GRB Jet Dynamics

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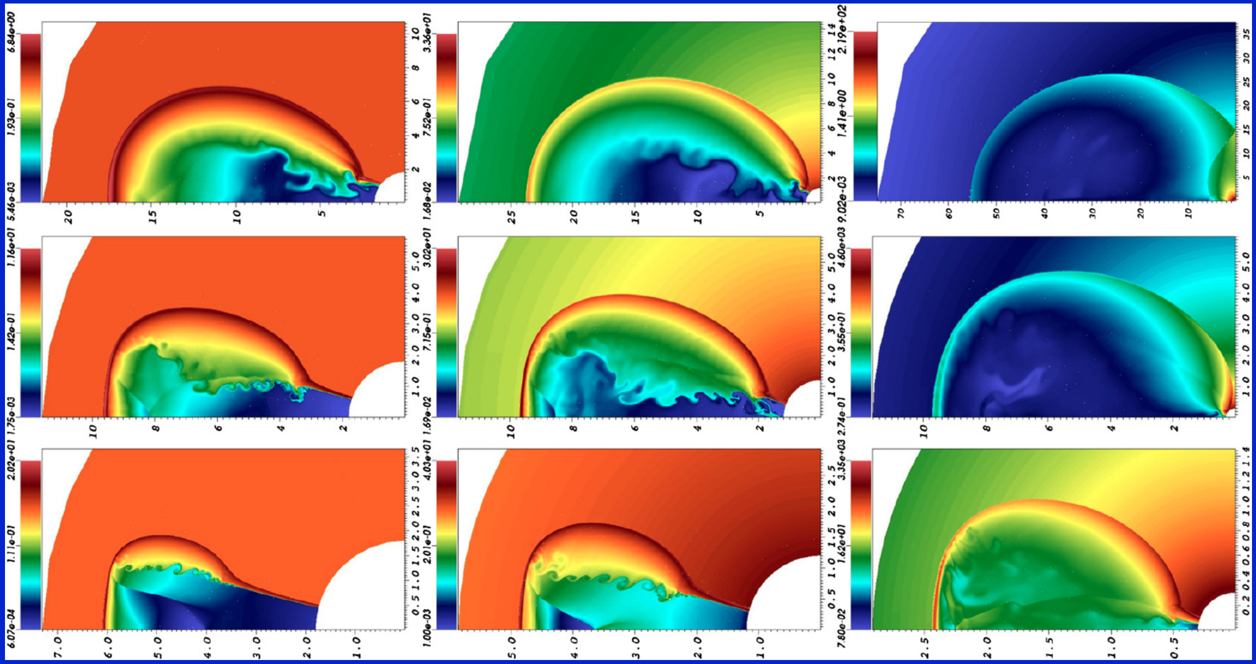


Gamma-Ray Burst Symposium, Marbella, Spain, Oct. 8, 2012

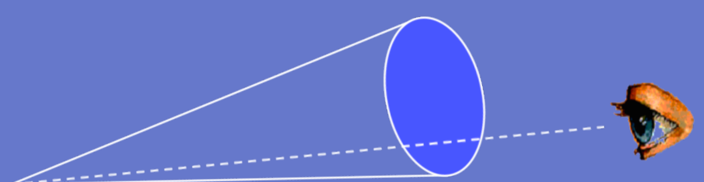
Outline of the talk:

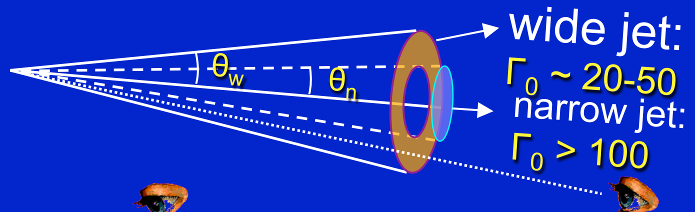
- Jet angular structure & evolution stages
- Magnetic acceleration: overview & recent results
- Jet dynamics during the afterglow: brief overview
- Analytic vs. numerical results: a discrepancy?
- Recent numerical & analytic results: finally agree
- Simulations of an afterglow jet propagating into a stratified external medium: $\rho_{ext} \propto R^{-k}$ for $k = 0, 1, 2$
- Implications for GRBs: **jet breaks, radio calorimetry**

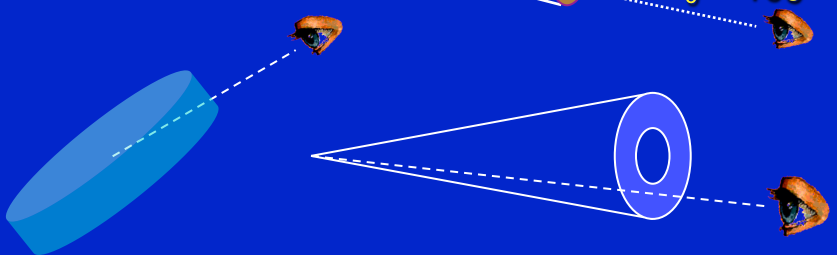
The **Angular Structure** of GRB Jets:

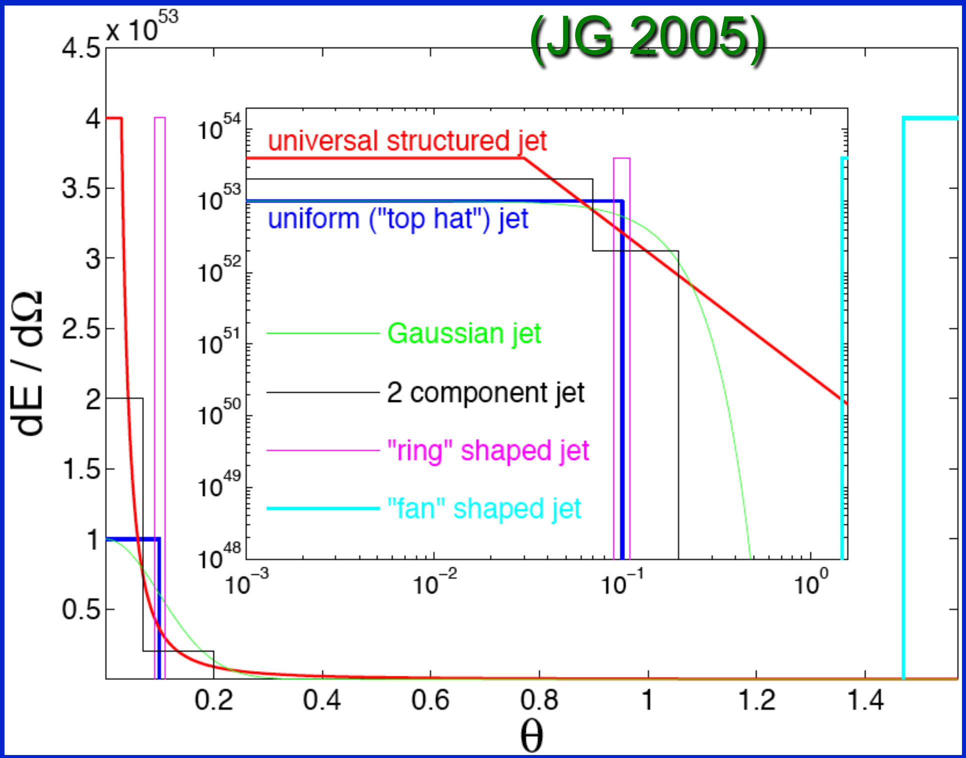
- **Jet structure: unclear** (uniform, structured, hollow cone,…)
  - Affects $E_{\gamma,iso} \rightarrow E_\gamma$ & observed GRB rate → true rate
  - Viewing-angle effects (afterglow & prompt - XRF)
  - Can also affect late time radio calorimetry

- Here I consider mainly a uniform "top hat" jet



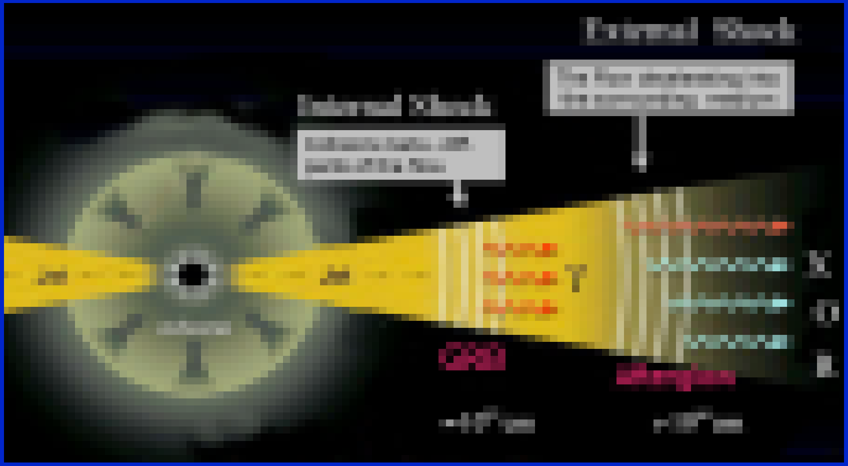






Stages in the **Dynamics** of GRB Jets:

- **Launching** of the jet: magnetic (B-Z?) neutrino annihilation?
- Acceleration: magnetic or thermal?
- For long GRBs: propagation inside progenitor star
- Collimation: stellar envelope, accretion disk wind, magnetic
- Coasting phase that ends at the deceleration radius R_{dec}
- At $R > R_{dec}$ most of the energy is in the shocked external medium: the composition & radial profile are forgotten, but the angular profile persists (locally: BM76 solution)
- Once $\Gamma < 1/\theta_0$ at $R > R_{jet}$ jet lateral expansion is possible
- Eventually the flow becomes spherical approaches the self-similar Sedov-Taylor solution



