Polynomiality for Bin Packing with a Constant Number of Item Types

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Abstract

Consider the bin packing problem with *d* different item sizes, *s_i*, and item multiplicities, *a_i*, where *d* is fixed and all numbers are given in binary encoding. This problem is also known as the 1-dimensional cutting stock problem. The complexity of this problem was open even for *d=3*. In this talk, I will provide an algorithm which, for any constant *d*, solves the problem in polynomial time.

Our approach is geometric, and the algorithm in fact solves the following problem for constant *d* in polynomial time: given two *d*-dimensional polytopes *P* and *Q*, find the smallest number of integer points in *P* whose sum lies in *Q*.

If time remains, applications to *high multiplicity* scheduling will also be discussed. In such problems, the number of copies of each job type is given in binary encoding and each type comes with certain parameters such as release dates, processing times and deadlines. A variety of high multiplicity scheduling problems can be solved in polynomial time if the number of job types is constant.

Joint work with Thomas Rothvoss.