Lessons from the first Magnetar Wind Nebula

Jonathan Granot
Open University of Israel


High Energy Astrophysics Workshop, February 28, 2017
Jerusalem
Outline of the Talk:

- **Introduction**: magnetars & Pulsar Wind Nebulae
- **Observations**: the 1\textsuperscript{st} MWN discovered around Swift J1834.9–0846
- Association with SNR W41 & MWN detectability
- **GeV/TeV Source**: next talk by Ramandeep Gill
- **Dynamics** of the Nebula + SNR – two main dynamical regimes
- **Internal Structure** of the Nebula: ideal MHD $\rightarrow$ non-ideal low-$\sigma$ flow
- X-ray synchrotron Nebula **Size**: electron advection, diffusion, cooling
- Steady-state X-ray emission: **energy balance** $\rightarrow$ $\dot{E}_{\text{rot}}$ is insufficient
- Alternative **energy source**: magnetar’s B-field decay
- **Conclusions**
Magnetars: differences from “normal” pulsars

Compared to “normal” radio pulsars, magnetars have:

- Long rotation periods: \( P \approx 2 \text{ } 12 \text{ s} \)
- Large period derivatives: \( \dot{P} \sim 10^{-13} \text{ } 10^{-10} \)
- Small spin-down ages
  \[ \dot{P} \propto P^{2-n} \quad P(t) = P_0 \left( 1 + \frac{t}{t_0} \right)^{1/(n-1)} \]
  \[ t \approx \frac{P}{(n-1)\dot{P}} \equiv \tau_c \quad (P_0 \ll P) \]
- Lower spin-down power
  \[ L_{sd} = -I \Omega \dot{\Omega} = \frac{4\pi^2 I \dot{P}}{P^3} \sim 10^{30} \text{ } 10^{34} \text{ erg s}^{-1} \]
- Higher inferred dipole surface magnetic fields
  \[ L_{sd} = f \frac{B_s^2 R_{NS}^6 \Omega^4}{c^3} \rightarrow B_s = 3.2 \times 10^{19} \sqrt{P \dot{P}} \text{ G} > B_Q = \frac{m_e^2 c^3}{e\hbar} = 4.4 \times 10^{13} \text{ G} \]

- High quiescent X-ray luminosities: \( L_X \sim 10^{33} \text{ } 10^{36} \text{ erg s}^{-1} > L_{sd} \)

\[(\text{Ng & Kaspi, 2010)}\]
Magnetars: differences from “normal” pulsars

- Magnetars (especially SGRs) show diverse bursting activity, from small bursts to giant flares.

- Giant flares are rare (only 3 observed so far) & extremely luminous bursts:
  \[ L_{pk} \sim 10^{44} - 10^{47} \text{ erg s}^{-1} \]

- These observations led to the Thompson & Duncan (1993, 1995, 1996, 2000) magnetar model, which posits that:
  
  - Magnetar bursts and quiescent emission is powered by the decay of the strong internal magnetic field \( B_{\text{int}} \gtrsim 10^{15} \text{ G} \)
  
  - Short bursts are related to stressing of the crust by the unwinding internal toroidal field.
  
  - Giant flares are produced by shearing and reconnection of the strong external magnetic field.
  
  - High magnetic fields in magnetars result from field amplification by a dynamo mechanism when \( P_0 \lesssim 3 \text{ ms} \)
Pulsar Wind Nebulae (PWNe)

- Cold ultra-relativistic MHD wind is launched from the pulsar, powered by $\dot{E}_{\text{rot}}$.
- This wind is decelerated & heated at the termination shock radius, $R_{TS}$, where its ram-pressure equals the pressure in the hot nebula that it inflates.
- The hot, high-pressure nebula is bounded by the SNR & performs work on it.

$$L_{sd} = 4\pi R^2 c P$$
$$P = \frac{e}{3} = \frac{E_{\text{neb}}}{4\pi R_{\text{neb}}^3}$$
$$R_{TS} = \sqrt{\frac{R_{\text{neb}}^3 L_{sd}}{c E_{\text{neb}}}}$$
The first-ever magnetar wind nebula

*Swift J1834.9-0846*

(Younes et al. 2016)

- **NuSTAR (3 – 30keV)**
- **XMM (0.5 – 10 keV)**
- **Fermi GeV**
- **H.E.S.S TeV**
- **Radio**