The σ-problem: for a "standard" steady ideal MHD axisymmetric flow

- $\Gamma_\infty \sim \sigma_0^{1/3}$ & $\sigma_\infty \sim \sigma_0^{2/3} \gg 1$ for a spherical flow; $\sigma_0 = B_0^2/4\pi\rho_0 c^2$
- However, PWN observations (e.g. the Crab nebula) imply $\sigma \ll 1$ after the wind termination shock – the σ problem!!!
- A broadly similar problem persists in relativistic jet sources
- Jet **collimation** helps, but not enough: $\Gamma_\infty \sim \sigma_0^{1/3}\theta_{jet}^{-2/3}$, $\sigma_\infty \sim (\sigma_0\theta_{jet})^{2/3}$ & $\Gamma\theta_{jet} \lesssim \sigma^{1/2}$ (~1 for $\Gamma_\infty \sim \Gamma_{max} \sim \sigma_0$)
- Still $\sigma_\infty \gtrsim 1 \Rightarrow$ inefficient internal shocks, $\Gamma_\infty\theta_{jet} \gg 1$ in GRBs
- Sudden drop in external pressure can give $\Gamma_\infty\theta_{jet} \gg 1$ but still $\sigma_\infty \gtrsim 1$ **(Tchekhovskoy et al. 2009)** $\Rightarrow$ inefficient internal shocks

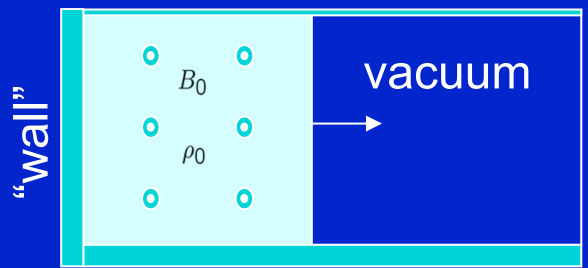
Alternatives to the "standard" model

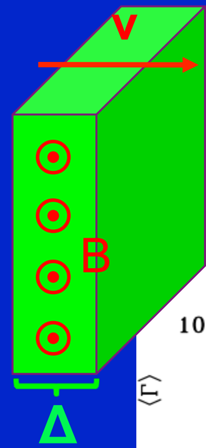
- ~~Axisymmetry~~: non-axisymmetric instabilities (e.g. the current-driven kink instability) can tangle-up the magnetic field (Heinz & Begelman 2000)
- If $\langle B_r^2 \rangle = \alpha \langle B_\phi^2 \rangle = \beta \langle B_z^2 \rangle \;;\; \alpha, \beta = \mathrm{const}$ then the magnetic field behaves as an ultra-relativistic gas: $p_{\mathrm{mag}} \propto V^{-4/3}$ $\Rightarrow$ magnetic acceleration as efficient as thermal
- ~~Ideal~~ MHD: a tangled magnetic field can reconnect (Drenkham & Spruit 2002; Lyubarsky 2010 - Kruskal-Schwarzschild instability (like R-T) in a "striped wind") **magnetic energy $\rightarrow$ heat (+radiation) $\rightarrow$ kinetic energy**
- ~~Steady-state~~: **effects of strong time dependence** (JG, Komissarov & Spitkovsky 2011; JG 2012a, 2012b)

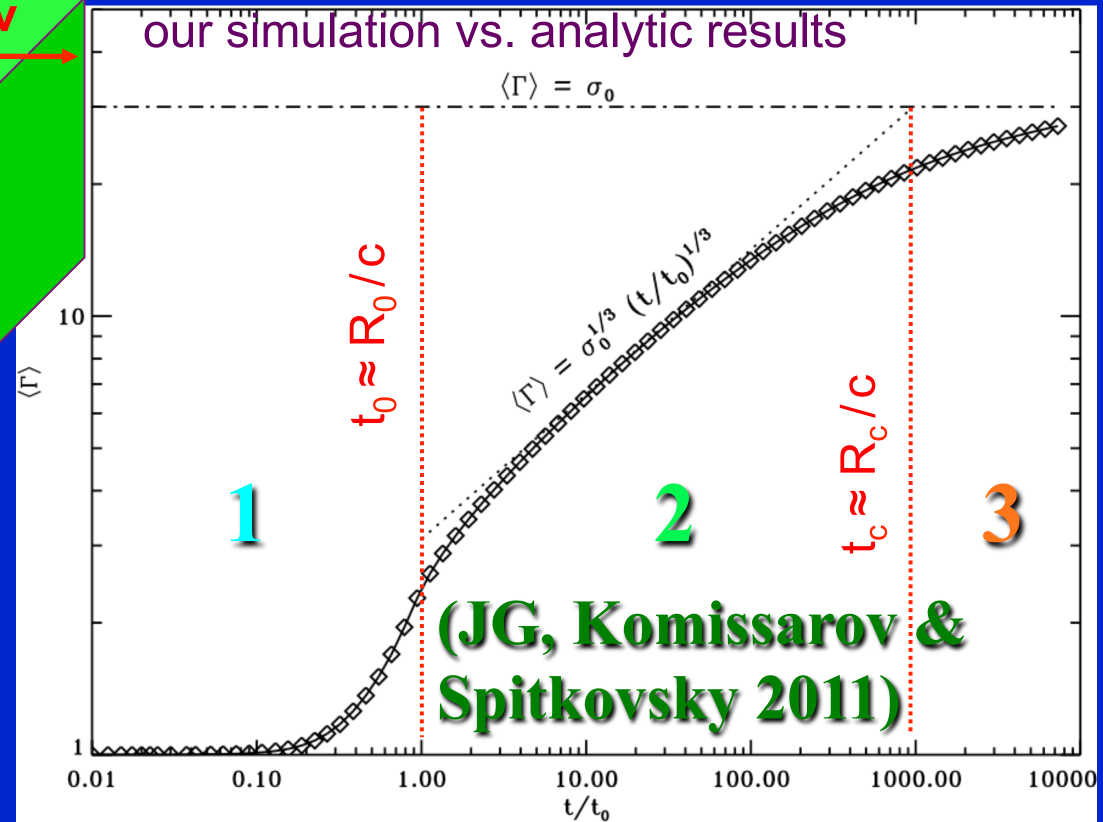
Impulsive Magnetic Acceleration: $\Gamma \propto R^{1/3}$

Useful **case study**:

Initial value of magnetization parameter: $\sigma_0 = \dfrac{B_0^2}{4\pi\rho_0 c^2} >> 1$





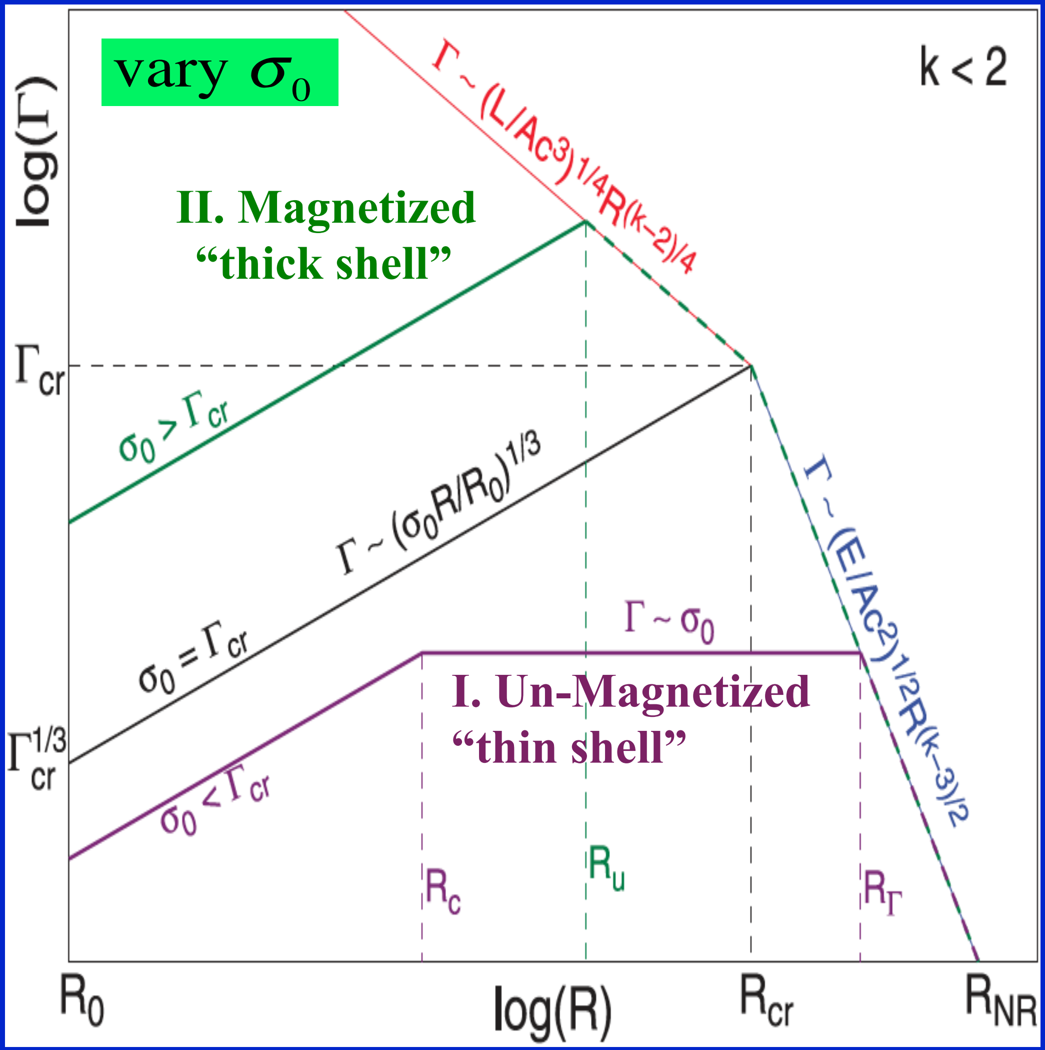


1. $\langle\Gamma\rangle_E \approx \sigma_0^{1/3}$ by $R_0 \sim \Delta_0$
2. $\langle\Gamma\rangle_E \propto R^{1/3}$ between $R_0 \sim \Delta_0$ & $R_c \sim \sigma_0^2 R_0$ and then $\langle\Gamma\rangle_E \approx \sigma_0$
3. At $R > R_c$ the sell spreads as $\Delta \propto R$ & $\sigma \sim R_c/R$ rapidly drops

