

Relativistic Jet Acceleration, Collimation & Stability

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Outline of the talk:

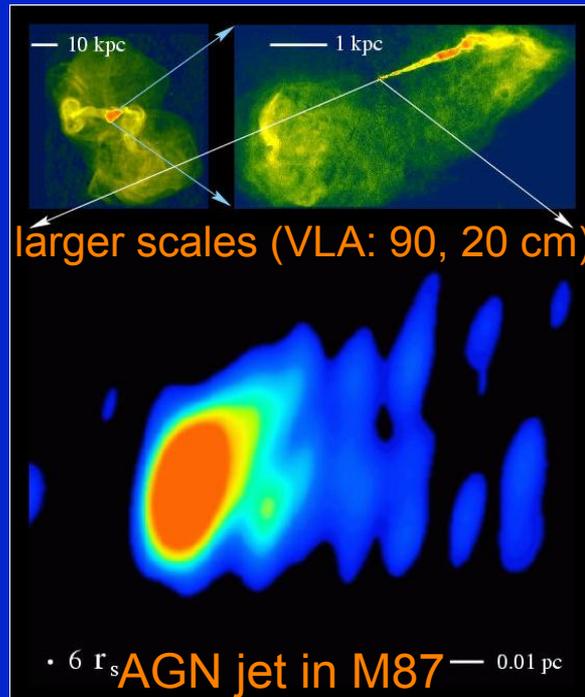
- Magnetic vs. thermal acceleration
- Role of external confinement
- Standard collimation-induced magnetic acceleration: steady, ideal MHD, axi-symmetric
- Problems with this standard picture \Rightarrow alternatives
- Impulsive magnetic acceleration
(JG, Spitkovsky & Komissarov 2011; JG 2012)
- Poynting dominated GRB jet propagating in a star
(Bromberg, JG, Lyubarsky & Piran 2014)
- Conclusions

Relativistic Magnetic Acceleration:

- Relativistic ($v \approx c$) outflows/jets are very common in astrophysics & involve strong gravity at the source: PWN (NS), GRBs, **AGN** (SMBH), **μ -quasars** (BH/NS)
- Most models assume a steady flow for simplicity, despite observational evidence for time variability

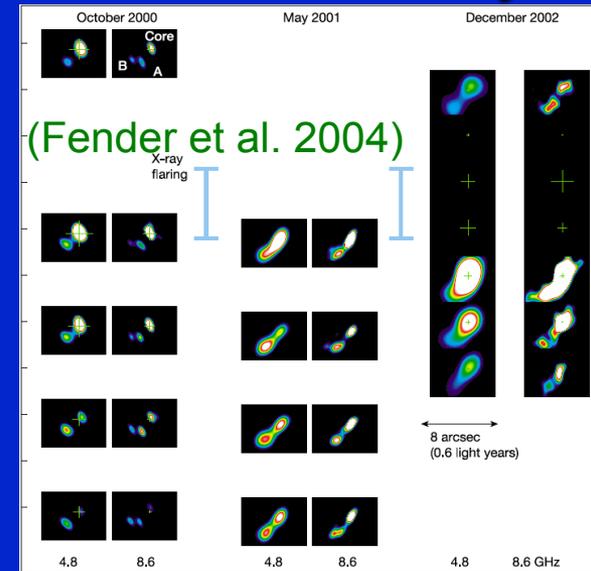


Crab Nebula: X-ray in blue, optical in red



larger scales (VLA: 90, 20 cm)

• 6 r_s AGN jet in M87 — 0.01 pc
(VLBA @ 43 GHz)



(Fender et al. 2004)

Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)

Relativistic Magnetic Acceleration:

Is the acceleration magnetic?

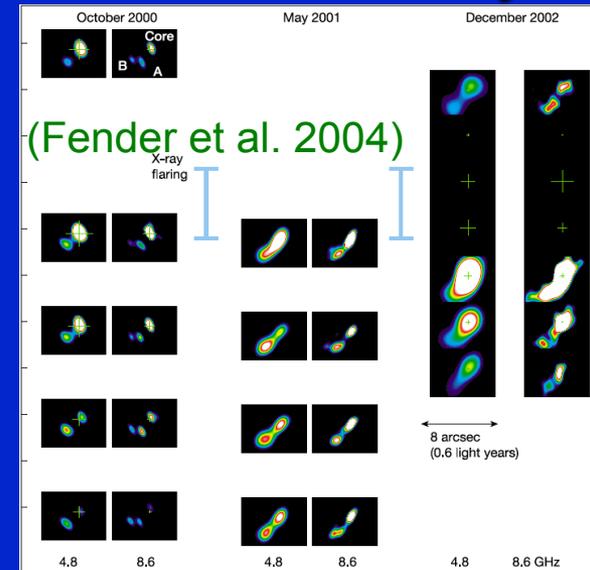
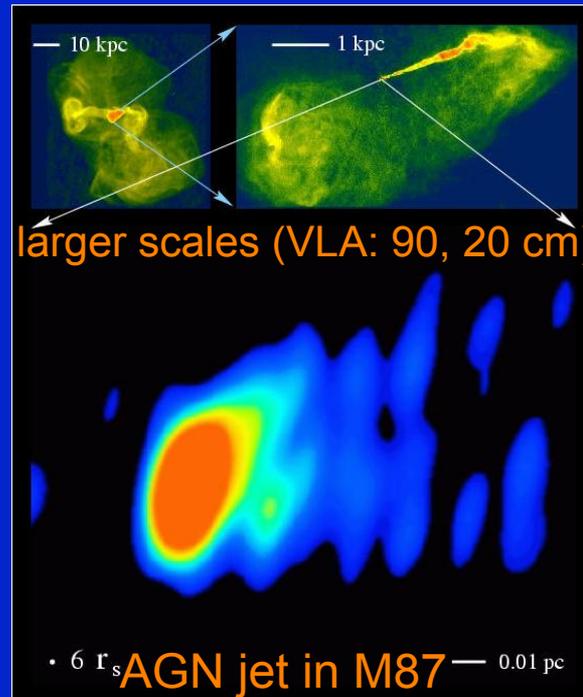


PWN (NS), GRBs, **AGN** (SMBH), **μ -quasars** (BH/NS)

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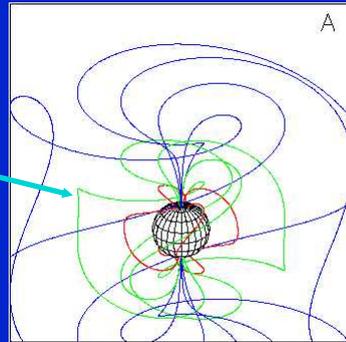
Crab Nebula: X-ray in blue, optical in red



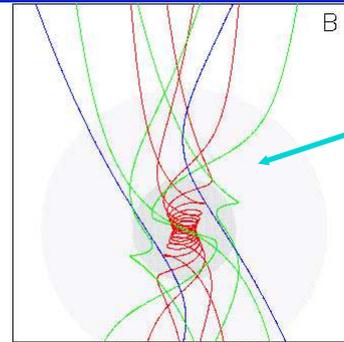
Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)

All these sources likely share a common basic mechanism, in which relativistic outflows are launched hydromagnetically

Pulsar magnetosphere

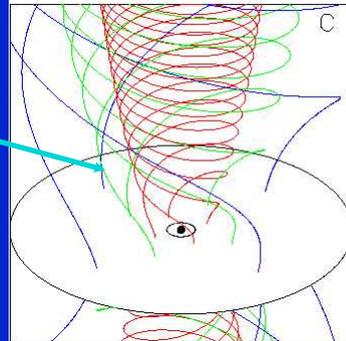


Collapsing, magnetized supernova core

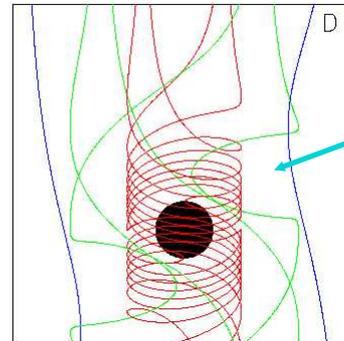


Courtesy to David Meier

Magnetized accretion disks around neutron stars and black holes



Magnetospheres of Kerr black holes



A rapidly spinning central body twists up the magnetic field into a toroidal component & plasma is ejected by the magnetic tension.

Magnetic vs. Thermal Acceleration:

- Hydromagnetic launching naturally helps avoid a high baryon loading, which can greatly limit the maximal possible asymptotic Lorentz factor Γ_{∞}

Key difference between thermal and magnetic steady state acceleration of relativistic supersonic flows:

- **Thermal:** fast, robust & efficient
- **Magnetic:** slow, delicate & less efficient

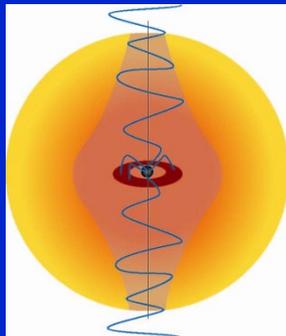
Force balance in Poynting dominated flows:

Does the huge tension of wound up magnetic field (hoop stress) compress the flow towards the axis?

