**Analysis and Probability of Boolean Functions**

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**Abstract**

Boolean functions (under various names) are important in combinatorics, probability theory, computer science, game theory, and other areas. In the talk I will discuss a few results and some problems concerning analysis and probability of Boolean functions. In the (self-contained and friendly for students) lecture I will describe first a few notions:

1) Influence: The definition and properties of the influence of a variable on a Boolean function. (Related to: the power of a voter for an election rule; the effect of malicious processors in a collective coin flipping.)

2) Noise-sensitivity: How sensitive is a Boolean function to noise? (Related to how likely it is that errors in counting the votes will change the outcome of an election.)

3) The Fourier Walsh expansion

Next, I will briefly describe a few results: the existence of influential variable (KKL), the tradeoff between total influence and noise sensitivity (BKS), and the majority is stablest (MOO).

Next, two conjectures will be presented. The first on the relation between expectation threshold and the actual threshold (with Jeff Kahn), and the second about the noise stability conjecture for monotone threshold circuits and the reverse Boppana-Hastad conjecture (with Itai Benjamini and Oded Schramm).

Finally two extensions will be mentioned: (a) Maps from Sigma^n to Sigma where Sigma is a larger alphabet and their thresholds (with Elchanan Mossel). ( b) maps from {0,1}^mn to {0,1}^m in the context of judgment aggregation (with Muli Safra and Moshe Tennenholtz).