- **Complete conversion of magnetic to kinetic energy!**
- **This allows efficient dissipation by shocks at large radii**

Impulsive Magnetic Acceleration: single shell propagating in an external medium

acceleration & deceleration are tightly coupled (JG 2012)



$$\rho_{\rm ext} = AR^{-k}$$

$$R_{\rm cr} \sim R_0\Gamma_{\rm cr}^2 \sim \left(\frac{ER_0}{Ac^2}\right)^{\frac{1}{4-k}}$$

I. "Thin shell", low-σ : strong reverse shock, peaks at $\gg T_{GRB}$

II. "Thick shell", high-σ: weak or no reverse shock, $T_{dec} \sim T_{GRB}$

III. like II, but the flow becomes independent of σ_0

IV. a Newtonian flow (if ρ_{ext} is very high, e.g. inside a star)

II*. if ρ_{ext} drops very sharply

Many sub-shells: acceleration, collisions

(JG 2012b)

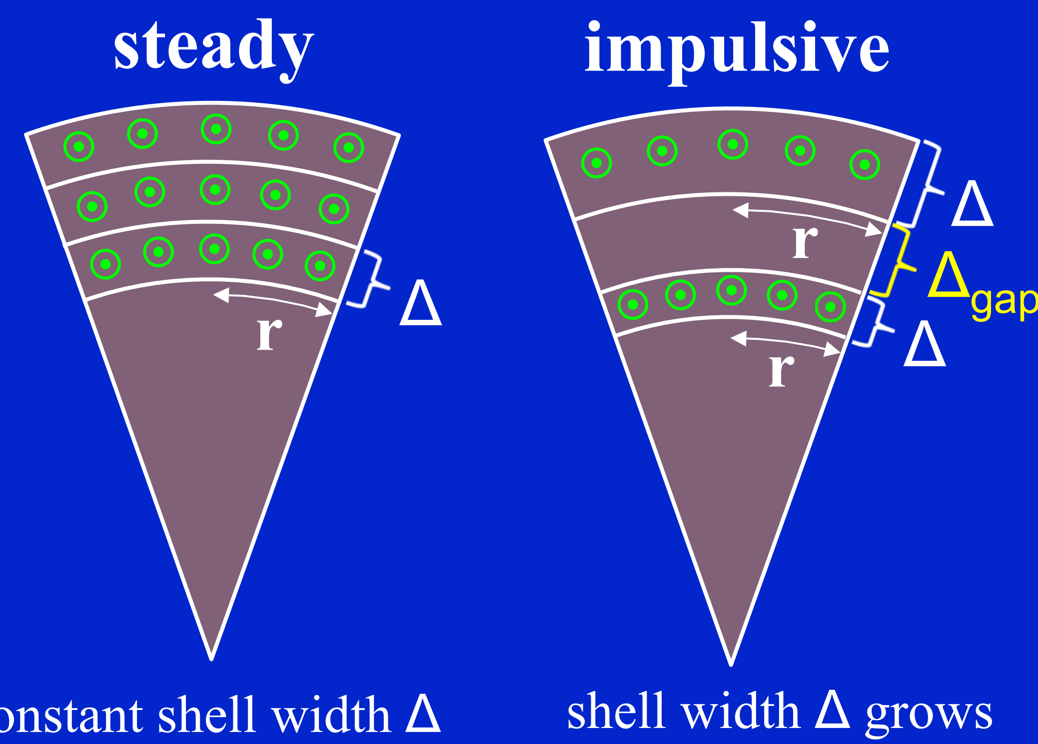
Flux freezing (ideal MHD):

$\Phi \sim B\, r\, \Delta = \textbf{constant}$

$E_{EM} \sim B^2 r^2 \Delta \propto 1/\Delta$

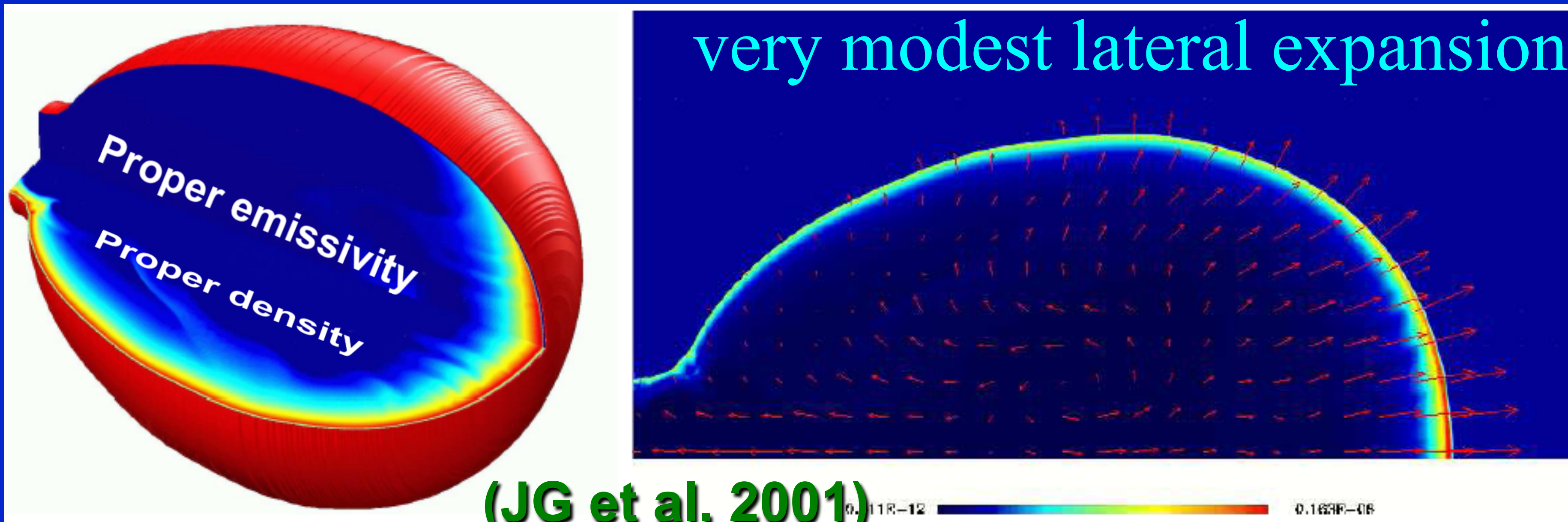
$$\frac{\text{total energy}}{\text{rest energy}} = (1+\sigma)\Gamma$$

