The one known MWN is a good start.

The spectrum reflects the electron distribution:
- single power law?
- Broken power law?

Synchrotron cooling time of X-ray emitting electrons (at 2$E_e^2$ keV) is $<<$ system's age $\Rightarrow$ quasi-steady state:

$$t_{\text{syn}} = \frac{6\pi m_e c}{\sigma_T B^2 \gamma_e} \approx 1.02B_{15}^{-3/2}E_2^{-1/2} \text{ kyr}.$$  

Other relevant quantities: wind/outflow composition & Lorentz factor or pair multiplicity, minimal & maximal Lorentz factor, power-law index/es

Total emitted power (0.5 – 30 keV)  
$\mathcal{L}_{x,\text{tot}} = 2.74 \times 10^{33}d_4^2 \text{ erg s}^{-1}$

The spin-down power  
$L_{sd} = 2.05 \times 10^{34} \text{ erg s}^{-1}$

X-ray efficiency of MWN  
$\eta_X = \frac{\mathcal{L}_{x,\text{tot}}}{L_{sd}} = 0.13d_4^2$

If energy injection is not dominated by spin-down:  
$\langle \dot{E} \rangle = gL_{sd} \Rightarrow \eta_{x,\text{true}} = \frac{\mathcal{L}_{x,\text{tot}}}{\langle \dot{E} \rangle} = \frac{\eta_X g}{\dot{E}} = \frac{0.0026d_4^2}{g_{50}} < 0.042 \frac{\sigma}{1 + \sigma} d_4^{-1}E_{M,30}^{-7/2} f_{-7/2}^{-7/2}$
Observed Size & Spectral Softening: Role of Diffusion

Synchrotron cooling time of X-ray emitting electrons (at 2E_2 keV) is \( \ll \) system’s age \( \Rightarrow \) quasi-steady state:

\[
\tau_{\text{syn}} = \frac{6\pi m_e c}{\sigma_T B^2 \gamma_e} \approx 1.02B_{15\mu G}^{-3/2}E_2^{-1/2} \text{ kyr}.
\]

- Diffusion dominates over advection in whole MWN

\[
r_{\text{c,dif}} \approx \sqrt{2\lambda_{\text{def}} c t_{\text{syn}}(\gamma_e)} \approx 1.57 B_{15\mu G}^{-3/2} \zeta^{1/2} \text{ pc}
\]

\( \zeta \equiv \lambda_{\text{def}}/R_L \gtrsim 1 \) (\( \zeta = 1 \) corresponds to Bohm diffusion)

- Resulting cooling length \( \sim \) observed nebula size \( R_X \) \( \Rightarrow \) may also explain the spectral softening
Is Swift J1846.9–0846 unique in any way (J1935?)? What helps produce a detectable MWN?

It is currently ~1-2 MWNe around ~30 known magnetars (small number statistics)

What makes the difference? Intrinsic vs. External properties:
- Initial spin period $P_0$ & rotational energy $E_0$
- Initial surface dipole field $B_0$
- Pair multiplicity & wind Lorentz factor
- Natal kick velocity

Small kick velocity: magnetar remains inside its SNR, which confines a MWN (traps the outflows & results in a relatively bright, easier to detect emission)

$$\text{offset} \lesssim (0.05 - 0.1)R_{\text{SNR}} \implies v_{\perp,SGR} \lesssim (30 - 60)d_4(t_{\text{SNR}}/10^{4.5}\text{ yr})^{-1} \text{ km s}^{-1}.$$ 

Large kick velocity: magnetar exits its SNR & forms a bow-shock containing much less energy (most of the outflow escapes, leading to weaker, harder to detect emission)

SGR 1806 – 20: $v_{\perp,SGR} \approx 580d_{15} \text{ km s}^{-1}$, SGR 1900 + 14 $v_{\perp,SGR} = 130 \pm 30 \text{ km s}^{-1}$

External medium density (SNR & MWN evolution) + composition (bow shock X-ray efficiency)