**Radio SNR**

**MWN**

**GeV/TeV source**

**Swift J1834.9-0846**

\[
P = 2.48 \text{ s}
\]

\[
\dot{P} = 7.96 \times 10^{-12} \text{ s s}^{-1}
\]

\[
\tau_c = 4.9 \text{ kyr} \quad (n = 3)
\]

\[
t = \tau_c - 10^5 \text{ yr}
\]

\[
B_s = 10^{14} \text{ G}
\]

\[
L_{sd} = 2 \times 10^{34} \text{ erg s}^{-1}
\]
Association with SNR W41 & Efficiency of MWN’s X-ray emission

\[ R_{\text{SGR}} \lesssim (0.05 - 0.1) R_{\text{SNR}} \]

\[ v_{\perp,\text{SGR}} \lesssim (30 - 60) \left( \frac{t_{\text{SNR}}}{10^{4.5} \text{ yr}} \right) d_4 \text{ km/s} \]

\[ d_4 \equiv d/4 \text{ kpc} \]

Total emitted power (0.5 – 30 keV)
\[ L_{x,\text{tot}} = 2.74 \times 10^{33} d_4^2 \text{ erg/s} \]

The spin-down power
\[ L_{\text{sd}} = 2.05 \times 10^{34} \text{ erg/s} \]

X-ray efficiency of MWN
\[ \eta_X = \frac{L_{x,\text{tot}}}{L_{\text{sd}}} = 0.13 d_4^2 \]
Association with SNR W41 & Efficiency of MWN’s X-ray emission

\[ R_{\text{SGR}} \lesssim (0.05 - 0.1) R_{\text{SNR}} \]

\[ v_{\perp,\text{SGR}} \lesssim (30 - 60) \left( \frac{t_{\text{SNR}}}{10^{4.5} \, \text{yr}} \right) d_4 \, \text{km/s} \]

\[ d_4 \equiv d/4 \, \text{kpc} \]

Total emitted power (0.5 – 30 keV)
\[ L_{X,\text{tot}} = 2.74 \times 10^{33} d_4^2 \, \text{erg/s} \]

The spin-down power
\[ L_{\text{sd}} = 2.05 \times 10^{34} \, \text{erg/s} \]

X-ray efficiency of MWN
\[ \eta_X = \frac{L_{X,\text{tot}}}{L_{\text{sd}}} = 0.13 d_4^2 \]
The Detectability of Magnetar Wind Nebulae

- Is Swift J1846.9–0846 really unique (J1935)? What helps make a MWN detectable?
- It is currently ~1-2 MWNe around ~30 known magnetars (small number statistics)
- What makes the difference? Intrinsic vs. External properties:
  - Current spin-down power $L_{sd}$
  - Initial spin period $P_0$ & rotational energy $E_0$
  - Initial surface dipole field $B_0$
  - Pair multiplicity & wind Lorentz factor
  - Natal kick velocity

Small kick velocity: magnetar remains inside its SNR, which confines a MWN (traps the outflows & results in a relatively bright, easier to detect emission)

\[
\text{offset} \lesssim (0.05 - 0.1) R_{SNR} \implies v_{\perp,SGR} \lesssim (30 - 60) d_4 (t_{SNR}/10^{4.5} \text{ yr})^{-1} \text{ km s}^{-1}.
\]

Large kick velocity: magnetar exits its SNR & forms a bow-shock containing much less energy (most of the outflow escapes, leading to weaker, harder to detect emission)

- SGR 1806–20: $v_{\perp,SGR} \approx 580 d_{15} \text{ km s}^{-1}$, SGR 1900+14 $v_{\perp,SGR} = 130 \pm 30 \text{ km s}^{-1}$
- SGR 0501+4516: $v_{\perp,SGR} \approx 3000 \text{ km s}^{-1}$, SGR 0526–66 $v_{\perp,SGR} \approx 1100 \text{ km s}^{-1}$

- External density (SNR & MWN evolution) + composition (bow shock X-ray efficiency)
Dynamics of the MWN + SNR

- The SN ejecta initially expands ballistically:
  \[ \rho_{\text{ej}}(r, t) = At^{-3}, \quad r < v_t t \]
  \[ \rho_{\text{ej}}(r, t) = Av_t^s r^{-s} t^{s-3}, \quad r > v_t t \]

  \[ m = \frac{n + 1}{n - 1} \]

Energy injection by the magnetar:
\[ L_{\text{sd}} = L_0 \left(1 + \frac{t}{t_0}\right)^{-m} \approx L_0 \times \begin{cases} 1 & t < t_0 \\ \left(\frac{t}{t_0}\right)^{-m} & t > t_0 \end{cases} \]