$\text{acceleration } (\Gamma\uparrow) \Leftrightarrow \sigma\downarrow$

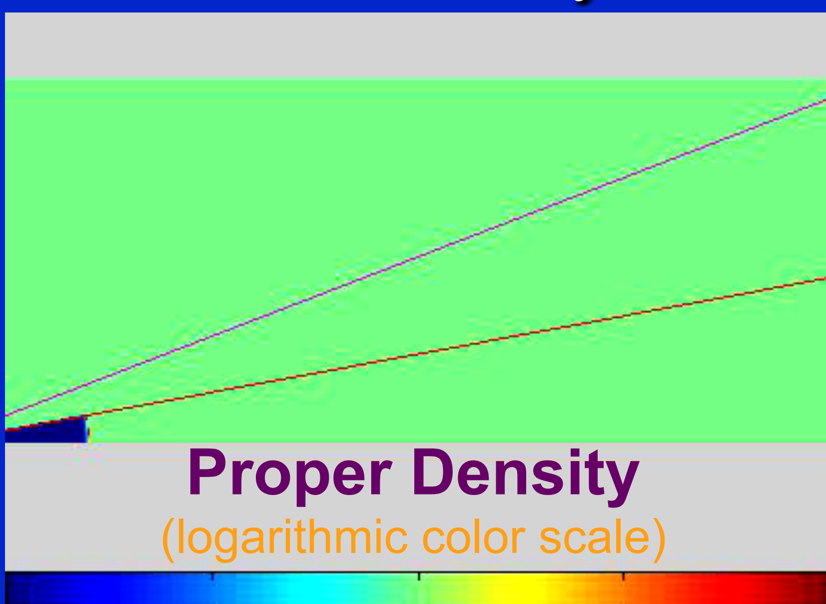


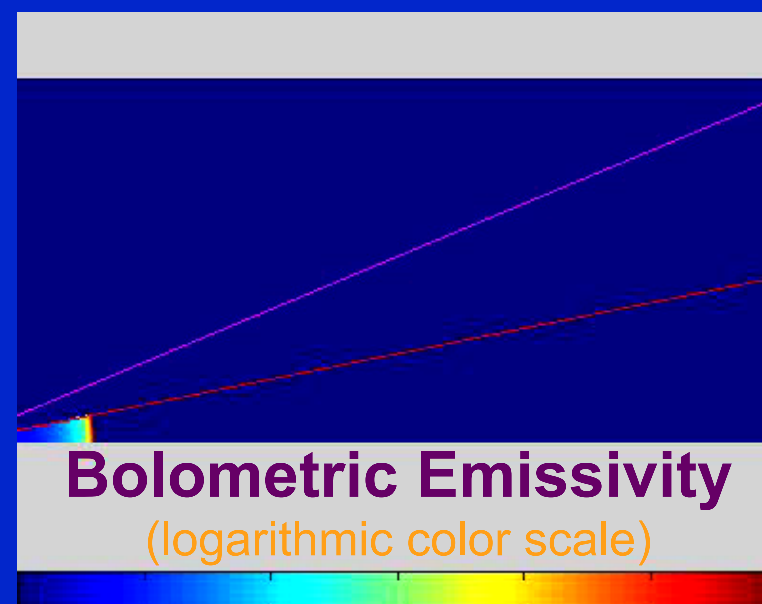
- For a long lived variable source (e.g. AGN), each sub shell can expand by $1+\Delta_{gap}/\Delta_0 \Rightarrow \sigma_\infty = (E_{total}/E_{EM,\infty} - 1)^{-1} \sim \Delta_0/\Delta_{gap}$
- For a finite # of sub-shells the merged shell can still expand
- Sub-shells can lead to a low-magnetization thick shell & enable the outflow to reach higher Lorentz factors

Afterglow Jet Dynamics: 2D hydro-simulations



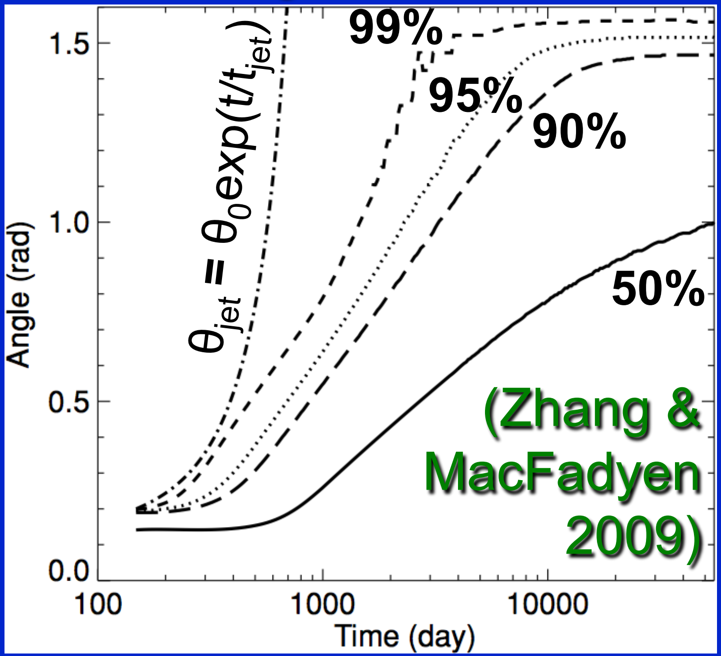
- Emission mostly from front, slow material at the sides

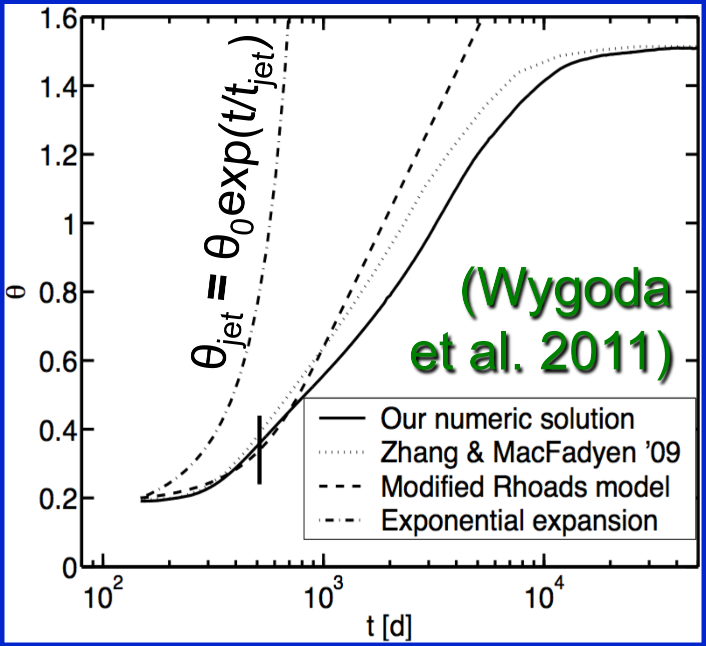




Analytic vs. Numerical results: a problem?

- **Analytic** results (Rhoads 1997, 99; Sari, Piran & Halpern 99): **exponential lateral expansion** at $R > R_{jet}$ e.g. $\Gamma \sim (c_s/c\theta_0)\exp(-R/R_{jet})$, $\theta_{jet} \sim \theta_0(R_{jet}/R)\exp(R/R_{jet})$
  - Supported by a self-similar solution (Gruvinov 2007)
- **Hydro-simulations**: very **mild** lateral expansion while jet is relativistic (also for simplified 2D → 1D)





Modest θ_0 ⇒ small region of validity

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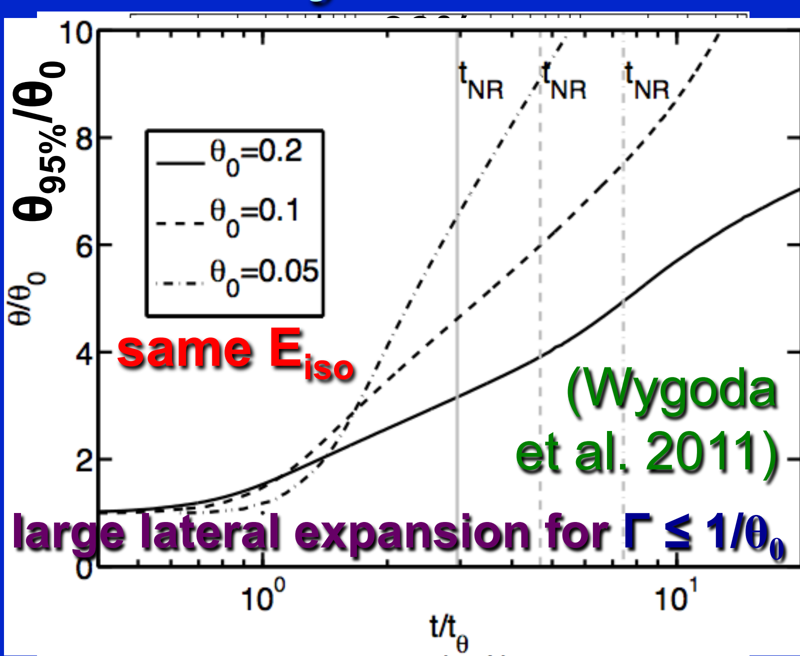
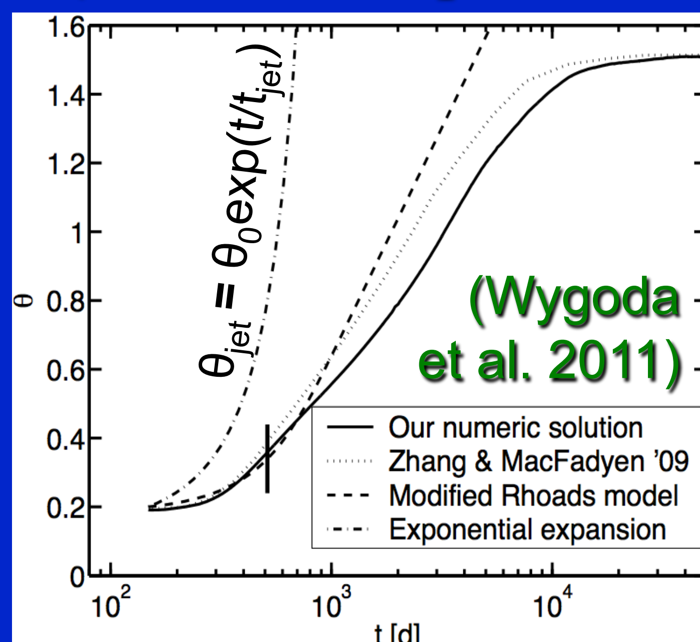


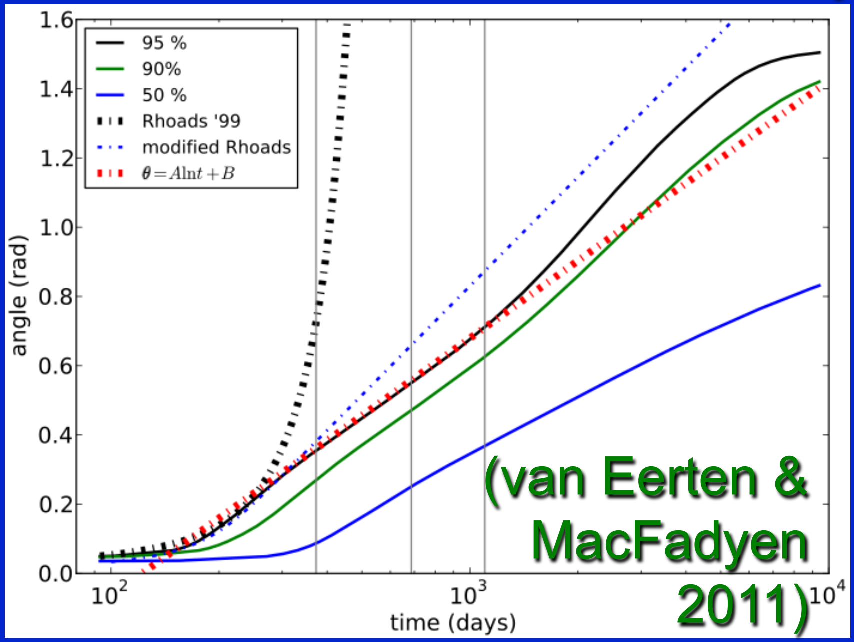


Modest θ_0 ⇒ small region of validity

Analytic vs. Numerical results: a problem?

