Expanders with respect to Metric Spaces

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Abstract

An expander is an infinite family of finite constant degree graphs G=(V,E) for which the following approximate equality holds true: for any mapping f of V into Hilbert space, the average over all pairs u,v in V of ||f(u)-f(v)||^2 is roughly equal to the average over (u,v) in E of ||f(u)-f(v)||^2.

It is a common practice in metric geometry to consider the above approximate equality where the Hilbert space is replaced by other metric spaces. It can be easily shown that there are metric spaces for which no expander satisfies the above approximate inequality. However, it was unknown whether there exists a metric space that differentiates between different expanders. In this talk we will answer this question in the affirmative, by constructing a 3-regular expander that satisfies the above approximate equality with respect to the shortest path metric of a random 3-regular graph. The construction uses the zigzag product, Aleksandrov spaces of non-positive curvature, martingale inequalities, nonlinear functional analysis, and random graph theory.

As a by product, following Barhum, Goldreich, and Shraibman, we obtain a linear time universal approximator for the average square distance in any subset of the vertices of random regular graphs.

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