Core crossing time by MWN: \( t_c \)
\[ t_0 < t_c : \quad P_0 > 4.1 \left[ \frac{n - 1}{2} E_{\text{SN,51}} \right]^{-1/2} \text{ ms} \]
\[ E_0 < 1.2 \times 10^{51} \left( \frac{n - 1}{2} \right) E_{\text{SN,51}} \text{ erg} \]

\[ t_{\text{ST}} = 519 M_3^{5/6} E_{\text{tot,51}}^{-1/2} n_0^{-1/3} \text{ yr} \]
\[ = 116 M_3^{5/6} E_{\text{tot,52.3}}^{-1/2} n_0^{-1/3} \text{ yr} \]

\[ R_{\text{ST}} = 3.07 M_3^{1/3} n_0^{-1/3} \text{ pc} \]

\[ E_{\text{tot}} = E_0 + E_{\text{SN}} \]
\[ E_0 = \frac{n - 1}{2} L_0 t_0 = \frac{1}{2} I \Omega_0^2 \approx 2 \times 10^{52} P_{0,\text{ms}}^{-2} \text{ erg} \]
\[ t_0 = 1.3 \times 10^5 \frac{2}{n - 1} f^{-1} B_{14}^{-2} P_{0,\text{ms}}^2 \text{ s} \]
Evolution of MWN & SNR Radii

\[(E_0 > E_{SN}), (E_0 < E_{SN})\]

\[R \quad R_{SNR}\]
Evolution of MWN & SNR Energies

\[ \begin{align*}
(E_0 > E_{SN}), \quad (E_0 < E_{SN}) & \quad E \quad \ldots \quad E_{SNR} \quad \ldots \quad (E_{SN} + E_{inj})
\end{align*} \]

\[ \begin{align*}
\text{Energy (erg)} \\
10^{52} & \quad 10^{51} & \quad 10^{50} \\
10^{49} & \quad 10^{48} & \quad 10^{47} \\
10^{-5} & \quad 10^{-3} & \quad 10^{-1} & \quad 10^1 & \quad 10^3 & \quad 10^5
\end{align*} \]

\[ t (\text{yr}) \]

\[ t_c, \quad t_0, \quad t_0, \quad t_{ST}, \quad t_{ST} \]
The MWN internal flow structure

- In contrast with the usual ideal MHD assumption (Kennel & Coroniti 84’) motivated by recent 3D RMHD simulations we assume a non-ideal low-σ flow (MHD → HD; BM76)

- In the inner-nebula there is a quasi-steady flow:

\[
\frac{\partial}{\partial t} (\tilde{n} \gamma) + \frac{c}{r^2} \frac{\partial}{\partial r} (r^2 \tilde{n} \gamma / \beta) = 0 , \quad \frac{d}{dt} P \tilde{n}^{4/3} = 0 , \quad \frac{d}{dt} (P \gamma^4) = \gamma^2 \frac{\partial P}{\partial t} \implies \beta(r) \approx \frac{1}{3} \left( \frac{r}{R_{TS}} \right)^{-2}
\]

- Velocity continuity with \(v_{SNR}\) at the outer radius \(R\) & equating the wind ram pressure to the nebula’s thermal pressure at \(R_{TS}\) implies a uniformly expanding outer region

\[
R_{TS,p}(t) = \sqrt{\frac{R(t)^3 L_{sd}(t)}{cE(t)}} \propto t^{(4a-m)/2}
\]

\[
\left( \frac{R_b(t)}{R(t)} \right)^3 = \frac{tL_{sd}(t)}{3aE(t)}
\]

\[
a = \frac{3}{2(5-k)} \implies 0.3 \ (t > t_{ST})
\]

\[\Delta t_{adv} \sim t \propto r^3\]
Synchrotron cooling time of X-ray emitting electrons (at 2E_2 keV) is \ll \text{system’s age} \Rightarrow \text{quasi-steady state:}

\[ t_{\text{syn}} = \frac{6\pi m_e c}{\sigma_T B^2 \gamma_e} \approx 1.02 B_{15 \mu G}^{-3/2} E_2^{-1/2} \text{ kyr} . \]

- Diffusion dominates over advection in whole MWN

\[ r_{c,\text{dif}} \approx \sqrt{2 \lambda_{\text{def}} c t_{\text{syn}} (\gamma_e)} \approx 1.57 B_{15 \mu G}^{-3/2} \zeta^{1/2} \text{ pc} \]

\[ \zeta \equiv \lambda_{\text{def}} / R_L \gtrsim 1 \quad (\zeta = 1 \text{ corresponds to Bohm diffusion}) \]