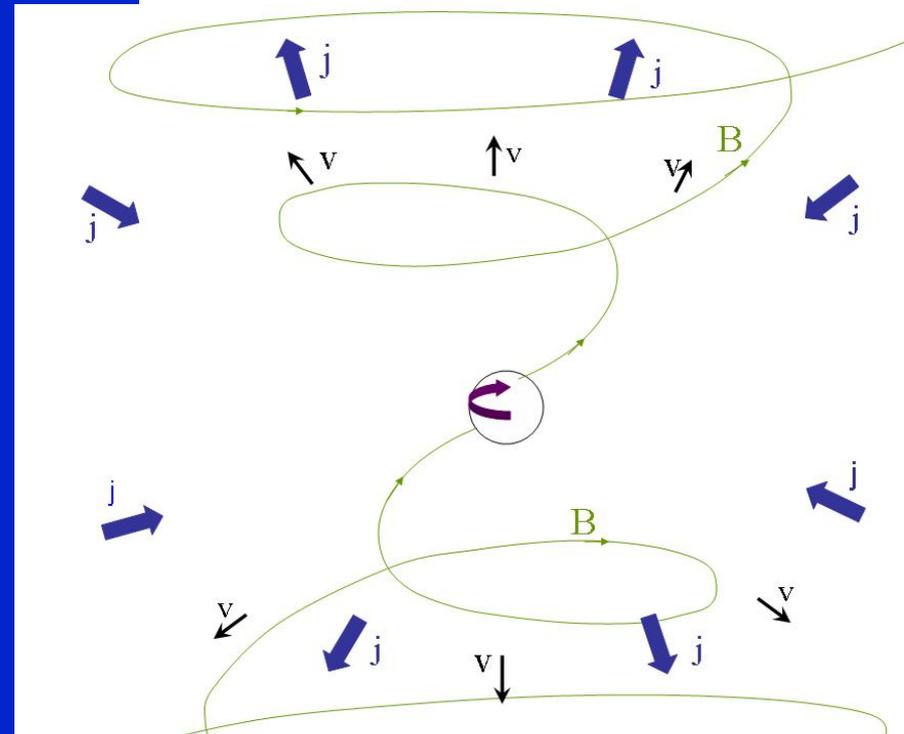
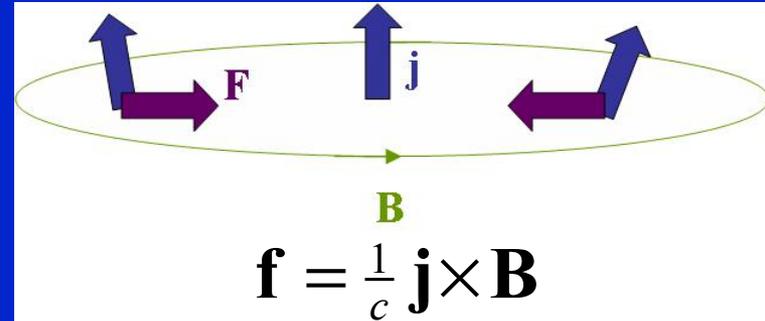
No!

In the current closure region, the force is decollimating.

The flow as a whole cannot be collimated without external confinement!



Magnetic hoop stress



Force balance in Poynting dominated flows:

Total EM force: $\mathbf{f} = \rho_e \mathbf{E} + \frac{1}{c} \mathbf{j} \times \mathbf{B} \approx 0$

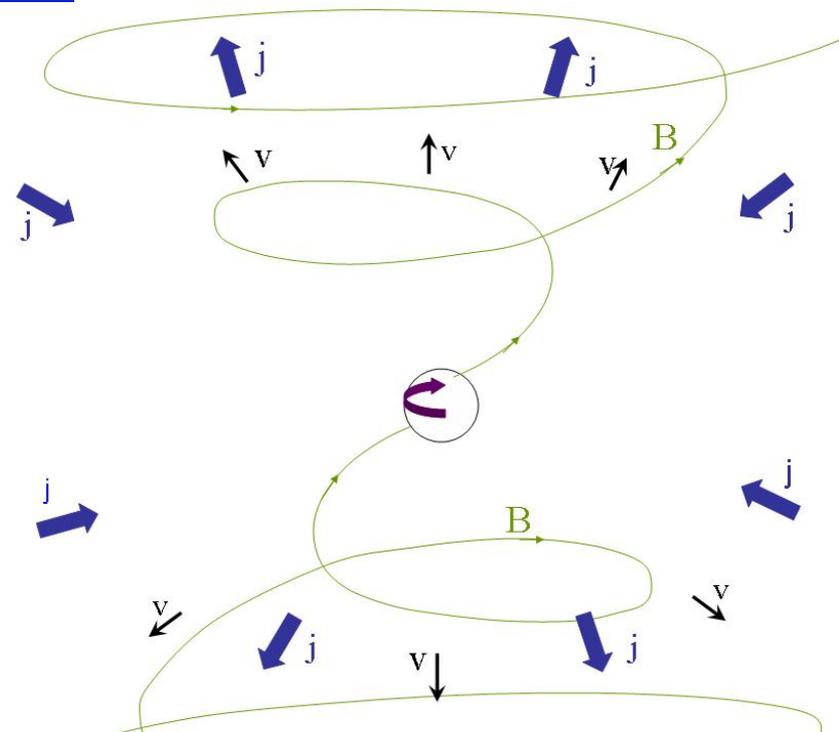
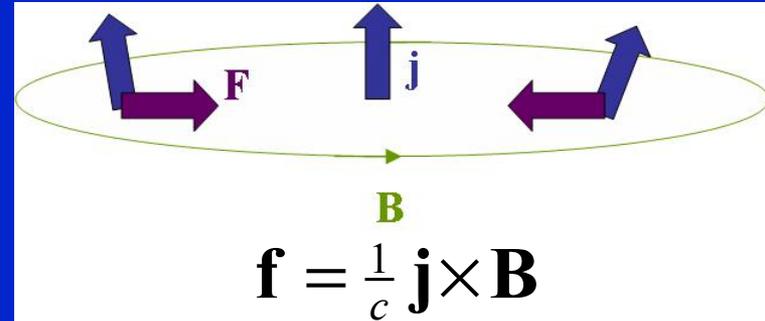
$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = 0 \quad \rho_e = \frac{1}{4\pi} \nabla \cdot \mathbf{E}$

In the far zone, $v \rightarrow c$ and $\mathbf{E} \rightarrow \mathbf{B}$.

In highly relativistic flows, the Lorentz and electric forces nearly cancel each other.

Acceleration & collimation are only due to a small residual force!

Magnetic hoop stress



Ideal MHD acceleration: numerical + analytic results (Komissarov+ 09; Lyubarsky 09; Tchekhovskoy+10)

- **Unconfined** flows quickly lose lateral causal contact, become radial & stop accelerating when

$$\Gamma_{\infty} \sim \sigma_0^{1/3} \quad \& \quad \sigma_{\infty} \sim \sigma_0^{2/3} \gg 1$$

(Goldreich & Julian 1970; Tomimatsu 1994; Beskin et al. 1998)

- **Weak confinement:** $p_{\text{ext}} \propto z^{-\alpha}$ with $\alpha > 2 \Rightarrow$ lose lateral causal contact, become conical & stop accelerating later:

loss of causal contact: $\Gamma_{\infty} \sim \sigma_0^{1/3} \theta_{\text{jet}}^{-2/3} \quad \sigma_{\infty} \sim (\sigma_0 \theta_{\text{jet}})^{2/3}$

efficient conversion: $\Gamma_{\infty} \theta_{\text{jet}} < 1$

- **Strong confinement:** $p_{\text{ext}} \propto z^{-\alpha}$ with $\alpha < 2 \Rightarrow$ stay in causal contact $\Gamma \propto z^{\alpha/4}$ and reach $\Gamma_{\infty} \sim \sigma_0, \quad \sigma_{\infty} \sim 1$ ($\Gamma_{\infty} \theta_{\text{jet}} \leq 1$)