van Eerten & MacFadyen 11'

- No exponential lateral expansion even for $\theta_0 = 0.05$
- Lateral expansion is instead only **logarithmic**
- Affects jet break shape + t_j & late time radio calorimetry



Lyutikov 2011

- Lateral expansion becomes significant only for $\Gamma \leq \theta_0^{-1/2}$
- Based on thin shell approx.

$$\tan\alpha = -\frac{\partial \ln R}{\partial \theta}$$ (Kumar & JG 2003)

$$\Rightarrow \beta_\theta \sim \frac{1}{\Gamma^2 \Delta\theta} \sim \frac{1}{\Gamma^2 \theta_j}$$

$r = R(\theta)$ $\rightarrow$ shock radius in spherical coordinates

$\alpha =$ angle between the shock normal $\hat{\boldsymbol{n}}$ and radial direction $\hat{\boldsymbol{r}}$

Generalized Analytic model (JG & Piran 2012)

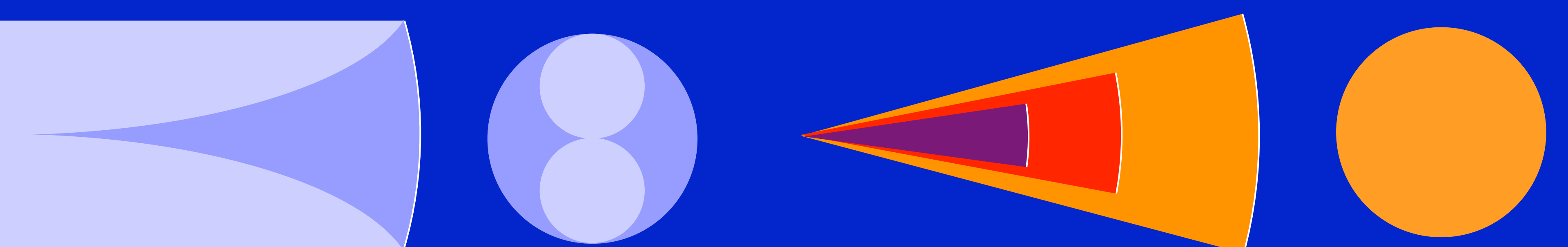
- **Lateral expansion**:

  1. new recipe: $\beta_\theta/\beta_r \sim 1/(\Gamma^2\Delta\theta) \sim 1/(\Gamma^2\theta_j)$ (based on $\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{n}}$)
  2. old recipe: $\beta_\theta = u_\theta/\Gamma = u'_\theta/\Gamma \sim \beta_r/\Gamma$ (based on $u'_\theta \sim 1$)

Generalized recipe: $$\frac{d\theta_j}{d\ln R} = \frac{\beta_\theta}{\beta_r} \approx \frac{1}{\Gamma^{1+a}\theta_j^a}\ ,\quad a = \begin{cases} 1 & (\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{n}}) \\ 0 & (u'_\theta \sim 1) \end{cases}$$

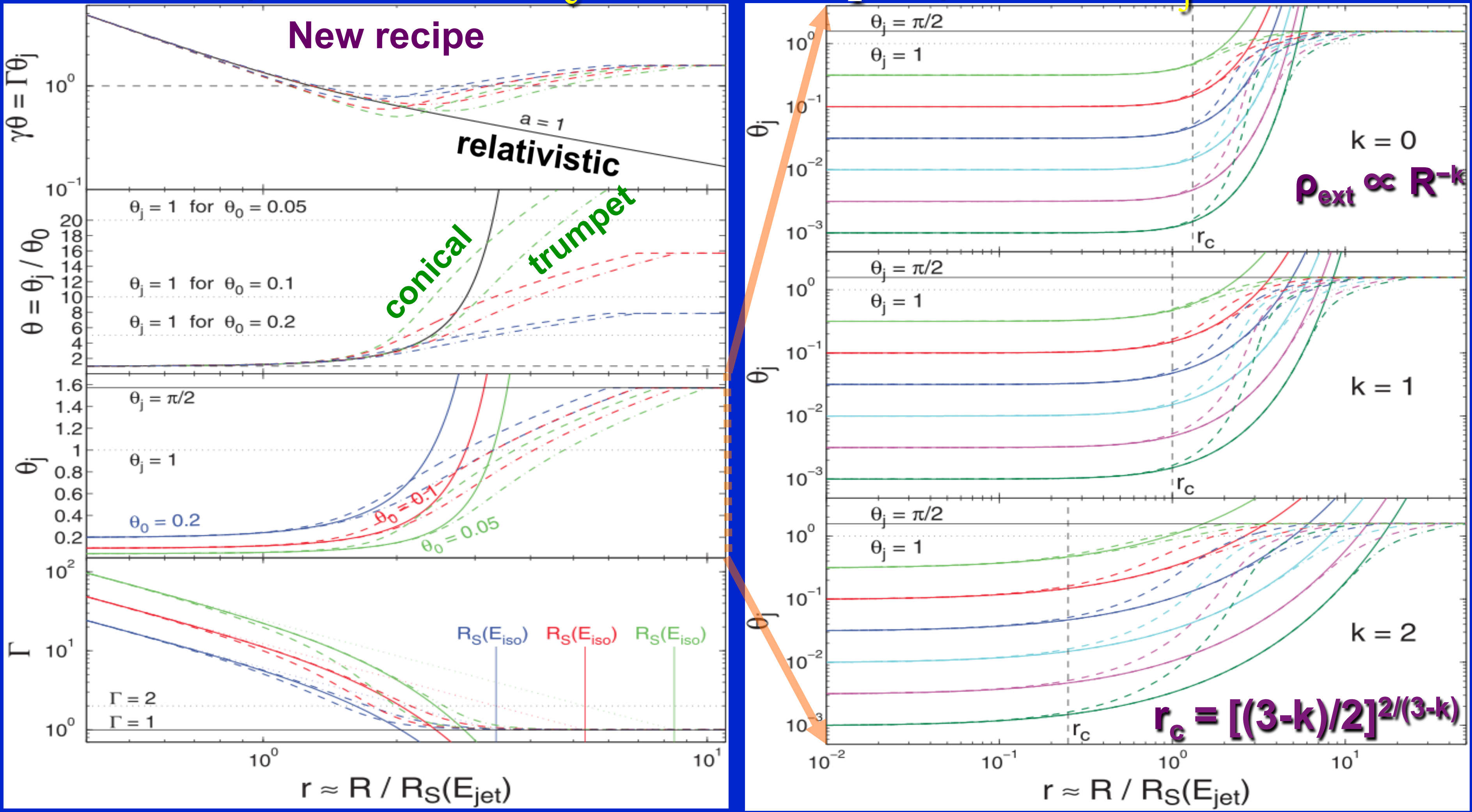
- ◆ New recipe: lower β_θ for $\Gamma > 1/\theta_0$ but higher β_θ for $\Gamma < 1/\theta_0$

- Does not assume $\Gamma \gg 1$ or $\theta_j \ll 1$ (& variable: Γ [illegible] $u = \Gamma\beta$)
- Sweeping-up external medium: trumpet vs. conical models



Generalized Analytic model (JG & Piran 2012)