- Resulting cooling length \sim observed nebula size \( R_X \Rightarrow \text{may also explain the spectral softening} \)

\[ (\Gamma_X = 1.41 \pm 0.12 \text{ in the inner nebula to } \Gamma_X = 2.5 \pm 0.2 \text{ in the outer nebula}) \]
The Synchrotron X-Ray Nebula around Swift 1834.9–0846

- Magnetic field in the X-ray emitting region is:

\[
B = \left( \frac{L_X \sigma_e}{\mathcal{A} V} \frac{\Gamma - 2}{\Gamma - 1.5} \frac{\nu_1^{1.5 - \Gamma} - \nu_2^{1.5 - \Gamma}}{\nu_m^{2 - \Gamma} - \nu_M^{2 - \Gamma}} \right)^{2/7}
\]

\[ \approx \begin{cases} 
4.0 x \sigma_e^{2/7} d_4^{-2/7} \mu \text{G} & \text{(whole nebula)}, \\
5.0 x \xi_{in} \sigma_e^{2/7} d_4^{-2/7} \mu \text{G} & \text{(inner nebula)}.
\end{cases} \]

Assuming:

- Power-law electron distribution: \( \frac{dN_e}{d\gamma_e} \propto \gamma_e^{-p} \quad \gamma_1 < \gamma_e < \gamma_2 \)
- Electrons emitting in observed range: \( \gamma_1 < \gamma_m < \gamma_e < \gamma_M < \gamma_2 \)
- Magnetization: \( \sigma = \frac{B^2}{4\pi \omega} = \frac{3}{2} \frac{B^2}{8\pi e} = \frac{3}{2} \frac{E_B}{E_m} = \frac{3}{2} \sigma_e \epsilon_e \)
- Emission volume: \( V = \frac{4}{3} \pi R_x^3 \)

\[ \xi^{7/2}(\nu_1, \nu_2) = \left( \frac{\nu_2^{1.5 - \Gamma} - \nu_1^{1.5 - \Gamma}}{\nu_M^{1.5 - \Gamma} - \nu_m^{1.5 - \Gamma}} \right) = \text{ratio of energy in all electrons to that in those radiating in the observed frequency range} \]

Observation: XMM (0.5 – 10 keV)

\[
L_x = 2.5 \times 10^{33} d_4^2 \text{ erg s}^{-1} \quad \Gamma = 2.2 \pm 0.2 \quad \Gamma_{in} = 1.3 \pm 0.3 \quad \Gamma_{out} = 2.5 \pm 0.2
\]

NuSTAR detected inner nebula up to 30 keV

XMM + NuSTAR (0.5–30 keV) joint fit results:

\[
L_{in} = 5.0 \times 10^{32} d_4^2 \text{ erg s}^{-1} \quad \Gamma_{in} = 1.41 \pm 0.12
\]

X-ray efficiency of MWN

\[
\eta_X = \frac{L_{X,tot}}{L_{sd}} = 0.13 d_4^2
\]

\[
E_e = \epsilon_e E_m
\]

\( E_m = \text{total energy in matter in the emission region} \)
Constraints From Maximum Electron Lorentz Factor

(De Jager & Harding 92)

Maximum injected L.F.: \( \gamma_{\text{max}} = \frac{e R_{\text{NS}}^3 \Omega^2 B_s}{m_e c^4} = \frac{e}{m_e c^2} \sqrt{\frac{L_{\text{sd}}}{f c}} = \frac{4.9 \cdot 10^8}{\sqrt{f}} \)

\( \gamma_{\text{max}} > \gamma_M \)

Max. obs. energy:

\( E_X = 30 E_{M,30} \text{ keV} \)

which further yields

\[ \sigma_e > 0.043 d_4 \left( \frac{f E_{M,30}}{\xi} \right)^{\frac{7}{2}}, \quad \xi > 16.1 f E_{M,30} \left( \frac{\epsilon_e d_4}{\sigma_{-2.5}} \right)^{\frac{2}{7}} \]

\[ \sigma_{e,\text{in}} > 0.055 d_4 \left( \frac{f E_{M,30}}{\xi_{\text{in}}/5} \right)^{\frac{7}{2}}, \quad \xi_{\text{in}} > 12.7 f E_{M,30} \left( \frac{\epsilon_e d_4}{\sigma_{\text{in},-2.5}} \right)^{\frac{2}{7}} \]