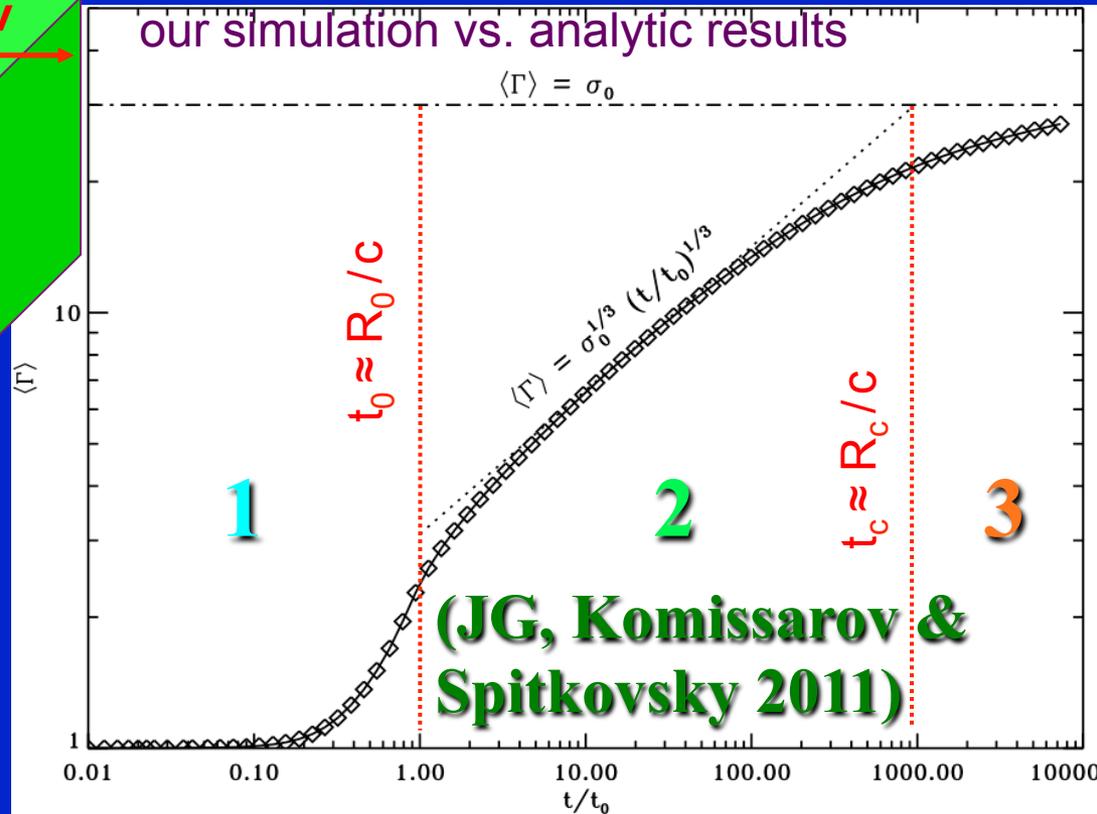
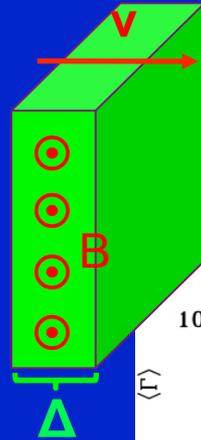
The “ σ -problem”: for a “standard” steady ideal MHD axisymmetric flow

- $\Gamma_\infty \sim \sigma_0^{1/3}$ & $\sigma_\infty \sim \sigma_0^{2/3} \gg 1$ for a spherical flow; $\sigma_0 = B_0^2 / 4\pi\rho_0 c^2$
- ◆ In PWN the solution is dissipation of the striped wind
- ◆ However, this doesn't work in relativistic jet sources
- Jet **collimation** helps, but not enough: $\Gamma_\infty \sim \sigma_0^{1/3} \theta_{\text{jet}}^{-2/3}$,
 $\sigma_\infty \sim (\sigma_0 \theta_{\text{jet}})^{2/3}$ & $\Gamma \theta_{\text{jet}} \lesssim \sigma^{1/2}$ (~ 1 for $\Gamma_\infty \sim \Gamma_{\text{max}} \sim \sigma_0$)
- Still $\sigma_\infty \gtrsim 1 \Rightarrow$ inefficient internal shocks, $\Gamma_\infty \theta_{\text{jet}} \gg 1$ in GRBs
- Sudden drop in external pressure can give $\Gamma_\infty \theta_{\text{jet}} \gg 1$ but still $\sigma_\infty \gtrsim 1$ (Tchekhovskoy et al. 2009) \Rightarrow inefficient internal shocks

Alternatives to the “standard” model

- ~~Axisymmetry~~: non-axisymmetric instabilities (e.g. the current-driven kink instability) can tangle-up the magnetic field & lead to significant dissipation (Begelman; Spruit; Eichler; Lyubarsky; Giannios;...)
- ~~Ideal MHD~~: a striped wind can dissipate its energy magnetic energy \rightarrow heat (+radiation) \rightarrow kinetic energy (Drenkham & Spruit 2002; Lyubarsky 2010)
- ~~Steady-state~~: **effects of strong time dependence** (JG, Komissarov & Spitkovsky 2011; JG 2012a, 2012b)

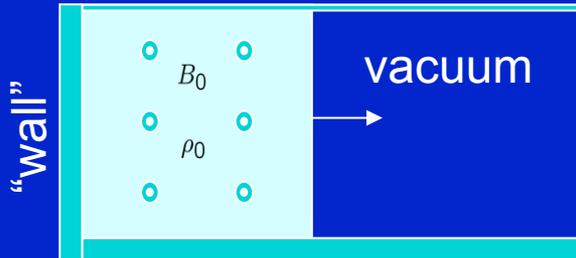
Impulsive Magnetic Acceleration: $\Gamma \propto R^{1/3}$



Useful case study:

Initial value of magnetization parameter:

$$\sigma_0 = \frac{B_0^2}{4\pi\rho_0 c^2} \gg 1$$

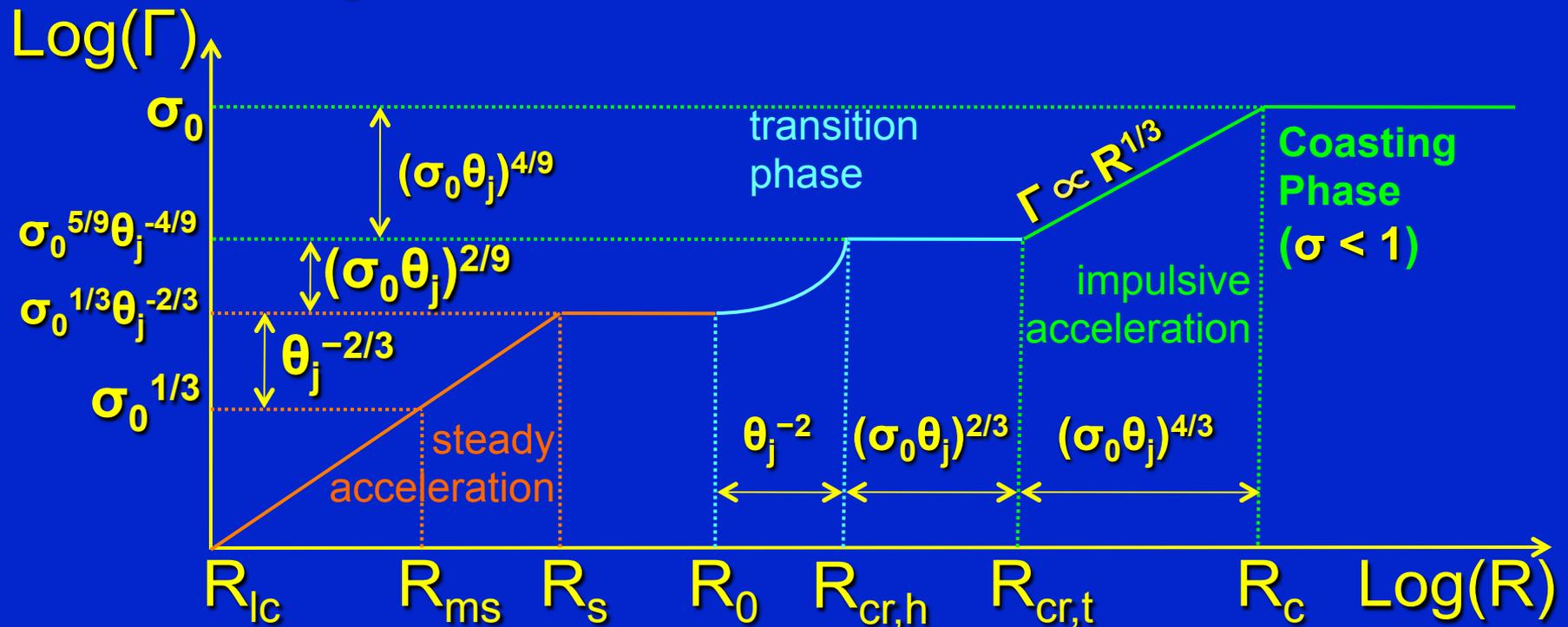


1. $\langle \Gamma \rangle_E \approx \sigma_0^{1/3}$ by $R_0 \sim \Delta_0$
2. $\langle \Gamma \rangle_E \propto R^{1/3}$ between $R_0 \sim \Delta_0$ & $R_c \sim \sigma_0^2 R_0$ and then $\langle \Gamma \rangle_E \approx \sigma_0$
3. At $R > R_c$ the shell spreads as $\Delta \propto R$ & $\sigma \sim R_c/R$ rapidly drops

