Outline of the talk:

- Background, jet angular structure & evolution stages
- Magnetic acceleration: overview & recent results
- Jet propagation inside the progenitor star
- Jet dynamics during the afterglow:
  - Recent numerical & analytic results: finally agree
  - Simulations of an afterglow jet propagating into a stratified external medium: $\rho_{ext} \propto R^{-k}$ for $k = 0, 1, 2$
  - Implications for GRBs: jet breaks, radio calorimetry
Differences between GRB jets & other Astrophysical Relativistic Jets:

- **GRB jets are not directly angularly resolved**
  - Typically at $z \geq 1 +$ early source size $\lesssim 0.1$ pc
  - Only a single radio afterglow (GRB 030329) was marginally resolved after 25 days (+monitored for years)
  - The jet structure is constrained indirectly

- **GRB jets are Impulsive**: most observations are long after the source activity

- GRBs are transient events, making the observations much more difficult
Observational Evidence for Jets in GRBs

- The energy output in $\gamma$-rays assuming isotropic emission approaches or even exceeds $M_\odot c^2$
  - difficult for a stellar mass progenitor
  - True energy is much smaller for a narrow jet

- Some long GRBs occur together with a SN
  - the outflow would contain $>M_\odot$ if spherical
  - only a small part of this mass can reach $\Gamma \gtrsim 100$
    & it would contain a small fraction of the energy

- Achromatic break or steepening of the afterglow light curves ("jet break")
Examples of Smooth & Achromatic Jet Breaks

Optical light curve of GRB 990510 (Harrison et al. 1999)

Optical light curve of GRB 030329 (Gorosabel et al. 2006)
The Angular Structure of GRB Jets:

- **Jet structure: unclear** (uniform, structured, hollow cone, …)
  - Affects $E_{\gamma,\text{iso}} \rightarrow E_{\gamma}$ & observed GRB rate $\rightarrow$ true rate
  - Viewing-angle effects (afterglow & prompt - XRF)
  - Can also affect late time radio calorimetry

- Here I consider mainly a uniform “top hat” jet

  - Wide jet: $\Gamma_0 \sim 20-50$
  - Narrow jet: $\Gamma_0 > 100$
Stages in the Dynamics of GRB Jets:

- **Launching** of the jet: magnetic (B-Z?) neutrino annihilation?
- **Acceleration**: magnetic or thermal?
- For long GRBs: propagation inside progenitor star
- Collimation: stellar envelope, accretion disk wind, magnetic
- Coasting phase that ends at the deceleration radius $R_{\text{dec}}$
- At $R > R_{\text{dec}}$ most of the energy is in the shocked external medium: the composition & radial profile are forgotten, but the angular profile persists (locally: BM76 solution)
- Once $\Gamma < 1/\theta_0$ at $R > R_{\text{jet}}$ jet lateral expansion is possible
- Eventually the flow becomes Newtonian & spherical: self-similar **Sedov-Taylor solution**
Relativistic Magnetic Acceleration:

- Relativistic ($v \approx c$) outflows/jets are very common in astrophysics & involve strong gravity at the source: PWN (NS), GRBs, AGN (SMBH), $\mu$-quasars (BH/NS)
- Most models assume a steady flow for simplicity, despite observational evidence for time variability

Crab Nebula: X-ray in blue, optical in red

AGN jet in M87 (VLBA @ 43 GHz)

Circinus X-1: an accreting neutron star (shows orbital modulation & Type I X-ray bursts)
Relativistic Magnetic Acceleration:

Is the acceleration magnetic?

✔ ? ✔ ?

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Thermal vs. Magnetic Acceleration:

- Most of the acceleration is in the supersonic regime

Key difference between thermal and magnetic steady state acceleration of relativistic supersonic flows:

- **Thermal**: fast, robust & efficient
- **Magnetic**: slow, delicate & less efficient
The $\sigma$-problem: for a “standard” steady ideal MHD axisymmetric flow

- $\Gamma_\infty \sim \sigma_0^{1/3}$ & $\sigma_\infty \sim \sigma_0^{2/3} \gg 1$ for a spherical flow; $\sigma_0 = B_0^2/4\pi \rho_0 c^2$

- However, PWN observations (e.g. the Crab nebula) imply $\sigma \ll 1$ after the wind termination shock – the $\sigma$ problem!!!

