	102.5	9.3 ± 0.50	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	104.5	9.1 ± 0.50	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	336.52	4.2 ± 0.30	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	338.42	4.1 ± 0.30	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	348.52	4.3 ± 0.30	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	350.48	4.1 ± 0.30	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	90.5	9.8 ± 0.50	ALMA	C43-3	2019.A.00006.T	Coppejans
10/29/2019	2796.0	92.5	9.9 ± 0.50	ALMA	C43-3	2019.A.00006.T	Coppejans
12/19/2019	2847.0	0.38	2.2 ± 0.30	GMRT	...	ddtC104	Coppejans
12/24/2019	2852.0	0.7	2.43 ± 0.20	GMRT	...	ddtC104	Coppejans
12/29/2019	2857.0	0.19	≤ 2.3	GMRT	...	ddtC104	Coppejans
12/29/2019	2857.0	1.25	2.79 ± 0.20	GMRT	...	ddtC104	Coppejans
7/1/2020	3042.0	1.5	3.2 ± 0.20	VLA	B	SC1074	Margutti
7/1/2020	3042.0	3.0	5.4 ± 0.30	VLA	B	SC1074	Margutti
7/1/2020	3042.0	6.05	9.2 ± 0.50	VLA	B	SC1074	Margutti
7/11/2020	3052.0	0.19	≤ 2.2	GMRT	...	ddtC104	Coppejans
7/11/2020	3052.0	11.0	11.8 ± 0.60	VLA	B	SC1074	Margutti
7/11/2020	3052.0	18.87	10.6 ± 0.50	VLA	B	SC1074	Margutti
7/11/2020	3052.0	21.13	10.6 ± 0.50	VLA	B	SC1074	Margutti
7/11/2020	3052.0	23.0	11.5 ± 0.60	VLA	B	SC1074	Margutti
7/11/2020	3052.0	25.0	9.9 ± 0.50	VLA	B	SC1074	Margutti
7/11/2020	3052.0	9.0	10.9 ± 0.50	VLA	B	SC1074	Margutti
7/13/2020	3054.0	225.0	4.8 ± 0.30	SMA	...	POETS	Berger
9/5/2020	3108.0	30.0	9.3 ± 0.50	VLA	B	SC1079	Margutti
9/5/2020	3108.0	32.0	8.8 ± 0.50	VLA	B	SC1079	Margutti
9/5/2020	3108.0	34.0	9 ± 0.50	VLA	B	SC1079	Margutti
9/5/2020	3108.0	36.0	9 ± 0.50	VLA	B	SC1079	Margutti
11/17/2020	3181.0	88.26	6.39 ± 0.60	NOEMA	...	ID D20AE	...
11/17/2020	3181.0	100.0	6.11 ± 0.60	NOEMA	...	ID D20AE	...
10/31/2023	4259.0	3.0	3.29 ± 0.44	VLA	D	SF151070	Margutti
10/31/2023	4259.0	5.0	6.12 ± 0.17	VLA	D	SF151070	Margutti
10/31/2023	4259.0	7.0	6.85 ± 0.10	VLA	D	SF151070	Margutti
10/31/2023	4259.0	9.0	7.15 ± 0.10	VLA	D	SF151070	Margutti
10/31/2023	4259.0	11.0	7.04 ± 0.08	VLA	D	SF151070	Margutti
11/16/2023	4275.0	225.5	2.9 ± 0.22	SMA	...	POETS	Berger
12/15/2023	4304.0	0.65	2.29 ± 0.24	GMRT	...	ddtC315	Nayana AJ
12/19/2023	4308.0	13.0	6.42 ± 0.19	VLA	D	23B-330	Nayana AJ
12/19/2023	4308.0	15.0	6.29 ± 0.15	VLA	D	23B-330	Nayana AJ
12/19/2023	4308.0	17.0	6.14 ± 0.11	VLA	D	23B-330	Nayana AJ
12/21/2023	4310.0	1.37	2.85 ± 0.28	GMRT	...	ddtC315	Nayana AJ
12/27/2023	4316.0	0.44	2.6 ± 0.26	GMRT	...	ddtC315	Nayana AJ

Table 3
(Continued)