Synchrotron cooling time of X-ray emitting electrons:

(at 2E_2 keV) is \(<\ll\) system’s age \(\rightarrow\) quasi-steady state

Electron energy balance:

\[ \langle \dot{E} \rangle = g L_{\text{sd}} = \frac{(1 + \sigma)}{\epsilon_e \epsilon_X} L_{X,\text{tot}} \]

In terms of the observed X-ray efficiency:

\[ g = \frac{L_{X,\text{tot}}}{L_{\text{sd}}} (1 + \sigma) \xi^{7/2} = \eta_X (1 + \sigma) \xi^{7/2} \]

Estimate of \( \xi_{\text{in}} \) from the inner nebula yields:

\[ \frac{g \sigma}{1 + \sigma} > 3.07 d_4^3 E_{M,30}^{7/2} f^{7/2} \]

\( t_{\text{syn}} = \frac{6\pi m_e c}{\sigma_T B^2 \gamma_e} \approx 1.02 B_{15}^{-3/2} E_2^{-1/2} \text{ kyr} \)

\((1 + \sigma)^{-1} = \text{fraction of the total energy injected into the nebula going to particles (the rest goes into the B-field)}\)

\( \epsilon_e = \text{fraction of that going into power-law energy dist. of electrons} \)

\( \epsilon_X = \xi^{-7/2} = \text{fraction of that going into electrons radiating observed X-rays} \)
What powers the MWN? Rotational energy is not enough

\[ g = \frac{\langle \dot{E} \rangle}{L_{sd}} \Rightarrow \eta_{X,\text{true}} = \frac{L_{X,\text{tot}}}{\langle \dot{E} \rangle} = \frac{\eta_X}{g} = \frac{0.0026d_4^2}{g_{50}} < 0.042 \frac{\sigma}{1 + \sigma} d_{4}^{-1} E_{M,30}^{-\frac{7}{2}} f^{-\frac{7}{2}} \]

**Lower X-ray efficiency:** \( \langle \dot{E} \rangle = gL_{sd} \Rightarrow \eta_{X,\text{true}} = \frac{L_{X,\text{tot}}}{\langle \dot{E} \rangle} = \frac{\eta_X}{g} = \frac{0.0026d_4^2}{g_{50}} < 0.042 \frac{\sigma}{1 + \sigma} d_{4}^{-1} E_{M,30}^{-\frac{7}{2}} f^{-\frac{7}{2}} \)

**Electron energy balance:**
\[ \langle \dot{E} \rangle = gL_{sd} = \frac{(1 + \sigma)}{\epsilon_{e}\epsilon_{X}} L_{X,\text{tot}} \]

In terms of the observed X-ray efficiency:
\[ g = \frac{L_{X,\text{tot}}}{L_{sd}} \frac{(1 + \sigma)\xi^{7/2}}{\epsilon_{e}} = \frac{\eta_X(1 + \sigma)\xi^{7/2}}{\epsilon_{e}} \]

Estimate of \( \xi_{in} \) from the inner nebula yields:
\[ \frac{g\sigma}{1 + \sigma} > 3.07d_4^3 E_{M,30}^{7/2} f^{7/2} \]

(1 + \sigma)^{-1} = fraction of the total energy injected into the nebula going to particles (the rest goes into the B-field)

\( \epsilon_{e} = \) fraction of that going into power-law energy dist. of electrons

\( \epsilon_{X} = \xi^{-7/2} = \) fraction of that going into electrons radiating observed X-rays

\[ \boxed{\text{Allowed region}} \]

\[ \boxed{\text{Excluded region}} \]
Can the decaying dipole field power the MWN?

- A potential energy source is the decay of the super-strong magnetar dipole magnetic field

Dipole Field Decay:

\[ B_s(t) = B_0 \left(1 + \frac{t}{t_B}\right)^{-1/\alpha} \equiv B_0 \tau^{-1/\alpha} \]

\[ \dot{E}_{\text{dip}} = \frac{d}{dt} \left( \frac{B_s^2 R_{\text{NS}}^3}{6} \right) = -\frac{2}{\alpha} \frac{E_{B,\text{dip}}(t)}{t_B \tau} \]

Since \( t_B \) is not well constrained, we find which decay timescale maximizes the the power

\[ |\dot{E}_{B,\text{dip}}|_{\text{max}} = \frac{2}{2 + \alpha} \frac{E_{B,\text{dip}}(t)}{t} \]

\[ t_B = 2t/\alpha \quad \tau_{\text{max}} = 1 + \alpha/2 \]