- A broadly similar problem persists in relativistic jet sources

- Jet collimation helps, but not enough: $\Gamma_\infty \sim \sigma_0^{1/3} \theta_{jet}^{-2/3}$, $\sigma_\infty \sim (\sigma_0 \theta_{jet})^{2/3}$ & $\Gamma \theta_{jet} \leq \sigma^{1/2}$ ($\sim 1$ for $\Gamma_\infty \sim \Gamma_{max} \sim \sigma_0$)

- Still $\sigma_\infty \geq 1 \Rightarrow$ inefficient internal shocks, $\Gamma_\infty \theta_{jet} \gg 1$ in GRBs

- Sudden drop in external pressure can give $\Gamma_\infty \theta_{jet} \gg 1$ but still $\sigma_\infty \geq 1$ (Tchekhovskoy et al. 2009) $\Rightarrow$ inefficient internal shocks
Alternatives to the “standard” model

- **Axisymmetry**: non-axisymmetric instabilities (e.g. the current-driven kink instability) can tangle-up the magnetic field (Heinz & Begelman 2000)

  - If $\langle B_r^2 \rangle = \alpha \langle B_\phi^2 \rangle = \beta \langle B_z^2 \rangle$; $\alpha, \beta = \text{const}$ then the magnetic field behaves as an ultra-relativistic gas: $p_{\text{mag}} \propto V^{-4/3}$

  - Magnetic acceleration as efficient as thermal

- **Ideal MHD**: a tangled magnetic field can reconnect (Drenkham & Spruit 2002; Lyubarsky 2010 - Kruskal-Schwarzschild instability (like R-T) in a “striped wind”) magnetic energy $\rightarrow$ heat (+radiation) $\rightarrow$ kinetic energy

- **Steady-state**: effects of strong time dependence (JG, Komissarov & Spitkovsky 2011; JG 2012a, 2012b)
Impulsive Magnetic Acceleration: $\Gamma \propto R^{1/3}$

**Useful case study:**

Initial value of magnetization parameter:

$$\sigma_0 = \frac{B_0^2}{4\pi \rho_0 c^2} \gg 1$$

1. $\langle \Gamma \rangle_E \approx \sigma_0^{1/3}$ by $R_0 \sim \Delta_0$
2. $\langle \Gamma \rangle_E \propto R^{1/3}$ between $R_0 \sim \Delta_0$ & $R_c \sim \sigma_0^2 R_0$ and then $\langle \Gamma \rangle_E \approx \sigma_0$
3. At $R > R_c$ the shell spreads as $\Delta \propto R$ & $\sigma \sim R_c/R$ rapidly drops

- **Complete conversion of magnetic to kinetic energy!**
- This allows efficient dissipation by shocks at large radii

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Our simulation vs. analytic results

$(JG, Komissarov & Spitkovsky 2011)$

$\sigma_0 = \frac{B_0^2}{4\pi \rho_0 c^2} > 1$
1st Steady then Impulsive Acceleration

- Our test case problem may be directly relevant for giant flares in SGRs (active magnetars); however:

- In most astrophysical relativistic (jet) sources (GRBs, AGN, \(\mu\)-quasars) the variability timescale \((t_v \approx R_0/c)\) is long enough (>\(R_{ms}/c\)) that steady acceleration operates & saturates (at \(R_s\)).

- Then the impulsive acceleration kicks in, resulting in \(\sigma < 1\).
Impulsive Magnetic Acceleration: single shell propagating in an external medium acceleration & deceleration are tightly coupled (JG 2012)

I. "Thin shell", low-\(\sigma\): strong reverse shock, peaks at \(\gg T_{GRB}\)

II. "Thick shell", high-\(\sigma\): weak or no reverse shock, \(T_{dec} \sim T_{GRB}\)

III. like II, but the flow becomes independent of \(\sigma_0\)

IV. a Newtonian flow (if \(\rho_{ext}\) is very high, e.g. inside a star)

II*. if \(\rho_{ext}\) drops very sharply
Impulsive Magnetic Acceleration: single shell propagating in an external medium. Acceleration & deceleration are tightly coupled (JG 2012)

\[ \rho_{\text{ext}} = AR^{-k} \]

through $A$
Dynamical Regimes:

I. “Thin shell”, low-$\sigma$: strong reverse shock, peaks at $\gg T_{GRB}$

II. “Thick shell”, high-$\sigma$: weak or no reverse shock, $T_{dec} \sim T_{GRB}$

III. like II, but the flow becomes independent of $\sigma_0$

IV. a Newtonian flow (if $\rho_{ext}$ is very high, e.g. inside a star)

II*: if $\rho_{ext}$ drops very sharply

$\sigma_0 = B_0^2 / 4\pi \rho_0 c^2$

$f_0 = \rho_0 / \rho_{ext}(R_0)$, $\rho_{ext} = AR^{-k}$

$\Gamma_{cr} \sim (f_0 \sigma_0)^{1/(8-2k)}$
Dynamical Regimes:

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\[ \sigma_0 = \frac{B_0^2}{4\pi\rho_0 c^2} \]
\[ f_0 = \frac{\rho_0}{\rho_{ext}(R_0)} \quad \rho_{ext} = AR^{-k} \]
\[ \Gamma_{cr} \sim (f_0\sigma_0)^{1/(8-2k)} \]
Many sub-shells: acceleration, collisions (JG 2012b)

Flux freezing (ideal MHD):

\[ \Phi \sim B r \Delta = \text{constant} \]

\[ E_{EM} \sim B^2 r^2 \Delta \propto 1/\Delta \]

For a long lived variable source (e.g. AGN), each sub shell can expand by \( 1 + \Delta_{\text{gap}}/\Delta_0 \Rightarrow \sigma_{\infty} = (E_{\text{total}}/E_{EM,\infty} - 1)^{-1} \sim \Delta_0/\Delta_{\text{gap}} \)

For a finite # of sub-shells the merged shell can still expand

Sub-shells can lead to a low-magnetization thick shell & enable the outflow to reach higher Lorentz factors
Jet propagation inside the progenitor star

- The Jet develops a slowly moving ‘head’
- At the head matter decelerates by a reverse shock & flows sideways forming a high-pressure cocoon that collimates the jet
- At the head there is a pressure balance between the shocked jet material & external medium
- The engine must continuously work until the jet breaks out
- After the jet breaks out it can freely accelerate & form the GRB

(B&L 2007)
Jet propagation inside the progenitor star

- The Jet develops a slowly moving ‘head’

- At the head matter decelerates by a reverse shock & flows sideways forming a high-pressure cocoon that collimates the jet

- At the head there is a pressure balance between the shocked jet material & external medium

- Breakout time (Bromberg et al. 2011)

\[ t_b \approx 15 \text{ sec} \left( \frac{L_{\text{iso}}}{10^{51} \text{ erg/s}} \right)^{-1/3} \left( \frac{\theta_0}{10^\circ} \right)^{2/3} \left( \frac{R}{5R_{\text{Sun}}} \right)^{2/3} \left( \frac{M}{15M_{\text{Sun}}} \right)^{1/3} \]
Jet propagation inside the progenitor star: highly magnetized vs. hydrodynamics jets

- The flow must decelerate to match its head velocity, but for high-$\sigma$ a shock can’t do it $\Rightarrow$ the jet converges near its head
- Narrower head $\Rightarrow$ larger head velocity $\Rightarrow$ faster jet breakout
- Relativistic head $\Rightarrow$ less energy into cocoon & supernova
- The head velocity is independent of the detailed jet structure $\Rightarrow$ simplifies the model & allows (semi-) analytic solutions

![Diagram of Cocoon (shocked stellar envelope) and Highly Magnetized Jet]

Jet’s Head
Jet propagation inside the progenitor star highly magnetized vs. hydrodynamics jets

- The flow must decelerate to match it’s head velocity, but for high-σ a shock can’t do it ⇒ the jet converges near its head
- Narrower head ⇒ larger head velocity ⇒ faster jet breakout
- Relativistic head ⇒ less energy into cocoon & supernova
- The head velocity is independent of the detailed jet structure ⇒ simplifies the model & allows (semi-) analytic solutions
- Levinson & Begelman (2013): current-driven instabilities dissipate most of the magnetic field → a hydrodynamic jet
- This is still unclear & strongly effects the jet dynamics
Afterglow Jet Dynamics: Analytic vs. Numerical

- **Analytic results** (Rhoads 1997, 99; Sari, Piran & Halpern 99): exponential lateral expansion at $R > R_{\text{jet}}$ e.g. $\Gamma \sim (c_s/c\theta_0)\exp(-R/R_{\text{jet}})$, $\theta_{\text{jet}} \sim \theta_0(R_{\text{jet}}/R)\exp(R/R_{\text{jet}})$
  - Supported by a self-similar solution (Gruvinov 2007)

- **Hydro-simulations**: very mild (logarithmic) lateral expansion while jet is relativistic (JG et al. 2001)

![Graph showing lateral expansion and theta values](image)

- Same $E_{\text{iso}}$
- Large lateral expansion for $\Gamma \leq 1/\theta_0$
- Modest $\theta_0$-small region of validity

(Wygoda et al. 2011)
Generalized Analytic model (JG & Piran 2012)

- Lateral expansion:
  
  1. new recipe: $\beta_0/\beta_r \sim 1/(\Gamma^2 \Delta \theta) \sim 1/(\Gamma^2 \theta_j)$ (based on $\hat{\beta} = \hat{n}$)
  