Start Date (UT) (MM/DD/YYYY)	Time ^a (days)	Frequency (GHz)	Flux Density ^b (mJy)	Telescope	Array Config.	Project Code	PI
12/27/2023	4316.0	19.0	6.02 ± 0.12	VLA	D	23B-330	Nayana AJ
12/27/2023	4316.0	21.0	5.67 ± 0.12	VLA	D	23B-330	Nayana AJ
12/27/2023	4316.0	23.0	5.55 ± 0.13	VLA	D	23B-330	Nayana AJ
12/27/2023	4316.0	25.0	5.51 ± 0.14	VLA	D	23B-330	Nayana AJ
10/27/2024	4621.5	3.0	4.421 ± 0.03	VLA	A	24B-311	Wiston
10/27/2024	4621.5	1.52	2.924 ± 0.05	VLA	A	24B-311	Wiston
10/27/2024	4621.5	9.0	5.807 ± 0.08	VLA	A	24B-311	Wiston
10/27/2024	4621.5	5.55	5.316 ± 0.04	VLA	A	24B-311	Wiston
10/27/2024	4621.5	7.0	5.514 ± 0.05	VLA	A	24B-311	Wiston
10/27/2024	4621.5	11.0	5.489 ± 0.09	VLA	A	24B-311	Wiston
10/27/2024	4621.5	19.0	4.9 ± 0.08	VLA	A	24B-311	Wiston
10/27/2024	4621.5	25.0	4.16 ± 0.12	VLA	A	24B-311	Wiston
10/27/2024	4621.5	21.0	4.72 ± 0.14	VLA	A	24B-311	Wiston
10/27/2024	4621.5	23.0	4.44 ± 0.13	VLA	A	24B-311	Wiston
10/27/2024	4621.5	1.26	2.444 ± 0.02	VLA	A	24B-311	Wiston
10/27/2024	4621.5	1.775	3.153 ± 0.07	VLA	A	24B-311	Wiston
11/17/2024	4642.5	0.4	4.45 ± 0.19	GMRT	...	47_021	Wiston
11/18/2024	4643.5	0.65	2.29 ± 0.20	GMRT	...	47_021	Wiston
2/13/2025	4730.5	220.0	2.2 ± 0.22	SMA	...	POETS	Berger

Notes.^a Since explosion (2012 March 3).^b Uncertainties are quoted at 1 σ , and upper limits are quoted at 3 σ .**Table 4**
CXO and NuSTAR X-Ray Observations of SN 2012au, with Inferred Count-rate and Flux Limits

Obs ID	δt (days)	Count-Rate ^a (10 ⁻⁴ counts s ⁻¹)	Photon Index ^b	Unabsorbed Flux ^c (10 ⁻¹⁵ erg s ⁻¹ cm ⁻²)	Exposure ^d (ks)	Spacecraft	PI
21160	2343.2	<1.5	2	<2.3	19.8	CXO	D. Patnaude
23316	3060.3	<1.5	2	<2.7	9.8	CXO	R. Margutti
23331	3060.9	...	2	...	10.1	CXO	R. Margutti
25187	3676.7	<1.6	2	<3.1	19.3	CXO	M. Stroh
90601521002	3034.9	<4.1	2	<24.5	23.8/23.6	NuSTAR	R. Margutti

Notes.^a 3 σ limit; background-subtracted; 0.5–8 keV for CXO, 8–20 keV for NuSTAR, both modules.^b Assumed.^c CXO: 0.3–10 keV and $N_{\text{H,int}} = 0 \text{ cm}^{-2}$. NuSTAR: 8–20 keV.^d For NuSTAR we report the exposure time of the A module and the B module.**Appendix D**
Synchrotron Constants

Here we report the constants defined in A. G. Pacholczyk (1970) that are used to compute the synchrotron emissivity and self-absorption. These constants are used in our treatment of synchrotron radiation and enter in our Equations (9) and (10). All constants are in cgs units:

$$c_1 = 6.27 \times 10^{18}, \quad (\text{D1})$$

$$c_5 = \frac{\sqrt{3} e^3}{16\pi m_e c^2} \frac{p + 7/3}{p + 1} \Gamma\left(\frac{3p + 2}{12}\right) \Gamma\left(\frac{3p + 7}{12}\right), \quad (\text{D2})$$

$$c_6 = \frac{\sqrt{3} \pi}{72} e m_e^5 c^{10} (p + 10/3) \Gamma\left(\frac{3p + 2}{12}\right) \Gamma\left(\frac{3p + 10}{12}\right). \quad (\text{D3})$$

Appendix E

Circumstellar Medium Interaction Plots

Here we present the plots relevant to the CSM-interaction models for late-time emission described in Section 5: the

corner plot for the edge-on torus model (Figure 18), the best fit model for the face-on torus (Figure 19), and the corner plots for the CSM clump models (Figure 20).