- Complete conversion of magnetic to kinetic energy!
- This allows efficient dissipation by shocks at large radii

1st Steady then Impulsive Acceleration

- Our test case problem has **no central engine**: it may be, e.g., directly applicable for giant flares in SGRs; however:
- In most astrophysical relativistic (jet) sources (GRBs, **AGN**, μ -quasars) the variability timescale ($t_v \approx R_0/c$) is long enough ($> R_{ms}/c$) that **steady acceleration** operates & saturates (at R_s)
- Then the **impulsive acceleration** kicks in & leads to $\sigma < 1$



Many sub-shells: acceleration, collisions

(JG 2012b)

Flux freezing (ideal MHD):

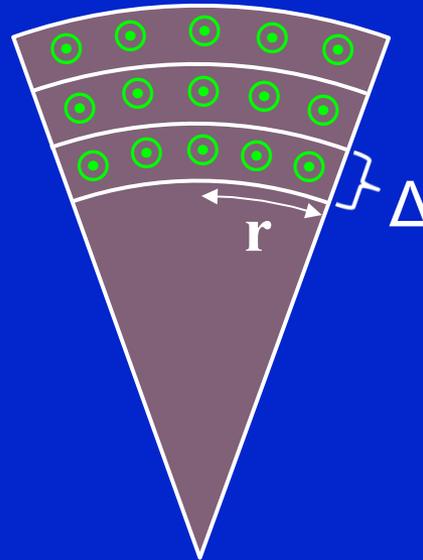
$$\Phi \sim B r \Delta = \text{constant}$$

$$E_{EM} \sim B^2 r^2 \Delta \propto 1/\Delta$$

$$\frac{\text{total energy}}{\text{rest energy}} = (1 + \sigma)\Gamma$$

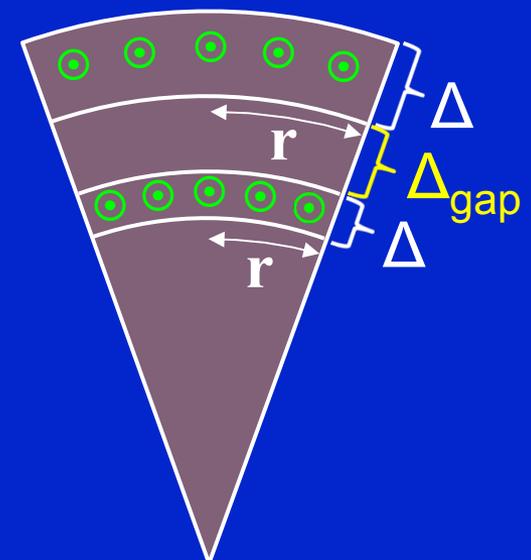
$$\text{acceleration } (\Gamma \uparrow) \Leftrightarrow \sigma \downarrow$$

steady



constant shell width Δ

impulsive

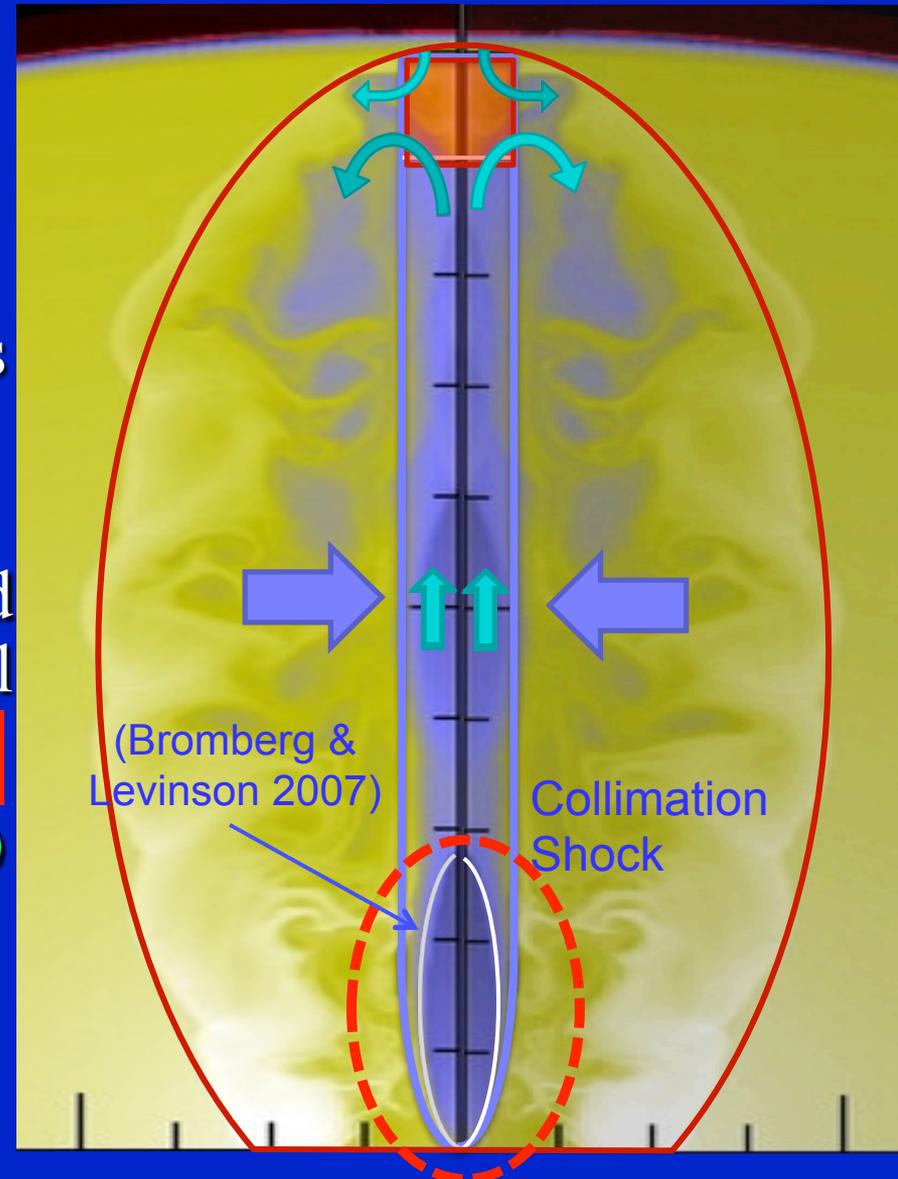


shell width Δ grows

- For a long lived variable source (e.g. AGN), each sub shell can expand by $1 + \Delta_{\text{gap}}/\Delta_0 \Rightarrow \sigma_{\infty} = (E_{\text{total}}/E_{EM,\infty} - 1)^{-1} \sim \Delta_0/\Delta_{\text{gap}}$
- For a finite # of sub-shells the merged shell can still expand
- Sub-shells in GRBs can lead to a low-magnetization thick shell & enable the outflow to reach higher Lorentz factors