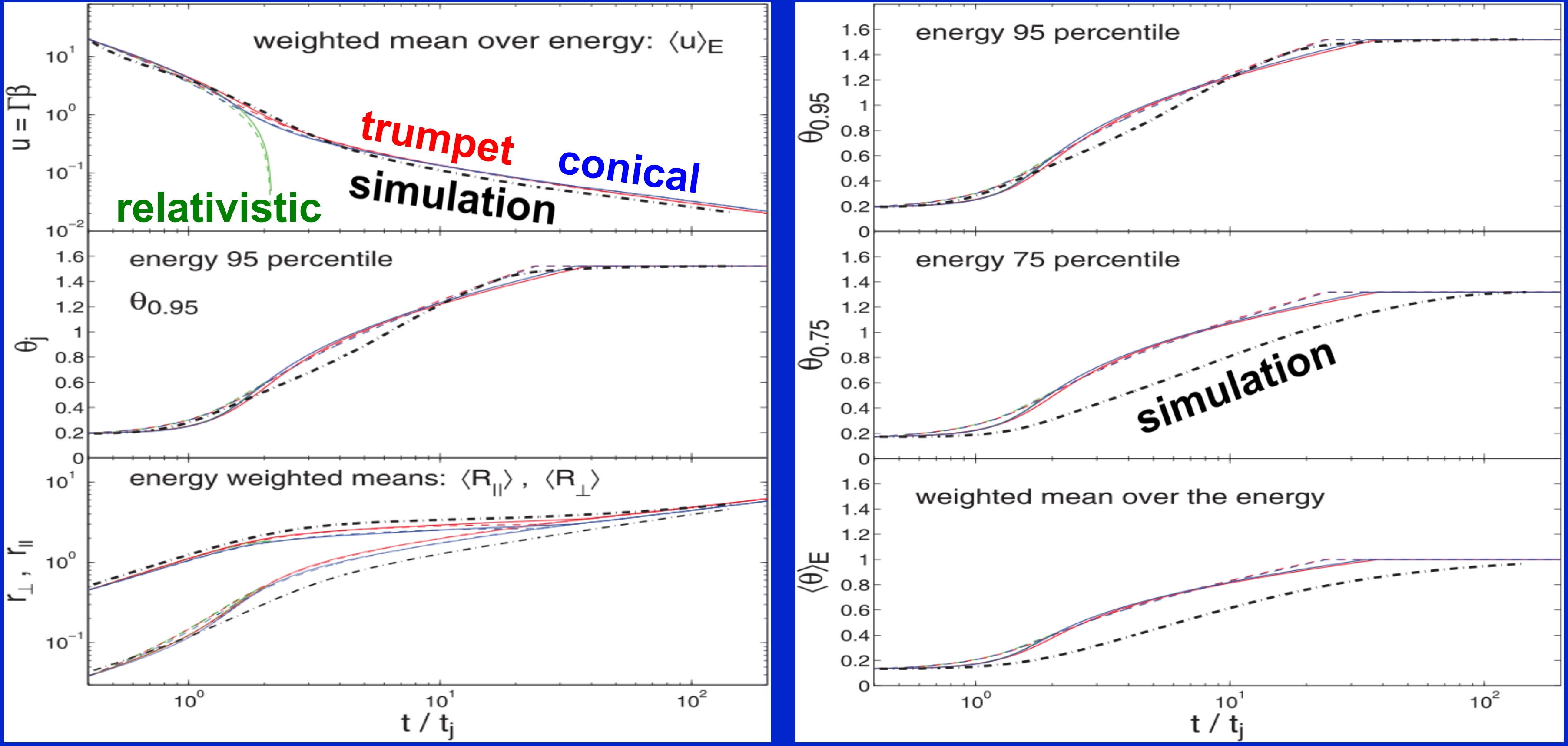
- Main effect of relaxing the $\Gamma \gg 1$, $\theta_j \ll 1$ approximation: quasi-logarithmic (~~exponential~~) lateral expansion for $\theta_0 \gtrsim 0.05$
- conical $\neq$ rel. for $r \gtrsim r_c$ while trumpet $\neq$ rel. for $\theta_j \gtrsim 0.2$



Comparison to Simulations (JG & Piran 2012)

- There is a reasonable overall **agreement** between the **analytic** generalized models and the hydro-**simulations**
- Analytic models: over-simplified, but capture the essence

2D hydro-simulation by F. De Colle et al. 2012, with $\theta_0 = 0.2$, $k = 0$



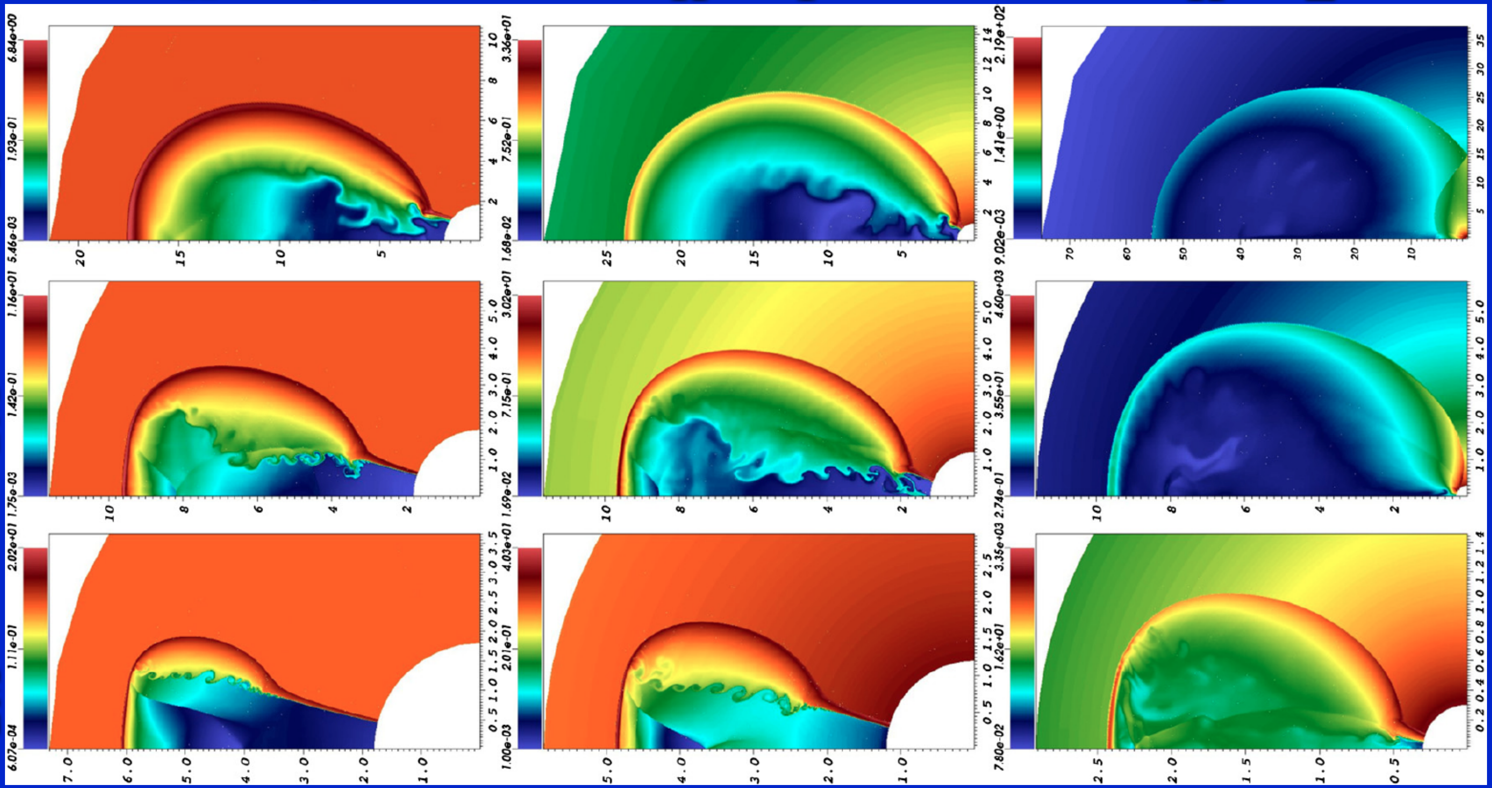
Afterglow jet in stratified external media

(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

- Previous simulations were all for $k = 0$ where $\rho_{ext} \propto R^{-k}$
- Larger k (e.g. $k = 1, 2$) are motivated by the stellar wind of a massive star progenitor for long GRBs