  2. old recipe: $\beta_0 = u_0/\Gamma = u'_0/\Gamma \sim \beta_r/\Gamma$ (assumes $u'_0 \sim \beta_r \sim c_s$)

Generalized recipe: 

$\frac{d\theta_j}{d \ln R} = \frac{\beta_0}{\beta_r} \approx \frac{1}{\Gamma^{1+a} \theta_j^a}$, $a = \begin{cases} 1 & (\hat{\beta} = \hat{n}) \\ 0 & (u'_0 \sim 1) \end{cases}$

- New recipe: lower $\beta_0$ for $\Gamma > 1/\theta_0$ but higher $\beta_0$ for $\Gamma < 1/\theta_0$

- Does not assume $\Gamma \gg 1$ or $\theta_j \ll 1$ (& variable: $\Gamma \rightarrow u = \Gamma \beta$)

- Sweeping-up external medium: trumpet vs. conical models
Comparison to Simulations (JG & Piran 2012)

- Main effect of relaxing the $\Gamma \gg 1, \theta_j \ll 1$ approximation: quasi-logarithmic (exponential) lateral expansion for $\theta_0 \geq 0.05$
- There is a reasonable overall agreement between the analytic generalized models and the hydro-simulations
- Analytic models: over-simplified, but capture the essence

2D hydro-simulation by F. De Colle et al. 2012, with $\theta_0 = 0.2, k = 0$
Afterglow jet in stratified external media
(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

- Previous simulations were all for \( k = 0 \) where \( \rho_{\text{ext}} \propto R^{-k} \)
- Larger \( k \) (e.g. \( k = 1, 2 \)) are motivated by the stellar wind of a massive star progenitor for long GRBs

\[ \Gamma_{BM} = \begin{cases} 2, & k = 0 \\ 5, & k = 1 \\ 10, & k = 2 \end{cases} \]

\[ \theta_0 = 0.2, \quad E_{iso} = 10^{53} \text{ erg}, \quad n_{\text{ext}}(R_{\text{jet}}) \sim 1 \text{ cm}^{-3} \]
Afterglow jet in stratified external media
(De Colle, Ramirez-Ruiz, JG & Lopez-Camara 2012)

- Previous simulations were all for $k = 0$ where $\rho_{\text{ext}} \propto R^{-k}$
- Larger $k$ (e.g. $k = 1, 2$) are motivated by the stellar wind of a massive star progenitor for long GRBs

At the same Lorentz factor larger $k$ show larger sideways expansion since they sweep up mass and decelerate more slowly (e.g. $M \propto R^{3-k}$, $\Gamma \propto R^{(3-k)/2}$ in the spherical case) and spend more time at lower $\Gamma$ (and $\beta_0$ decreases with $\Gamma$)
The shape of the jet break

- Jet break becomes smoother with increasing $k$ (as expected analytically; Kumar & Panaitescu 2000 – KP00)
- However, the jet break is significantly sharper than found by KP00 ⇒ better prospects for detection
- Varying $\theta_{\text{obs}} < \theta_0$ dominates over varying $k \leq 2$

**Lightcurves**

**Temporal index**
Late time Radio emission & Calorimetry

- The bump in the lightcurve from the counter jet is much less pronounced for larger $k$ (as the counter jet decelerates & becomes visible more slowly) $\Rightarrow$ hard to detect
- The error in the estimated energy assuming a spherical flow depends on the observation time $t_{\text{obs}}$ & on $k$

Radio Lightcurves

Flux Ratio: $2D/1D(E_{\text{jet}})$
Conclusions:

- **Magnetic acceleration**: likely option worth further study.
- **Jet propagation in star**: can help probe jet magnetization.
- **Jet lateral expansion**: analytic models & simulations agree.
  - For $\theta_0 \geq 0.05$: quasi-logarithmic (exponential) lateral expansion.
  - For $\theta_0 \ll 0.05$: an early exponential lateral expansion phase.
    (but such narrow GRB jets appear to be rare).
  - Jet becomes first sub-relativistic, then (slowly) spherical.
- **Jet in a stratified external medium**: $\rho_{\text{ext}} \propto R^{-k}$ for $k = 0, 1, 2$.
  - Larger $k$ jets sweep-up mass & slow down more slowly.
    ⇒ sideways expansion is faster at $t < t_j$ & slower at $t > t_j$.
    ⇒ become spherical slower; harder to see counter jet.
  - Jet break is smoother for larger $k$ but possibly detectable.
  - Jet break sharpness affected more by $\theta_{\text{obs}} < \theta_0$ than $k \leq 2$.
  - Radio calorimetry accuracy affected both by $t_{\text{obs}}$ & $k$. 