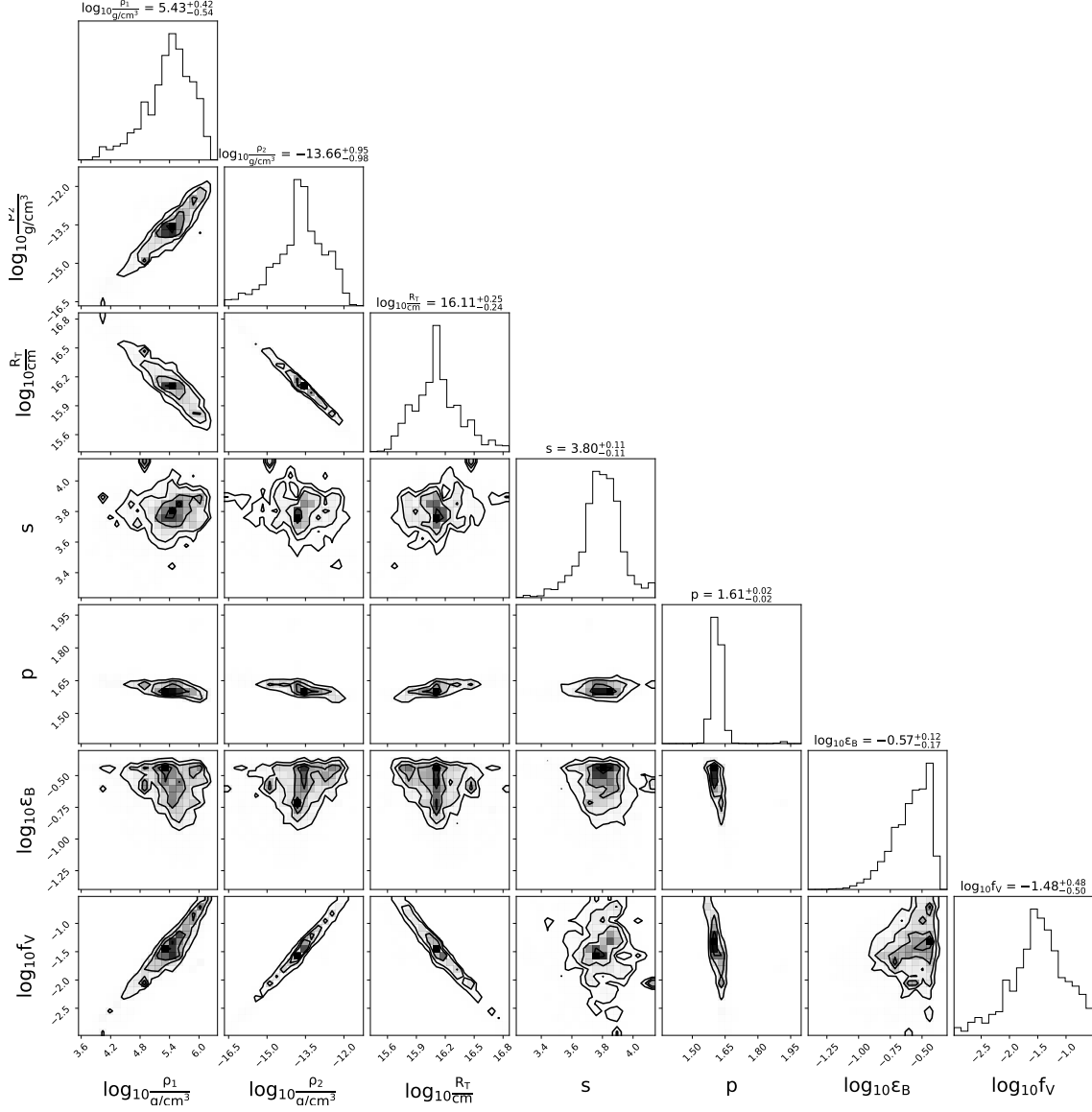


Figure 18. Corner plot for the edge-on torus-like dense-CSM model in Section 5.1.1. The region at $R < R_T$ is characterized by extremely high densities ($\rho_1 \approx 10^5 \text{ g cm}^{-3}$), needed to significantly decelerate the shock. At $R = R_T$, there is a large drop in density to $\rho_2 = 10^{-12.56} \text{ g cm}^{-3}$ that declines in radius with a steeper than wind density profile: $\rho_{\text{CSM}}(R) \propto R^{-3.29}$. All chains have converged with all parameters' ESS $> 10^5$ and $\hat{R} \approx 1$.