$k = 0$ $k = 1$ $k = 2$

$\Gamma_{BM} = 2$ $\Gamma_{BM} = 5$ $\Gamma_{BM} = 10$



Logarithmic color map of ρ

$\theta_0 = 0.2$, $E_{iso} = 10^{53}$ erg, $n_{ext}(R_{jet}) \sim 1\ cm^{-3}$

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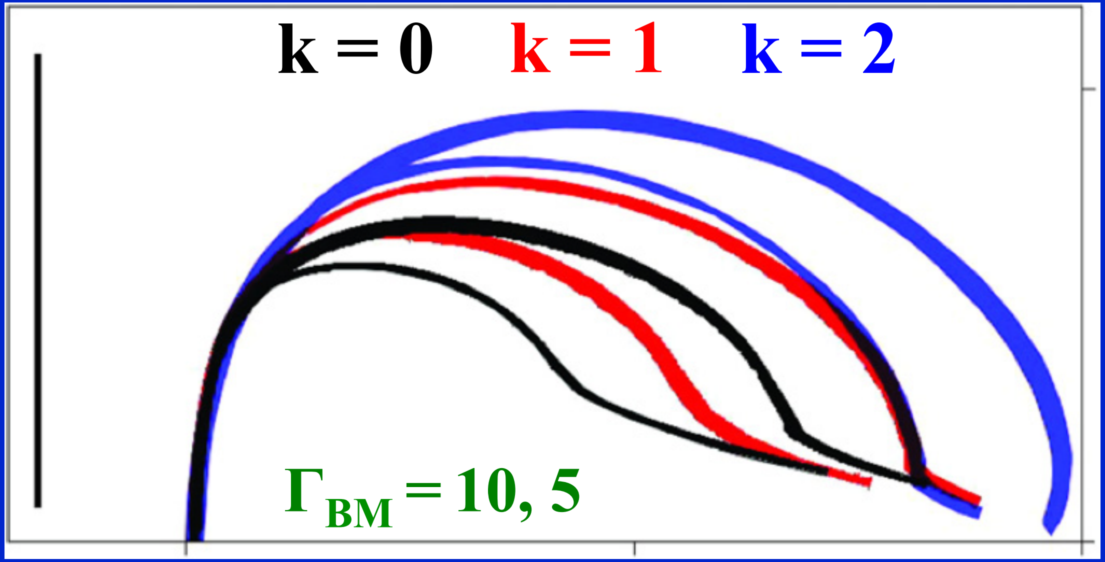
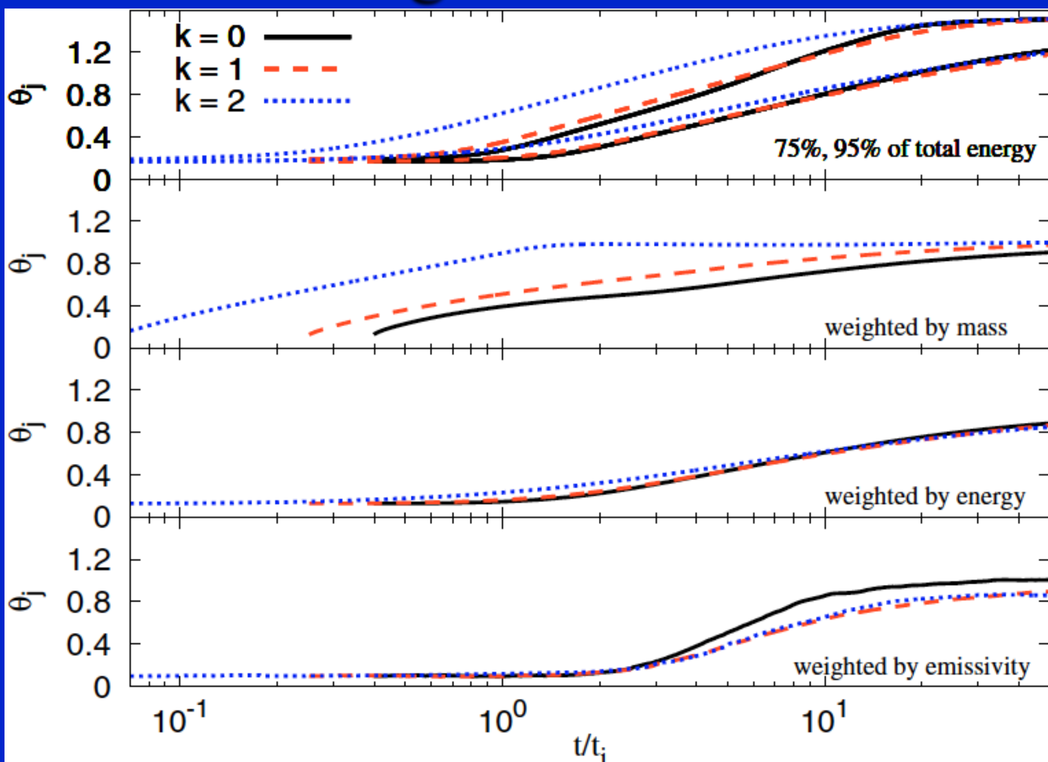


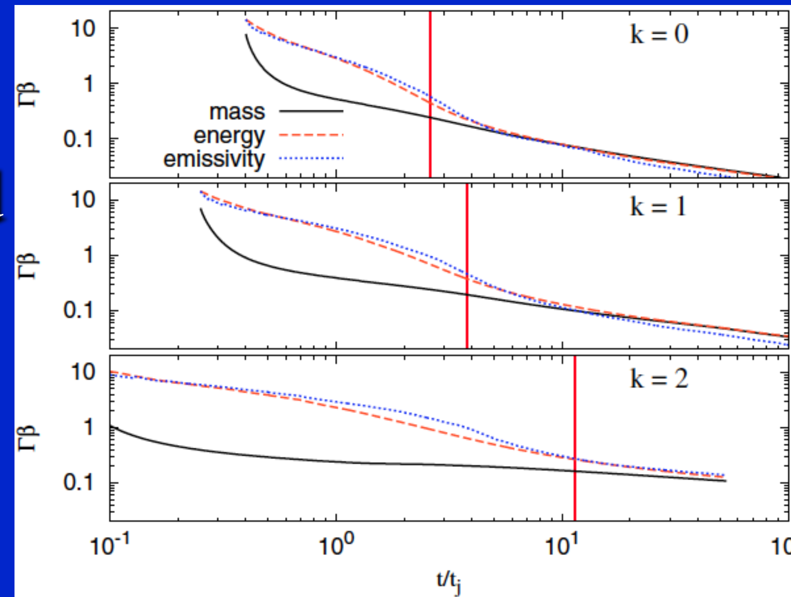
- At the same Lorentz factor larger k show larger sideways expansion since they sweep up mass and decelerate more slowly (e.g. $M \propto R^{3-k}$, $\Gamma \propto R^{(3-k)/2}$ in the spherical case) and spend more time at lower Γ (and β_θ decreases with Γ)

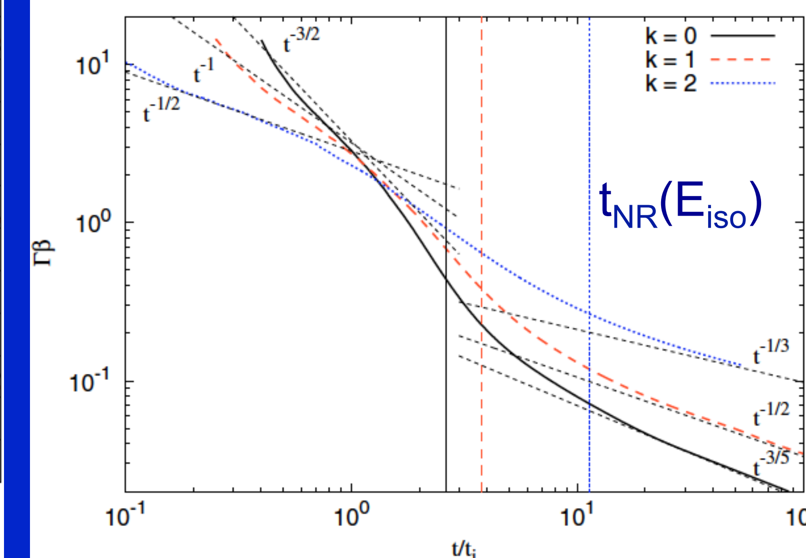
Afterglow jet in stratified external media

(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

- Swept-up mass: a lot at the sides of the jet at large angles
- Energy, emissivity: near the head
- Spherical symmetry approached later for larger k