Comparison of this power with that required to power the nebula gives

\[ \frac{|\dot{E}_{B,\text{dip}}|_{\text{max}}}{gL_{\text{sd}}} = 1.25 \times 10^{-3} g_{50}^{-1} f^{-1} t_{4.5}^{-1} \]

\( g = 50g_{50} \quad t_{\text{SNR}} = 10^{4.5} t_{4.5} \text{ yr} \quad \alpha = 3/2 \)

Decay of dipole field alone cannot supply the requisite power \( \langle \dot{E} \rangle = gL_{\text{sd}} \)

(motivated by Dall'Osso, JG & Piran 2012)
Decay of the stronger internal magnetic field $B_{\text{int}}$ is needed

Internal Field Decay: We demand that the maximum field decay power slightly exceeds the required power due to inefficiencies in power transfer to particles

$$\left| \dot{E}_{B,\text{int}} \right|_{\text{max}} = \frac{2}{2 + \alpha} \frac{E_{B,\text{int}}(t)}{t} \geq g L_{sd} \quad \Rightarrow \quad B_{\text{int}}(t) \geq 3.3 \times 10^{15} g_{50}^{1/2} t_{4.5}^{1/2} \, \text{G}$$

- Stability requires that $B_{\text{int}}/B_{\text{dip}}$ cannot be too large; Simulations find (Braithwaite 2009):
  
  (I) the configuration is stable when both poloidal & toroidal components exist
  (II) the ratio of the two components is constrained

$$\frac{AE_{B,\text{int}}/E_G}{\simeq 10^{-3}} \lesssim \frac{E_{B,\text{dip}}/E_{B,\text{int}}}{\lesssim 0.8}$$

- The lower limit yields the maximum internal field

$$B_{\text{int}} \lesssim B_{\text{int, max}} = \left( \frac{6E_G B_{\text{dip,0}}^2}{AR_{\text{NS}}^3} \right)^{1/4} = 6.6 \times 10^{15} B_{\text{dip,0,15}}^{1/2} \, \text{G}$$

- Consistent with the results of Dall’Osso et al. (2012), which favor a young age $t \lesssim 10^{4.5}$ yr

$B_{\text{int,0}} \sim (1 - 3) \times 10^{16} \, \text{G} \, , \quad \alpha_{\text{int}} \sim 1 - 1.5 \, , \quad t_{B_{\text{int}}} \sim 7 - 10 \, \text{kyr}$
Assuming the fiducial age: $t_{\text{SNR}} = 10^{4.5}$ yr

\[ g = \frac{\langle E \rangle}{L_{\text{sd}}} \]

- Excluded region
- Allowed region

$B_{\text{int, max}} = 10^{16.5}$ G

$B_{\text{int, max}} = 10^{16}$ G
Energy injection through bursts & flares

- A natural mechanism for additional energy injection into the MWN is through bursts and giant flares, but how many bursts are required?

- Energy distribution of magnetar bursts appear to follow a power-law:

  \[ E^2 \frac{dN}{dE} \propto E^{-s+2} \]

  \[ s = 1.4 - 1.8 \]

- Similar distribution is found for Earth quakes and in simulations of stressed elastic medium, which suggests that magnetar bursts are “star quakes” and may indeed be a self-critical phenomena.

  \[ \frac{E}{\langle \dot{E} \rangle} \sim 100g_{50}^{-1}E_{45.5} \text{ yr} \]
Conclusions:

- A small natal kick might help MWN detectability.
- Possible new internal nebula flow structure (for non-ideal low-\( \sigma \) flow).
- X-ray Nebula Size: may be \( \sim \) the diffusion dominated cooling length.
- Steady-state X-ray emission: energy balance \( \Rightarrow \dot{E}_{\text{rot}} \) is insufficient.
  \[
g = \frac{\langle \dot{E} \rangle}{L_{\text{sd}}} > g_{\text{min}} = 3.07 \left( \frac{1 + \sigma}{\sigma} \right) a_4^3 E_{M,30}^{7/2} f^{7/2}
\]
- An alternative energy source is needed:
  - magnetar’s dipole B-field decay is not enough \( \times \)
  - magnetar’s internal B-field decay is enough \( \checkmark \)
- Energy from the B-field decay may be injected into the MWN via outflows associated with regular busts or giant flares.