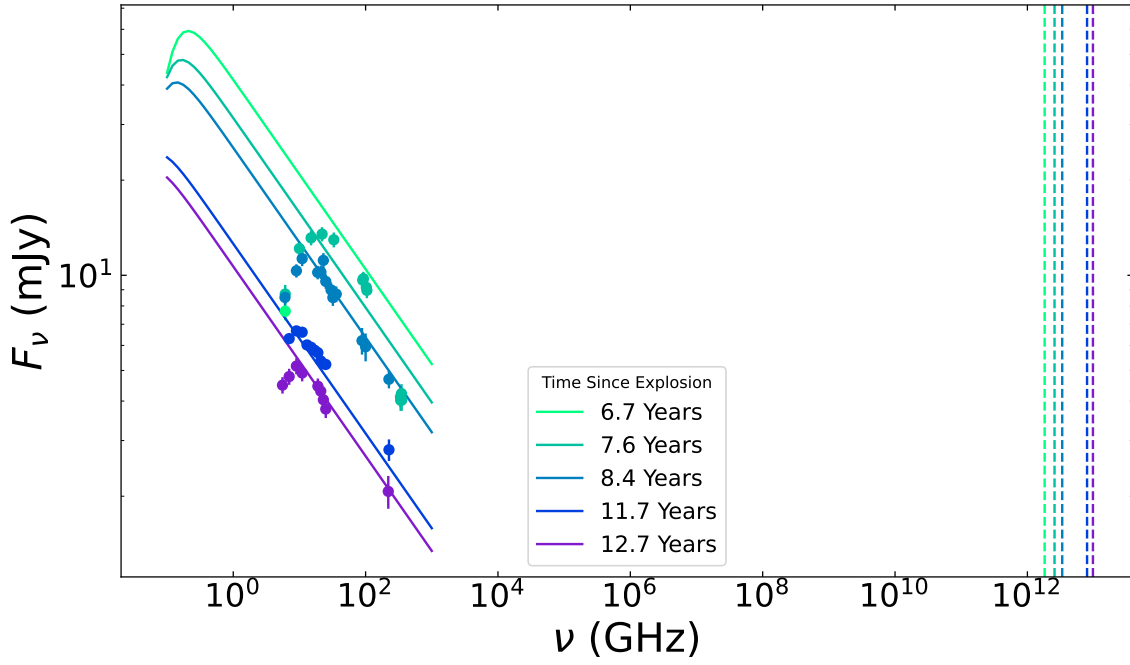


Figure 19. The best model fit for the dense torus viewed from a face-on perspective, described in Section 5.1.2. While the optically thin side of the observed SEDs is well modeled, the optically thick side is significantly overestimated by the model. In the face-on perspective, there is not significant FFA, which would otherwise shift the observed peak to higher ν_{pk} and lower F_{pk} .

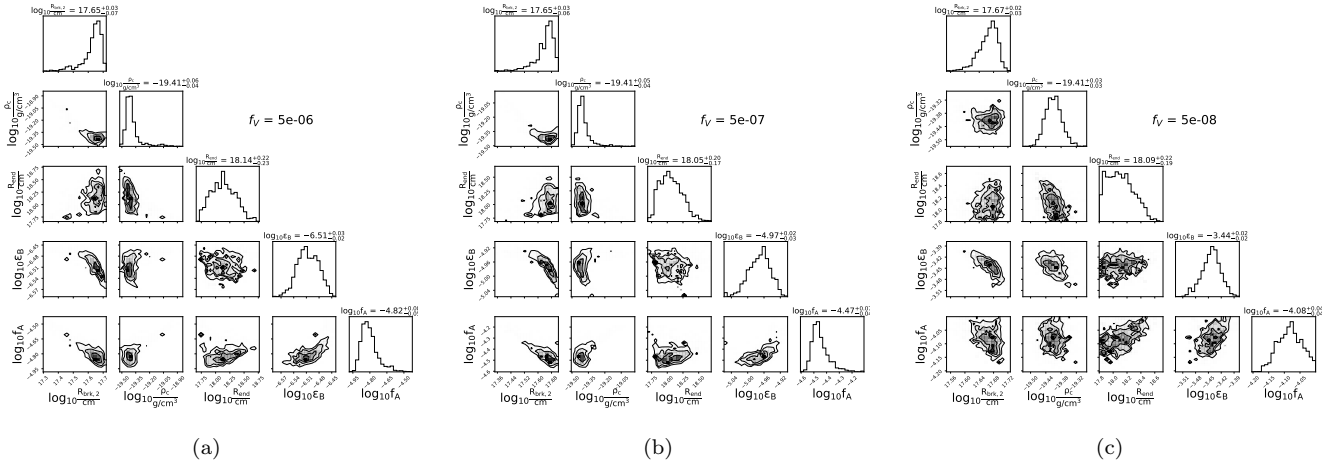


Figure 20. Corner plots for the small clump of dense-CSM models in Section 5.2. Plots (a)–(c) show the parameter space for $f_v = 5 \times 10^{-6}$ through 5×10^{-8} , respectively. The factors related to the dynamics of the shock $R_{\text{brk},2}$, ρ_c , and R_{end} are all constant across different filling factors. The only variable that changes with considerably with f_v is ϵ_B , which increases with decreasing filling factor. This compensates for the lower emitting volume to maintain the same observed luminosity. These models consequently do not have the same B -field strength, like the torus models, because they are not matching the observed cooling break. All chains have converged with all parameters' ESS $> 10^5$ and $\hat{R} \approx 1$.

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