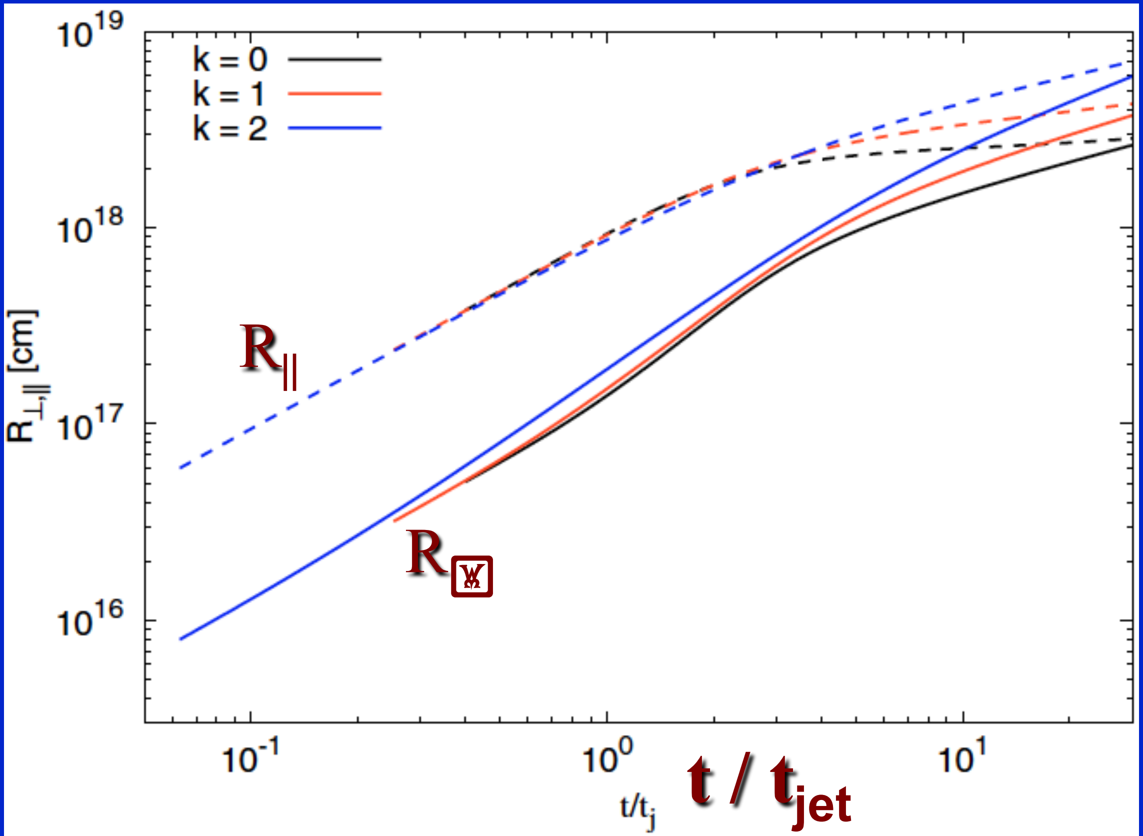




Afterglow jet in stratified external media

(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

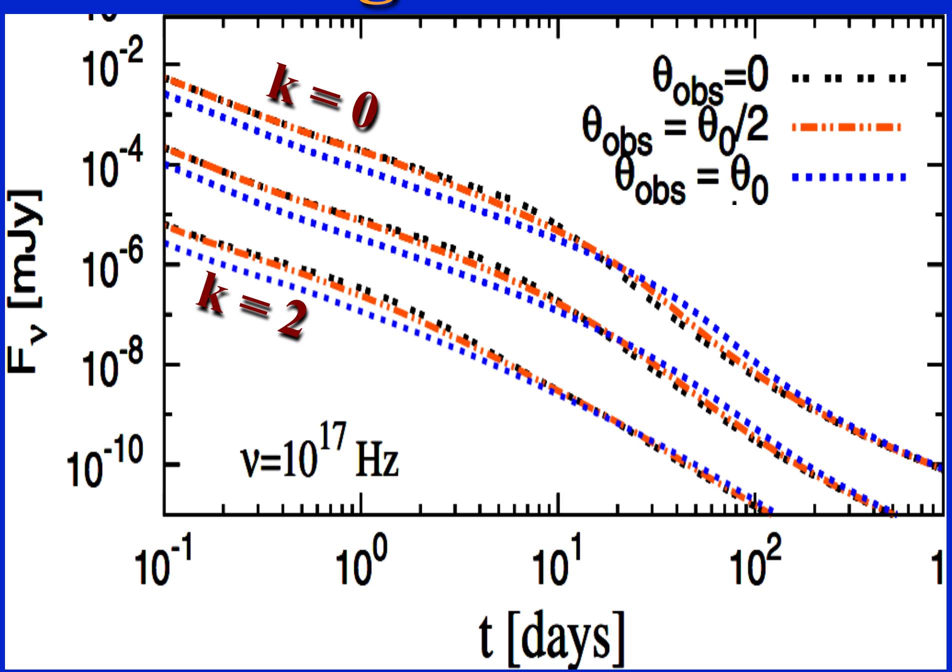
- For $k = 0$ the growth of $R_{||}$ is stalled at $t_{NR}(E_{iso})$ while $R_{\perp}$ continues to grow ➔ helps approach spherical symmetry
- Less pronounced for larger k as the slower accumulation of mass enables $R_{||}$ to grow more ➔ become spherical more slowly



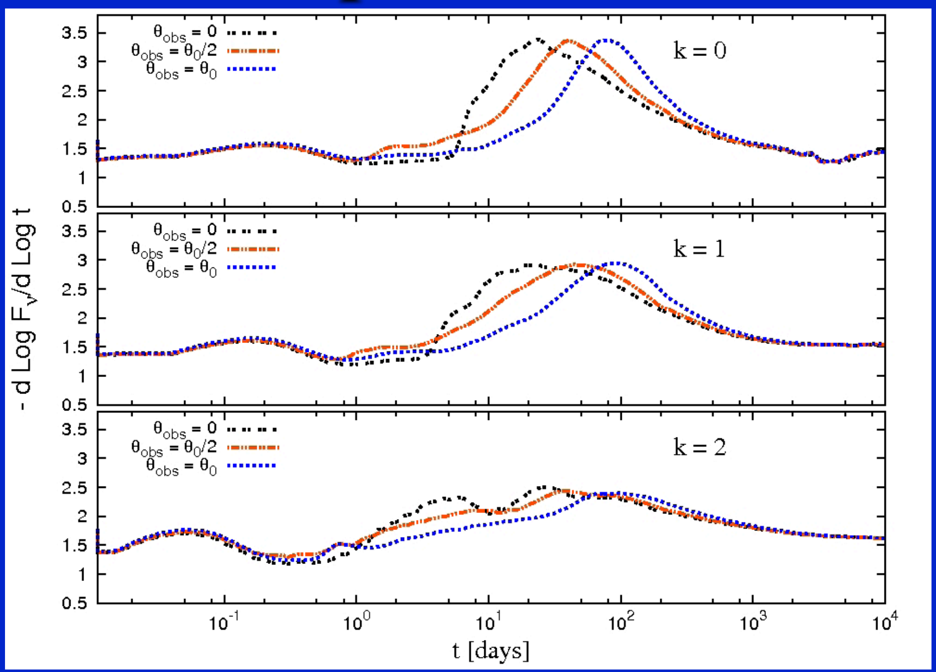
The shape of the jet break

- Jet break becomes smoother with increasing k (as expected analytically; Kumar & Panaitescu 2000 – KP00)
- However, the jet break is significantly sharper than found by KP00 ➔ better prospects for detection
- Varying $\theta_{obs} < \theta_0$ dominates over varying $k \lesssim 2$

Lightcurves



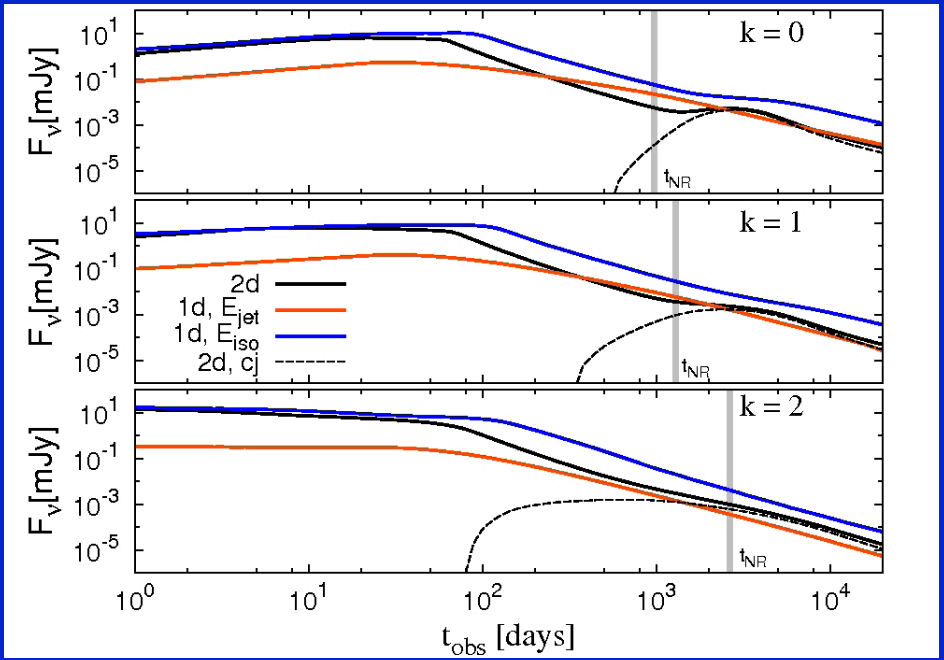
Temporal index



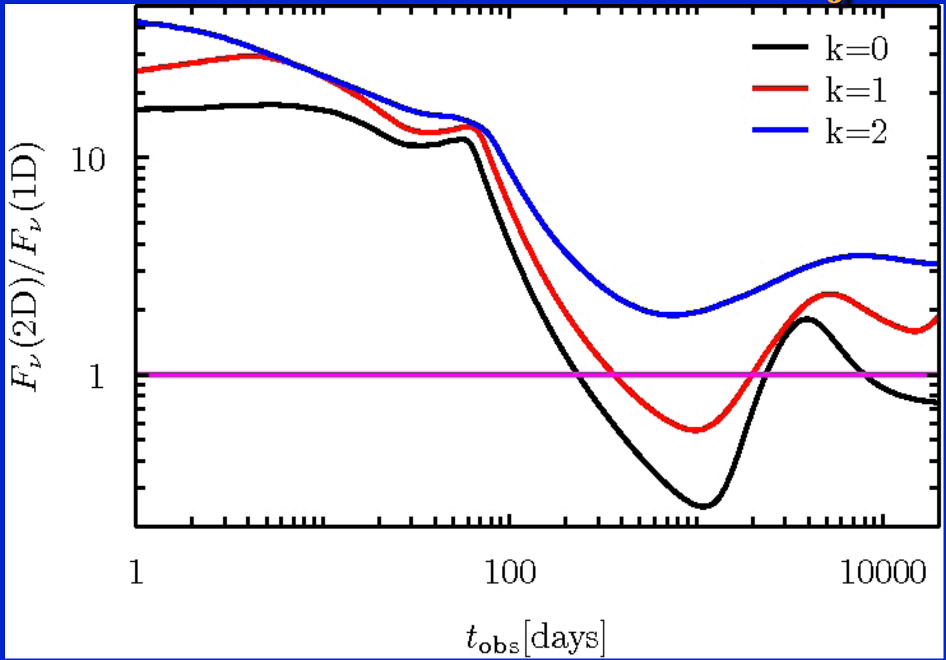
Late time Radio emission & Calorimetry

- The bump in the lightcurve from the counter jet is much less pronounced for larger k (as the counter jet decelerates & becomes visible more slowly) ➔ hard to detect
- The error in the estimated energy assuming a spherical flow depends on the observation time t_{obs} & on k

Radio Lightcurves



Flux Ratio: 2D/1D(E_{jet})



Conclusions:

- Magnetic acceleration: likely option worth further study
- Jet lateral expansion: analytic models & simulations agree
  - For $\theta_0 \gtrsim 0.05$ the lateral expansion is quasi-logarithmic (~~exponential~~), due to small dynamic range $1/\theta_0 > \Gamma \gg 1$
  - For $\theta_0 \ll 0.05$ there is an exponential lateral expansion phase early on (but such narrow GRB jets appear rare)
  - Jet becomes first sub-relativistic, then (slowly) spherical
- Jet in a stratified external medium: $\rho_{ext} \propto R^{-k}$ for $k = 0, 1, 2$
  - larger k jets sweep-up mass & slow down more slowly
    - ➔ sideways expansion is faster at $t < t_j$ & slower at $t > t_j$
    - ➔ become spherical slower; harder to see counter jet
  - Jet break is smoother for larger k but possibly detectable
  - Jet break sharpness affected more by $\theta_{obs} < \theta_0$ than $k \lesssim 2$
  - Radio calorimetry accuracy affected both by t_{obs} & k