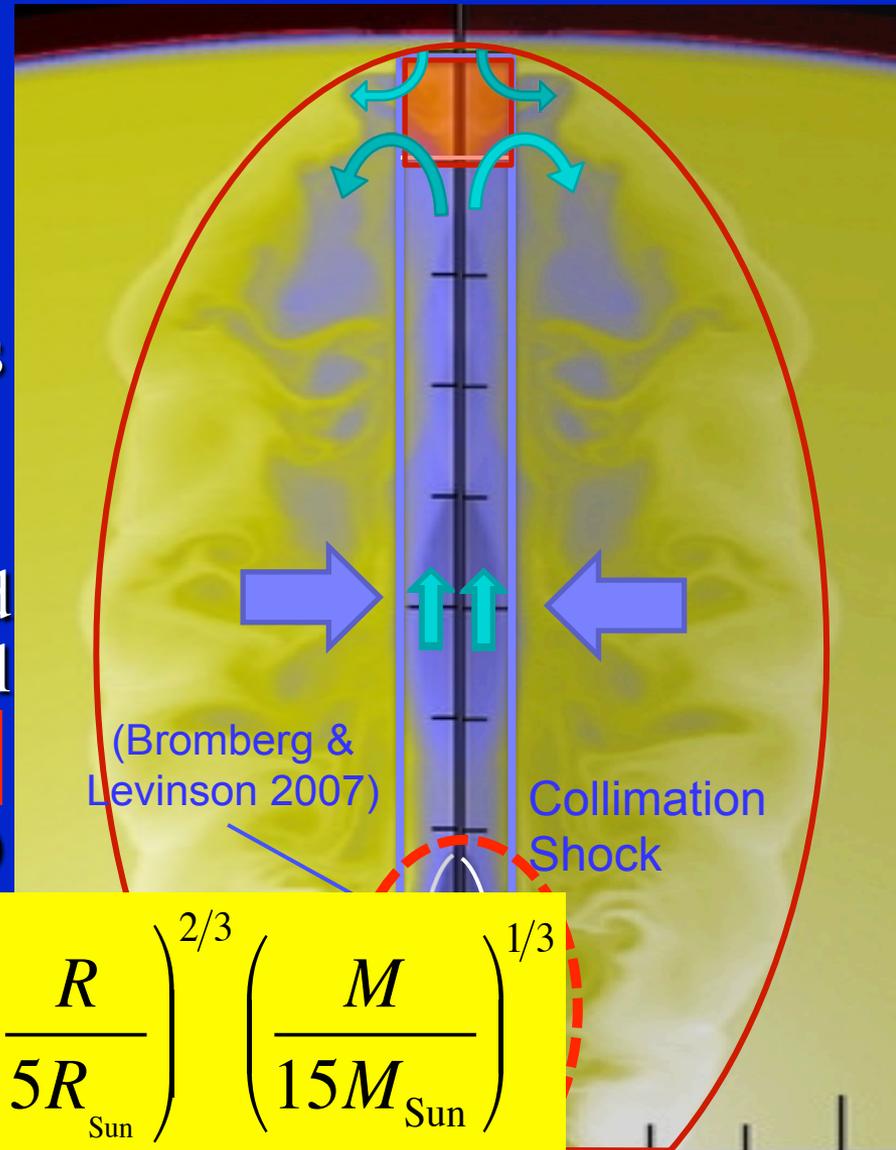
Hydrodynamic GRB Jet in its parent star

- The Jet develops a slow-moving ‘head’, where there is a pressure balance between the shocked jet material & external medium
- At the head jet matter decelerates by a reverse shock, flows sideways & forms a high-pressure cocoon that collimates the jet
- To propagate the head must be fed by jet material & the jet would fail if engine stops before $z_h \cong R(1 - \beta)$
- Breakout time (Bromberg et al. 2011)



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$$t_b \cong 15 \text{ sec} \left(\frac{L_{\text{iso}}}{10^{51} \text{ erg / s}} \right)^{-1/3} \left(\frac{\theta_0}{10^\circ} \right)^{2/3} \left(\frac{R}{5R_{\text{Sun}}} \right)^{2/3} \left(\frac{M}{15M_{\text{Sun}}} \right)^{1/3}$$

GRB Jet propagation in its parent star: highly magnetized vs. hydrodynamic jets

- The flow must decelerate to match its head velocity, but for high- σ a shock can't do it \Rightarrow the jet converges near its head
- Narrower head \Rightarrow larger head velocity \Rightarrow faster jet breakout
- Relativistic head \Rightarrow less energy into cocoon & supernova
- The head velocity is independent of the detailed jet structure \Rightarrow simplifies the model & allows (semi-) analytic solutions

(Bromberg, JG,
Lyubarsky & Piran
2014)

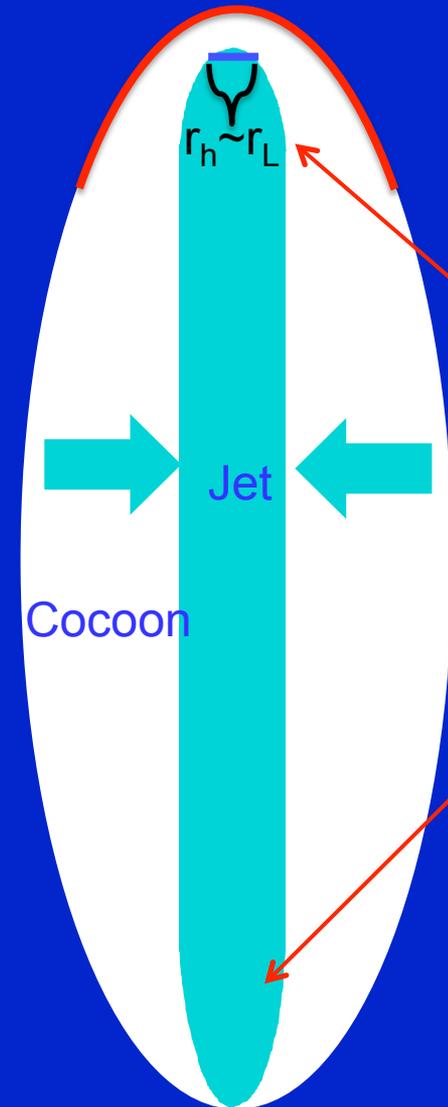
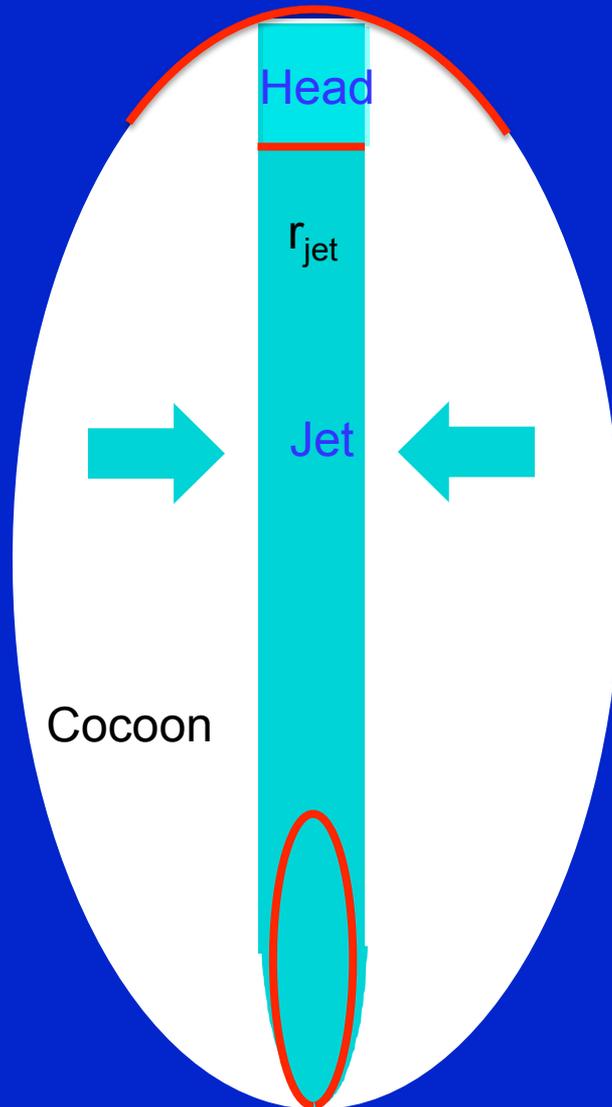


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- The head velocity is independent of the detailed jet structure \Rightarrow simplifies the model & allows (semi-) analytic solutions
- **Levinson & Begelman (2013)**: current-driven instabilities dissipate most of the magnetic field \rightarrow a hydrodynamic jet
- This is still unclear & strongly affects the jet dynamics

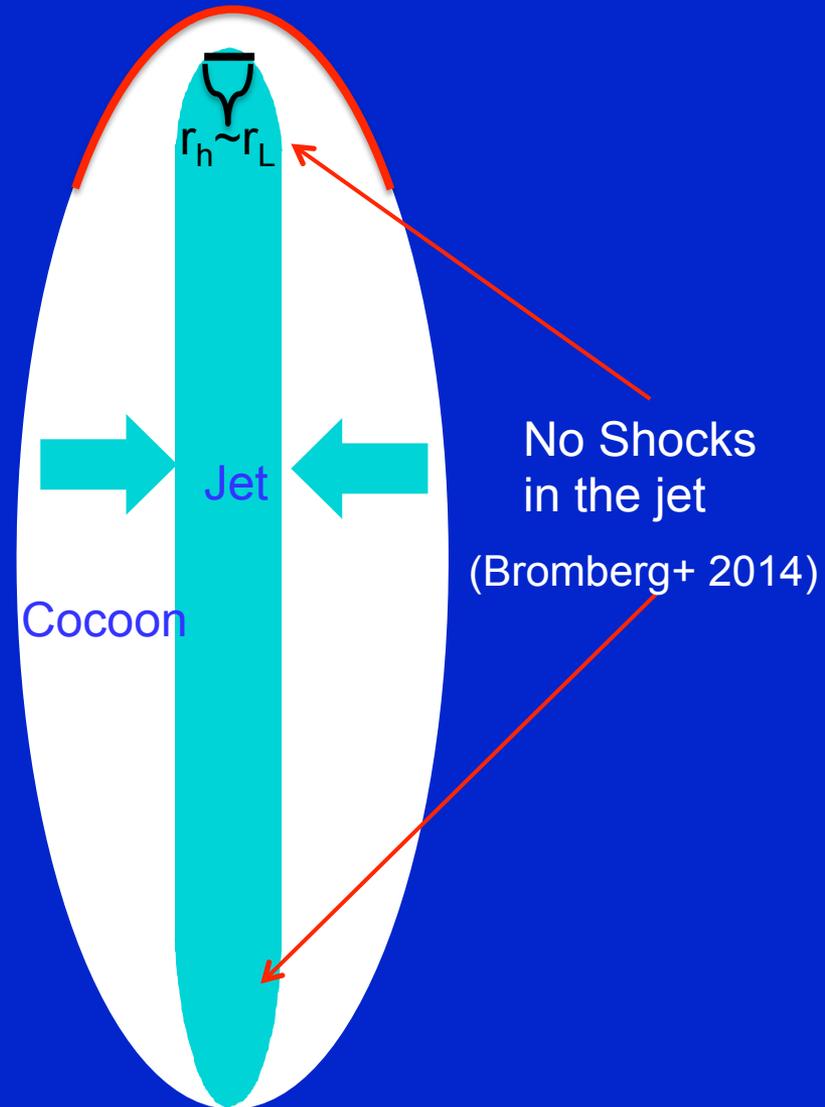
Magnetic Jet Propagation

(Bromberg, JG,
Lyubarsky & Piran
2014)



No Shocks
in the jet

Magnetic Jet Propagation



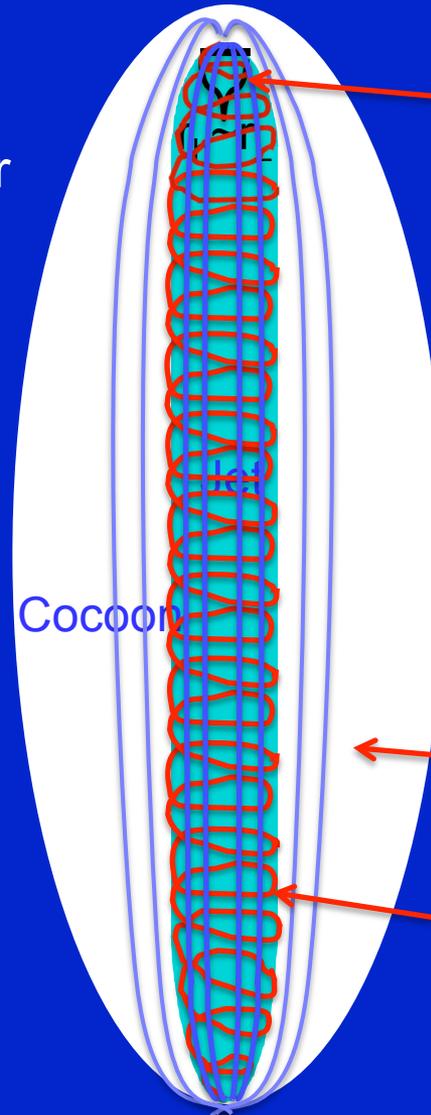
Magnetic Jet Propagation

Stability of the jet

Purely toroidal fields suffer from the kink instability

But as the jet is in lateral equilibrium, $B'_\theta \sim B'_p$ and this helps to make it more stable

Our analysis shows the the jet is at most only mildly unstable and is likely crosses the star largely intact.



$$r_h \cong r_L \left(\frac{L_j}{\pi r_L^2 \rho c^3} \right)^{1/6}$$

Bromberg+ 2014 ; Levinson & Begelman 2012

The cocoon radius evolves like a cylindrical BM / ST solution.
Uniform pressure in r direction

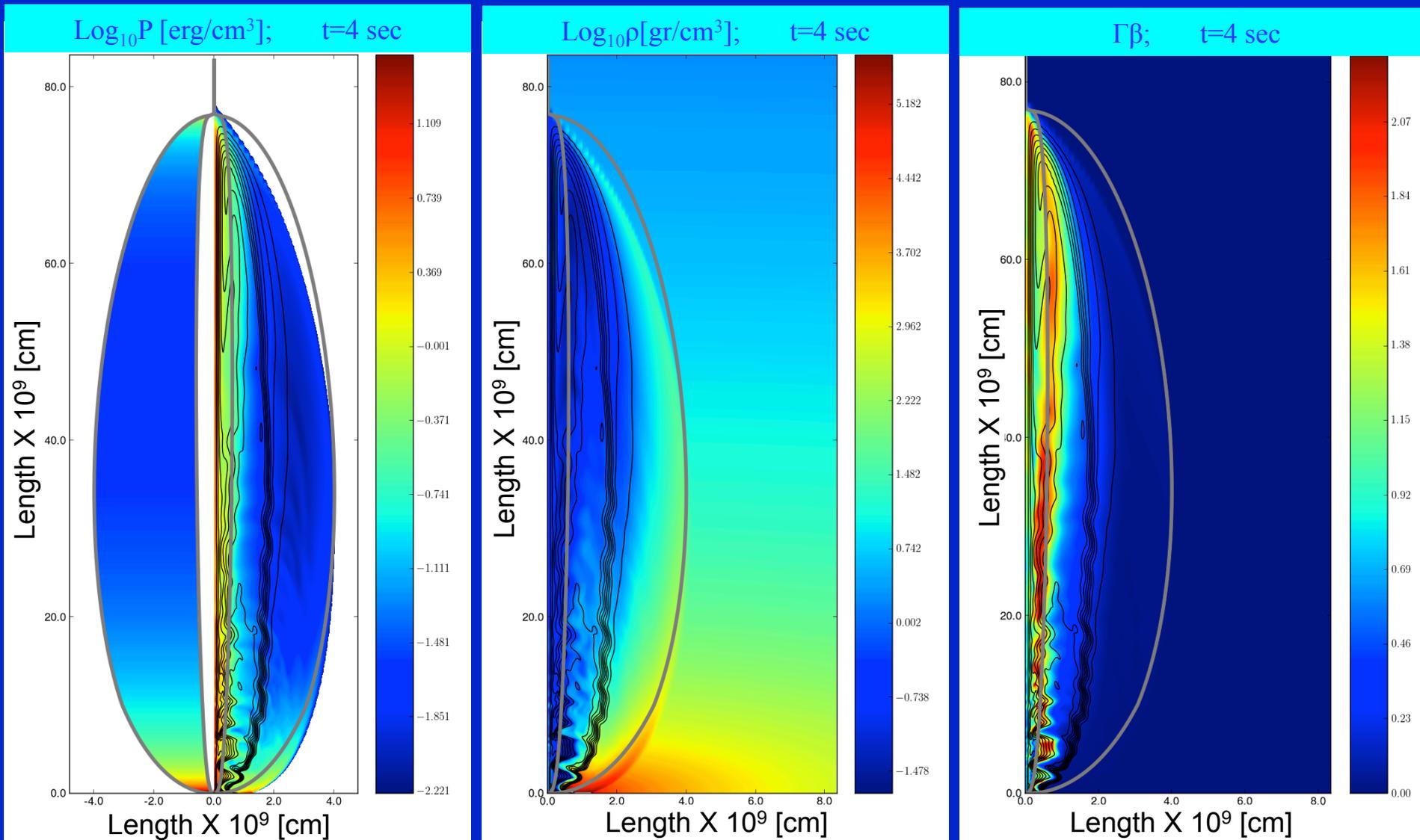
$$r_c(z) \sim \rho(z)^{-1/6} \sqrt{z_h - z}$$

$$P_c(z) \sim \rho(z)^{2/3} (z_h - z)^{-1}$$

$$r_j \sim P_c(z)^{-1/4}$$

Comparison with Simulations

(Bromberg & Tchekhovskoy 2014 in prep)



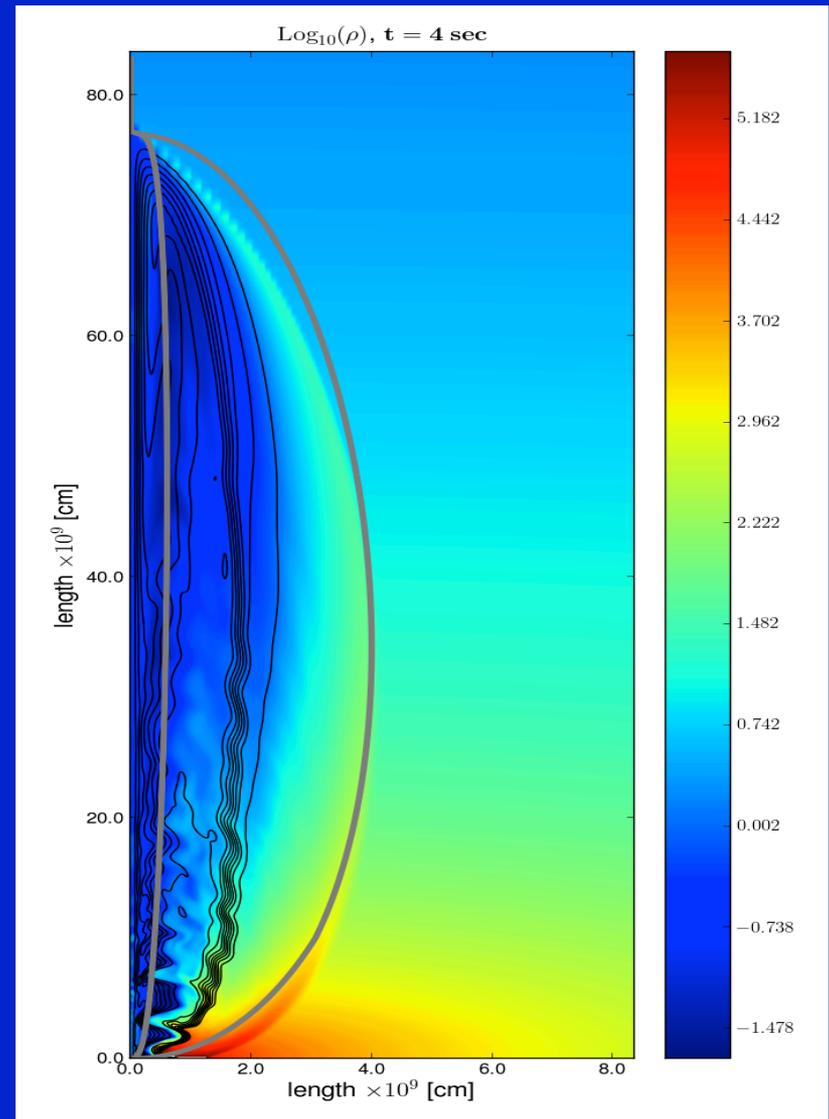
Magnetic jet breakout time

- The jet becomes relativistic deep in the star.
- It crosses the star at a time comparable to R/c
- The engine minimal activity time in this case:

$$t_b \cong \left(\frac{R}{\beta c} \right) (1 - \beta) \cong \frac{R}{2\Gamma^2 c}$$

(Bromberg, JG, Lyubarsky & Piran 2014b)

(Bromberg & Tchekhovskoy 2014 in prep)



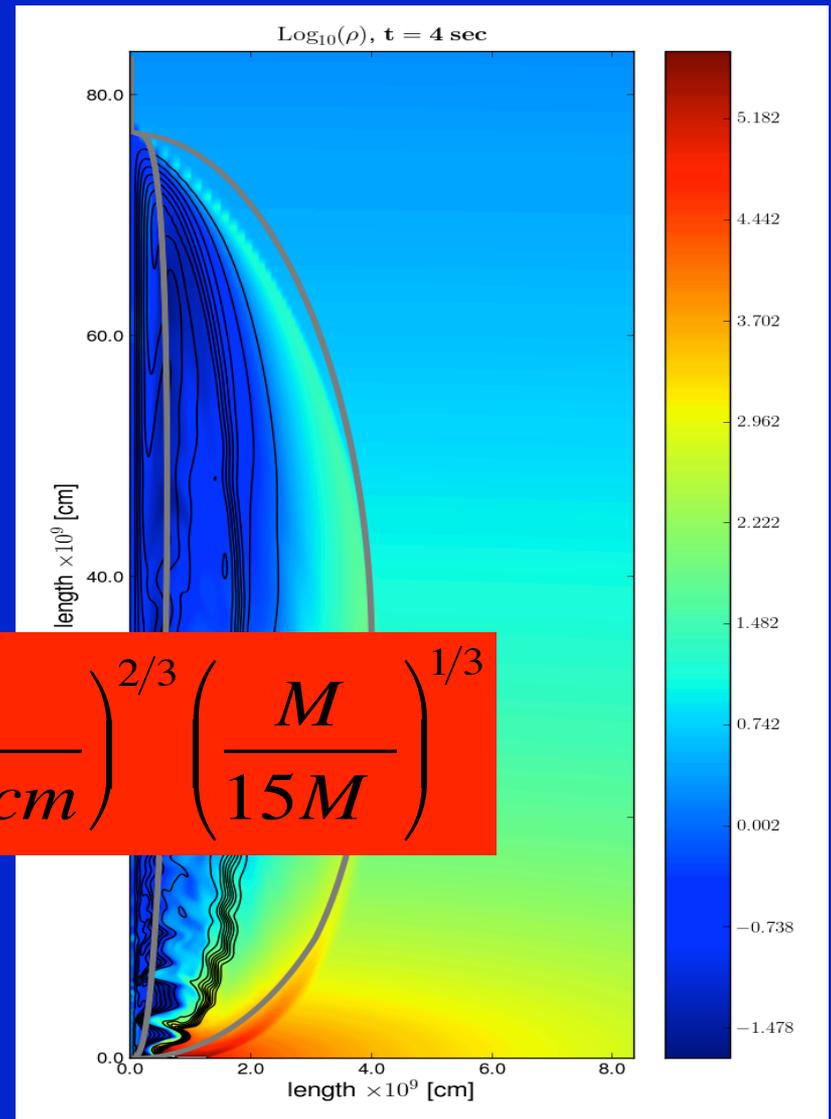
Magnetic jet breakout time

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$$t_b \cong 2 \text{ sec} \left(\frac{L_{iso}}{10^{51} \text{ erg / s}} \right)^{-1/3} \left(\frac{r_L}{5 * 10^7 \text{ cm}} \right)^{2/3} \left(\frac{M}{15M} \right)^{1/3}$$

(Bromberg, JG, Lyubarsky & Piran 2014b)

(Bromberg & Tchekhovskoy 2014 in prep)



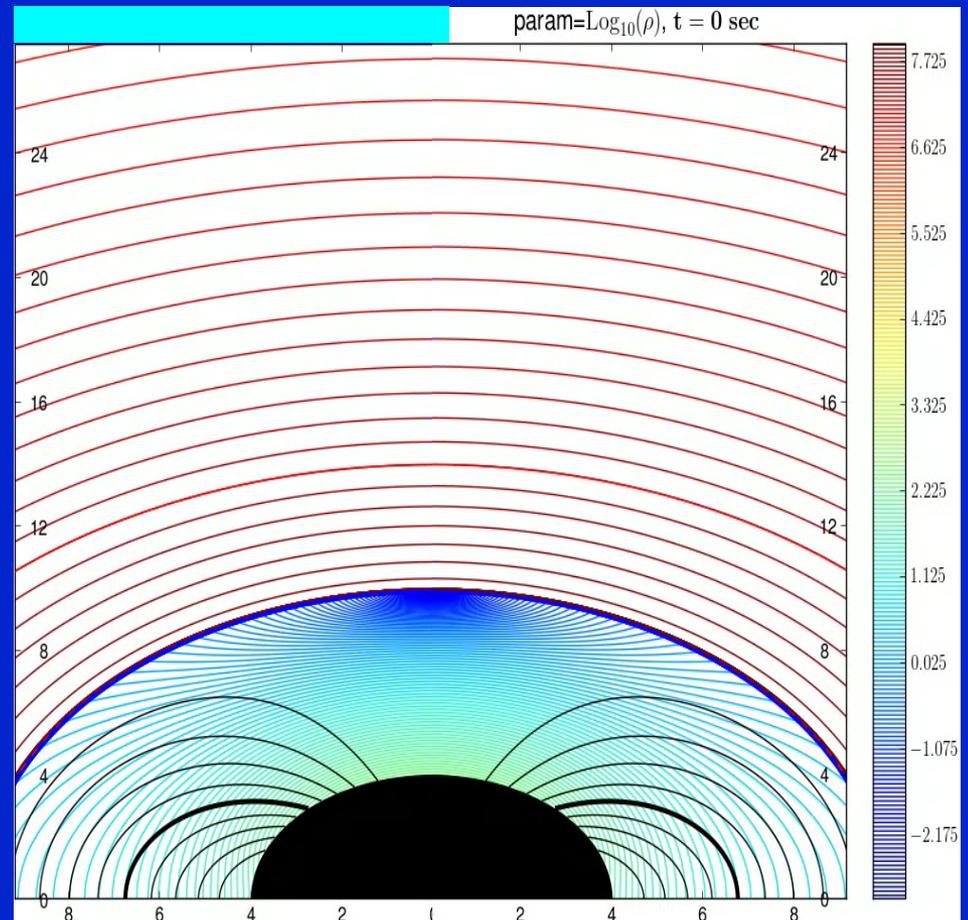
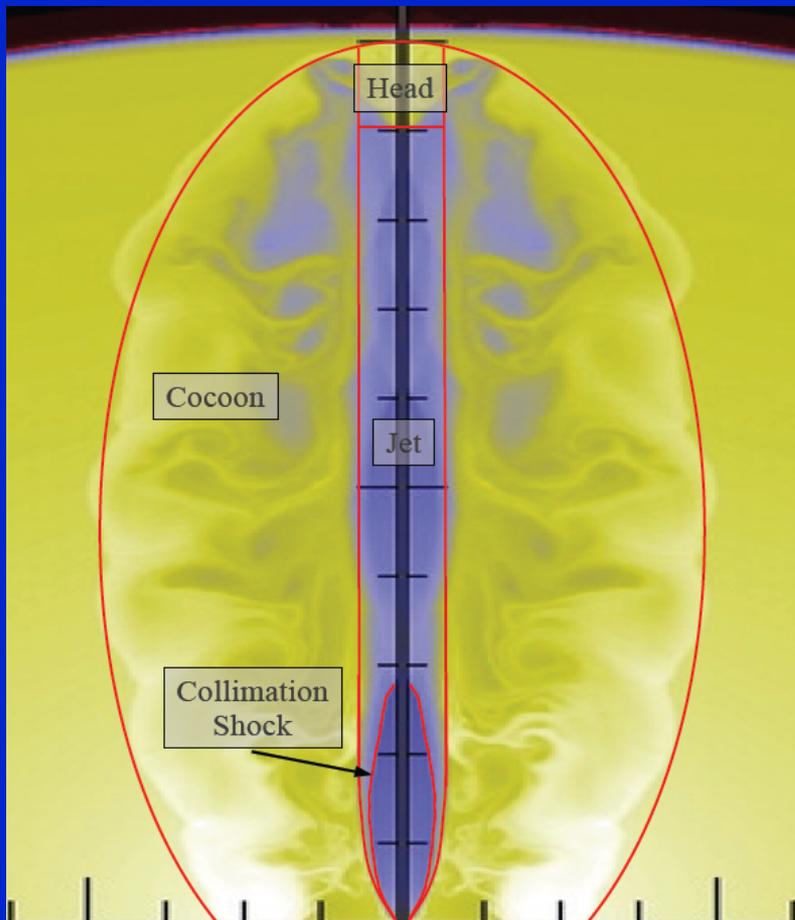
Breakout times comparison:

Bromberg et al 2011a

$$t_b \cong 15 \text{sec} \left(\frac{Liso}{10^{51} \text{erg/s}} \right)^{-1/3} \left(\frac{\theta_0}{10^\circ} \right)^{2/3} \left(\frac{R}{5R_\odot} \right)^{2/3} \left(\frac{M}{15M_\odot} \right)^{1/3}$$

Bromberg+ 2014

$$t_b \cong 1.8 \text{sec} \left(\frac{Liso}{10^{51} \text{erg/s}} \right)^{-1/3} \left(\frac{r_L}{5 \cdot 10^7 \text{cm}} \right)^{2/3} \left(\frac{M}{15M_\odot} \right)^{1/3}$$



Bromberg & Tchekhovskoy 2014 in prep

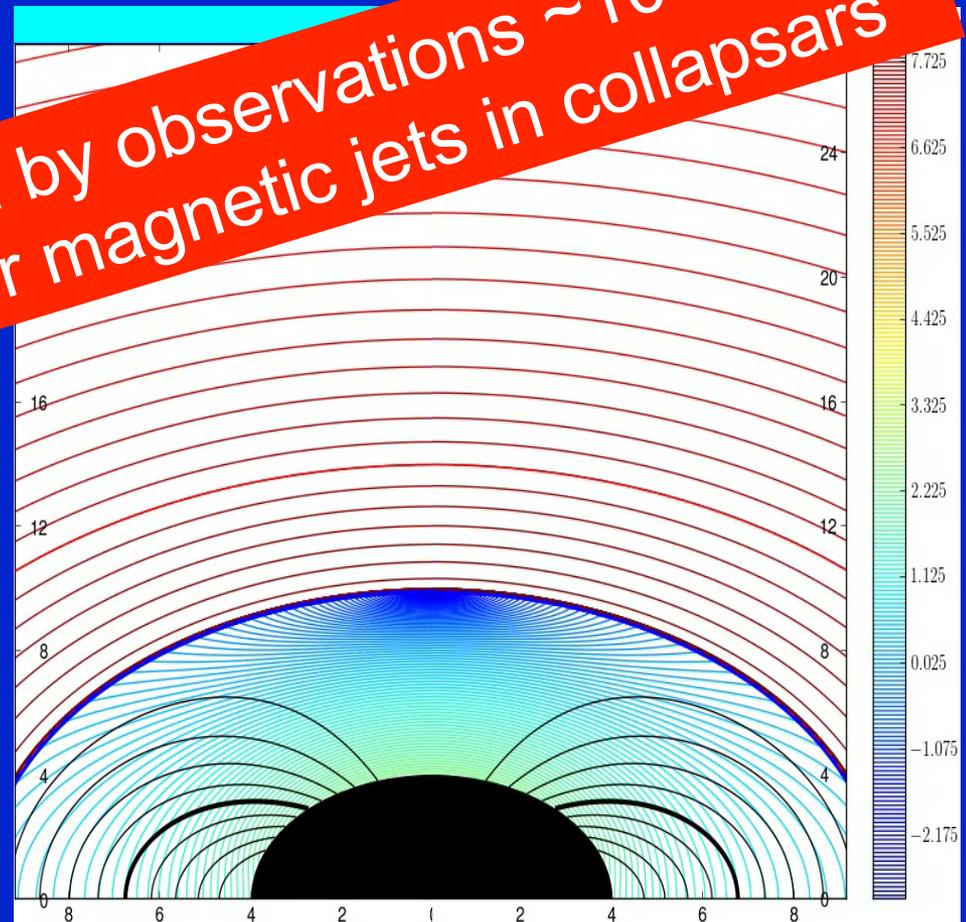
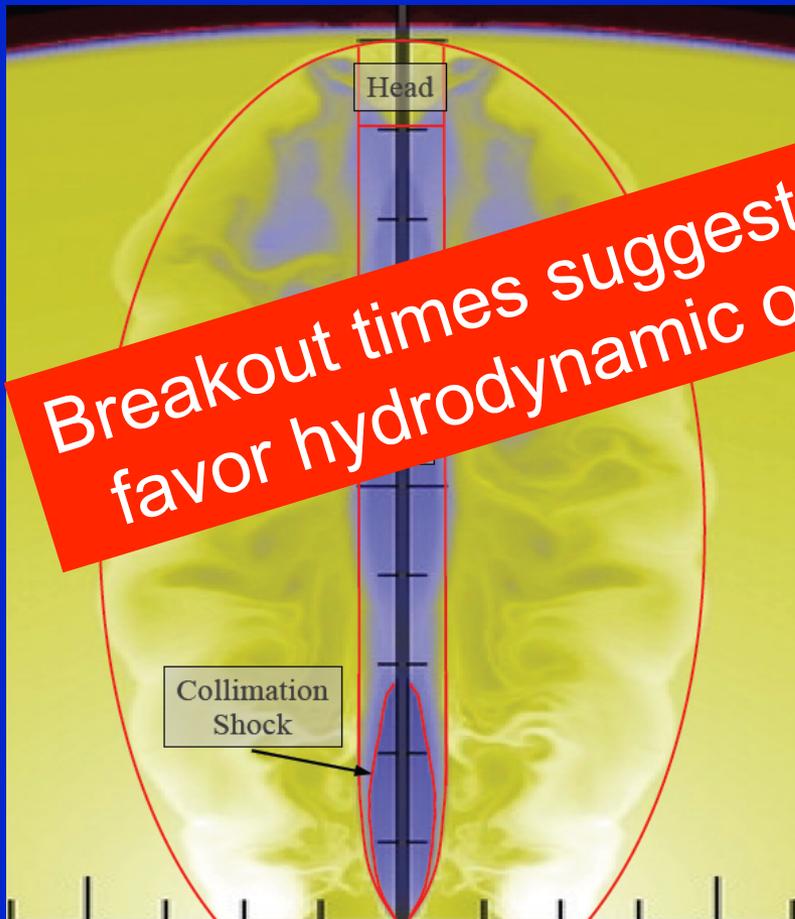
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Breakout times suggested by observations ~10-15 sec favor hydrodynamic over magnetic jets in collapsars

Conclusions:

■ Magnetic acceleration:

- ◆ Helps avoid large baryon loading
- ◆ Requires external confinement
- ◆ Is tightly related to the jet collimation

■ Impulsive magnetic acceleration:

- ◆ Can help reach kinetic energy dominance
- ◆ Allows efficient dissipation in internal shocks

■ Poynting dominated GRB jet propagating in a star

- ◆ Analytic solution: the jet's head is relativistic throughout most of the star
- ◆ Smaller breakout time, less energy injected into cocoon
- ◆ Observational implication: long GRB durations, the SN